

TRICKY ISSUES OF COMPOUNDING

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ABSTRACT: *The corporate finance, which is one of the most important field of financial management, is based on principles of time value of money, compounding. This subject is very important for the business decision making, and at the same time very interesting and challenging. There are a lot of specific topics in compounding and in this article we will discuss and analyze three different tricky issues.*

KEYWORDS: Compounding, Time Value of Money, Corporate Finance

INTRODUCTION

Tricky Issues of Compounding

Let's start with a very simple question: "how much is $1 + 1$ ". According to my research, more than 93% of adults answer with a smile: "of course, it is 2". It means that more than 93% of respondents are wrong.

As it happens to many other straight questions in business, the correct answer to the above equation is "it depends". If we consider time value of money concept, which is such an important part of the financial management, we understand that timing of the first and second "1" is crucial. If timing is different, we cannot sum them without understanding their values on the same date and after it, their sum can be less, equal, or more than 2 (more discussion on this issue is given in the appendix).

In the country where I am from, there is a very popular question. "What do you prefer, the egg delivered today, or the chicken delivered tomorrow"? Everybody has their own solution and arguments, but from financial point of view this question has only one correct answer: it depends on your required return¹.

Compounding calculations have many tricky issues and here we will discuss just three of them on the basis of concrete examples:

- 1. Question: Suppose you are analyzing two strategies: Strategy A will increase the CPI (Customer Price Index) by 11% during first 5 years and by 5% during next 5 years. Strategy B will perform vice versa (5% during first 5 years and 11% during next five years). Your goal is to have the minimum possible CPI after 10 years. Please, describe which strategy would you choose and why?**

Answer: Many people answer to this kind of questions intuitively, that if the rate for the early years is more, compared with the final years, the result will be higher (and vice versa) because of compounding. But whoever thinks in this way, is wrong.

¹ As "egg" is your Present Value, and "Chicken" – Future Value.

The final result will be absolutely the same. And the detailed description of this, and other two questions are indicated in the appendix.

2. **Question:** You are going to make the “child deposit” for your amazing boy, George. How much will be on the deposit account after 18 years, if you put 50 USD at the end of every month and deposit earns 5% yearly.

Answer: I have asked this kind of question to my 32 students. To be perfectly honest, 5 of them did not have any idea how it was possible to calculate it. But all the remaining 27 thought that it was very simple, just by calculating the FV of Annuity with the following characteristics:

$$PV = 0$$

$$PMT = 50\$$$

$$\text{Number of Periods} = 18 \times 12$$

$$\text{Interest Rate} = 5\% \div 12$$

From these 32 students only those 5 were perfectly correct who thought that they did not know how to calculate it.

Because accrued interest is added to the deposit only once a year², first we need to calculate yearly amounts and only after that it is possible to use the annuity formula.

3. **Question:** Suppose you are analyzing two bonds and gathered the given information:

	Bond 1	Bond 2
Years to Maturity	5	5
Coupon Rate	11%	11%
Face Value	10,000 USD	10,000 USD
Required Rate	12%	12%
Payments are done	Annually	Semiannually

Please calculate the maximum price you should pay for “Bond 1” and “Bond 2”?

Answer: Generally, people calculate easily with the following estimations:

	Bond 1	Bond 2
Number of Periods	5	10
PMT	1,100 USD	550 USD
FV	1,000 USD	1,000 USD
Interest Rate	12%	6%

² For long term deposits, like “child deposit”, it is almost always the case that accumulated interest is accrued on the deposit amount only at the end of the year period.

And find out the following values respectfully:

	Bond 1	Bond 2
PV	9,639.52 USD	9,632.00 USD

Which means that you are ready to pay more for the bond with annual payments, and not for the bond with exactly the same characteristics, but with semiannual (so, early) payments. And this is nonsense ...

The problem is that in the above calculations for “Bond 2” we discounted the face value also for 10 periods and its negative effect in the present value was more than the positive effect of early semiannual payments.

The correct calculation, which I have seen very rare for this kind of situations, is to discount the face value on the annual basis (besides the frequency of coupon payments) and the payments according the relevant number of periods.

APPENDIX

Detailed Answer for the Statement № 1 –

There are many other solutions, too, to this simple question, e.g. people with a good knowledge in information science will mention that if we calculate this equation in binary number system, then it is equal to 10 (not ten, but one and zero, which is equal to 2 in the decimal number system). If we look at this question from the business point of view, we can say, that in the business world, some of the effects of first activity (for example a marketing campaign) and the effect of exactly the same campaign held just second time, can be less, equal, or more to

„the effect of the first campaign“ X 2

And so on ...

Detailed Answer for the Question № 1 –

If we want to find out how much our money will be after one year, we will calculate it in the following way:

$$FV_1 = PV \times (1 + I)$$

Where:

FV = Future Value

PV = Present Value

I = Interest Rate

If our timeline is more than one year, it means that for the second year the interest will be earned on the amount which was accumulated at the beginning of the second year. So:

$$FV_2 = FV_1 \times (1 + I)$$

$$FV_2 = PV \times (1 + I) \times (1 + I)$$

The same logic is true for the third, fourth and ... nth years:

$$FV_n = PV \times (1 + I)^n$$

Now, let's use this equation for the purpose of calculation the CPI. For the first scenario we will have:

$$FV_{10} = [CPI_0 \times (1 + 11\%)^5] \times (1 + 5\%)^5$$

While for the second scenario the equation will be the following:

$$FV_{10} = [CPI_0 \times (1 + 5\%)^5] \times (1 + 11\%)^5$$

And, of course, these two scenarios will give absolutely the same result.

Detailed Answer for the Question № 2 –

We all know that annuity is the stream of cash flows with two main criteria:

1. Cash flows are equal amounts
2. Time periods between each and every following cash flows are equal

Besides these two criteria are both correct for our cash flow stream (50 USD, paid at the end of each month for next 18 years), we cannot initially use the annuity formula because the interest earned is accrued on our balance only once a year. So, compounding is happening only on the yearly basis.

To calculate the correct Future Value for this deposit, first we need to calculate what the future value of monthly payments will be after one year (FV_1), when the interest earned will be added on our account. Then we will calculate the final value with the following data:

$$PV = 0$$

$$PMT = FV_1$$

$$\text{Number of Periods} = 18$$

$$\text{Interest} = 5\%$$

Detailed Answer for the Question № 3 –

With the right understanding, we will calculate the correct present value with very simple approach:

	Bond 1	Bond 2
Find out the PV of PMTs		
Number of Periods	5	10
PMT	1,100 USD	550 USD
FV	0	0
Interest Rate	12%	6%
PV of PMTs	3,965.25 USD	4,048.05 USD
Find out the PV of Face Value		
Number of Periods	5	5
PMT	0	0
FV	10,000 USD	10,000 USD
Interest Rate	12%	12%
PV of Face Value	5,674.27 USD	5,674.27 USD
Present Value of Bond	9,639.52 USD	9,722.32 USD

Of course, the bond with the option of semiannual coupon payments is more attractive and worth more than the one with annual payments.