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PHYSICAL MEANING OF IRRATIONAL NUMBERS AND IMAGINARY NUMBERS

Lu Shan
Zhuhai Entry-Exit Inspection and Quarantine Bureau

ABSTRACT*: Numbers are used to count the quantities of things and to indicate the sequence of things in physics. Since neither could irrational numbers and imaginary numbers break away their dependence on the theory based on geometric intuition nor could they be established on the continuous orderly foundation logically, it is therefore impossible to derive their interdependence and relationship. With number axis model of the* 2^k *th power, the author finds that the continuous orderly relationship of the* 2^k *th powers of the rational numbers, irrational numbers and imaginary numbers can be derived because numbers of the* 2^k *th power are conjugate rational numbers and irrational numbers of the* 2^k *th power quantified by the* 2^k *th power of the integral ratio and their opposite numbers are conjugate imaginary numbers of the* 2^k *th power of the equal absolute value.*

KEYWORDS: value; quantity numbers; irrational number; imaginary number
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INTRODUCTION

The physical quantity in physics is expressed by numerical value which in turn is expressed by the number of the same value. For example, quantities of length, speed, weight, heat, electric current and energy, etc. are all measured in this way. Therefore, numerical values must be of continuous orderly numbers and are quantified by the same value. Thus, the interdependence and relationship among different quantities of things could be derived by the numerical calculation. The first numerical system that human beings perceive is the “natural numerical system” as is usually called whose value is confined to express the integral multiples of the same value, but not other non-integral parts, which gives rise to the segmentation of quantity from integer to integral ratio (fraction), hence “rational numbers” come into being. Since any rational numbers

Corresponding author’s E-mail: zhuhailushan@outlook.com Tel: 0756-3219063, Fax: 0756-3219538
Address: No.1144, East of Jiuzhou Avenue, City of Zhuhai 519015, Guangdong Province P. R. China

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of no same value can be compared in magnitude, the process of using value to quantify things becomes the origin of continuous orderly numbers. Pythagorean School found irrational numbers based on the geometrical theory in 500 BC. As the interdependence relationships of irrational numbers could not be expressed by arithmetically segmentation like rational numbers, i.e. irrational numbers could not be quantified by integral ratio like rational numbers, irrational numbers therefore could not be fully understood. The French mathematician Rene Descartes coined the word “imaginary number” under the circumstance of not totally understanding irrational numbers in 17th century. Hence, affected by the theory based on geometric intuition, Greek mathematicians proposed that horizontal direction and vertical direction represent real numbers and imaginary numbers respectively. From 18th century, the Swiss mathematician Leonhard Euler began to use the symbol $i=\sqrt{-1}$ to express the imaginary number unit. German mathematician C·F·Gauss used such symbol systematically and proposed number couple (a,b) to express $a+bi$, called complex numbers and later the concept of complex plane [1-6]. Since neither could irrational numbers and imaginary numbers break away their dependence on the theory based on geometric intuition nor could they be established on the foundation of continuous order logically, it is therefore impossible to describe physical quantities, even to explain the simplest root of a physical quadratic equation. However, with number axis model, we find that we can not only break away the traditional concept based on geometric intuition but also provide thinking for the derivation of continuous relations among quantities of irrational numbers and imaginary numbers with the help of numbers of the 2^k th power on the number axis to express the interdependence and relationship among different quantities of things as axis is the abstract model of any possible value of continuous orderly transforming quantities.

Continuous Orderly Structure of Irrational Numbers and Imaginary Numbers

Numbers are used to count quantities of things and to indicate a sequence of things .The straight line is seamless, thus, continuous orderly geometrical figures become the basic and original numerical model of continuity. According to Descartes, the straight line along the positive direction of the origin “0” is the positive axis $+X_1$ if an origin “0” is selected on a straight line. Hence numbers along the positive axis $+X_1$ can be expressed by the following. See Fig. 1 a .

$$+X_1=+x_1=x \qquad (1)$$

Let any two integral points on the positive axis $+X_1$ $A=a$, $B=b$, $b\neq 0$, and $a<b$, a and b are relatively prime, then the integral ratio of these two points A and B on the positive axis $+X_1$ is

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$$m_1 = \frac{a}{b} \qquad (2)$$

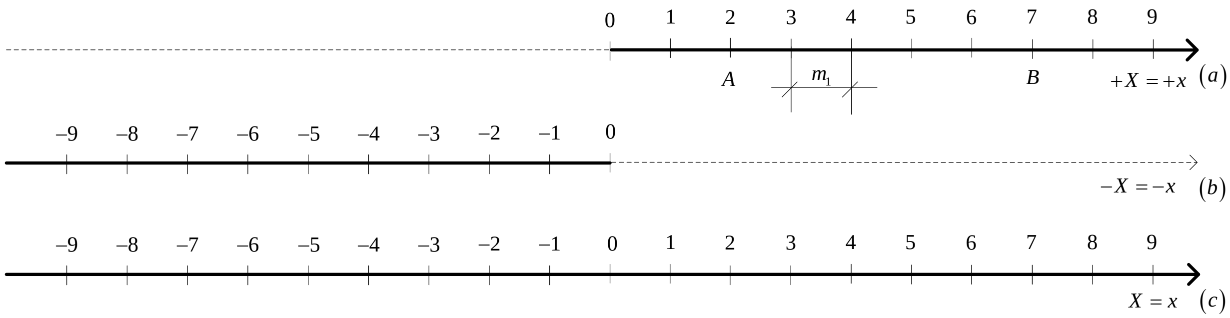

Fig.1 the First Power Axis

Then the negative axis $-X_1$ is added along the opposite extending direction to the positive axis $+X_1$ from the origin "0". Since the opposite number of the positive number is negative number, see Fig. 1b, numbers on the negative axis $-X_1$ can be expressed in the following equation.

$$-X_1 = -x_1 = -x \qquad (3)$$

As $a = m_1 b$, there exist numbers segmented by m_1 on axis X_1, i.e. the interdependence relationship among numerical values on axis X_1 could be described by the value and the quantity numbers of the same integral ratio. See Fig. 1c. Such numbers whose segmentation of quantities on the number axis is described as the same integral ratio are named numbers of the first power, namely rational numbers as are usually called.

Further we can find that if we fold the negative axis $-X_1$ from the origin "0" of the axis X_1 to the positive axis $+X_1$, we'll obtain the number $+x_2$ on the positive axis $+X_2$ which is the overlap of the number $-x_1$ on the negative axis $-X_1$ and the number $+x_1$ on the positive axis $+X_1$. As the square roots of the positive number are two numbers equal in absolute value, but opposite in sign, i.e. $x_1 = \pm\sqrt{x_2}$, numbers on the positive axis $+X_2$ can be described in the following equation. See Fig. 2a.

$$+X_2 = +x_2 = x_1^2 = x^2$$

Let any two integral points on the positive axis $+X_2$ $A = a^2$, $B = b^2$, $b \neq 0$, and $a < b$, a^2 and b^2 are relatively prime, the integral ratio of point A and point B on the positive axis $+X_2$ is

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$$m_2 = \frac{a^2}{b^2}$$

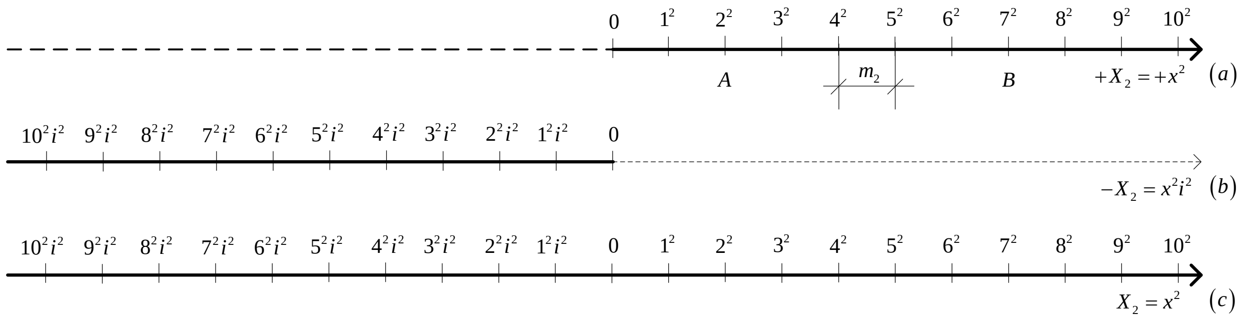

Fig. 2 the Second Power Axis

Then the negative axis $-X_2$ is added along the opposite extending direction to the positive axis $+X_2$ from the origin “0”. Since the opposite number of the positive number is negative number, $i^2 = -1$, see Fig. $2b$, numbers on the negative axis $-X_2$ can be described in the following equation.

$$-X_2 = -x_2 = x^2 i^2$$

Since $a^2 = m_2 b^2$, numbers segmented by m_2 appear on the axis X_2, namely the interdependence relationship among numerical values on axis X_2 can be described by the value and quantity numbers of the second power of the same integral ratio. See Fig. $2c$. Such numbers whose segmentation of quantities on the number axis is described as the second power of the same integral ratio are named numbers of the second power.

Similarly, if we fold the negative axis $-X_2$ from the origin “0” of the axis X_2 to the positive axis $+X_2$, we’ll obtain the number $+x_4$ on the positive axis $+X_4$ which is the overlap of the number $-x^2$ on the negative axis $-X_2$ and the number $+x^2$ on the positive axis $+X_2$. As the square roots of the positive number are two numbers equal in absolute value, but opposite in sign, i.e. $x_2 = \pm\sqrt{x_4}$, numbers on the positive axis $+X_4$ can be described in the following equation. See Fig. $3a$.

$$+X_4 = +x_4 = +x_2^2 = x^4$$

Let any two integral points on the positive axis $+X_4$ $A = a^4, B = b^4$, $b \neq 0$, and $a < b$, a^4 and b^4 are relatively prime, the integral ratio of point A and point B on the positive axis $+X_4$ is

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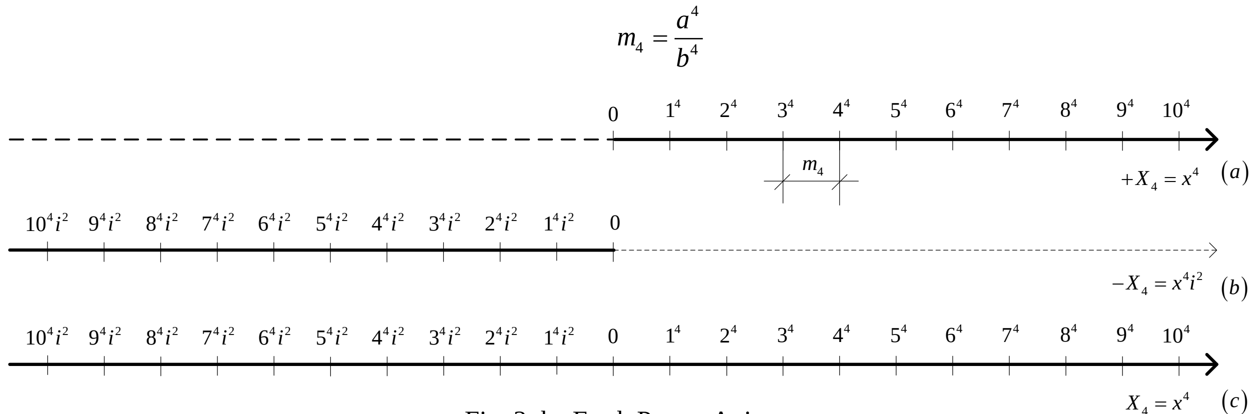

Fig. 3 the Forth Power Axis

Then the negative axis $-X_4$ is added along the opposite extending direction to the positive axis $+X_4$ from the origin "0". Since the opposite number of the positive number is negative number, $i^2 = -1$, see Fig. $3b$, numbers on the negative axis $-X_4$ can be described in the following equation.

$$-X_4 = -x_4 = x^4 i^2$$

Since $a^4 = m_4 b^4$, numbers segmented by m_4 appear on the axis X_4, namely the interdependence relationship among numerical values on axis X_4 can be described by the value and quantity numbers of the fourth power of the same integral ratio. See Fig. $3c$. Such numbers whose segmentation of quantities on the number axis is described as the fourth power of the same integral ratio are named numbers of the fourth power.

By analogy, let numbers on the positive axis $+X_{2^k}$ be the numbers of the 2^k th power ($k = 1, 2, 3 \cdots\cdots$), as the square roots of the positive number are two numbers equal in absolute value, but opposite in sign, each point on the positive axis $+X_{2^k}$ can be expressed by the 2^k th power of two numbers equal in absolute value, but opposite in sign. Numbers on the positive axis $+X_{2^k}$ can be described in the following equation.

$$X_{2^k} = +x_{2^k} = x^{2^k} \qquad (4)$$

Let any two integral points on the positive axis $+X_{2^k}$ $A = a^{2^k}$, $B = b^{2^k}$, $b \neq 0$, and $a < b$, a^{2^k} and b^{2^k} are relatively prime, the integral ratio of point A and point B on the positive axis $+X_{2^k}$ is

$$m_{2^k} = \frac{a^{2^k}}{b^{2^k}} \qquad (5)$$

Then the negative axis $-X_{2^k}$ is added along the opposite extending direction to the positive axis $+X_{2^k}$ from the origin "0". Since the opposite number of the positive number is negative number,

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$i^2 = -1$, numbers on the negative axis $-X_{2^k}$ can be described in the following equation.

$$-X_{2^k} = -x_{2^k} = x^{2^k} i^2 \quad (6)$$

Since $a^{2^k} = m_{2^k} b^{2^k}$, numbers segmented by m_{2^k} appear on the axis X_{2^k}, namely the interdependence relationship among numerical values on axis X_{2^k} can be described by the value and quantity numbers of the 2^kth power of the same integral ratio. Such numbers whose segmentation of quantities on the number axis is described as the 2^kth power of the integral ratio are named numbers of the 2^kth power.

It can be seen from Eq. (4) that if both sides of the equation are extracted the 2^kth root, then $\sqrt[2^k]{x_{2^k}} = x$,

(I) When $x_{2^k} > 0$, it can be shown by reduction to absurdity that if $\sqrt[2^k]{x_{2^k}}$ is rational number, letting $\sqrt[2^k]{x_{2^k}} = \frac{a}{b}$, where $\frac{a}{b}$ is the fraction in lowest term, i.e. a and b are relatively prime, the 2^kth power of both sides, then we obtain

$$x_{2^k} = \frac{a^{2^k}}{b^{2^k}} \quad (7)$$

1. When $k = 0$, $2^k = 1$, then $a^{2^k} = a$, $b^{2^k} = b$, $x_{2^k} = x_1$. The following can be obtained from Eq. (7)

$$a = x_1 b$$

Since x_1 is the multiple of the integral ratio, x_1 is the rational number. When $b = 1$, $x_1 = x = a$, namely x is integer. When $b > 1$, $x_{2^k} = x_1 = a/b$, namely x is rational number.

2. When $k = 1, 2, 3\cdots\cdots$, 2^k is even number. We can obtain Eq. (8) from Eq. (7)

$$a^{2^k} = x_{2^k} b^{2^k} \quad (8)$$

It can be seen that b is the factor of a^{2^k}. When $b = 1$, $x_{2^k} = a^{2^k}$, i.e. x_{2^k} is the number of perfect 2^kth power, i.e. x is rational number. When $b > 1$, according to the fundamental theorem of arithmetic, there exists a prime number p which is the factor of b, i.e. p is also the factor of b^{2^k}. It can be seen from Eq. (8) that p is also the factor of a^{2^k}, i.e. p is the common factor of both a^{2^k} and b^{2^k}. However a and b are relatively prime, so are a^{2^k} and b^{2^k} which is contradicted with the statement that p is the common factor of a^{2^k} and b^{2^k}. In such a case, x is irrational number.

(II) When $x_{2^k} < 0$, $\sqrt[2^k]{x_{2^k}}$ is imaginary number.

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Therefore, x_{2^k} is the number quantified by m_{2^k}. As to k of different value, m_{2^k} varies accordingly. Each x_{2^k} has its own specific value m_{2^k}. The number x_{2^k} on the number axis is hard to be quantified by the same vale m_{2^k} unless k is specified. When $x_{2^k} > 0, k = 0$, x_{2^k} is rational number. When $k = 1,2,3\cdots\cdots$, x_{2^k} is the number of perfect 2^k th power, x_{2^k} is the rational number of 2^k th power. When x_{2^k} is the number of imperfect 2^k th power, x_{2^k} is the irrational number of the 2^k th power. When $x_{2^k} < 0$, as $-1 = i^2$, x_{2^k} is the imaginary number of the 2^k th power. Thus, numbers on the positive axis are real numbers of the 2^k th power composed of conjugate rational numbers and irrational numbers of the 2^k th power. Numbers on both positive and negative axes are complex numbers of the 2^k th power composed of real numbers of the 2^k th power and imaginary numbers of the 2^k th power.

Calculation

With knowledge of continuous orderly structure among quantities of complex numbers on the number axis X_{2^k}, we can calculate complex numbers. As numbers on the number axis of the 2^k th power are continuous orderly, with any two selected points A and B being known on the number axis of the 2^k th power, then

1. When A plus B of the same sign

(1)If $A > 0$, $B > 0$, then $A = a^{2^k}$, $B = b^{2^k}$. Let $a < b$,

since $A + B = a^{2^k} + b^{2^k} > 0$, let $C = A + B$, $C = c^{2^k}$, then we can get

$$c = \pm\sqrt[2^k]{a^{2^k} + b^{2^k}} \quad (9)$$

When $k = 0$, $c = \pm(a+b)$; when $k = 1$, $c = \pm\sqrt{a^2 + b^2}$; when $k = 2$, $c = \pm\sqrt[4]{a^4 + b^4}$; $\cdots\cdots$

(2)If $A < 0$, $B < 0$, then $A = a^{2^k} i^2$, $B = b^{2^k} i^2$. Let $a < b$,

since $A + B = a^{2^k} i^2 + b^{2^k} i^2 = -\left(a^{2^k} + b^{2^k}\right) < 0$, let $C = A + B$, $C = c^{2^k} i^2$, then we can get

$$c = \pm\sqrt[2^k]{a^{2^k} + b^{2^k}}\, i \quad (10)$$

When $k = 0$, $c = \pm(a+b)i$; when $k = 1$, $c = \pm\sqrt{a^2 + b^2}\, i$; when $k = 2$, $c = \pm\sqrt[4]{a^4 + b^4}\, i$; $\cdots\cdots$

2. When A plus B of the opposite sign

(1)If $A > 0, B < 0$, then $A = a^{2^k}$, $B = b^{2^k} i^2$. Let $a < b$,

since $A + B = a^{2^k} + b^{2^k} i^2 = a^{2^k} - b^{2^k} < 0$, let $C = A + B$, $C = c^{2^k} i^2$, then we can get

$$c = \pm\sqrt[2^k]{b^{2^k} - a^{2^k}}\, i \quad (11)$$

When $k = 0$, $c = \pm(b-a)i$; when $k = 1$, $c = \pm\sqrt{b^2 - a^2}\, i$; when $k = 2$, $c = \pm\sqrt[4]{b^4 - a^4}\, i$; $\cdots\cdots$

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(2)If $A<0$, $B>0$, then $A=a^{2^k}i^2$, $B=b^{2^k}$. Let $a<b$,

since $C=A+B=a^{2^k}i^2+b^{2^k}=b^{2^k}-a^{2^k}>0$, let $C=A+B$, $C=c^{2^k}$, then we can get

$$c=\pm\sqrt[2^k]{b^{2^k}-a^{2^k}} \qquad (12)$$

When $k=0$, $c=\pm(b-a)$; when $k=1$, $c=\pm\sqrt{b^2-a^2}$; when $k=2$, $c=\pm\sqrt[4]{b^4-a^4}$; ……

3. When A multiply B of the same sign

When $A>0$, $B>0$ or $A<0$, $B<0$, then $A=a^{2^k}$, $B=b^{2^k}$ or $A=a^{2^k}i^2$, $B=b^{2^k}i^2$.

Since $AB=a^{2^k}b^{2^k}=\left(a^{2^k}i^2\right)\left(b^{2^k}i^2\right)=(ab)^{2^k}>0$, let $C=AB$, $C=c^{2^k}$, then we can get

$$c=\pm\sqrt[2^k]{(ab)^{2^k}}=\pm ab \qquad (13)$$

c is irrelevant to k.

4. When A multiply B of the opposite sign

When $A>0, B<0$ $A<0$, $B>0$, then $A=a^{2^k}$, $B=b^{2^k}i^2$ or $A=a^{2^k}i^2$, $B=b^{2^k}$.

Since $AB=a^{2^k}\left(b^{2^k}i^2\right)=\left(a^{2^k}i^2\right)b^{2^k}=-(ab)^{2^k}<0$, let $C=AB$, $C=c^{2^k}i^2$, then we can get

$$c=\pm\sqrt[2^k]{-(ab)^{2^k}}=\pm abi \qquad (14)$$

c is irrelevant to k.

Obviously, calculations of complex numbers are similar to those of rational numbers, while what is different is that rational numbers use "positive" and "negative" to differentiate two opposite numbers, while complex numbers use "real" and "imaginary" to differentiate two opposite numbers.

DISCUSSION

Number axis is the abstract model of the possible value of all continuous orderly transforming quantities. Position of numerical values on the number axis must be specified if we physically describe the interdependence and relationship of these values. Hence, such value, in nature, can be quantified by the same value and can express the sequence of quantities of things. However, in the history of numerical system expansion, the evolution of number value is a long and hard process for people to understand and accept. When irrational numbers were found, due to their interdependence, unlike that of rational numbers which can be described by arithmetic segmentation, irrational numbers are considered as numbers which could not be quantified by integral ratio, unlike rational numbers. Just like relations between "2" and $\sqrt{2}$, the equation $1+1=2$ expresses the adding of two segments of "1" on a line, and the equation $\sqrt{1^2+1^2}=\sqrt{2}$ expresses the adding of two sides of "1" in a square. As a result, irrational numbers were not

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totally understood due to the neglect of logic by people so that irrational numbers could not be logically established on the basis that the numbers are quantified by the same value, resulting in the missing of continuity and orderliness when the numerical system expanded from rational numbers to irrational numbers. When the imaginary numbers appear as numbers, unlike positive and negative numbers, imaginary numbers seem not to have the orderly sequence referring to such expressions as $x>0$ or $x<0$. In fact, if x and y both are rational numbers, and $x>0$, $y>0$, then their product $xy>0$. However, if such relation is put into imaginary numbers, there would be troubles. One simple way to understand this is to set a counter example. In other words, if we make imaginary numbers have orderly sequence relations, as to i, in particular, either $i>0$, or $i<0$. Set $i>0$, then $-1=i\cdot i>0$. Obviously, this is invalid, making it hard to derive relations among imaginary numbers. Hence, the practical use of imaginary numbers is not known and there is no such quantity expressed by imaginary number in our real life, which leads to various doubts and misunderstandings toward imaginary numbers for a long time[1-6]. Nevertheless, with number axis model of the 2^k th power, it can be found that the continuous orderly relationship of rational numbers, irrational numbers and imaginary numbers of the 2^k th power can be derived because numbers of the 2^k th power are the conjugate rational numbers and irrational numbers of the 2^k th power quantified by the 2^k th power of integral ratio and their opposite numbers are conjugate imaginary numbers of the 2^k th power of the equal absolute value.

In the past researches, rational numbers are continuous orderly, i.e. the position of any point on the number axis can be described by numerical value x, the magnitude of which is quantified by integral ratio and near which there are limitless numbers of points whose position could be described by numerical value x_1. x_1 can be close to x of any possible value, because of which rational numbers are continuous orderly. Similarly, the position of any point on the number axis may also be described by numerical value x^{2^k}. What is different is that the magnitude of this number is quantified by 2^k th power of integral ratio. As to the position of each point, the adjacent point can be selected infinitely, with an infinitesimal quantity from this point, the value being $x_1^{2^k}$ to the initial considered point with the value of x^{2^k}. Thus x^{2^k} is also continuous orderly like x. When $k=0$, numbers on the number axis are rational numbers. When $k=1,2,3\cdots\cdots$, numbers on the number axis of the 2^k th power expand first from conjugate rational numbers of the 2^k th power to conjugate irrational numbers of the 2^k th power, and then to conjugate imaginary numbers of the 2^k th power.

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We can simply and intuitively understand from the number axis that description of the number of any position is relative to “0”. Hence, the standard selected to describe the quantity and sequence is “0”. Different “0” expresses the different quantity and sequence of the same point. Consequently, the “0” should be specified in describing the quantity and sequence of numbers. In this sense, it can be seen that the position of “0” on the number axis is necessary. What is more important is that the meaning of “0” varies with the expansion of numerical system. “0” differentiates something and nothing in natural numbers; “0” differentiates positive and negative concept in rational numbers; “0” differentiates real and imaginary concept in complex numbers. Therefore, “0” is mainly referred to the reference point relative to the quantity and consequence of things, which coincide with the physical reference point in meaning.

The above statement of the continuous orderly structure of rational numbers, irrational numbers and imaginary numbers is confined to the case that the integral ratio is regarded as number of the 2^kth power, but such insufficient statement could provide a definite concept for the value of numbers, without which rational numbers, irrational numbers and imaginary numbers could not express the quantity and sequence of things in various physical process. In addition, from Eq. (6), we can see that when $k=1$, the caculation of irrational numbers is in accord with Pythagoras' findings that we can divide one square number into the sum of two square numbers, i.e. $a^2+b^2=c^2$. Meanwhile, we can also provide new thinking for the method to prove the Fermat's Last Theorem, i.e. $a^N+b^N=c^N$ (When $N>2$, there is no integer solution.)[7]

Since number axis is the abstract model of the possible value of all continuous orderly transforming quantities, with the number axis model of the 2^kth power, we can find rational numbers, irrational numbers and imaginary numbers can be regarded as continuous orderly numbers for rational numbers, irrational numbers and imaginary numbers on the number axis can be quantified by the 2^kth power of integral ratio, which is incredible for the number dependent on the theory based on geometric intuition. Reunderstanding the continuous orderly structure of rational numbers, irrational numbers and imaginary numbers can help us find that not only can the conjugate rational numbers, irrational numbers and imaginary numbers of the same 2^kth power be regarded as numbers that could be quantified, but also their relations could be derived. It's worth noting that the values of different 2^kth powers of conjugate rational numbers, irrational numbers and imaginary numbers differ, each having its own specific value. Therefore, it is the nature that conjugate rational numbers, irrational numbers and imaginary numbers of the 2^kth power can be quantified by the 2^kth power of integral ratio that enables the above mentioned numbers be logically established on the continuous orderly basis, making us realize that our consideration about the interdependence and relationship of quantities of numbers in the expansion of numerical system is obviously not appropriate. However, it is a pity that mathematics did not realize that the 2^k th power of integral ratio in nature is divisible quantity. Instead, it simply and hastily expanded rational numbers to real numbers, even to complex numbers without logical consideration. As a result, it is impossible to unify physical quantities when physics comes across issues of irrational numbers and imaginary numbers, which is the cause of the missing of continuity and orderliness in irrational numbers and imaginary numbers.

Consequently, understanding the continuous orderly structure among quantities of irrational numbers and imaginary numbers helps to put the calculation of physical quantities down to the unification of numbers

CONCLUSION

It can be seen with number axis model of the 2^k th power that numbers of the 2^k th power are conjugate rational numbers and irrational numbers of the 2^k th power quantified by the 2^k th power of the integral ratio and their opposite numbers are conjugate imaginary numbers of the 2^k th power of the equal absolute value. Thus the continuous orderly relationship of rational numbers, irrational numbers and imaginary numbers of 2^k th power can be derived accordingly.

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