

## SURVEY OF 4WS AUTOMOTIVE MOVEMENT FUND WITH THE EFFECTS OF TIRE STIFFNESS

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**ABSTRACT:** *This paper proposes a 4WAS (4 Wheel Active Steer) system assists drivers by automatically controlling the steering angle of a vehicle's four wheels according to speed. By controlling the steering angle of all four wheels, this active steering system helps improve stability and response at high speed and helps reduce driver's steering workload at low speed. The study compared the operation of the system with and without PID controller. Computer simulations demonstrate good maneuverability of the proposed system.*

**KEYWORDS:** Active four wheel steering, 4 Wheel Active Steer, PID controller, stiffness in the tire

### INTRODUCTION

In recent years, active steering systems (Ackermann, 1997; Shimada et al., 1997) [1] have been studied and developed to improve safety. The author (Hiraoka et al, 2001; .. Hiraoka et al, 2002) [3] also proposed an active front steering system (AFS-Active Front Steering) mainly based on the acceleration of the body rotation and lateral acceleration. By using the control rules, the acceleration at center of gravity can adjust the wheel rotation angle without being affected by different system working conditions. However, the AFS system as well as the conventional 2WS (2 Wheel Steer) steering system still has certain disadvantages that need to be improved: when the motion on the slippery road behind the wheel has tendency to slip when rotating. In the research framework of this paper, the author will introduce 4WAS system - this system is a combination of AFS and ARS. The system, which overcomes the inherent disadvantages inherent in AFS and 2WS systems and improves the turning radius significantly.

### 4WAS BICYCLE MODEL

Consider the bicycle model with the hypothetical wheel lying between the wheels and rolling off with the mean deviation rolling angle of the right and left wheels so that the hypothetical wheel direction passes through the intersection of the wheel deviation. left and right side. The model consists of 3 degrees of freedom: vertical displacement  $x$ , horizontal displacement  $y$  and the angle of rotation around the vertical axis passing through the center of the car  $\psi$  according to the XCY mobile coordinate system mounted at the center of C cell bowl. [3]

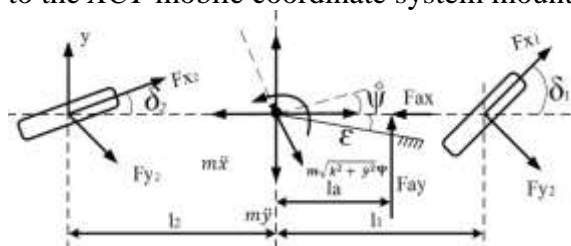


Figure 1. Model of a 4WAS steering system

Inside:

Front and rear guide wheels rotate by an angle on average  $\delta_1, \delta_2$ . At the contact area between the wheel and the road surface, there are jet components in the vertical and horizontal planes of the front and rear wheels, respectively  $F_{x1}, F_{y1}, F_{x2}, F_{y2}$ . At the center of the car there are longitudinal inertial force components  $m_v \ddot{x}$ , horizontal  $m_v \ddot{y}$ , centrifugal inertia force  $m_v \dot{\psi} \sqrt{\dot{x}^2 + \dot{y}^2}$  and moment of inertia  $J_v \ddot{\psi}$ . Components of longitudinal wind resistance  $F_{ax}$ , horizontal  $F_{ay}$  placed a distance from the center of the car  $l_a$

Ignore wind resistance, air resistance :

**Equation of the force in the direction x :**

$$-m\ddot{x} + m\sqrt{\dot{x}^2 + \dot{y}^2}\dot{\psi} \sin\Psi + F_{x1}\cos\delta_1 + F_{y1}\sin\delta_1 + F_{x2}\cos\delta_2 + F_{y2}\sin\delta_2 - F_{ax} = 0 \quad (1)$$

**Equation of the force in the direction y :**

$$-m\ddot{y} - m\sqrt{\dot{x}^2 + \dot{y}^2}\dot{\psi} \cos\Psi + F_{x1}\sin\delta_1 - F_{y1}\cos\delta_1 + F_{x2}\sin\delta_2 - F_{y2}\cos\delta_2 + F_{ay} = 0 \quad (2)$$

**The torque balance equation:**

$$-I\ddot{\psi} + l_1(F_{x1}\sin\delta_1 - F_{y1}\cos\delta_1) - l_2(F_{x2}\sin\delta_2 - F_{y2}\cos\delta_2) + F_{ay}l_a = 0 \quad (3)$$

(1), (2), (3) transforming us is:

$$\begin{cases} m\ddot{x} = m\dot{\psi}\dot{y} + F_{x1}\cos\delta_1 + F_{y1}\sin\delta_1 \\ \quad + F_{x2}\cos\delta_2 + F_{y2}\sin\delta_2 - F_{ax} \\ m\ddot{y} = -m\dot{x}\dot{\psi} + F_{x1}\sin\delta_1 - F_{y1}\cos\delta_1 \\ \quad + F_{x2}\sin\delta_2 - F_{y2}\cos\delta_2 + F_{ay} \\ I\ddot{\psi} = l_1(F_{x1}\sin\delta_1 - F_{y1}\cos\delta_1) \\ \quad - l_2(F_{x2}\sin\delta_2 - F_{y2}\cos\delta_2) + F_{ay}l_a \end{cases}$$

Inside  $F_{x1}, F_{x2}, F_{y1}, F_{y2}$  are the force components interacting between the wheel and the road surface, using a linear model :

$$\begin{aligned} F_{x1} &= C_{x1}\lambda_1 & ; & & F_{x2} &= C_{x2}\lambda_2 \\ F_{y1} &= C_{y1}\alpha_1 & ; & & F_{y2} &= C_{y2}\alpha_2 \end{aligned}$$

$$\alpha_1 = \text{atan}\left(\frac{\dot{y} + \dot{\psi}l_1}{\dot{x}}\right) - \delta_2$$

$$\alpha_2 = \text{atan}\left(\frac{\dot{y} - \dot{\psi}l_2}{\dot{x}}\right) - \delta_2$$

$$\lambda_1 = \frac{\omega_1 R_{bx} - (\dot{x}\cos\delta_1 + \dot{y}\sin\delta_1 + \dot{\psi}l_1\sin\delta_1)}{\omega_1 R_{bx}}$$

Watch the car move evenly  
 $\dot{x} = v_0$ ;  $\delta_1$ ;  $\delta_2$  small:

$$\begin{cases} m\ddot{y} = -mv_0\dot{\psi} - F_{y1} - F_{y2} + F_{ay} \\ I\ddot{\psi} = -l_1F_{y1} + l_2F_{y2} + F_{ay}l_a \end{cases}$$

$$\begin{aligned}
F_{y1} &= C_{y1}\alpha_1 = C_{y1}\left(\frac{\dot{y} + \dot{\Psi}l_1}{v_0} - \delta_1\right) \\
F_{y2} &= C_{y2}\alpha_2 = C_{y2}\left(\frac{\dot{y} - \dot{\Psi}l_2}{v_0} - \delta_2\right) \\
\leftrightarrow \begin{cases} \dot{y} = -v_0\dot{\Psi} - \frac{1}{m}C_{y1}\left(\frac{\dot{y} + \dot{\Psi}l_1}{v_0} - \delta_1\right) \\ -\frac{1}{m}C_{y2}\left(\frac{\dot{y} - \dot{\Psi}l_2}{v_0} - \delta_2\right) + \frac{F_{ay}}{m} \\ I\ddot{\Psi} = -l_1C_{y1}\left(\frac{\dot{y} + \dot{\Psi}l_1}{v_0} - \delta_1\right) \\ + l_2C_{y2}\left(\frac{\dot{y} - \dot{\Psi}l_2}{v_0} - \delta_2\right) \end{cases} \\
\leftrightarrow \begin{cases} \ddot{y} = -\left(\frac{C_{y1} + C_{y2}}{mv_0}\right)\dot{y} - \left(\frac{C_{y1}l_1 - C_{y2}l_2}{mv_0} + v_0\right)\dot{\Psi} \\ + \frac{C_{y1}}{m}\delta_1 + \frac{C_{y2}}{m}\delta_2 + \frac{F_{ay}}{m} \\ I\ddot{\Psi} = -\left(\frac{C_{y1}l_1^2 + C_{y2}l_2^2}{v_0}\right)\dot{\Psi} - \left(\frac{C_{y1}l_1 - C_{y2}l_2}{v_0}\right)\dot{y} \\ + C_{y1}l_1\delta_1 - C_{y2}l_2\delta_2 + F_{ay}l_a \end{cases} \\
\leftrightarrow \begin{cases} \ddot{y} = -\frac{C_{y1} + C_{y2}}{mv_0}\dot{y} - \left(\frac{C_{y1}l_1 - C_{y2}l_2}{mv_0} + v_0\right)\dot{\Psi} \\ + \frac{C_{y1}}{m}\delta_1 + \frac{C_{y2}}{m}\delta_2 + \frac{F_{ay}}{m} \\ \ddot{\Psi} = -\frac{C_{y1}l_1^2 + C_{y2}l_2^2}{Iv_0}\dot{\Psi} - \frac{C_{y1}l_1 + C_{y2}l_2}{Iv_0}\dot{y} \\ + \frac{C_{y1}l_1}{I}\delta_1 - \frac{C_{y2}l_2}{I}\delta_2 + \frac{F_{ay}l_a}{I} \end{cases}
\end{aligned}$$

Write in the matrix we have:

$$\begin{aligned}
\begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\Psi} \\ \ddot{\Psi} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{C_{y1} + C_{y2}}{mv_0} & 0 & \left(v_0 + \frac{C_{y1}l_1 - C_{y2}l_2}{mv_0}\right) \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{C_{y1}l_1 - C_{y2}l_2}{Iv_0} & 0 & -\frac{C_{y1}l_1^2 + C_{y2}l_2^2}{Iv_0} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \Psi \\ \dot{\Psi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{C_{y1}}{m} \\ 0 \\ C_{y1}l_1 \end{bmatrix} \delta_1 \\
&+ \begin{bmatrix} 0 \\ \frac{C_{y2}}{m} \\ 0 \\ -C_{y2}l_2 \end{bmatrix} \delta_2 + \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{l_a}{I} \end{bmatrix} F_{ay}
\end{aligned}$$

The condition that the equation above stabilizes is all individual values  $\lambda_i$  of the matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{C_{y1} + C_{y2}}{mv_0} & 0 & \left( v_0 + \frac{C_{y1}l_1 - C_{y2}l_2}{mv_0} \right) \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{C_{y1}l_1 - C_{y2}l_2}{Iv_0} & 0 & -\frac{C_{y1}l_1^2 + C_{y2}l_2^2}{Iv_0} \end{bmatrix}$$

Have real negative parts. Moment of inertia  $I$  can be approximated by formula:

$$I = ml_1l_2$$

### DESIGNING THE PID CONTROLLER

Consider the model in ideal conditions [3]

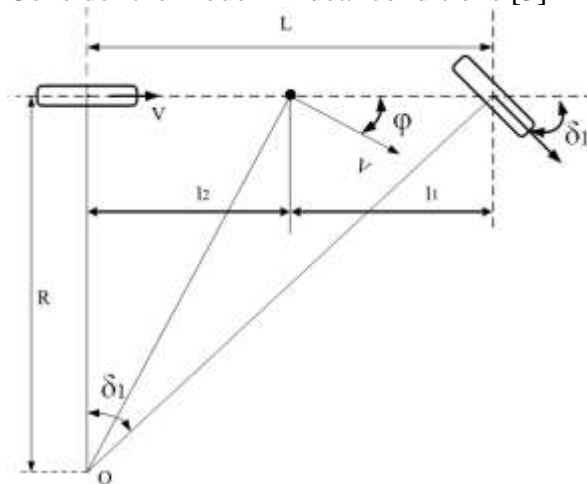


Figure 2. Turn around in ideal conditions

In the picture we have a turning radius:  $R = \frac{l_1 + l_2}{\tan \delta_1}$

Centrifugal acceleration:

$$a_{yd} = \frac{v^2}{R} = \frac{v^2 \tan \delta_1}{l_1 + l_2}$$

Realistic

centrifugal

acceleration:

$$a_y = \frac{v^2}{R} = v \frac{v}{R} = v \dot{\Psi}$$

With  $\dot{\Psi}$  is the angular velocity of the vehicle body. Finally, we get the difference between actual and theoretical centrifugal acceleration [4]

$$e = a_y - a_{yd}$$

Table 1. Parameters of the simulation model

| Symbol | Value | Unit                 |
|--------|-------|----------------------|
| $J_v$  | 2100  | [kg.m <sup>2</sup> ] |
| $l_1$  | 1,1   | [m]                  |
| $l_2$  | 1,3   | [m]                  |
| $C_y$  | 90624 | [N/rad]              |
| $l_a$  | 0,4   | [m]                  |
| $d$    | 1,4   | [m]                  |

Choose the parameters :  $K_p=1$ ;  $K_i=1$ ;  $K_d=0$  for the PID controller .

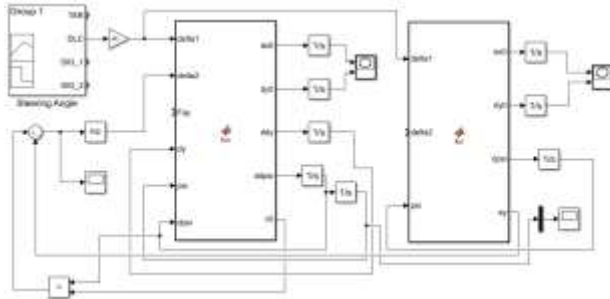


Figure 3. Simulation of a PID controller in Matlab Simulink

**MODEL AND SURVEILLANCE OF AUTOMOTIVE MOVEMENT FUND WHEN USING PID CONTROLLER**

**Case 1 when the car turns around with the guide wheel rotation angle unchanged**

Front tire stiffness  $C_{y1} = 0,5C_y$ ;  $C_{y2} = C_y$ ,  $F_{ay} = 0$ . The driver steers from the start to the 5th second to achieve a steering wheel angle of 30 degrees then keep the wheel rim

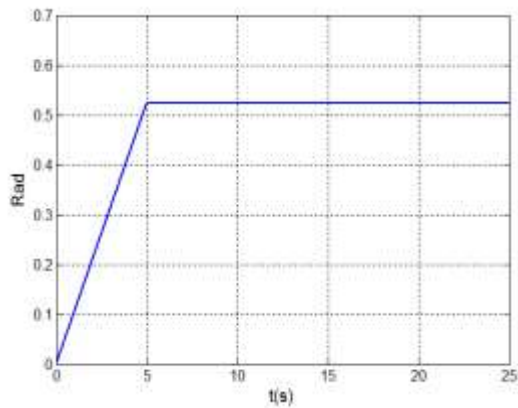


Figure 4. a. Steering angle

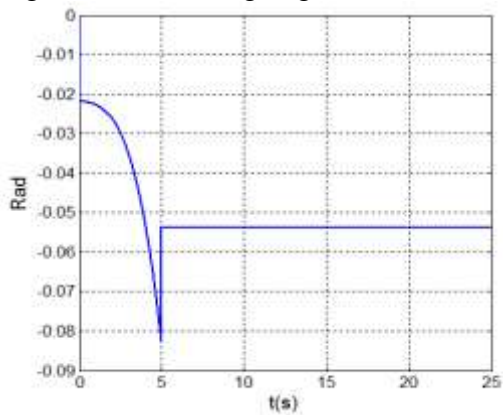


Figure 4. b. Rear wheel offset angle

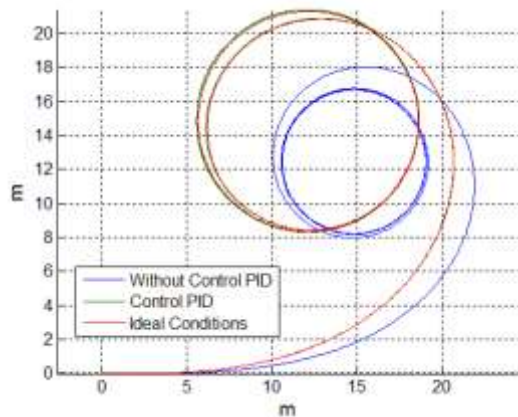


Figure 5. The trajectory of a car's movement when turning around

Result: At the beginning of the simulation, the car's center of gravity is at the origin, the motion trajectory of the car changes when it is influenced by the steering wheel. When there is a PID controller, the radius of rotation is greater than that of a wheel without a controller, thus making the car more stable. At the same time, the trajectory of the car with PID control adheres to the ideal road conditions.

**Case 2, when linear motion is influenced by horizontal wind:**

Considering a car in straight motion that has a sudden impact of horizontal wind in a period of time. The figure shows that the transverse wind impacts the horizontal wind force with a value of 200 [N] during the simulation

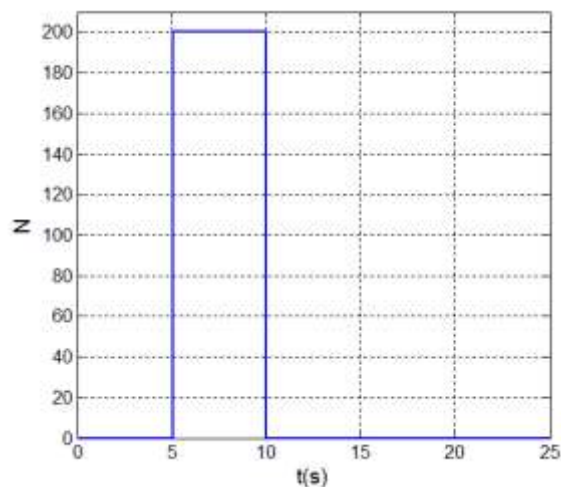


Figure 6.a. Horizontal wind force

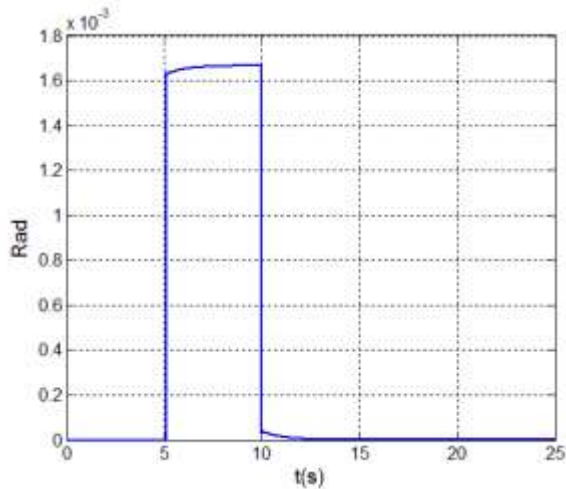


Figure 6 .b. Rear wheel offset angle

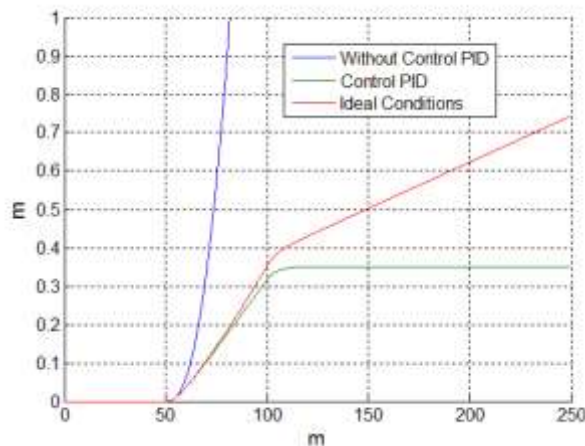


Figure 7. The trajectory of the car's motion when there is a horizontal wind effect

In the period of 5[s] to 10[s], which corresponds to a longitudinal displacement of about 53[m] to 100[m], the trajectory of the car is deviated by about 0.35 [m] then the car moves in a straight line parallel to the original direction. To ensure that the car retains its original motion, the driver must have the action of adjusting the direction of motion of the car through the steering system. The car's trajectory of motion with PID controller is better in the absence of PID control and ideal conditions because the car retains its direction of motion and the vehicle only deviates a short distance of 0.35m horizontal. If the driver wants to change the car to coincide with the movement and direction before the impact of the horizontal wind, just steer lightly.

### Case of 3 vehicles changing lanes

Select the following tire hardness parameters:  
 $C_{y1}=0,4C_y$  ;  $C_{y2}=C_y$ ;

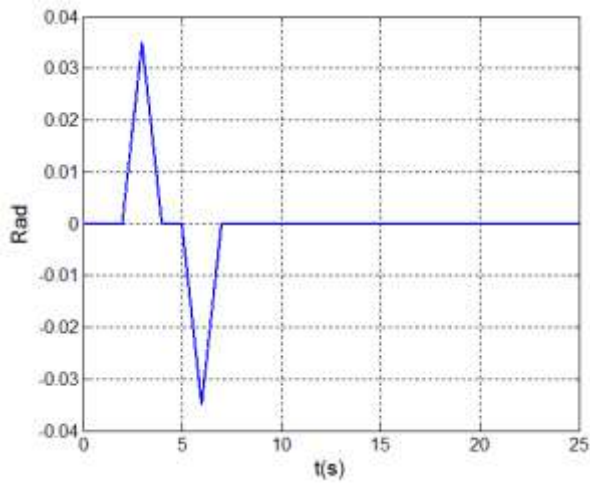


Figure 8. a. Steering angle

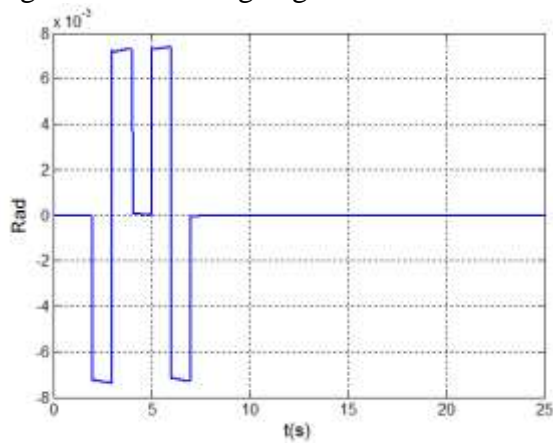


Figure 8. b. Rear wheel offset angle

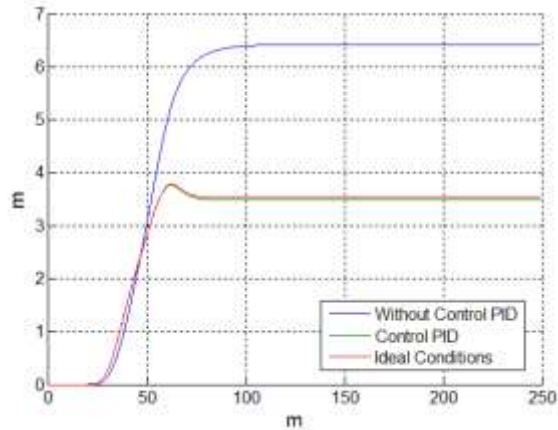


Figure 9. The trajectory of car movement when changing lanes

The distance of changing lanes of cars without a PID controller is about 6.45 [m]. If a car changes lanes on a road with 1 lane width of about 4 [m], the ability to change lanes is almost impossible but with PID controller it is about 3.3 [m] compared to ideal conditions of 3,5 [m]. Therefore, the vehicle changing lanes is more stable with PID control.



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## CONCLUSION

On the basis of the built mathematical model, the author uses MATLAB / Simulink software to simulate the rotation dynamics of a car with a set of parameters of a particular vehicle. Perform simulation of the revolving trajectory with different cases when using PID controller and not using PID controller such as : rotation with constant angle of rotation of guide wheel; movement simulation of cornering, cornering - out cornering; Simulates the motion trajectory of a car while in straight motion with horizontal winds. The simulation results show the law and suitability of the research model.

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