WEAK AND STRONG CONVERGENCE THEOREMS BY USING MATLAB

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ABSTRACT: *Regarding the research work "Use of Matlab for teaching and learning of linear algebra with engineering students from the Universidad de Caldas", research endorsed by the Vice-rectory of Research from this university, different activities were planned with the purpose of increasing the academic performance and improving linear algebra's learning (second semester's engineering subject). To reach this objective, the design and development of an innovative methodology based on laboratory exercises was proposed for each one linear algebra's topics, by means of mathematical software. The article explains the design of the laboratory "Solution of systems of linear equations using Matlab".*

KEYWORDS: *Education, Learning, Linear algebra, Systems of linear equations, Status, Matlab.*

INTRODUCTION

Linear Algebra is a subject in the second half of the race and Food Systems Engineering at the University of Caldas. The academic background [1], as previous poor knowledge of students, led us to give a different methodological approach, as was the incorporation of mathematical software [2]. From this perspective, it identifies a need for change in the methodologies that have traditionally characterized the teaching of mathematics in higher education institutions, bringing to the stage a new form [3] computer- assisted mathematics

This idea is mainly based on learning sessions using the laboratory method [3], [4] supplemented with computer work in a learning environment characterized by exploration, discovery, the approach of conjecture and testing results. The teacher designs a sequence of workshops [9] so that the student will exercise her ability to analyze, prove, verify, interpret and guess the different concepts linear algebra directly related to the Matlab tool. This paper presents a case report is presented as a product of the research, the design of an innovative methodological approach in teaching and learning systems of linear equations using Matlab. For the development of this project, and according to the history of this type of research, a methodology in which the preliminary concepts about solving linear equations are inserted, the same geometry using Matlab, and the numerical solution was created. Building activities are essential and complementary part to enhance learning and guessing mathematical laws. Finally, an application of systems of linear equations is presented and solved using the software.

Background
The following work arises from the need to respond to one of the main questions related to difficulties encountered in learning linear algebra [12]. At the University of Santiago del Estero Argentina Dr. Maria Ines Morales asserts that the use of software as a mathematical tool in computational linear algebra courses strongly favors the processes of teaching and learning, allowing the student to manipulate mathematical objects, make conjectures about the properties

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that characterize and validate or reject as you go in their exploration thus is the student who discovers, thus appropriating knowledge, which leads to meaningful learning.

In the Autonomous University of Madrid professor Pedro Pulido Ortega analyzes the educational characteristics of a teaching strategy that incorporates the use of mathematical software in teaching and learning of linear algebra. It also states that the role of power allowing students to recognize the essential contents and facilitates the simplification of the numerical calculations.

METHODOLOGY

We worked according to the following parameters. The methodology based on the active participation of the student as the main actor of his own learning. Students worked in a computer lab that has enabled one student per machine. At first use Matlab as a digital computer with built-in functions of linear algebra, and eventually use as a programming language and a symbolic language. Basic Matlab, matrix algebra, vectors, polynomials and their roots, graphs of functions in one and several variables, linear systems of equations, range computer workshops for the review and management of the following topics were conducted of a matrix, determinant, inverse matrix, vector spaces, linear transformations and diagonalization. The activities were developed aimed at building knowledge from problem situations, the appropriation of concepts and activities reinforced the conjecture's own mathematical laws. New technologies allow to save time on routine calculations in the geometric interpretation of the problem, and formulate and solve problems with many variables and equations, which could not be performed in a traditional course. The project was developed in two phases. The first was made in the second half of 2008 with 30 students in systems engineering, in which two theoretical hours worked and two-machine shop legal Matlab software in the computer room Faculty of Engineering; for this activity mathematical seven laboratories were developed:

1. Matlab Basics
2. Vectors and matrices with Matlab
3. Charts in two and three dimensions
4. Systems of linear equations
5. Vector spaces
6. Vector product, lines and planes
7. Linear transformations and eigenvalues

The second phase corresponds to the first half of 2009, with 45 engineering students in which a traditional linear algebra course it worked without technological aids; course results and comparison of the two methodologies is the end of this document. This new methodology in teaching linear algebra is built upon four moments: theoretical [16], numeric [11], graphic, and interpretive and applicative [9], which can be interleaved.

Linear Equation

Identify a linear equation, Graph a linear equation in two variables. Graph a linear equation in three variables. A linear equation with variables $x_1, x_2, \ldots, x_n$. It is an equation that can be written as:

$$a_1, x_1 + \ldots\ldots + a_n x_n = b$$

The constant coefficients a_1, a_2,….a_n are fixed by the real numbers.The algebra and geometry for special cases can be related; the linear equation [1] for n = 2 represents a straight line in the plane, and for n = 3 a plane in three dimensional space. Matlab is powerful in the graphics: the trace command >> plot the graph of a sequence of points in the plane, as follows:

Geometry of the Straight

Write-file in the Matlab window and interpret each line of Listing 1
% List No. 1
x = [-2,5]; % Generate the vector x = (-2.5)
y = x/2-1; % Generate the vector y = (-2, 1.5)
plot (x, y)% Draw a line segment from the point (-2, -2) to (5; 1.5)
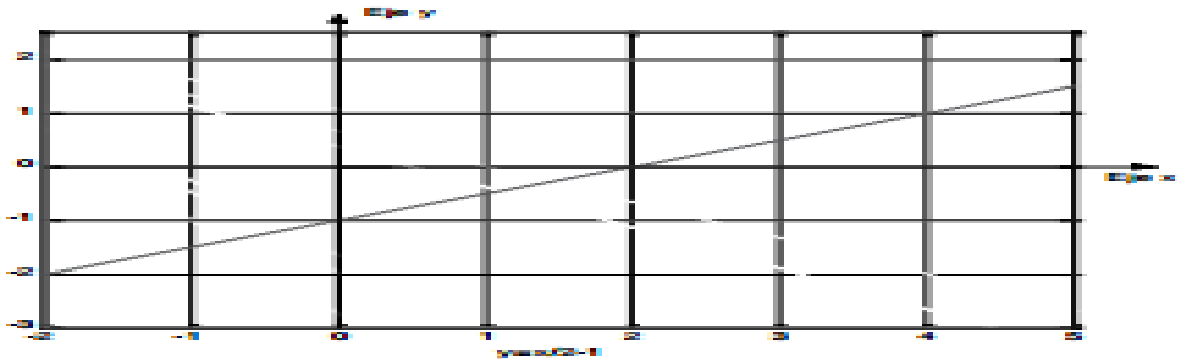grid% Places a grid axis equal to the xy plane

Fig 2.1 Geometry of the line source

In Listing 1 shows four linear which are written in the window of Matlab Editor / Debugger, which are stored in archivos.my can be called from the main >> sales Matlab named student has decided to burn the-file . Figure 1 represents the graph of the linear equation y = x/2-1. The response of the students is 90% correct; what they had was more difficulty in finding the slope of the line because they did not remember the definition. The reaction of the student at the computer was positive since the degree of understanding was satisfactory. At this point we were asked to graphed many straight, changing the equation in Listing 1 and answering questions 1.

PARALLEL LINES

Type in the Matlab window Listing 3.
Listing 3%
x = [0,3];
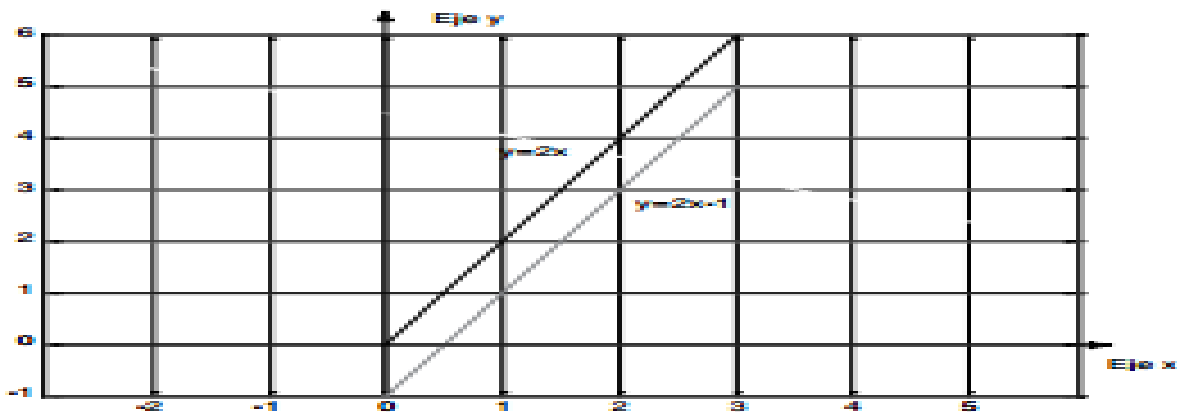y1 = 2 * x;
y2 = 2 * x-1;
plot (x, y1, x, y2)
grid
axis equal

Fig 2.2 Source Parallel Lines

Interpretation: Listing 3 generates two lines whose equations are y = 2x and y = 2x-1. Interpretation: Listing 3 generates two lines whose equations are y = 2x and y = 2x-1. The student responses are immediate and correct, and write your first conjecture: Conjecture 1: "Two lines are parallel if and only if their slopes are equal."

PERPENDICULAR LINES

Type in Listing 4 Matlab; interpret each line of the list.
% List No.4
x = -0:0.1:5;
y1 = - (2/3) * x +2;
y2 = (3/2) * x-3;
plot (x, y1, x, y2)
grid
axis equal

Fig 2.3 Source Perpendicular Lines

Type in the Matlab window and one-file using the tutor interprets each line of the list.

Listing 4%
[X, Y] = meshgrid (0:0.1:3); % Produces a mesh
points where X, Y vary from 0 to 3 with an increment of 0.1
Z1 = 2 * X * Y-3 +2;% generates a vector Z1
mesh (X, Y, Z1, 'Edgecolor', 'black')% plots the Z1 plane in black and white
hold on% plot allows another figure in the same plane
Z2 = -2 * X * Y +3;
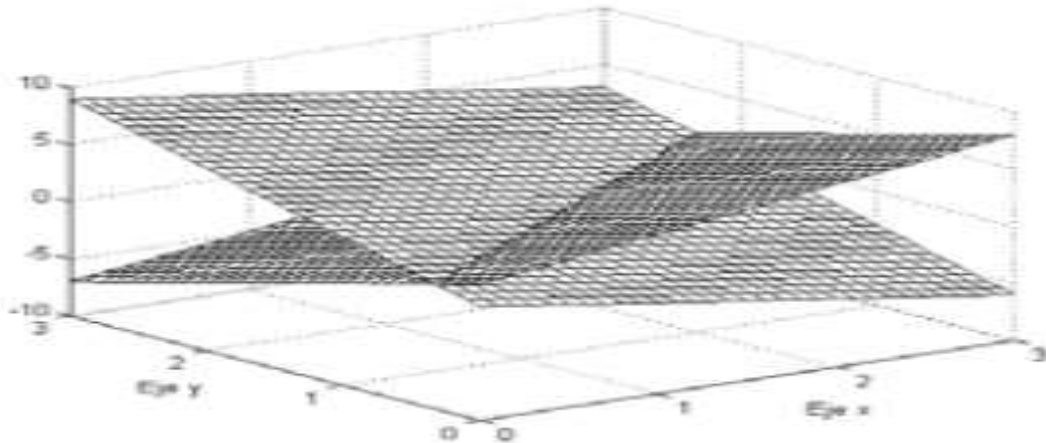mesh (X, Y, Z2, 'Edgecolor', 'black')

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Fig 2.4 Representation planes Source

Interpretation of the graph: in Figure 2.4 the graph product is presented in Listing 4.
The planes of the figure are respectively:
- $2x + 3y + z = 2$, $2x - 3y + z = 0$, they intersect in a line, therefore there are infinite points of intersection.

SUPPLEMENTARY ACTIVITY

Enter the one-file Matlab window. Interpret the product of Listing 5.
Listing 5%
```
[X, Y] = meshgrid (-3:0.1:3);
Z1 = X + Y;
mesh (X, Y, Z1, 'Edgecolor', 'black')
hold on
Z2 = X + Y +5;
mesh (X, Y, Z2, 'Edgecolor', 'black')
```
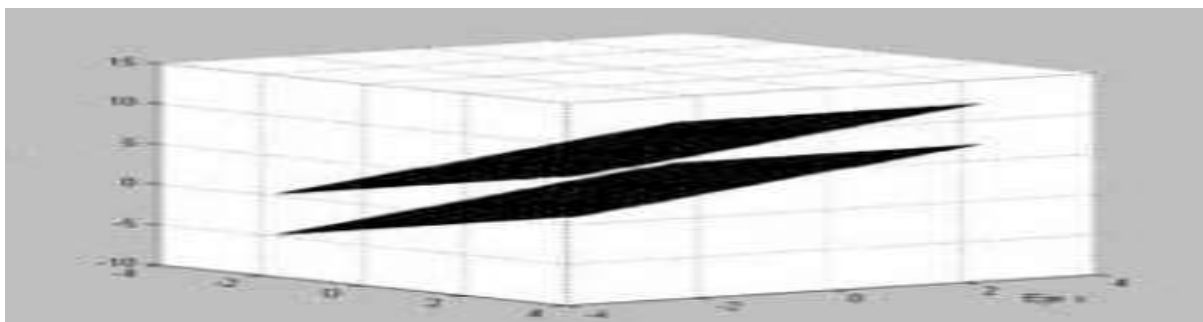

Fig 2.5 Two parallel planes
Further interpretation of the activity
Two parallel planes are observed in Figure 5 whose equations are:
$z - x - y = 0$
$z - x - y = 5$
There are no points of intersection. The normal vectors are equal. The system is inconsistent.

SYSTEM OF LINEAR EQUATIONS

The method of Gaussian elimination Jordan says a procedure for solving a linear system, which consists of the following: elementary transformations by the given system AX = b easier to solve linear system is reduced; to this augmented matrix transforms [A b] equivalent to echelon form and reduced row, using the command >> rref ([A b]).

Rank of a matrix: The rank of a matrix is the number of nonzero rows of the matrix A in echelon form and reduced row. The >> rank (A) command is used to find the rank of the matrix A.

the rank of a matrix is the number of nonzero rows of the matrix A in its echelon form and reduced row. The >> rank (A) command is used to find the rank of the matrix A.

if rank (A) = rank ([A b]) = n then the system has a unique solution if rank (A) = rank ([A b]) <n the system has infinite solutions.

Listing 8 helps solve the system of linear equations:...

AU =

$$\begin{array}{rrrr} 1 & 2 & -1 & 2 \\ 2 & 3 & 5 & 5 \\ -1 & -3 & 8 & -1 \end{array}$$

R1 = 2
R2 = 2

The letter z is a free variable that we denote by z = t, where t is any real number. The figure shows that the intersection of the three planes is a straight parametric equations which are:

x = 4-13 t
y = -1-7t
z = t, t is any real number

The graphical solution shown in Listing 9

Listing 9%
[x, y] = meshgrid (-5:0.5:5);
z = (x +2 * y-2);
surface (x, y, z)
z = (5-2 * 3 * x-y) / 5
hold on
surface (x, y, z)
z = (-1 + x +3 * y) / 8
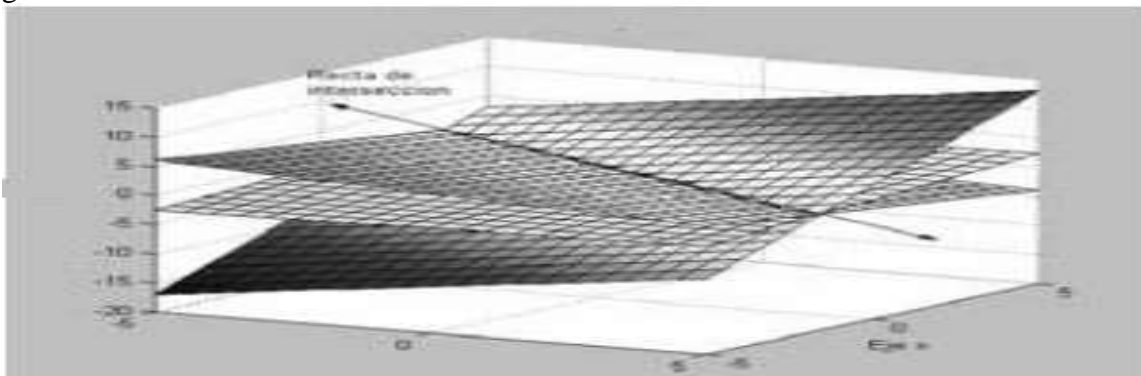surface (x, y, z)
grid

Fig 2.6 Intersection of three planes

EXAMPLE : Find the polynomial of degree 3 which satisfies the points (-1,0), (0,1), (1,0) and (2,0) and plot the polynomial.
Solution.
The polynomial is of the form:
p (x) = c1x3 + c3x2 + c3x1 c4

$$\begin{bmatrix} -1 & +1 & -1 & +1 \\ +0 & +0 & +0 & +1 \\ +1 & +1 & +1 & +1 \\ +8 & +4 & +2 & +1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

To find the coefficients c1 we use the points that satisfy the graph and a system of equations of the form AC = Y. A get is the Vandermonde matrix and is obtained with the command >> vander (X), X is the vector of the abscissas of the points satisfying the polynomial, the vector Y is the ordinate, and C is the matrix of coefficients of the polynomial.
Using V = A, the VC system is obtained = Y.

The system has no solution because the ranges are the same, press Enter The system has a unique solution, since the rank of the augmented matrix is equal to the dimension of the matrix V

Resolution and reduced system using phased array, enter
0.5000
-1.0000
-0.5000
1.0000
The polynomial is sought:
p (x) = 0.5 x3 - x2 - 0.5 x + 1
graph
X = [-1, 0, 1, 2]; Y = [0, 1, 0, 0];
A = vander (X);
C = inv (A) * Y;
s = min (X): 01: max (X);
YY = polyval (C, s);
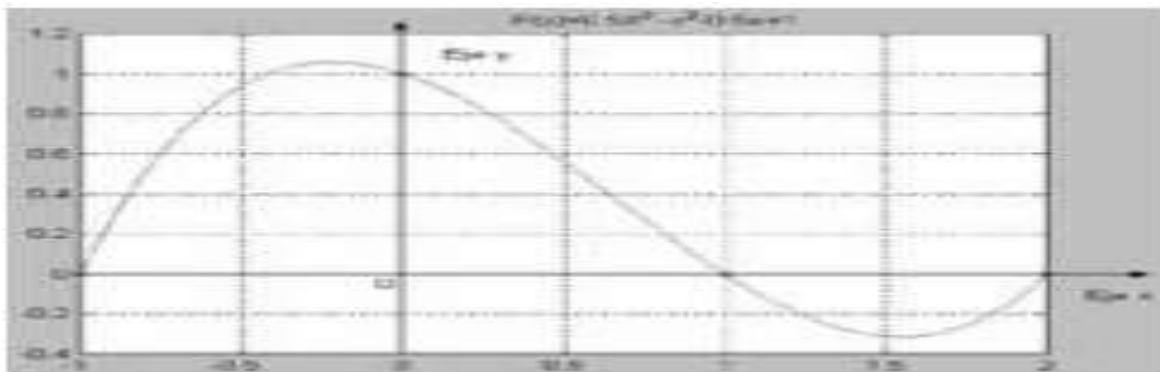plot (X, Y, '*' s, YY)
grid

Fig 2.7 Graph the polynomial cubic

Reinforcement activity: for P (0,5), Q (-1, -2), R (3,3) and S (4, -2), find the cubic polynomial satisfies the four points. Graph the polynomial cubic and find the zeros of the polynomial using the Command >> roots.

Results
Second semester 2008
Subject: Linear Algebra
Number of students: 30
Race Systems Engineering
Teaching aid: Matlab Software
They approved the matter: 70%
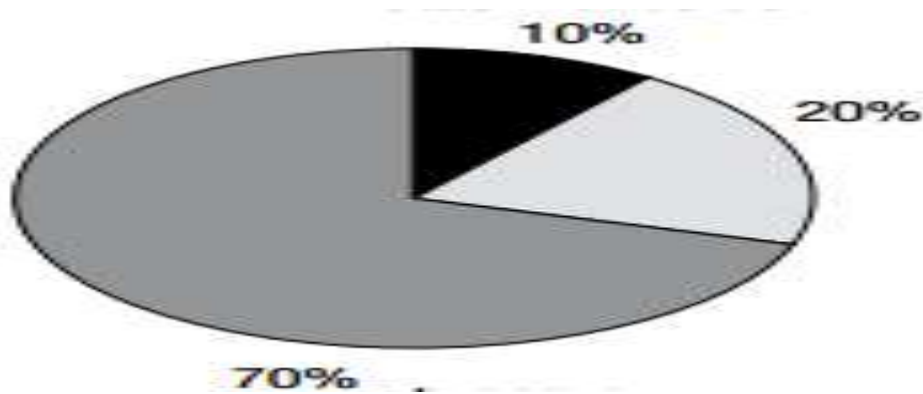They failed the field: 30%
Notes less than 2

Fig 2.8 Final results with Matlab Linear

DISCUSSION AND CONCLUSION

The management of computational tools to those referred to above, in addition to facilitating the learning of the subject Linear Algebra, will enable students to acquire knowledge in the field of engineering they deserve and that they ultimately assist in their professional development. Importantly, the computational tools do not replace atter as such, but constitute material support for better assimilation and integration of the concepts that are inherent to the subject of linear algebra. The student achievement has improved following the introduction of the program Linear Algebra with Matlab, which currently has a better approval in the subject. He refused to recognize, too, that further efforts are needed to maintain and, if possible, enhance the program level in order to provide students with a quality education to meet the challenges of the future. It is well known that the subject represents an abstract discipline and often becomes an insurmountable barrier for students done. This is because much of its content is difficult to conceptualize. Matlab support for subjects where graphical representations are required to facilitate the learning of a concept.

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