

The Uniqueness Theorem

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ABSTRACT: *The Pauli Exclusion Principle may be derived from the Heisenberg Uncertainty Principle by using Planck's constant.*

KEYWORDS: Atom, Alpha Particle, Binding Energy Theorem, Electron, Heisenberg Uncertainty Principle, Pauli Exclusion Principle, Planck's Constant, Photoelectric Effect, Quantum Mass, Quantum Space, Quantum Wave.

INTRODUCTION

The Heisenberg Uncertainty Principle and Pauli Exclusion Principle have their origin in quantum mechanics, or the study of the atom and its component particles, which forms an important part of physics.

Where the Heisenberg Uncertainty Principle is often thought of as a limit on the resolving power of light, it may be viewed as a statement about the identity of a particle which uses Planck's constant. To arrive at this idea, it may be helpful to review some major highlights that have occurred in the study of the atom.

The atom

The idea of the atom is quite old. The first man we know of, who thought of the atom, belonged to a group of philosopher-scientists from ancient Greece, who, centuries before the birth of Christ, had begun to reason about the physical world around them. (Haber, page 28)

Before these philosopher-scientists, it was thought that nearly everything was the work of gods, demigods, or demons, which man looked on with awe and superstition. Rather than be frightened or awed by these gods or superstitions, the Greek thinkers began to reason systematically, using logic to understand and explain the natural world and its laws. (Haber, page 28)

This period of scientific and philosophical enlightenment started with Thales of Miletus, a city in Asia Minor. Thales was one of the first of these Greek philosopher-scientists. During his travels, it is thought he visited the Babylonians, who were diligent observers of the sky, and he inspected their tables that recorded eclipses of the Sun and Moon. (Haber, pages 28-29)

A story says that rather than believe eclipses were the work of a huge Babylonian dragon, Thales reasoned they are a natural phenomenon. Since they occurred regularly in the past, he reasoned they would occur regularly in the future, and predicted that an eclipse of the Sun would occur on May 28, 585 B.C., which took place as predicted and became an event of history. (Haber, page 29)

Around 465 B.C., about 80 years after the death of Thales, the philosopher-scientist Democritus was born in Abdera, a little town in Thrace, a province of ancient Greece. To the best of our knowledge, Democritus was the first to think about the atom, and gave us the word "atom,"

which comes from the Greek word atomos, meaning something that cannot be cut. (Haber, page 30)

While some historians believe another Greek philosopher, Leucippus, had the idea of the atom as early as 500 B.C., Democritus went beyond an idea to develop a theory about the atom, which contained remarkable insight, and is often called the father of the atom. (Haber, page 30)

Democritus saw the universe as a vast void in which atoms reside. He thought atoms were indestructible with an indivisible hardness. But through their incessant motion and ever changing arrangements, they weave the tapestry of the physical world in the form of matter. (Haber, page 32)

Democritus believed there are many different kinds of atoms. Some are little smooth spheres; others are sharp edged cubes; others are irregular with rough surfaces. If a large number of rough atoms were packed together, they could stick to each other, which could explain the toughness of metals. Other atoms are smooth and heavy, and slide freely over each other with hardly any friction between them, making them fluid like water. (Haber, page 32)

Other atoms are light and smooth, and float, moving in all directions to give air and fire. Democritus thought atoms represented a universal, unchanging world and law of nature. But atoms were always active, arranging themselves in patterns, only to break up and rearrange themselves in new patterns. (Haber, page 33)

Despite their brilliance, the ideas of Democritus were nearly lost. His writings vanished, with only a few fragments relayed through the centuries. His theory about the atom was practically forgotten after the writings of the famous philosopher Aristotle became ascendant, who was born in 384 B.C. (Haber, page 33)

In contrast to Democritus, Aristotle thought if air and fire consisted of small, solid particles, or atoms, they would fall to the ground like pebbles. To explain the nature of the universe, Aristotle thought that the physical world consisted of the four elements of earth, water, air, and fire. (Haber, page 34)

But pebbles can be ground up into fine dust, and float in air. Fresh water and salt water are different in taste. Water can support tiny solids in a mixture or solution, which do not precipitate, but remain suspended.

Air can support tiny particles like a solution. Even though these tiny particles are invisible to the human eye, they may be detected by the human nose like the smoke of fire, or fragrance of a flower.

In 1827 the motion of tiny particles in water was observed directly by the English botanist Robert Brown. Using a microscope, Brown saw tiny particles move around in water in random motion, which became known as Brownian motion or movement. (Haber, pages 62-67)

Brown had observed the type of motion Democritus reasoned would occur between different types of atoms. Scientists would use this type of motion to explain how gases can expand as heat is added.

Starting in the 19th century, the study of the atom advanced with the introduction of electron tubes where metal wires are placed at each end of a tube of glass, which is sealed, and a voltage

is applied to the wires. As air is pumped out, a discharge of light appeared, but weakened and disappeared as more air is removed. (Haber, page 72)

But opposite the negative wire, called the cathode, the glass wall of the electron tube glowed with a flickering pale green color. Since a magnet could deflect the path of the glowing discharge, scientists reasoned the glow was created by tiny charged particles, which emerged from the negatively charged wire. The tiny particles were called electrons. (Haber, pages 72-73)

However, the tiny charged particles emitted by the electron tube could be interpreted to represent waves. The amplitude or size of the wave would increase with increases in the size or volume of current, while the speed of the wave would reflect the amount of voltage applied. Indeed, the idea of expressing electricity as current reflects how electricity displays the property of a wave.

While experimenting with the light produced by an electron tube that was projected onto a fluorescent screen, in 1895 William Roentgen in Germany discovered that when he put his hand between the light and fluorescent screen, he could see the bones in his hand on the screen. The discovery caused a sensation, and quickly led to use of X-rays to detect broken bones. (Haber, pages 74-75)

The discovery of X-rays was followed by the discovery of how uranium emits radiation, including X-rays. The idea of radioactivity, where a heavy substance or metal could emit rays, was new. Radioactivity seemed to involve neither light nor electricity. (Haber, pages 76-79)

Shortly thereafter, Marie Curie and her husband Pierre, who lived in Paris, processed a large amount of uranium ore, and discovered polonium, which is more radioactive than uranium. Continuing to refine the ore, they also discovered radium, which is so radioactive that it glows in the dark. (Haber, pages 76-83)

Radium became popular, used to paint watch hands so they would glow in the dark, and as medicine. Scientists used radium to study radioactivity, which was new to them, by placing a tiny sample inside a hollow block of lead, which absorbed its rays, which had a small hole from which radium rays emerged in a thin, straight beam. To study the beam, they used a magnet to deflect its rays, and projected the beam onto a fluorescent screen. (Haber, page 84)

When a magnet was placed near the beam, some of the rays bent to the side, which showed they carried electric charge. The rays that bent in a positive direction were called alpha rays. The rays that bent in a negative direction were called beta rays, and the rays that traveled in a straight line were called gamma rays. (Haber, page 85)

While the beta rays turned out to be electrons, they traveled much faster, at nearly at the speed of light, than the electrons scientists observed in their electron tubes. The gamma rays were a powerful type of X-ray. The positively charged alpha rays were harder to identify. (Haber, page 85)

It was not until the early part of the 20th century that scientists identified the composition of the alpha particle. Four times heavier than a hydrogen atom, its charge was equivalent to two electrons. At around the same time, Robert Millikan in the United States weighed the electron, showing it was extremely light, nearly 2,000 times lighter than a hydrogen atom, which consists of a single proton and electron. (Haber, page 86)

In 1903 the English physicist Sir Ernest Rutherford and his co-worker, Frederick Soddy, explained how the alpha particle appears. As an atom of radium, which weighs 226 times more than an atom of hydrogen, emits an alpha particle, which weighs 4 times more than an atom of hydrogen, the radium turns into radon, a rare, heavy gas, which weighs 222 times more than an atom of hydrogen. (Haber, page 86)

In other words, radioactive elements decay into products like the alpha particle and other elements that are lighter in their atomic number or weight, and the sum of the radioactive decay products equals the atomic weight of the original element.

For example, radium, which has an atomic weight of 226, emits an alpha particle, which has an atomic weight of 4, as it turns into radon, which has an atomic weight of 222, and $226 = 4 + 222$.

Technically, the atomic number of an element equals its number of protons, which determines its number of electrons. In contrast, the atomic weight of an element gives its total weight in atomic units, which includes its protons, neutrons, and electrons.

An electron weighs 9.109×10^{-31} kg (kg stands for kilograms)

A proton weighs 1.6726×10^{-27} kg

A neutron weighs 1.6749×10^{-27} kg

Since the electron is so light, and a proton weighs almost as much as a neutron, the atomic weight of an element often appears as a whole number, which basically equals its number of protons and neutrons.

For example, the atomic weight of the alpha particle is often stated as four since it is four times larger than an atom of hydrogen, whose nucleus consists of a single proton, and no neutrons. However, hydrogen has two isotopes, deuterium and tritium, whose nucleus includes one and two neutrons, respectively.

Technically, the atomic weight of an element also reflects the weight of its electrons, as well as its protons and neutrons, and the slight difference in weight between a neutron and proton, and the binding energy that holds the nucleus together. Binding energy affects the mass of the nucleus by slightly reducing its mass. Atomic weights are sometimes expressed in decimals to account for these adjustments.

Another clue to the atom came in 1905 when Albert Einstein published his paper that showed the equivalence of matter and energy, relating them to each other in the famous equation of science of $E = mc^2$ where E stands for energy, m stands for mass, and c stands for the speed of light. (Haber, pages 90-93)

1905 was a productive year for Einstein, with the publication of his papers explaining Brownian motion, the photoelectric effect, and special relativity, as well as his paper on the equivalence of matter and energy, making him one of the great theoretical physicists of all time. His paper on the equivalence of matter and energy was crucial to understanding how an atom exhibits radioactive decay. (Wikipedia)

Since the alpha particle plays such an important role in the decay of radioactive elements, it may be helpful to examine it in more detail.

The Alpha Particle

Emitted as a product of the radioactive decay of radon and radium and other elements, the alpha particle consists of two protons and two neutrons, and equivalent to the nucleus of a helium atom. In other words, the alpha particle is essentially an atom of helium with its two electrons stripped away.

Helium is the second element after hydrogen, and first of the noble gases, which include argon, neon, krypton, xenon, and radon. The noble gases are all chemically inert. Since helium is chemically inert and not radioactive, it may be inferred that the alpha particle represents a stable structure for a nucleus, whether in terms of its tendency to exhibit radioactive decay, or the chemical reactivity of its associated electron shell.

In contrast, radium and radon exhibit radioactivity. They emit the alpha particle due to an apparent instability in their nucleus with its arrangement of a large number of neutrons and protons. Elements with high atomic numbers tend to be radioactive.

Paradoxically, radium and radon, which are radioactive, emit the alpha particle, which represents a stable nucleus for the helium atom, a building block that is made up of two protons and two neutrons, which is easier to emit than emitting the two protons and neutrons individually.

In other words, an atomic nucleus breaks apart, or relieves itself of the pressure of radioactive decay within its nucleus by emitting building blocks such as the alpha particle. In other words, radioactive elements with high atomic numbers tend to emit components such as the alpha particle since they break apart at their joints, rather than emitting individual protons and neutrons.

From another point of view, radioactive decay represents a type of atomic erosion. Radium turns into radon and the nucleus of a helium atom, or two simpler arrangements of its protons and neutrons. Since elements with high atomic numbers or weights tend to be radioactive, it may be inferred that one of the principle causes of radioactive decay is the structural strain within a nucleus arising from an unbalanced or overly complex arrangement of its protons and neutrons.

In contrast, the alpha particle represents a stable geometrical arrangement between two protons and two neutrons. This geometrical arrangement involves the areas that connect its protons and neutrons, which use binding energy.

Binding energy, which is calculated according to Einstein's equation of $E = mc^2$, is the amount of energy equivalent to the mass consumed out of a proton or neutron in order for them to bind together, where a neutron binds with a proton, and a neutron binds with another neutron.

Protons do not bind with each other since like charges repel. Instead, protons bind in a nucleus by binding with neutrons, which, being electrically neutral, are able to use their mass for binding instead of carrying electric charge.

To understand the role of binding energy in constructing a nucleus, it may be helpful to review the hydrogen atom. The first element, hydrogen has the lowest atomic number of any element and the simplest nucleus, a single proton, and its electron shell consists of a single electron.

Hydrogen shows how a nucleus can consist of a single proton, and does not require a neutron. It also shows how symmetry in electric charge dominates the construction of an atom. The number of electrons in an atom's electron shell is evenly matched with the number of protons in its nucleus.

In other words, atoms are constructed to be electrically neutral, where the negative charge in an atom's electron shell equals the positive charge in its nucleus. Chemically speaking, most atoms are able to form molecules with other atoms by sharing charge within their electron shell. Electron shells that are lacking symmetry are prone to capture or share electrons in order to keep an atom electrically neutral.

Hydrogen has two isotopes, defined by the addition of neutrons to its nucleus. While neutrons do not change the charge of an atom, they tend to affect its chemistry, and may affect the stability of its nucleus since they change its structure.

The first isotope of hydrogen, which is called deuterium, has a nucleus that includes a single neutron. The second isotope of hydrogen, which is called tritium, has a nucleus that includes two neutrons.

Since hydrogen is more abundant than its two isotopes of deuterium and tritium, and is by far the most abundant element in the universe, nature shows a preference for atoms with a simple nucleus, presumably because they are easier to construct, and are arguably more durable.

Deuterium teaches that a neutron binds with a proton. Since deuterium is not radioactive, it may be inferred that binding a neutron with a proton results in a stable nucleus. On the other hand, since tritium is mildly radioactive, it may be inferred that a nucleus that consists of two neutrons and one proton is mildly unstable.

To explain the instability of tritium, recall that deuterium shows a neutron binds with proton since it does not carry an electric charge, and has slightly more mass than a proton. In other words, a neutron has more mass or structure available for conversion into binding energy than the proton, which carries electric charge.

Since the neutron binds, it may be inferred that tritium forms by binding a second neutron to the first neutron. Since the proton has bonded, its binding with the first neutron is kept intact while the two neutrons bind together since they are associative, able to form binding energy relationships.

On the other hand, since the proton is bound, binding it to another neutron would either require a double binding, or a reallocation of its original binding to two neutrons. Since a double binding would be unnecessary and binding energy transactions seem to require a reason, it may be inferred that the original binding of the proton stays intact, and so the second neutron binds with the first neutron.

Moreover, binding two neutrons together may be inferred to require a lesser amount of binding energy than binding a neutron to a proton, whose binding must be strong enough to offset the force of electric repulsion between like charges at close range, at least in atoms that have a nucleus with two or more protons.

In other words, as a rule, atoms minimize their use of binding energy and any changes in their binding energy relationships. Binding energy transactions require a good reason for the

conversion of mass into energy. As a result, in tritium, the second neutron binds with the first neutron since this avoids disturbing the existing binding energy relationship between the proton and first neutron.

The binding energy relationships in tritium may be depicted by listing the components of its nucleus linearly, like the elements in a set, as PNN or NNP, where P stands for proton and N stands for neutron.

This list builds from the principle that the second neutron binds to the first neutron, and the binding of a neutron with a proton in deuterium, which may be depicted as PN or NP. In other words, tritium adds a second N to the existing N in deuterium, which produces PNN or NNP.

In other words, since deuterium is stable, additions to its nucleus are made through its neutron, which is naturally associative, instead of disturbing the existing binding energy relationship between its proton and first neutron.

In other words, there is no obvious reason for a proton to be doubly bonded to two neutrons or for its binding energy relationship with the first neutron to change with a reallocation between two neutrons.

On the other hand, since a neutron is associative, able to bind with a proton, its trait or characteristic of being associative makes it likely to bind with another neutron, especially since the binding between two neutrons requires less binding energy than binding a neutron to a proton.

Moreover, listing the components of the tritium nucleus as PNN or NNP suggests how tritium is mildly radioactive since it is geometrically unbalanced. In other words, since tritium is unbalanced in its geometry, defined in discrete terms of its components that consist of a proton and two neutrons, its binding energy interfaces are unbalanced, making it radioactive.

The idea of a geometrical imbalance in binding energy interfaces as a factor in making a nucleus radioactive may be reasoned to occur in other elements, especially since elements with a high atomic weight are typically radioactive, and the elements with the highest atomic weights are radioactive.

In other words, an atom's nucleus begins to break down or emit products of radioactive decay when it has an imbalanced geometry like tritium, or an overly complex geometrical arrangement of its protons and neutrons that is imbalanced. With this in mind, it may be argued that the alpha particle is constructed to where a proton is bound with a neutron, and a second proton is bound to a second neutron like two pairs of deuterium nuclei, which bind together through their neutrons.

In other words, from a geometrical point of view, or a listing of objects in a set, the alpha particle is configured as PNNP. It consists of two deuterium nuclei, which bind together through their neutrons instead of their protons since like charges repel.

A configuration of PNNP reflects how like charges repel. The protons are placed away from each other, while the neutrons bind with each other, as well binding with a proton. In other words, two neutrons may bind with each other, in addition to each of them binding with a proton.

With this in mind, a binding energy theorem may be suggested.

Binding Energy Theorem

The alpha particle binds in the geometrical configuration of PNNP.

As a preliminary step, it may be argued that the binding energy between a neutron and proton is greater than the binding energy between a pair of neutrons, or

$BE (P+N) > BE (N+N)$ where BE stands for binding energy.

The greater binding energy between a proton and neutron than between a neutron and neutron may be argued by observing how, in the alpha particle and atoms with more than two protons, binding a proton with a neutron must offset the force of electric repulsion between two protons at close range, a requirement that does not exist for binding two neutrons together. And this point may be argued by observing how an atom seeks to minimize its binding energy.

Moreover, deuterium has a binding energy relationship between a proton and neutron that results in a stable nucleus. In contrast, tritium incorporates a binding energy relationship with a second neutron that results in a nucleus that is mildly radioactive, which implies the binding between two neutrons is weaker than the binding between a proton and a neutron since it uses a lesser amount of binding energy.

In other words, since deuterium has a stable nucleus, its proton and neutron are tightly bound together. This implies it uses a larger amount of binding energy than binding two neutrons together in tritium, which exhibits mild radioactive decay.

Returning to the alpha particle, since like electric charges repel each other, it may be argued that the geometrical configuration of NPPN is prohibited since it requires binding two protons directly to each other.

In other words, binding two protons directly together, where they are touching, requires significantly more energy than binding a proton with a neutron since binding two protons together must offset their force of electric repulsion without any intervening distance between them.

From another point of view, protons do not bind with each other because particles with the same charge repel each other, and because its structure is not designed to both carry an electric charge and bind with another proton. As a result, a proton binds with a neutron, as seen in the nucleus of deuterium.

In other words, since the proton and neutron are both particles of matter, and the neutron has slightly more mass than a proton, it may be argued that the proton is designed to carry charge, while the neutron is designed to bind with other particles, including binding with a proton and another neutron.

In contrast, since the proton carries charge, it does not bind with another proton due to force of repulsion and the design of its mass structure. While an electron carries the same amount of charge with very little mass compared to the proton, so it may be thought that a proton only uses a small part of its mass structure to carry charge, an electron occupies a relatively large volume of space, and in some ways resembles a wave function.

In other words, a proton uses more of its mass structure to carry charge than an electron. As a result, it may be inferred that binding energy relationships used the neutron, just as it may be

argued that deuterium and tritium show how a neutron binds with a proton and a neutron may bind with another neutron.

Since binding energy transactions require a good reason, it may be argued that a nucleus minimizes its use of binding energy. An atom minimizes its use of binding energy so that it may retain its protons and neutrons in the form of matter instead of converting their mass into binding energy.

In other words, in the world of matter, the atom represents a key design, largely indivisible, that seeks to maintain its components of protons, electrons, and neutrons in the form of matter by minimizing its use of binding energy relationships.

While, except for hydrogen, a nucleus requires the conversion of mass into binding energy, a nucleus arranges its components in a geometry that minimizes its use of binding energy, or keeps them in the form of matter.

While it may be argued the alpha particle uses a linear geometry of PNP or NPN, it was already argued that the nucleus of tritium uses a linear geometry of NN or PN, which does not lead to a configuration of NPN.

In particular, if the alpha particle uses a linear geometry of PNP, this requires a neutron to bind twice with two protons, without necessarily requiring that it binds with the second proton at a lesser rate, so the neutron may be required to bind extra with two protons.

Moreover, a configuration of PNP results in an unsymmetrical use of neutrons, where one neutron binds with two protons, while the other neutron binds with a single proton at a level that is unclear, requiring a reallocation of binding energy relationships, or possessing a binding at the same level as PN.

In other words, a configuration of PNP requires three binding energy relationships between a neutron and proton when it is reasonable to argue that a nucleus minimizes its use of binding energy, and the configuration of NPN requires only two binding energy relationships between a neutron and proton.

If the alpha particle uses a linear geometry of NPN, this only reverses the configuration of PNP to put a neutron on the other side. The same arguments can be used to where a configuration of NPN is an unsymmetrical use of neutrons, and requires three bindings between a neutron and proton when a configuration of PNP requires only two bindings between a neutron and proton, and a single binding between two neutrons, which does not require as much binding energy.

Moreover, a configuration of PNP or NPN would mean that as one of the neutrons binds with two protons, either the other neutron becomes extraneous to the nucleus, as its binding is no longer needed to maintain a nucleus of two protons, or the binding of a proton to a neutron is reallocated without good reason or a clear mechanism.

While it may be said that PNP shares its binding where the interior neutron binds with two protons, where its binding with the interior proton is shared with the exterior neutron, presumably in a half energy binding, it is simpler to view the neutron as engaging in a limited number or type of binding energy transactions with a proton instead of half bindings.

While this linear representation of the alpha particle as PNNP is one dimensional, the many depictions of an alpha particle or a nucleus that show protons touching together do not seem to account for how protons do not bind together, or the role of the neutron in binding a nucleus.

Since the charge of the proton means that protons repel each other, the binding between a pair of deuterium nuclei, or a proton and neutron, and another proton and neutron, occurs through their neutrons.

In particular, the neutrons bind together to keep the protons away from the center of their binding since this results in a larger distance of separation between the two protons, and reduces the amount of binding energy required.

In other words, the nucleus arranges its geometry to minimize its use of binding energy. The nucleus uses a point of balance in its geometrical center, or binding energy interfaces, to minimize its radioactive decay.

A similar idea of geometric symmetry appears in electron shells. As a rule, the number of electrons that inhabit a given orbit represents a balance in their spacing since electrons repel each other. When the number of electrons in an orbit exceeds a limit, the force of electric repulsion pushes the extra electron into an outer orbit.

But if an electron in a given orbit is lonely or the orbital slot is imbalanced, the atom becomes chemically reactive, sharing electrons in its shell in order to achieve a state of equilibrium. But when its shell becomes full, the atom loses its tendency to be chemically reactive.

Ernest Rutherford

Ernest Rutherford continued his investigations into the atom. To trace the movement of alpha particles from the radioactive decay of radium, he used a fluorescent screen. As an alpha particle struck the screen at a speed of thousands of miles per second, it caused a tiny flash or scintillation he could see. (Haber, page 97)

Scientists were excited that they could see an alpha particle strike a fluorescent screen. Geiger and Marsden were the first to explore the interior of the atom using atomic bullets such as the alpha particle. Their results encouraged Rutherford to pursue this line of research. (Haber, page 98)

Rutherford built an atomic shooting range, using a small sample of radium encased in a block of lead that had a tiny hole for the rays to escape. To avoid collisions of the radium rays with air molecules, he put the lead block inside a glass jar and pumped the air out, and put a fluorescent screen in the line of sight of the rays with a microscope attached behind the screen to detect the impact of the rays. (Haber, pages 98-99)

On the screen, Rutherford saw a constant sparkling or scintillation from the radium rays. Then he placed a thin gold foil, about 2,000 atoms deep, between the line of sight of the radium rays and the screen to see if it would stop the rays since gold is a fairly dense metal. (Haber, pages 98-99)

To his amazement, Rutherford saw the same sparkling on the screen as before. The radium rays or alpha particles continued to pour through the gold foil as if it did not exist. He had discovered that the atom is not solid, and its mass is unequally distributed within its structure. (Haber, pages 99-100)

As Rutherford kept observing, he saw a tiny flash, far away from the line of sight of the beam. Then he saw a flash at the edge of his field of view, and another flash outside the beam. The flashes were unexpected. To get a better look, he moved the screen and microscope to the side of the beam, and saw another flash. (Haber, page 100)

To look directly across the beam, Rutherford placed his microscope and screen at a right angle to the gold foil, and saw a flash. Then he moved his microscope and screen behind the lead block, and again saw a flash. A ray must have bounced directly off the gold foil at a hundred and eighty degrees. After repeating his experiment many times, Rutherford counted the flashes that went straight through the gold foil and compared them to the number of flashes that he counted outside the beam. Only one out of more than 8,000 flashes bounced or ricocheted. (Haber, pages 100-102)

Rutherford reasoned that the atoms in the gold foil were like a stack of nearly empty shells, which let nearly all the alpha particles go through them. But something was deep inside each atom, which deflected the alpha particles. Calling it the nucleus, Rutherford deduced that the nucleus is ten to fifty thousand times smaller than the shell of the atom, and very dense. (Haber, pages 102-103)

Rutherford concluded that the nucleus of the atom is not only extremely tiny and dense, but positively charged. He reasoned the force of electric repulsion between the nucleus and alpha particle had deflected the alpha particle rather than a kinetic energy collision.

Rutherford had inferred this since the outstanding feature of the alpha particle was its positive charge, and as an alpha particle was deflected, the tiny flash he saw was the same as the tiny flash he saw as the alpha particles passed directly through the gold foil, which implied that the alpha particle retained its structure, and the deflection was not kinetic.

Indeed, an atom of gold has a positive electric charge of 79 units, or 79 protons in its nucleus, giving it plenty of charge to repel the alpha particle that has a positive charge of 2 units. Rutherford made this discovery in 1911. (Haber, pages 104-105)

Rutherford had discovered that atoms are not solid, but are like a deep well. The entrance to the well has a porous cover, before a bucket drawn into it reaches a dense, tiny nucleus that is positively charged.

However, a positively charged nucleus raised the question of where an atom's negative charge is located, which must be located elsewhere. The shell of the atom, or its exterior became the likely candidate since Rutherford's experiments had showed the interior of the atom is basically empty, except for the nucleus.

Moreover, locating an atom's negative charge on its shell would give the shell some substance. While very porous to the alpha particle, the shell would have a clear electrical definition, giving an atom a clear sense of symmetry in the location of its electric charge between its shell and nucleus.

Moreover, as the shell did not present a barrier to the alpha particle, it may be inferred the shell has a low density of both mass and charge. In other words, the charge of an electron shell is distributed around it, instead of being concentrated into tiny particles, which could interact with an alpha particle.

This diffusion of negative charge around a shell suggests that an electron is more complex than a tiny particle, or its electrical structure is spread out, so an electron may be pictured as having a tail.

An extended electrical structure or tail could help account for the orbit of an electron as it would not necessarily need to orbit at a high speed to avoid falling in toward the nucleus as its body is more complex than a single point.

Niels Bohr

In 1913 the Danish physicist Niels Bohr proposed that an electron orbits the nucleus at a high speed to offset the force of electrical attraction toward it. He viewed the electron as a tiny particle whose orbital mechanics resemble a planet orbiting a star under the influence of gravity. (Haber, pages 110-111)

Bohr also proposed that the orbit of an electron is quantized, or orbits the nucleus in a series of discrete steps. His idea that the orbit of an electron is quantized was based on the discovery in 1900 by Max Planck that a heated blackbody emits light in discrete packets of energy, or that light is quantized.

Planck had discovered that light is emitted in tiny packets of energy, which are multiples of a physical constant, called Planck's constant or h . The energy in an individual packet of light is equal to the frequency of the light, which is the reciprocal of its wavelength, multiplied by Planck's constant and the speed of light.

Bohr's idea that the orbit of an electron is quantized also built on spectral line analysis, which identifies an element by analyzing the light it emits. The light an element emits is not continuous like sunlight, but appears as a series of lines or wavelengths, with one wavelength usually dominant. (Still, page 20)

To explain how each element emits light at different wavelengths or spectral lines, Bohr realized each element has a nucleus that is different from another element as measured by the number of its protons or amount of charge, which along with its count of neutrons also determines its size.

The amount of charge in a nucleus affects the orbit of its electrons. For example, since helium has two protons in its nucleus, it exerts twice the force upon an electron than the nucleus of hydrogen, which has a single proton, so their electrons orbit at different distances.

Bohr also realized multiple electrons may inhabit the same orbit, dividing the orbit into equal pieces. For example, two electrons in the same orbit will orbit at opposite sides due to the force of repulsion between them. And while an orbit may accommodate more than two electrons, at some point, adding another electron to the orbit will force the additional electron into a higher orbit, further away from the nucleus due to the force of repulsion between electrons.

Moreover, an electron seems to require a certain amount of space in order to operate its electrical structure. As a result, atoms with many protons develop shells where electrons inhabit different orbits, which are quantized, according to their level of energy. When an element is heated, its atoms absorb energy, causing their electrons to shift orbit, and emit light in order to return to a state of equilibrium.

Since light is emitted in discrete packets of energy, Bohr reasoned an atom emits light as a result of a change in the orbit of one of its electrons, due to its gain or loss of energy in a discrete unit. He reasoned electron orbits are quantized by their level of energy since an electron operates much like a musical string, which vibrates in discrete steps to generate sound waves. (Still, page 21)

Bohr's idea of using the vibration of a string to represent the different energy levels for an electron, which are quantized by their orbit, gave a powerful explanation for how an electron functions like a wave. An electron orbits at a certain distance according to the length and energy level of its vibrating string, or electrical structure.

In contrast to the electron, the proton, which is nearly 2,000 more times more massive than the electron, carries its charge within its mass structure. With this in mind, the atom reflects a natural division due to how its positively charged nucleus and electrons carry their charge in different ways.

Max Planck

In 1900 Max Planck discovered that a heated blackbody emits light in discrete packets of energy. The energy contained within a packet is equal to the product of Planck's constant, denoted as h , the frequency of the light, which is the reciprocal of its wavelength, and the speed of light, which amplifies its force. Planck's equation is sometimes written as

$$E = h \cdot c \cdot \nu, \text{ or } E = h c \nu, \text{ where the dot for multiplication is omitted}$$

In this equation, E stands for energy, h stands for Planck's constant, c stands for the speed of light, and ν stands for the frequency of the light, which is the reciprocal of its wavelength.

Planck's constant is often given as $h = 6.626 \times 10^{-34}$ joules · seconds.

When Planck's constant is multiplied by the frequency of a wave, its units become joules, a unit of energy, multiplied by seconds per meter. Multiplied by the speed of light, these units become joules since the speed of light is a velocity whose units, when expressed in meters per second, cancel the other units of seconds per meter.

Planck had discovered that light is transmitted in individual packets, which may be called quantum waves, identified by their frequency or wavelength. Similar to how matter may be broken down into atoms but no further without becoming a mix of particles, binding energy, and wave functions, light may similarly be broken down into waves that are smaller, but no smaller than the quantum wave.

As a wave, a quantum wave requires both a wave front and depth to transmit energy. In effect, a quantum wave may be viewed as a tiny length of string or spring, which imparts energy.

Since a quantum wave occupies space, it takes on some of the attributes of a particle, although not possessing mass. As a result, light is sometimes treated as exhibiting the attributes of both a wave and particle.

From another point of view, Planck discovered how the light is emitted using quantum waves as a type of monetary currency. Just as financial transactions take place in dollars or other currencies as a measure of their economic value, atomic transactions involving light take place in quantum waves.

Photoelectric Effect

In his 1905 paper on the photoelectric effect, Einstein argued that light transmits energy like a particle when interacting with electrons, and introduced the term photon to describe an individual packet of light as a particle without mass.

Building on Planck's discovery that light was transmitted in individual packets, Einstein treated light as a particle so it could interact directly with an electron, jostling it out of its shell so the electron could appear in the form of electricity.

When Einstein wrote his paper in 1905, the atom was not well understood. Quantum mechanics had yet to be developed. The photon would let light interact directly with the electron, although this interaction would be specialized due to the photon's lack of mass or charge.

In 1923 Young's interference experiments showed that light passes through a pair of slits like a wave, subject to a probability function. While most scientists had viewed light as a wave, Planck's discovery that light was quantized had upset that view. Young's experiment reinforced the view of light as a wave. (Still, page 23)

In 1923 Louis De Broglie suggested that particles such as electrons, protons, and neutrons behave like light by displaying the attributes of both a wave and particle. But since matter waves are so small, they generally assume importance at the level of particles. (Still, page 23)

Proof of the existence of matter waves came in 1927 when George Thomson in England, and Clinton Davisson and Lester Germer in the United States observed Young's interference patterns for a beam of electrons passing through a pair of slits. (Still, page 23)

Using De Broglie's idea of "matter waves," Erwin Schrodinger, Werner Heisenberg, Max Born, and Pascual Jordan independently developed the theory to calculate the probability of a subatomic particle being observed at a particular location at a given level of energy. This theory became known as quantum mechanics since it dealt with the movement of particles. (Still, page 24)

Regarding the photoelectric effect, it may be suggested that since light travels in quantum waves, it may interact directly with an electron since electrons also exhibit some of the attributes of a wave, as seen in Young's interference experiments. Since waves of the same general type may interact with each other, a quantum wave may impart energy directly to an electron through its electrical structure.

As a quantum wave imparts energy to an electron, the electron may break loose from its orbital shell to appear in the form of electricity. Semiconductor materials such as silicon, which are on an intermediate section in the periodic table as being in between a metal or other material, tend to exhibit the photoelectric effect.

As an electron absorbs energy from a quantum wave, the electron retains the integrity of its electrical structure or tail, rather than losing mass from it like a comet approaching the Sun. Since the electron retains its structural integrity, it may be used again and again to flow as electricity.

This direct interaction between a quantum wave and electron avoids the interaction between a photon and electron, when the photon is a practically a contradiction in terms since a particle

is a tiny unit of matter, which implies the possession of mass, while the photon does not possess mass.

But since a quantum wave has a mass equivalent, letting it be treated as a particle, using Einstein's equation of $E = mc^2$ to convert its energy into mass, a photon may be viewed as being able to interact directly with an electron.

However, since using particles to transfer energy implies a transfer of momentum where both particles possess mass, using a photon as a particle without mass to transfer energy to an electron that has mass does not seem entirely logical.

On the other hand, when a quantum wave strikes an electron's extended electrical structure or tail, it may exert force directly against the electron as a similar type of energy, which is electromagnetic in nature.

Moreover, the interaction of a quantum wave with an electron's electrical structure or tail allows their interaction to occur over a relatively large area, letting the probability of their interaction be expressed in terms of area.

In contrast, the probability of a tiny photon striking a tiny electron is tiny.

Pauli Exclusion Principle

The Pauli Exclusion Principle states that two similar particles such as protons, neutrons, and electrons cannot exist in the same state, or have the same position and same velocity, within the limits of the Heisenberg Uncertainty Principle.

The Heisenberg Uncertainty Principle states that the uncertainty in a particle's position, which is often denoted since Δx since x is a variable commonly used for distance and Δ is often used to express a difference interval, multiplied by the uncertainty in its momentum, which is often denoted as Δp as p is often used to denote the momentum of a particle, is equal to or greater than $\hbar/2$, where \hbar is a derivative of Planck's constant, sometimes called the reduced Planck's constant or h bar.

The value of \hbar or h bar is equal to $h/2\pi$. It is used when h is divided by a parameter for a circular orbit such as an electron that fills its orbit with a wave function, or in calculations involving the quantization of angular momentum.

Where $h = 6.626 \times 10^{-34}$ joules · seconds, $\hbar = 1.054 \times 10^{-34}$ joules · seconds

Using Δx , Δp , and \hbar , the Heisenberg Uncertainty Principle may be restated as

$$\Delta x \cdot \Delta p \geq \hbar/2$$

In other words, the Heisenberg Uncertainty Principle states that the uncertainty in the position of a particle multiplied by the uncertainty in its momentum is equal to or greater than h bar divided by 2.

Since the momentum of a particle is equal to the product of its mass times its velocity and velocity is the rate of change in its position, and the mass of a particle is usually constant, the Heisenberg Uncertainty Principle may be restated where the standard deviation in the position of a particle is equal to or greater than h bar divided by 2, using an adjustment for units.

The use of the standard deviation in location rather than $\Delta x \cdot \Delta p$ may be argued if the measurement of time to determine velocity is accurate, which leaves the uncertainty in a particle's momentum largely a function of the uncertainty in its position.

From another point of view, the Pauli Exclusion Principle restates the principle of the atom, applied to its components of electrons, protons, and neutrons. The principle of the atom states that matter may be subdivided into smaller and smaller pieces but no smaller than the atom, which is the building block of all matter, and appears in the form solids, liquids, or gases.

As a corollary, the principle of the atom states that an atom inhabits a volume of space, which is not shared with other atoms, except in sharing electrons in their electron shell as atoms combine to form molecules.

Likewise, electrons, protons, and neutrons do not inhabit or occupy the same space as other particles, although they may pass by or collide with each other within the limits of the Heisenberg Uncertainty Principle. When particles pass by or collide in the same space, the particles have different velocities.

From an economic point of view, as atoms are aggregated, they form solids, liquids, and gases in a macroeconomic system. In contrast, electrons, protons, and neutrons analyze the atom at a microeconomic level. For electrons, protons, and neutrons to reach a macroeconomic level of activity, they must be aggregated into atoms.

From a mathematical point of view, the Pauli Exclusion Principle says that two similar particles inhabit different spaces. A particle inhabits a space that is identifiable, and is different from the space inhabited by other particles, where the boundaries of these spaces are set by the Heisenberg Uncertainty Principle.

But since particles are always in motion, two particles may inhabit the same space as they pass by or collide. So while two particles may occupy the same space, they do so in only a brief visit.

From an economic point of view, the Pauli Exclusion Principle may be viewed as a type of Exclusive Economic Zone that surrounds a particle, using the Heisenberg Uncertainty Principle to define its boundaries.

The Pauli Exclusion Principle applies to particles, not waves. Unlike particles, similar elements of energy can add or subtract in the same space, just as waves may add to create a wave of greater amplitude, and use the same medium for transmission like waves traveling through water.

However, like particles, waves also possess a sense of identity or uniqueness in their wavelength or frequency, amplitude or level of energy, and direction and general location. Waves use a different set of parameters to establish their identity than particles.

The Pauli Exclusion Principle is sometimes stated as applying to particles that possess a non-zero spin, or have mass. In contrast, particles that possess a spin of zero are usually seen as having no mass, and are used to transmit energy.

Particles that possess mass also possess structure. In other words, a particle with spin occupies a certain volume of space with a structure to house its mass, so that its structure may be observed.

If the entire structure of a particle may be observed by rotating the particle three hundred and sixty degrees in a circle, the particle is said to have a spin of one. A particle that has a structure that repeats itself within a three hundred and sixty degree rotation is said to have a spin greater than one.

For example, a particle with a spin of two repeats its structure two times within a three hundred and sixty degree rotation. On the other hand, a particle with a spin of one half has a structure that moves during the first rotation, so a second viewing is needed to observe the entire structure.

Electrons are sometimes said to have a spin of one half, and electrons have two types of spin, a top and bottom spin where the direction of the spin is identified by using a magnet. This suggests that their structure is more complex than a tiny particle.

Heisenberg Uncertainty Principle

While it is often seen as an obstacle to the observation of particles, the Heisenberg Uncertainty Principle may be treated as a statement about the identity of a particle, which identifies a particle in terms of its occupation of a minimum space. In other words, where Planck's constant describes the energy of an individual packet of light or quantum wave, it also describes a minimum space, or a quantum space that a particle occupies.

The space a particle occupies affects the determination of its velocity or momentum, and is implied in the description of the space it occupies. In other words, a particle may be identified by its position, and velocity or momentum as an indication of its mass, by using Planck's constant.

In effect Planck's constant gives a discrete measurement of a particle in terms of occupying a minimum space, or quantum space, at a given velocity or momentum, which is related to its level of energy. In terms of a wave, this minimum space and energy level define the intrinsic vibration of the particle. The distance over which a particle is located and its momentum define the space the particle has to vibrate in, and its level of energy.

In effect, the Heisenberg Uncertainty Principle may be stated to say that a particle may resemble a wave by having a certain energy level, which is associated with its momentum, and a wavelength or frequency, which is associated with its position, for which Planck's constant defines its limits or boundaries. In terms of resembling a wave, the uncertainty in the position of a particle becomes a wavelength, for which the particle's velocity or momentum is related to the measurement of its energy.

Just as Einstein showed how matter and energy are equivalent, Planck's constant may be used to explain how light is transmitted in discrete packets, and the quantization of matter in terms of a particle that occupies a minimum space, or quantum space. In other words, just as light is discrete, matter is discrete, identifiable in terms of the space a particle occupies, and related to its level of energy.

The space a particle occupies is unique. Other particles are excluded from occupying the same space, although they may pass through or collide with it, so that the Pauli Exclusion Principle appears as a corollary of Planck's constant.

The idea of a quantum space may be viewed by restating the units of Planck's constant, which is often expressed as the product of joules multiplied by seconds, or joules seconds and express the energy of a quantum wave as the product of Planck's constant multiplied by the frequency of the light, and the speed of light.

The units for Planck's constant may be rewritten since a joule is equal to the energy contained in a mass of one kilogram, which is accelerated over a distance of one meter, at a rate of one meter per second squared. In other words,

1 joule = $1 \text{ kg} \cdot \text{m} \cdot \text{m}/\text{seconds}^2$ and substituting this expression for a joule into Planck's constant gives

$$h = 6.626 \times 10^{-34} \text{ joules} \cdot \text{seconds} = 6.626 \times 10^{-34} \text{ kg} \cdot \text{m} \cdot \text{m}/\text{seconds}^2 \cdot \text{seconds}, \text{ or}$$

$$6.626 \times 10^{-34} \text{ m} \cdot \text{kg m/s}$$

The units of meters, which measure distance, multiplied by the product of the units of kilograms times meters per second, which measure mass and velocity, gives a value for Planck's constant in terms of units that are suitable for particles.

In particular, the uncertainty in the position of a particle may be expressed in terms of distance or meters; and the uncertainty in the particle's momentum may be expressed in terms of its mass multiplied by its velocity. As a result, the units of Planck's constant may be expressed using units of distance for the uncertainty in a particle's position, multiplied by units for the uncertainty in its momentum, or $\Delta x \cdot \Delta p$.

This is the usual way to express the Heisenberg Uncertainty Principle as the product of the uncertainty in the position of a particle multiplied by the uncertainty in its momentum, which is set equal to or greater than the reduced Planck's constant, divided by two as a measure of an average value.

In other words, Planck's constant can unfold into the Heisenberg Uncertainty Principle since it describe the quantum space of a particle, which measures its approximate location in terms of distance, and its energy level in terms of momentum.

Alternate Derivation

When in 1905 Albert Einstein published his paper showing the equivalence of matter and energy, relating them in the equation of $E = mc^2$ where E stands for energy, m stands for mass, and c stands for the speed of light, he gave a powerful tool for applying Planck's constant to particles.

In other words, there is the complete convertibility of matter into energy and of energy into matter to where matter follows the same laws of physics as energy, including key principles of light, such as Planck's law.

In other words, particles follow Planck's law, where light is emitted in discrete packets of energy, or quantum waves, and describes the energy of an individual packet of light or quantum wave, as $E = h c \nu$, where ν is the frequency of the light, which is the reciprocal of its wavelength.

Planck's law may be rewritten as $E = h c / \lambda$, using λ for the wavelength of the light. Using Einstein's equation of $E = mc^2$, Planck's law has an analogy in the world of matter, where the energy in a quantum wave has an analogy as a quantum mass.

Since $E = mc^2$ and $E = h c / \lambda$, setting both expressions for energy equal to each other gives the result that $mc^2 = h c / \lambda$, or $mc^2 = h c / \lambda$. Dividing both sides of the equation by c or the speed of light, gives $mc = h / \lambda$.

Again dividing both sides of the equation by c to isolate m , lets a quantum mass be expressed as a function of the frequency of wavelength of light, or $m = (h/c) / \lambda$.

Using a value for c of 3×10^8 m/s gives

$$m = (h/c) / \lambda = (6.626 \times 10^{-34} \text{ m} \cdot \text{kg m/s}) / (3 \times 10^8 \text{ m/s}) \cdot 1/\lambda (1/\text{m}), \text{ or}$$

$$m = (6.626 \times 10^{-34} \text{ m} \cdot \text{kg}) / (3 \times 10^8) \cdot 1/\lambda (1/\text{m}), \text{ or}$$

$$m = 2.208 \times 10^{-42} \text{ kg m} \cdot 1/\lambda (1/\text{m}), \text{ or}$$

$$m = 2.208 \times 10^{-42} \text{ kg m} \cdot 1/\lambda (1/\text{m})$$

The units of this expression for a quantum mass result in kilograms when the wavelength of the light is expressed in meters.

While the idea of a quantum mass does not answer current investigations into tiny particles of matter that form building blocks for subatomic particles, a quantum mass that is dependent upon a wavelength or frequency of vibration may help link the world of matter to the world of energy waves.

Using a wavelength of one meter as a nominal value to test this concept, the value of a quantum mass becomes 2.208×10^{-42} kg, a value that is in line with the weight of various tiny subatomic particles, or their magnitude, so that this idea may merit further investigation.

In other words, Planck's constant gives a workaround to the mass of light, to express its energy in terms of momentum. The frequency of the light is the inverse of its wavelength, which may be said to identify its position, or the amount of space it takes to achieve a vibration of energy.

From another point of view, since matter is quantized, using Planck's law, a quantum unit of matter has a wavelength of frequency of vibration, which requires a minimum space or length for it to vibrate freely. This minimum space or length requires the quantum mass to keep itself separate from other quantum masses, which also need to vibrate freely, without interference from other masses.

Aggregating quantum masses into particles results in the Pauli Exclusion Principle, which requires particles to maintain a separate identity from other particles by not occupying the same space or space of vibration except when other particles are passing through at a different velocity.

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