# PLANNING PROBLEMS OF NURSES: CASE OF A MOROCCAN HEALTHCARE UNIT 

Touria BENAZZOUZ ${ }^{\text {a }}$, Abdelwahed ECHCHATBI $^{\text {a }}$, Adil BELLABDAOUI ${ }^{\text {b }}$<br>${ }^{\text {a }}$ Laboratory of Mechanical, Industrial Management and Innovation, FST Settat, Settat, Morocco.<br>${ }^{\mathrm{b}}$ Team of Information Technology and Management of Enterprises, ENSIAS, University Mohammed V Souissi, Rabat, Morocco.


#### Abstract

Healthcare systems are facing the human resources planning's problem as a result of the budgets restrictions and the productivity improvement's need, while ensuring care services' quality provided to patients. In this paper, we focus on the nurses' schedule problem in a hospital center in Morocco. Different works in the last years indicate that this assignment to the different activities must comply with many constraints (professional rules, workload, individual preference, skills, etc.) while optimizing one or several objectives (equity in the work, the costs etc.). We present a description of the studied service as well as professional rules adopted and a mathematical formulation with a mixed integer linear program (MILP). The problem is solved using Ceplex software and the nurses' schedule is presented which achieve the expected balancing of their workloads.


KEYWORDS: hospital management, decision support, mathematical modeling, the nurses' assignment.

## INTRODUCTION

Managers within the hospital organizations are interested to the personnel management in the short term as well as the long term. It is a complex task which involves not only to plan and schedule the personnel but also to meet their requirements, the patient needs and manage all hardware resources. We aim in this research to provide aid to the manager in order to improve the planning for polyvalent nurses. As nurses are a key resource in healthcare systems, the service quality they provide depends on their overall satisfaction, especially with respect to the flexibility of schedules they perform.

Several literature reviews have been published on the modeling of the personal schedules' problem and particularly nurses ([1-4]). They present syntheses allowing a characterization of the problems associated with the obtained results and a classification of the used tools. Also, [5] and [6] have presented a decision support system for the nurse's allocation to patients in order to minimize the work overload. The problem has been modeled using a linear program resolved on CPLEX.

Thus [7] and [8] offer a problem modeling in a mixed integer linear program (MILP), whose objective is to minimize the arduousness gap between the nurses and to find the best compromise between: the cost, the service quality and the social satisfaction. In [9] has treated the same problem but in allocation children to the nurses.

In emergency home service, the nurses' assignment problem is always present. So [10] considers a set of constraints and nurses' assignment rules, in particular those related to the needs for daily jobs and holydays. Then [11] based on the development of a mathematical model using mixed integer linear programming (MILP). The model assigns the nurses to 7 posts and 4 teams according to their number available per day.

So, we can see a very large variety of constraints, objectives and methods employed in the various works depending on the viewpoint specificity of each author. Our model attempts to perform this planning for a healthcare service in a Moroccan hospital. Scheduled and not scheduled days off that can disrupt a previous planning are considered. Thus, the model integrates the workloads history made by the nurses in order to meet the workloads balance.This work is presented as follows. The second section provides the management process adopted in the medicine service of the hospital Ibn Sina as well as current practices for staff management. Section 3 proposes a mathematical formulation with a mixed integer linear program (MILP). The last section presents the numerical results using Ilog Cplex software.

## Service Description

We focus our study on one of the most important services in a hospital in Morocco. Its main activities are centered on the infectious diseases, the hematology clinic, infection with HIV-Aids, the inflammatory disease and the chronic pathology. The nurse staff management at the service is based on simple rules without worrying about the good governance of the nurse resources. Each nurse may perform all activities and be assigned to all posts, teams and days. The normal load per week is 36.5 hours and the extra times are paid. The staff work at service is organized into 4 teams: the morning, evening, night and weekend. Managers are often facing the lack of the nurses and can't provide the service with the adequate workforce. This is meeted mainly in the case of the evening team, the nights team and days off or sickness periods.

So, it is crucial to make an efficient nurses' assignment on these different posts and teams in order to balance their workloads. The following table (Table1) shows the nurses distribution in the teams and posts by associating a workload in the range [0,2] depending on the available nurses number. Thus, the notation $n(p)$ means that each of $(\mathrm{n})$ nurse.

## Table 1.

The nurses' distribution in different teams and posts according to the available nurses' number and their associated workloads.

| Nurses' number | Teams | Post 1 | Post 2 | Post 3 | Post 4 | Post 5 | Post 6 | $\begin{gathered} \text { Post } \\ 7 \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}=6$ | Morning | 1 (2) |  |  |  | 1 (2) |  | 1(1) |
|  | Evening/weekend | 1 (2) |  |  |  | $1(1,6)$ |  | - |
|  | Night | 1 (2) |  |  |  |  |  | - |
| $\mathrm{N}=7$ | Morning | $1(1,6)$ |  | 1 (2) |  |  | 1 (2) | 1 (1) |
|  | Evening/weekend | 1 (2) |  |  |  | $1(1,6)$ |  | - |
|  | Night | 1 (2) |  |  |  |  |  | - |
| $\mathrm{N}=8$ | Morning | $1(1$ |  |  |  | 1(1) | 1 (2) | 1 (1) |
|  | Evening/weekend | 1 (2) |  |  |  | $1(1,6)$ |  | - |
|  | Night | 1 (2) |  |  |  |  |  | - |
| $\mathrm{N}=9$ | Morning | $1(1$ |  |  |  | 1(1) | 2(1) | 1 (1) |
|  | Evening/weekend | 1 (2) |  |  |  | 1 1,6) |  | - |
|  | Night | 1 (2) |  |  |  |  |  | - |
| N=10 | Morning | 1 (1 |  | 1(1) | 1 (1) | 1 (1) | 2 (1) | 1 (1) |
|  | Evening/weekend | 1 (2) |  |  |  | $1(1,6)$ |  | - |
|  | Night | 1 (2) |  |  |  |  |  | - |
| N=11 | Morning | 1 (1) | 1 (1) | 1(1) | 1 (1) | 1 (1) | 2 (1) | 1 (1) |
|  | Evening/weekend | 1 (2) |  |  |  | $1(1,6)$ |  | - |
|  | Night | 1 (2) |  |  |  |  |  | - |

Several constraints must be observed in the studied service. This is to ensure the availability of daily capacities in terms of nurses by recognizing the associated workload. These constraints achieve the regulations of work which limits to 12 hours the daily nurse load, and 48 hours the weekly one and ensures a day off after a night shift. It is also expected to reduce the nurses' number during the holidays, one for each team, while associating a workload factor equal 2. For fairness, the planning must take into account the workloads history and the planned days off presented in the table 2 for each nurse during the planning horizon. The time arranged for teams is fixed: morning team: 7 hours, tonight team: 5 hours, night team: 12 hours, weekend team: 9 hours. Therefore, the problem's purpose is to generate the nurses' schedule that complies with all the constraints while minimizing the workload deviations. We structure our model in the sub-programs called according to the nurses' availability. The mathematical formulation presented below, takes into account a number of nurses equal $11,10,9,8,7$ and 6 . The example shown in this work considers a minimum of 6 nurses available to working when the total number is 12 nurses.

Table 2: The planned days off for each nurse during three weeks.

| Day/Nur <br> se | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Nurse1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| Nurse 2 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| Nurse 3 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| Nurse 4 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| Nurse 5 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| Nurse 6 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| Nurse 7 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| Nurse 8 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| Nurse 9 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| Nurse 10 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| Nurse 11 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| Nurse 12 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |

## The mathematical programming model

The variables description.
The notations in our model are set out in Table 3. The model uses a mixed integer linear program with four decision variables as it is shown in tables 4 and 5.

Table 3. Notations.

| N : | The nurses' number. |
| :---: | :---: |
| D | The planning day number. |
| V : | The teams' number. |
| P | The posts' number. |
| i : | Index representing the nurses. |
| j | Index representing the planning days. |
| p | Index representing the posts. |
| V : | Index representing the teams. |
| T : | The work time for the week. |
| $\mathrm{T}_{\text {max }}$ | Maximum permissible working time. |
| $\mathrm{N}_{\mathrm{V}}$ : | The hours' number worked during the team v. |
| $\mathrm{N}_{\mathrm{j}}$ : | The nurses' number available for the day j . |
| JO: | The working days. |
| JF: | The holidays. |
| JW : | The weekends. |
| $\mathrm{PH}_{\mathrm{i}}$ : | The total nurses workloads during the previous planning days. |
|  | The planned days off for each nurse $i$ during the day $j$ ( 0 if the nurse on day off and 1 for Otherwise) (Table 2). |

Table 4: Binary Variables
1: The nurse i assigned to the post p , team v during the day j .
$X_{i p v j}=0$ 0: Otherwise.

## Continuous Variables

$P_{\max }$ : The total workloads of the busiest nurses among all nurses.
$P_{\min }$ : The total workloads of the lightest nurses among all nurses.
$P n_{i p v j}$ : The workload of the nurse i to the post p to the team v during the day j .

## The constraints description

The constraints (C1), (C3), (C4) and (C5) are considered as the mandatory constraints. The schedule must respect these constraints for a nurse number available equal $11,10,9,8,7$ and 6 . The constraints C1.1 to C1.25 propose that for each day j and team v , the post p must contain as many nurses as necessary. The constraints C3.1 to C3.22 present the workloads distribution assigned to various posts, teams and days. The constraints C 4.1 to C 4.4 ensure that the working time per day should not exceed 12 hours per nurse. The working time per week must not exceed 48 hours per week (C5).These constraints shall formalize the management rules in the table 1 which establishes the requirements for each post, team and day given in the case of $11,10,9,8,7$ and 6 nurses available.

The constraint C 2 ensures that any nurse assigned to the night, the next day is a day off.C6 treats the days off and the constraints C7.1 to C7.4 model the holidays. The constraint C8 ensures that the nurse workloads during the horizon D are limited by an upper and lower bound.
$1-\mathrm{M}\left(8-\mathrm{N}_{\mathrm{j}}\right)\left(8-\mathrm{N}_{\mathrm{j}}\right) \leq \sum_{\mathrm{i}=1}^{\mathrm{N}} \sum_{\mathrm{p}=3}^{4} \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq 1+\mathrm{M}\left(8-\mathrm{N}_{\mathrm{j}}\right)\left(8-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{j} \in \mathrm{J}$
$1-\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) \leq \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq 1+\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{j} \in \mathrm{J}, \mathrm{p}=6,7$
$1-\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) \leq \sum_{\mathrm{i}=1}^{\mathrm{N}} \sum_{\mathrm{p}=1}^{2} \mathrm{X}_{\mathrm{i}, \mathrm{p}, \mathrm{i}, \mathrm{j}} \leq 1+\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{j} \in \mathrm{J}$
$1-\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) \leq \sum_{\mathrm{i}=1}^{\mathrm{N}} \sum_{\mathrm{p}=3}^{5} \mathrm{X}_{\mathrm{i}, \mathrm{p}, \mathrm{i}, \mathrm{j}} \leq 1+\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{j} \in \mathrm{J}$
$1-\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) \leq \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{X}_{\mathrm{i}, 7,1, \mathrm{j}} \leq 1+\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{j} \in \mathrm{J}$
$1-\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) \leq \sum_{\mathrm{i}=1}^{\mathrm{N}} \sum_{\mathrm{p}=1}^{4} \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq 1+\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{j} \in \mathrm{J}$
$1-\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) \leq \sum_{\mathrm{i}=1}^{\mathrm{N}} \sum_{\mathrm{p}=5}^{6} \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq 1+\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{j} \in \mathrm{J}$
$\sum_{\mathrm{p}=1}^{4} \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{X}_{\mathrm{i}, \mathrm{p}, \mathrm{v}, \mathrm{j}}=1 ; \quad \forall \mathrm{j} \in \mathrm{J}, \mathrm{v}=2,4$
$\sum_{\mathrm{p}=5}^{6} \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{X}_{\mathrm{i}, \mathrm{p}, \mathrm{v}, \mathrm{j}}=1 ; \quad \forall \mathrm{j} \in \mathrm{J}, \mathrm{v}=2,4$
$\sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{X}_{\mathrm{i}, 7, \mathrm{vj}}=0 \quad ; \quad ; \forall \mathrm{j} \in \mathrm{J}, \mathrm{v}=2,4$
$\sum_{\mathrm{p}=1}^{6} \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{X}_{\mathrm{i}, \mathrm{p}, 3, \mathrm{j}}=1 ; \quad \forall \mathrm{j} \in \mathrm{J}$
$\sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{X}_{\mathrm{i}, 7,3, \mathrm{j}}=0 \quad ; \quad \forall \mathrm{j} \in \mathrm{J}$
$\sum_{\mathrm{p}=1}^{\mathrm{P}} \mathrm{X}_{\mathrm{i}, \mathrm{p}, 3, \mathrm{j}}+\sum_{\mathrm{p}=1}^{\mathrm{P}} \sum_{\mathrm{v}=1}^{\mathrm{V}} \mathrm{X}_{\mathrm{i}, \mathrm{p}, \mathrm{v},(\mathrm{j}+1)} \leq 1 ; \quad \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in\{1, \ldots, 7\}$
$\mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}-\mathrm{M}\left(11-\mathrm{N}_{\mathrm{j}}\right)\left(11-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(11-\mathrm{N}_{\mathrm{j}}\right)\left(11-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in$ $\mathrm{J}, \mathrm{p}=1 . .7$
$\mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}-\mathrm{M}\left(10-\mathrm{N}_{\mathrm{j}}\right)\left(10-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(10-\mathrm{N}_{\mathrm{j}}\right)\left(10-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in$ (C3.2)

J,p=3.. 7
$1.6 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}-\mathrm{M}\left(10-\mathrm{N}_{\mathrm{j}}\right)\left(10-\mathrm{N}_{\mathrm{j}} \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq 1.6 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(10-\mathrm{N}_{\mathrm{j}}\right)\left(10-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in\right.$
(C3.3)
$\mathrm{I}, \forall \mathrm{j} \in \mathrm{J}, \mathrm{p}=1 . . .2$
$X_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}-\mathrm{M}\left(9-\mathrm{N}_{\mathrm{j}}\right)\left(9-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(9-\mathrm{N}_{\mathrm{j}}\right)\left(9-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}, \mathrm{p}=5 . .7$
$1.6 * X_{i, p, 1, j}-M\left(9-N_{j}\right)\left(9-N_{j}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq 1.6 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(9-\mathrm{N}_{\mathrm{j}}\right)\left(9-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}$, $\mathrm{p}=1 . .2$
$2 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}-\mathrm{M}\left(9-\mathrm{N}_{\mathrm{j}}\right)\left(9-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq 2 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(9-\mathrm{N}_{\mathrm{j}}\right)\left(9-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in$
$\mathrm{J}, \mathrm{p}=3 . .4$
$\mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}-\mathrm{M}\left(8-\mathrm{N}_{\mathrm{j}}\right)\left(8-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(8-\mathrm{N}_{\mathrm{j}}\right)\left(8-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}, \mathrm{p}=5,7$
$1.6 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}-\mathrm{M}\left(8-\mathrm{N}_{\mathrm{j}}\right)\left(8-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq 1.6 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(8-\mathrm{N}_{\mathrm{j}}\right)\left(8-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}$, $\mathrm{p}=1 . .2$
$2 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}-\mathrm{M}\left(8-\mathrm{N}_{\mathrm{j}}\right)\left(8-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 1 \mathrm{j}, \mathrm{j}} \leq 2 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(8-\mathrm{N}_{\mathrm{j}}\right)\left(8-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in$
(C3.9)
J,p=3..4
$2 * X_{i, 6,1, \mathrm{j}}-\mathrm{M}\left(8-\mathrm{N}_{\mathrm{j}}\right)\left(8-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, 6,1, \mathrm{j}} \leq 2 * \mathrm{X}_{\mathrm{i}, 6,1, \mathrm{j}}+\mathrm{M}\left(8-\mathrm{N}_{\mathrm{j}}\right)\left(8-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}$ $X_{i, 7,1, j}-M\left(7-N_{j}\right)\left(7-N_{j}\right) \leq \mathrm{pn}_{\mathrm{i}, 7,1, \mathrm{j}} \leq \mathrm{X}_{\mathrm{i}, 7,1, \mathrm{j}}+\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}$
$1.6 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}-\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq 1.6 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, \mathrm{i}, \mathrm{l}}+\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in$ $\mathrm{J}, \mathrm{p}=1 . .2$
$2 * X_{i, p, 1, j}-M\left(7-N_{j}\right)\left(7-N_{j}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}} \leq 2 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in$
(C3.13)
$\mathrm{J}, \mathrm{p}=3 . .5$
$2 * \mathrm{X}_{\mathrm{i}, 6,1, \mathrm{j}}-\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, 6,1, \mathrm{j}} \leq 2 * \mathrm{X}_{\mathrm{i}, 6,1, \mathrm{j}}+\mathrm{M}\left(7-\mathrm{N}_{\mathrm{j}}\right)\left(7-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}$ $2 * X_{i, p, 1, \mathrm{j}}-\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, \mathrm{i}, \mathrm{j}} \leq 2 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}, \mathrm{p}=$
(C3.14)
(C3.15) 1. 4
$2 * X_{i, p, 1, \mathrm{j}}-\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, \mathrm{p}, \mathrm{j}, \mathrm{j}} \leq 2 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 1, \mathrm{j}}+\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}, \mathrm{p}=$ 5. . 6
$\mathrm{X}_{\mathrm{i}, 7,1, \mathrm{j}}-\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) \leq \mathrm{pn}_{\mathrm{i}, 7,1, \mathrm{j}} \leq \mathrm{X}_{\mathrm{i}, 7,1, \mathrm{j}}+\mathrm{M}\left(6-\mathrm{N}_{\mathrm{j}}\right)\left(6-\mathrm{N}_{\mathrm{j}}\right) ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}$
(C3.17)
$\mathrm{pn}_{\mathrm{i}, \mathrm{p}, \mathrm{v}, \mathrm{j}}=2 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, \mathrm{v}, \mathrm{j}} ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}, \mathrm{p}=1 . .4, \mathrm{v}=2,4$
$\mathrm{pn}_{\mathrm{i}, \mathrm{p}, \mathrm{v}, \mathrm{j}}=1.6^{*} \mathrm{X}_{\mathrm{i}, \mathrm{p}, \mathrm{v}, \mathrm{j}} ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}, \mathrm{p}=5,6, \mathrm{v}=2,4$

$$
\begin{aligned}
& \mathrm{pn}_{\mathrm{i}, \mathrm{p}, \mathrm{v}, \mathrm{j}}=0 \quad ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{~J}, \mathrm{v}=2,4 \\
& \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 3, \mathrm{j}}=2 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, 3, \mathrm{j}} ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{~J}, \mathrm{p}=1 . .6 \\
& \mathrm{pn}_{\mathrm{i}, \mathrm{p}, 3, \mathrm{j}}=0 \quad ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{~J} \\
& X_{\text {ip1j }}+X_{\text {ip2j }} \leq 1 \quad ; \forall i \in I, \forall j \in J, \quad \forall p \in P \\
& \mathrm{X}_{\mathrm{ip} 2 \mathrm{j}}+\mathrm{X}_{\mathrm{ip} 3 \mathrm{j}} \leq 1 \quad ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{~J}, \quad \forall \mathrm{p} \in \mathrm{P} \\
& X_{i p 1 j}+X_{i p 3 j} \leq 1 \quad ; \forall i \in I, \forall j \in J, \quad \forall p \in P \\
& X_{i p 3 j}+X_{i p 4 j} \leq 1 \quad ; \forall i \in I, \forall j \in J \\
& \sum_{\mathrm{j}=1}^{7} \mathrm{n}_{\mathrm{v}} \mathrm{X}_{\mathrm{ipvj}} \leq \mathrm{T}_{\max } \quad ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{p} \in \mathrm{P} \\
& \sum_{\mathrm{v}=1}^{V} \mathrm{X}_{\mathrm{ipvj}} \leq \mathrm{C}_{\mathrm{i}, \mathrm{j}} ; \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{~J} \\
& \sum_{\mathrm{i}=1}^{\mathrm{I}} \mathrm{X}_{\mathrm{ipvj}}=1 \quad ; \forall \mathrm{j} \in \mathrm{JF}, \forall \mathrm{v} \in \mathrm{~V}, \mathrm{p}=1 . .6 \\
& \sum_{i=1}^{I} X_{i 7 p 3 j}=0 \quad ; \forall j \in J F \\
& \mathrm{pn}_{\mathrm{i}, \mathrm{p}, \mathrm{v}, \mathrm{j}}=2 * \mathrm{X}_{\mathrm{i}, \mathrm{p}, \mathrm{v}, \mathrm{j}} \quad ; \forall \mathrm{j} \in \mathrm{JF}, \mathrm{v}=1,2,3 ; \mathrm{p}=1 . .6 \\
& \mathrm{pn}_{\mathrm{i}, 7, \mathrm{v}, \mathrm{j}}=0 \quad ; \forall \mathrm{j} \in \mathrm{JF}, \mathrm{v}=1,2,3 ; \\
& \mathrm{P}_{\text {min }} \leq \sum_{\mathrm{v}=1}^{\mathrm{V}} \sum_{\mathrm{p}=1}^{\mathrm{P}} \sum_{\mathrm{j}=1}^{\mathrm{J}} \mathrm{C}_{\mathrm{i}, \mathrm{j}} \mathrm{p} \mathrm{n}_{\mathrm{ipvj}}+P H_{i} \leq \mathrm{P}_{\max } \quad ; \forall \mathrm{i} \in \mathrm{I}
\end{aligned}
$$

The objective function is to minimize the difference between the upper and lower bound (Pmax and Pmin).
Minimize $Z=P_{\text {max }}-P_{\text {min }}$

## Numerical results

The study example proposed the allocation of 12 nurses on a three weeks horizon when the days off, the holidays and the workloads history are known.

This model has 14451 variables and 288588 constraints resolved with ILOG CPLEX 9.0 solver on a PC Core i5 1.70 GHz with 4 GB of RAM and the Windows 7 operating system. This data set provides a feasible solution equal 0.2 , this after a run time of 10 minutes. Table 6 presents the results of the same data with the execution times ranging from 10 minutes at 8 hours. We found that:

- The optimal solution is not obtained within a reasonable time; this is justified by the importance of the variable number in our model.
- The feasible solution obtained in term of Xi,p,v,j and Pni,p,v,j is invariant relative to the execution time.
Table 7 presents the assignment results of each nurse in each post, team and day of the week as well as their workloads. The notation adopted in this table $\mathrm{p}(\mathrm{pn})$ specifies that the nurse held the post p with a load pn.
The days $6,7,13,14,20$ and are the weekends and the 10 th day is a public holiday.

Table 6. Results with different execution time

| Execution Time | Iterations <br> Number | Gap | Pmin | Pmax |
| :---: | :---: | :---: | :---: | :---: |
| 10 Min | 1,728,633 | (0.2 | 13.4 | 13.6 |
|  |  | ) |  |  |
|  |  | (0.2 |  |  |
| 2 H | 44,573,074 | ) | 13.4 | 13.6 |
|  |  | (0.2 |  |  |
| 4 H | 63,437,568 | ) | 13.4 | 13.6 |
|  |  | (0.2 |  |  |
| 5 H | 75,000,508 | ) | 13.4 | 13.6 |
|  |  | (0.2 |  |  |
| 8 H | 93,994,523 | ) | 13.4 | 13.6 |

Table 7. The assignment of each nurse for each post, team and day as well as their workloads on a horizon of three weeks

| Nur | teams | J1 | J2 | J3 | J4 | J5 | J6 | J7 | J8 | J9 | J1 | J11 | J12 | J13 | J14 | J15 | J16 | J17 | J18 | J19 | J20 | J21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Morni | 5 | 5 | 1( | 7 | 3 |  |  | 5 | 6 |  |  |  |  |  |  |  |  | 6 |  |  |  |
|  | $\underline{\mathrm{ng}}$ | (1) | (1) | 1.6) | (1) | (2) |  |  |  |  |  |  |  |  |  |  |  |  | (2) |  |  |  |
|  | Eveni |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | $\underline{\mathrm{ng}}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Night |  |  |  |  |  |  |  |  |  |  |  |  | (2) |  | (2) |  |  |  | (2) |  |  |
|  | Week |  |  |  |  |  | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 5(1 |
|  | end |  |  |  |  |  | (2) |  |  |  |  |  |  |  |  |  |  |  |  |  |  | .6) |
|  | Morni |  | 1(1) |  | 3 |  |  |  |  | 6 |  | 6 |  |  |  |  | 7 | 3 | 1(1 |  |  |  |
|  | $\underline{\mathrm{ng}}$ |  | .6) | 3 (2) | (2) |  |  |  | (1) | (1) |  | (1) |  |  |  |  | (1) | (2) | .6) |  |  |  |
|  | Eveni |  |  |  |  |  |  |  |  |  |  |  | 5(1) |  |  |  |  |  |  | $6(1$ |  |  |
| 2 | ng |  |  |  |  |  |  |  |  |  |  |  | .6) |  |  |  |  |  |  | .6) |  |  |
|  | Night |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Week |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | end |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Morni | 3 |  |  |  | 1(1 |  |  |  | 5 |  |  |  |  |  | 7 |  |  |  |  |  |  |
|  | ng | (2) |  | 5 (1) |  | .6) |  |  |  | (1) |  |  |  |  |  | (1) |  |  |  |  |  |  |
|  | Eveni |  | 6(1) |  |  |  |  |  | 5(1 |  |  |  |  |  |  |  | $6(1$ |  |  |  |  |  |
| 3 | ng |  | .6) |  |  |  |  |  | .6) |  |  |  |  |  |  |  | .6) |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  |
|  | $\underline{\text { Night }}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | (2) |  |  |  |
|  | Week |  |  |  |  |  |  |  |  |  |  |  |  |  | 5(1) |  |  |  |  |  |  |  |
|  | end |  |  |  |  |  |  |  |  |  |  |  |  |  | .6) |  |  |  |  |  |  |  |
|  | Morni |  | 7 |  |  |  |  |  | 3 |  |  |  |  |  |  |  |  |  |  | 1(1 |  |  |
|  | ng |  |  | 7 (1) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | .6) |  |  |
| 4 | Eveni | 1 |  |  |  |  |  |  |  |  | 1 (2 |  |  |  |  |  | 3 | 1 |  |  |  |  |
|  | ng | (2) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

International Journal of Scientific Research in easy and case studies
Vol.2, No.1, pp.32-42, 2021
Published by ECRTD-UK



## CONCLUSION

In this work, we have presented nurses' assignment problem in a care service from the hopital Ibn Sina in Morocco; we used a mathematical model with a mixed integer linear program (MILP). The proposed model would like account of the days off, holidays and the loads history executed on a predefined horizon. The numerical results illustrate the feasibility of the problem as well as the observance of different constraints.

The improvement's ways exist to bring this work to the reality. In particular, we intend to validate the results on the other hospital and integrate the constraints with take into account the nurse's preferences and skills as well as for the taking into account of the random aspect of the care request.

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