International Journal of Mathematics and Statistics Studies
Vol.10, No.3, pp.28-38, 2022
Print ISSN: 2053-2229 (Print),
Online ISSN: 2053-2210 (Online)

Stochastic Analysis of Asset Price Returns for Capital Market Domain

Amadi, Innocent Uchenna[1] and Charles, Anthony[2]

[1]Department of Mathematics/Statistics Captain Elechi Amadi polytechnic, Rumuola Port Harcourt.
[2]Department of Mathematics/Statistics, Ignatius Ajuru University of Education , Rumuolumeni, Port Harcourt

Citation: Amadi, Innocent Uchenna and Charles, Anthony (2022) Stochastic Analysis of Asset Price Returns for Capital Market Domain, *International Journal of Mathematics and Statistics Studies,* Vol.10, No.3, pp.28-38

ABSTRACT: *The solution of differential equations and stochastic differential equations of time-varying investment returns is considered. Precise conditions are obtained which govern asset price return rate through multiplicative and multiplicative inverse trends series. From the analysis of the proposed model showed that multiplicative inverse trend series is efficient and reliable than multiplicative trend in both deterministic and stochastic systems. Furthermore, the analytical solutions were explicitly verified graphically and discussed accordingly.*

KEYWORDS: differential equations, stochastic differential equation, asset returns.

INTRODUCTION

Market price of stock is the most recent price at which the stock was traded. It is the result of traders, investors and dealers interacting with each other in a market. The influence of stock market operations and performance has gained a significant role in financial development of markets as investors gain their normal returns to earn a living. However, stock trading is normally known in making high returns. Return on investment for capital market is a measure to properly evaluate gain of an investment. It is a major parameter often used by a trader or investors to regulate profitability of expenditure.

In stock market the dynamics of stock prices are reflected by uncertain movement of their value over time. One possible reason for the random behavior of the asset price is the efficient market (EMH) hypothesis. The EMH basically states two things: (a) the past history of a stock price is fully reflected in present prices. (b) The markets respond immediately to any new information about the stock. These two assumptions imply that changes in the stock price are a markov process. These two assumptions imply that changes in the stock depend only on its current price. Markov process is a class of stochastic processes in which the probability of the process being in a given state at a particular time is related only to the immediately preceding state of that process.

Nevertheless, scholars has written extensively on stock market price changes and its return rates, for instance,[1].A stochastic analysis of stock market expected returns and Growth-rates were investigated. The precise conditions for obtaining the drifts, volatilities and Growth-rates of four different stocks were also considered. In the same vain,[2] studied the problem of stock price fluctuations using stochastic differential equations, principal component analysis and KS goodness of fit test. The analytical solutions were detailed; the computational and graphical results were presented and discussed respectively; as if it was not enough, [3] studied the stochastic analysis of stock market expected returns for investors. The detailed conditions for obtaining the drifts, volatilities and variances of four different stocks were considered. We compared the variances of four different stocks using criteria for selection and the result showed that stock (1) is the best among the stocks of different companies.

Still in modeling [4] considered the unstable nature of stock market forces using proposed differential equation model. In the work of [5] studied stability analysis of stochastic model of price change at the floor of a stock market. In their research précised conditions are obtained which determines the equilibrium price and growth rate of stock shares.

[6] Considered stochastic analysis of the behavior of stock prices. Results reveal that the proposed model is efficient for the prediction of stock prices. In the same vain, [7] studied the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in their research the drift and volatility coefficients for the stochastic differential equations were determined and the Euler-Maruyama method for system of SDE'S was used to stimulate the stock prices. [8], built the geometric Brownian Motion and studied the accuracy of the model with detailed analysis of simulated data.

More so [3] worked on stochastic model of the fluctuation of stock market price is considered. Here conditions for determining the equilibrium price, sufficient conditions for dynamic stability and convergence to equilibrium of the growth rate of the value function of shares. On the other hand, [9] considered a stochastic model of price changes at the floor of stock market. In their research the equilibrium price and the market growth rate of shares were determined. See [10] for considerable extensions and constrains subsequently in this particular area of study.

Amadi et al.(2022) Investigated the stochastic analyses of Markov chain in finite states. The data was subjected to 5-step transition matrix for independent stocks where transition matrix replicated the use of 3-states transition probability matrix which enables them to proffer precise condition of obtaining expected mean rate of return of each stock.

In this paper, differential equations and stochastic differential equation were solved analytical for time varying investment return. These equations were solved to study the behavior of deterministic and stochastic systems in predicting different commodity price processes using multiplicative and multiplicative inverse trends series as major parameter in the model.

The aim of this paper is to propose an investment equations whose rate of return follows multiplicative and multiplicative inverse trends series respectively in time-varying investment returns. To the best of our knowledge these novel contributions is unique and reliable predicting return rates.

This paper is arranged as follows: Section 2.1 presents the mathematical formulation of the problem, method of solution is seen in Subsection 2.1.1, Results are seen in Section 3.1, while the discussion of results is presented in Section 3.1.1 and paper is concluded in Section 4.1.

Mathematical Formulation of the Problem

Considering a complete probability space $(\Omega,,\mathrm{F}\wp)$ endowed with a Wiener processes such that a finite time investment horizon $T>0$ .Investors are always geared towards maximizing investment returns. Therefore, this paper is built on the premise that an investor's rate of return grows either multiplicative and multiplicative inversely trends respectively at time t . This assertion is good because investors observes prices and takes action in discrete time periods $t = 0,1,2,3,4,...,T$,the factors underlying price changes are very uncertain and are described in probability terms,[13].Hence the rate of return is defined as follows:

$$R_t := \lambda_1\lambda_2,...\text{where } t=1,2... \tag{1.1}$$

$$R_t := (\lambda_1\lambda_2)^{-1},..,\text{where } t=1,2.. \tag{1.2}$$

Thus the stochastic process describing the process is of the form:

$$dS(t) = \lambda dt + \beta dZ^{(1)}(\mathrm{t}) \tag{1.3}$$

where λ is an expected rate of returns on stock, β is the volatility of the stock , dt is the relative change in the price during the period of time and $Z^{(1)}$ is a Wiener process . Using (1.1)-(1.3) gives the following mathematical structure:

$$\frac{dS_1(t)}{dt} = (\lambda_1\lambda_2)S_1(t) \tag{1.4}$$

$$dS_2(t) = (\lambda_1\lambda_2)\mathrm{S}_2(t)dt + \beta S_2(t)dZ^{(1)}(\mathrm{t}) \tag{1.5}$$

$$\frac{dS_3(\mathrm{t})}{dt} = (\lambda_1\lambda_2)^{-1} S_3(t) \tag{1.6}$$

$$dS_4(\mathrm{t}) = (\lambda_1\lambda_2)^{-1} S_4(t)dt + \beta S_4(t)dZ^{(1)}(\mathrm{t}) \tag{1.7}$$

Where $S_1(\mathrm{t}), S_2(\mathrm{t}), S_3(\mathrm{t})\ and\ S_4(\mathrm{t})$ are underlying stocks with the following initial conditions:

$$S_1(0) = S_1 0 \, , t > 0 \tag{1.8}$$

$$S_2(0) = S_2 0 \, , t > 0 \tag{1.9}$$

$$S_3(0) = S_3 0 \, , t > 0 \tag{1.10}$$

$$S_4(0) = S_4 0 \, , t > 0 \tag{1.11}$$

Though, the price evolution of a risky assets are usually modeled as the trajectory of a risky assets that are usually of a diffusion process defined on some underlying probability space ,with the geometric Brownian motion the paramount tool used as the established reference model, [13].

Theorem 1.1: (Ito's formula) Let $(\Omega, \beta, \mu, \mathrm{F}(\beta))$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, \mathrm{F}(\beta))$ processing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t)$$

$t \in \Re$ and for $u = u(t, X(t) \in C^{1\times 2}(\Pi \times \square))$

$$du(t, X(t)) = \left\{ \frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2} f^2 \frac{\partial^2 u}{\partial x^2} \right\} d\tau + f\frac{\partial u}{\partial x} dW(t)$$

METHOD OF SOLUTION

The model (1.4)-(1.7) consist of a system of variable coefficient differential equations and stochastic differential equations whose solutions are not trivial. We adopts the methods of variable separable and Ito's lemma in solving for $\mathrm{S}_1(t),\ S_2(t),\ S_3(t),\ and\ S_4(t)$ To tackle this problem we note that

$\mathrm{S}_1(t),\ S_2(t),\ S_3(t)$ and $S_4(t) < \infty$ for all $r \in [0,1)$

From (1.4), this equation is solved using separable variable, hence

$$\frac{dS_1(t)}{S_1(t)} = (\lambda_1 \lambda_2)dt$$

Integrating both sides gives

$$\int \frac{dS_1(t)}{S_1(t)} dS = \int (\lambda_1 \lambda_2) dt + \phi \ , \ \ln S_1(t) = \lambda_1 \lambda_2 t + \ln \phi$$

$$\ln\left(\frac{S_1(t)}{\phi}\right) = \lambda_1 \lambda_2 t$$

Taking log of the both sides the following

$$S_1(t) = \phi e^{\lambda_1 \lambda_2 t} \tag{1.12}$$

Applying the initial condition in(1.8) yields

$$S_1(t) = S_0 e^{\lambda_1 \lambda_2 t} \tag{1.13}$$

From (1.5) , Thus, generalizing and considering a function $f\left(S_2(t), t\right)$,hence it has to do with partial derivatives. Expansion of $f\left(S_2(t), \mathrm{dS}_2(\mathrm{t}), t+dt\right)$ in a Taylor series about $\left(S_2(t), t\right)$ gives

$$df = \frac{\partial f}{\partial S_2(t)} dS_2(t) + \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S_2{}^2(t)} dS_2^2(t) + \ldots \tag{1.14}$$

Substituting(1.5) in (1.14) gives

$$df = \frac{\partial f}{\partial S_2(t)} \left\{ \beta S_2(t) dZ^{(1)}(\mathrm{t}) + (\lambda_1 \lambda_2) S_2(t) dt \ \right\} + \frac{1}{2} \frac{\partial^2 f}{\partial S_2{}^2(t)} dS_2^2(t) \mathrm{dS}_2^2(\mathrm{t})$$

$$df = \beta S_2(t) \frac{\partial f}{\partial S_2(t)} dZ^{(1)}(t) + \left((\lambda_1 \lambda_2) S_2(t) \frac{\partial f}{\partial S_2(t)} + \frac{1}{2} \beta^2 S_2^2(t) \frac{\partial^2 f}{\partial S_2{}^2(t)} dS_2^2(t) \right) dt$$

Now considering the SDE in(1.5)

Let $f\left(S_2(\mathrm{t})\right) = \ln S_2(t)$, the partial derivatives becomes

$$\frac{\partial f}{\partial S_2(t)} = \frac{1}{S_2(t)}, \ \frac{\partial^2 f}{\partial S_2^2(t)} = -\frac{1}{S_2^2(t)}, \ \frac{\partial f}{\partial t} = 0 \tag{1.15}$$

According to theorem 1.1(Ito's),substituting(1.14) and simplifying gives

$$df = \left\{ (\lambda_1 \lambda_2) - \frac{1}{2} \beta^2 \right\} dt + \beta dZ^{(1)} \tag{1.16}$$

Since the RHS of (1.16) is independent of $f\left(S_2(t)\right)$,the stochastic is computed as follows:

$$f(S_2(t) = f_0 + \int_0^t \left\{(\lambda_1\lambda_2) - \frac{1}{2}\beta^2 \quad \right\} dt + \int_0^t \beta dZ^{(1)}(t)$$

$$= f_0 + \left\{(\lambda_1\lambda_2) - \frac{1}{2}\beta^2\right\} t + \beta dZ^{(1)}(t), \text{Since } f\left(S_2(t)\right) = \ln S_2(t) \text{ a found solution for } S_2(t) \text{ becomes}$$

$$\ln S_2(t) = \ln S_o + \left\{(\lambda_1\lambda_2) - \frac{1}{2}\beta^2\right\} t + \beta dZ^{(1)}(t), \ln S_2(t) - \ln S_2 0 = \left\{(\lambda_1\lambda_2) - \frac{1}{2}\beta^2\right\} t + \beta dZ^{(1)}(t)$$

$$\ln\left(\frac{S_2(t)}{S_0}\right) = \left\{(\lambda_1\lambda_2) - \frac{1}{2}\beta^2\right\} t + \beta dZ^{(1)}(t)$$

$$S_2(t) = S_0 \exp^{\left\{\left((\lambda_1\lambda_2) - \frac{1}{2}\beta^2\right) t + \beta dZ^{(1)}(t)\right\}} \tag{1.17}$$

This is the complete solution stock market prices when the return rate is multiplicative trend

From (1.6), this equation is solved using separable variable ,hence

$$\frac{dS_3(t)}{S_3(t)} = \left(\lambda_1\lambda_2\right)^{-1} dt$$

Integrating both sides gives

$$\int \frac{dS_3(t)}{S_3(t)} dP = \int \left(\lambda_1\lambda_2\right)^{-1} dt + K_1 \ , \ln S_3(t) = \left(\lambda_1\lambda_2\right)^{-1} t + \ln K_1$$

$$\ln\left(\frac{S_3(t)}{K_1}\right) = \left(\lambda_1\lambda_2\right)^{-1} t$$

Taking ln of both sides gives

$$S_3(\mathrm{t}) = \mathrm{K}_1\, e^{(\lambda_1\lambda_2)^{-1} t} \tag{1.18}$$

Applying the initial condition in (1.10) yields

$$S_3(\mathrm{t}) = S_0 e^{(\lambda_1\lambda_2)^{-1} t} \tag{1.19}$$

where S_0 is the initial price of asset at time, t .

considering a function $f\left(S_4(t),t\right)$,hence it has to do with partial derivatives. Expansion of $f\left(S_4(t),\mathrm{dS}_4(\mathrm{t}),t+dt\right)$ in a Taylor series about $\left(S_3(t),t\right)$ gives

$$df = \frac{\partial f}{\partial S_4(\mathrm{t})}dS_4(\mathrm{t}) + \frac{\partial f}{\partial t}dt + \frac{1}{2}\frac{\partial^2 f}{\partial {S_4}^2(\mathrm{t})}dS_4^2(t) + \ldots \tag{1.20}$$

Putting (1.6) in (1.20) gives

$$df = \frac{\partial f}{\partial S_4(\mathrm{t})}\left(\beta S_4(t)\,\mathrm{dZ}^{(1)}(\mathrm{t}) + \left(\lambda_1\lambda_2\right)^{-1} S_4(\mathrm{t})\,\mathrm{dt}\right) + \frac{\partial f}{\partial t}dt + \frac{1}{2}\frac{\partial^2 f}{\partial {S_4}^2(\mathrm{t})}dS_4^2(t) \tag{1.21}$$

$$= \beta S_4(t)\frac{\partial f}{\partial S_4(t)}dZ^{(1)}(t) + \left(\left(\lambda_1\lambda_2\right)^{-1} S_4(t)\frac{\partial f}{\partial S_4(t)} + \frac{1}{2}\beta^2 S_4^2(t)\frac{\partial^2 f}{\partial {S_4}^2(t)}dS_4^2(t)\right)dt$$

Let $f\left(S_4(\mathrm{t})\right) = \ln\mathrm{S}_4(t)$, the partial derivatives becomes

$$\frac{\partial f}{\partial S_4(\mathrm{t})} = \frac{1}{S_4(\mathrm{t})},\ \frac{\partial^2 f}{\partial S_4^2(\mathrm{t})} = -\frac{1}{{S^2}_4(\mathrm{t})},\ \frac{\partial f}{\partial t} = 0 \tag{1.22}$$

According to theorem 1.1(Ito's),substituting(1.21) and simplifying gives

$$= \left(\left(\lambda_1\lambda_2\right)^{-1} - \frac{1}{2}\beta^2\right)dt + \beta dZ^{(1)} \tag{1.23}$$

Since the RHS of (1.22) is independent of $f\left(S_4(t)\right)$,the stochastic is computed as follows:

$$S_4(t) = S_4 0\exp\left\{\left(\left(\lambda_1\lambda_2\right)^{-1} - \frac{1}{2}\beta^2\right)t + \beta dZ^{(1)}(\mathrm{t})\right\} \tag{1.24}$$

Equations (1.19) and (1.24) is an investment of stock (3) and of stock(4) whose rate of return is inversely multiplicative respectively at time, t

RESULTS

This Section presents the graphical results for the problems in (1.4)-(1.7) whose solutions are in (1.13)-(1.24), Hence the following parameter values were used in the simulation study:

$\lambda_1 = 2.00, \lambda \ = 4.00, S_1 0 = 50.72, \ S_2 0 = 50.72, \quad S_3 0 = 80.57, S_4 0 = 80.57,$
$\beta = 1.00, \ dZ = 1.00, \ t = 1$

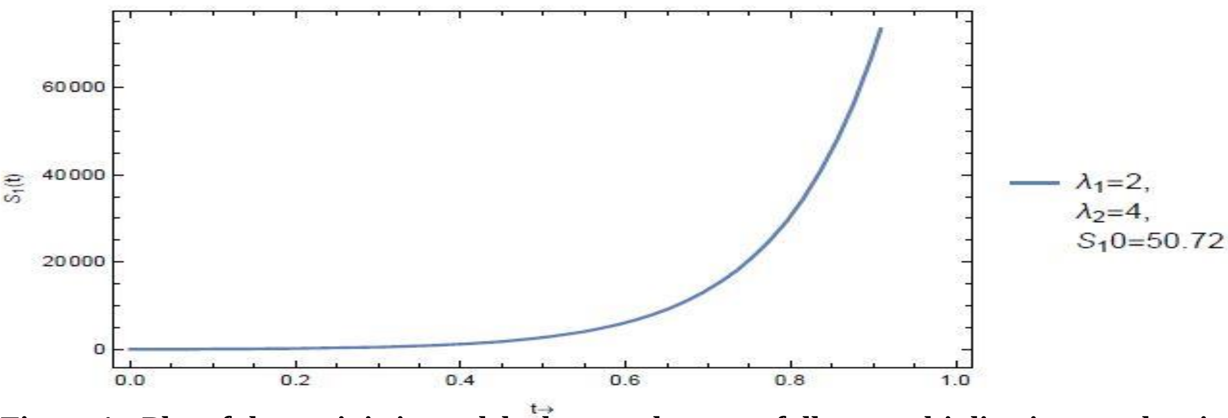

Figure 1: Plot of deterministic model when stock return follows multiplicative trend series.

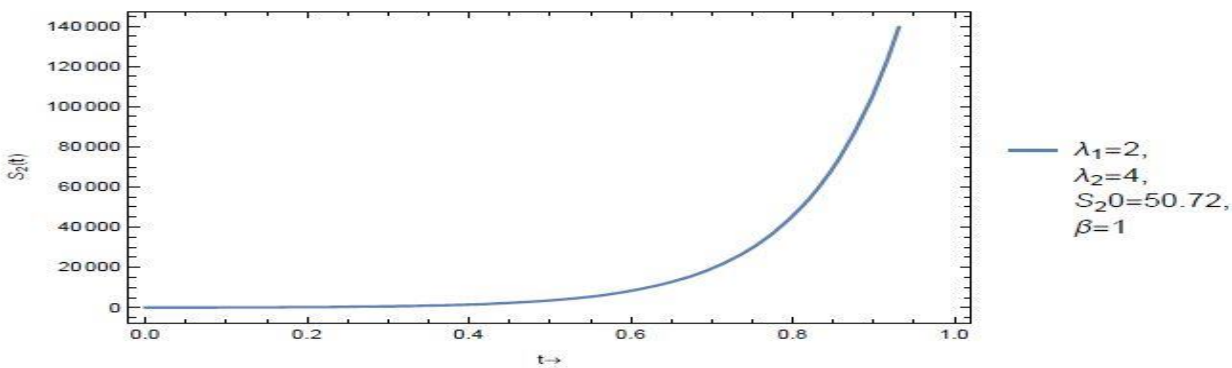

Figure 2: Plot of stochastic model when stock return follows multiplicative trend series.

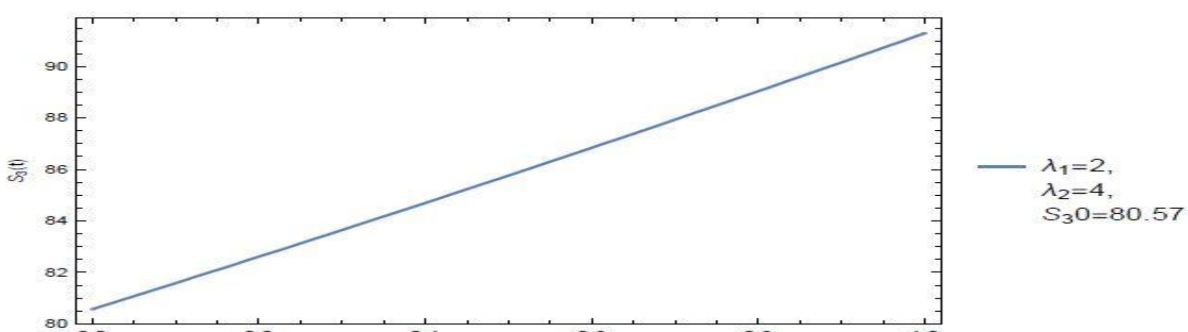

Figure 3: Plot of deterministic model when stock return follows multiplicative inverse trend series

.

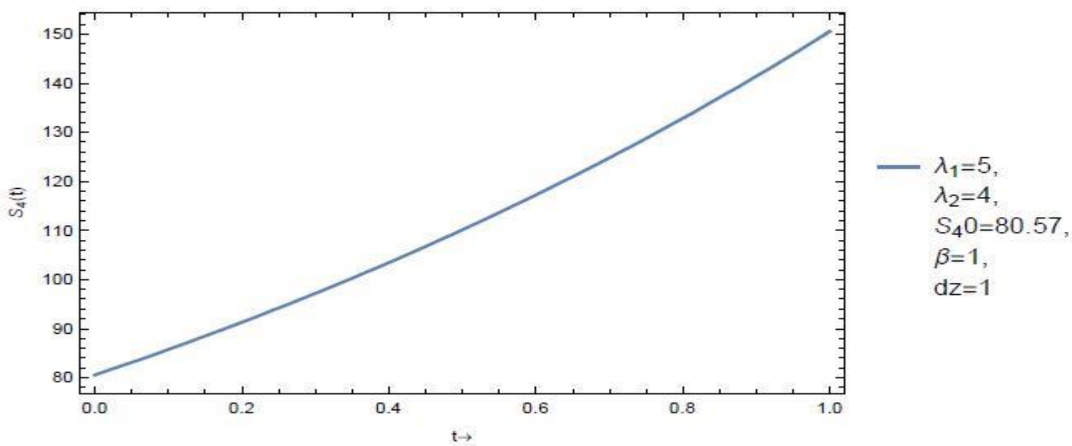

Figure 4: Plot of stochastic model when stock return follows multiplicative inverse trend series.

DISCUSSION OF RESULTS

In Figures 1 and 2, it can be seen that the two plots are exponential growth of asset returns of time-varying investment which grows unboundedly. The trends stipulate that as time of trading days increases, the value of asset return approaches to infinity. In the same vain, time decreases the value of asset return approaches to zero. This result agrees with the concept of multiplicative trend which indicates a non-linear trend (curved trend line). This informs investors that when return rates follow multiplicative trend it grows unboundedly.

As seen in Figures 3 and 4 respectively that the plots of multiplicative inverse is linear; this means that there's a linear relationship between return rates and time of trading. Carefully examination of the plots shows that asset return increases linearly as time increases, with the tendency of being 1.These results also agree with what the principle of multiplicative inverse and correlation as it affects trend line. Multiplicative inverse of trend of a number is a number by which the multiplication result is 1. Therefore, in terms of prediction multiplicative inverse is more efficient and reliable than multiplicative trend.

However, deterministic and stochastic systems are both effective in modeling asset returns for long and short term investments plans.

CONCLUSIONS

The concepts of differential equations and stochastic differential equations towards time-varying investment returns have been demonstrated. Precise conditions are obtained which regulate asset return rates through multiplicative and multiplicative inverse trend series. The analytical solution of problem shows:

(i) As time of trading day's increases, the value of asset return approaches to infinity.
(ii) Asset return increases linearly as time also increases.
(iii) Asset return rate is correlated with time of trading.
(iv) Multiplicative inverse trend series is more effective and reliable than multiplicative trend series based on the linearity of the trading activities of the capital market.
(v) The deterministic and stochastic systems are good and unique in modeling stock returns; no one of them is better than each other.

Finally, solving the deterministic and stochastic systems as coupled systems will be an interesting study.

Reference

[1] Amadi, I.U, Igbudu, R. and Azor, P.A(2022**).**Stochastic Analysis of the Impact of Growth-Rates on Stock Market Prices in Nigeria ,Asian journal of Economics, Business and Accounting,21(24),9-21.

[2] Wokoma, D.S.A(Bishop),Amadi, I.U and Aboko,I.S (2022).A Stochastic Model for Stock Market Price Variation ,*Asian research journal of Mathematics,*18(4),41-49.

[3]Osu B.A and Amadi, I.U(2022).A Stochastic Analysis of Stock Market Price Fluctuations for Capital Market, *journal of Applied Mathematics and Computation* 6(1),85-95,

[4].Davies, I. Amadi, I.U and Ndu, R.I(2019).Stability Analysis of Stochastic Model for Stock Market Prices. *International journal of Mathematics and Computational Methods ,4,*79-86.

[5].Osu, B.O,Okoroafor,A.C. and Olunkwa, C.(2009). Stability Analysis of Stochastic model of Stock market price. *African journal of Mathematics and Computer Science 2(6),98-103*

[6]. Adeosun,M.E., Edeki,S.O. and Ugbebor,O.O(2015).Stochastic Analysis of Stock Market Price Models: A case Study of the Nigerian Stock Exchange(NSE*). WSEAS transactions on Mathematics,14,*353-363.

[7].Ofomata,A.I.O.,Inyama,S.C.,Umana,R.A . and Omane,A.O(2017).A stochastic Model of the Dynamics of Stock Price for Forecasting. *Journal of Advances in Mathematics and Computer Science.25(6*),1-24.

[8].Dmouj,A.(2006).Stock price modeling:Theory and practice.*BIM Paper*.

. [9].Ugbebor,O.O,Onah,S.E. and Ojowo, O.(2001). An Empirical Stochastic Model of Stock Price Changes. *Journal Nigerian Mathematical Society,20,*95-101.

[10].Osu, B.O. and Okoroafor, A.C.(2007).On the measurement of random behavior of stock price changes. *Journal of mathematical science Dattapukur,18(2),*131-141.

[11].Wets, R. J. B. and Rios, I. (2012). Modeling and estimating commodity prices: copper prices. *JEL classification C53*.
[12] Amadi, I.U ,Ogbogbo, C.P and Osu, B.O(2022).Stochastic Analysis of Stock Price changes as Markov Chain in Finite States, *Global journal of pure and applied sciences,*28,91-98.
[13].Osu, B.O. (2010). A Stochastic Model of the Variation of the Capital Market Price. *International journal of of Trade,Economics and Finance,1,3, 297-302.*
[14]Abodolsadeh, N. and Moslem, P. (2011). Financial modeling by ordinary and stochastic differential equations. *World Applied Sciences Journal*, 13 (11): 2288-2295.
[15]Aitssahlia, F. Goswam M. and Guha, S. (2016). *American options under stochastic volatility: Parameter Estimation and pricing efficiency.*
[16]Andersen, T. G., Chung, H. J. and Sorensen, B. E. (1999). Efficient method of moments estimation of a stochastic volatility model: A monte Carlo Study. *Journal of Econometrics.* 91,pp 61 – 87.
[18]Bachelier, L. (1964). *Theories de la speculative paris: Gauthier-Villar. Transplanted in cooler.*