ON THE COMPUTATION OF PHONON SPECTRUM IN THE SOLID SATE CAVITY QED SYSTEM

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ABSTRACT: *Cavity quantum electrodynamics (QED) provides an important platform to study the interaction between light and matter for quantum information processing. In the solid-state cavity QED system, a semiconductor quantum dot serves as an artificial atom. Electrons in the quantum dot interacts with the phonon reservoir in the solid environment besides interacting with the cavity mode. Three effects can be induced by the phonon bath: the coupling renormalization, the off-resonance assisted feeding and the pure dephasing. Phonon spectrum functions are essential in the calculation of this three effects which are the focus of this study. Spherical-Gaussian wave function is used to model the confinement of electrons and holes in the quantum dot. Analytical expressions of phonon spectrum has been developed including both deformation potential coupling and piezoelectric coupling. The confinement volume of quatnum dot plays a dominating role in the phonon spectrum and it's dependent dynamic parameters.*

KEYWORDS: Quantum Dot, Cavity QED, Phonon Spectrum, Feeding Rate, Pure dephasing rate

INTRODUCTION

In the traditional cavity quantum electrodynamics (QED) system, an atom is placed in a cavity with reflecting walls [1]. While in a solid-state cavity QED system, a quantum dot serves as an artificial atom with two energy levels [2]. Solid-state system has a small size which is an advantage to reach strong light-matter interaction. And it also can be potentially integrated into a network for quantum computation on a chip. Typical microcavities in solid-state cavity QED systems includes micropillars, photonic crystal and microdisk [3]. Solid systems provide a promising platform for quantum information schemes.

Electrons in the semiconductor quantum dot interacts with the phonon reservoir in the solid environment besides interacting with the cavity mode [4,5]. Acoustic phonons have considerable impact on the dynamic parameters in both strong and week coupling regimes [6]. Three effects can be induced by the phonon bath: the coupling renormalization, the off-resonance assisted feeding and the pure dephasing.

Phonon spectrum functions are essential in the calculation of this three effects which are the focus of this study. Spherical-Gaussian wave function is used to model the confinement of electrons and holes in the quantum dot [7]. Analytical expressions of phonon spectrum has been developed including both deformation potential coupling and piezoelectric coupling. The confinement volume of quatnum dot plays a dominating role in the phonon spectrum and it's dependent dynamic parameters.

In this paper, the physics and applications of cavity QED is introduced first. Then the theoretical model including the Hamiltonian of the solid-state cavity QED systems and the Gaussian wave functions were developed. The difference between electron and hole in the confinement length has been taken into consideration. Spherical Gaussian wave functions were used to simulate the size of the quantum dot. Two coupling mechanisms between the electron (hole) and the phonon bath were investigated, and the corresponding phonon spectrum functions were calculated analytically. The phonon-introduced pure dephasing rates were calculated numerically from the phonon bath correlation function.

THOERY AND MODEL

The Solid-State Cavity QED System

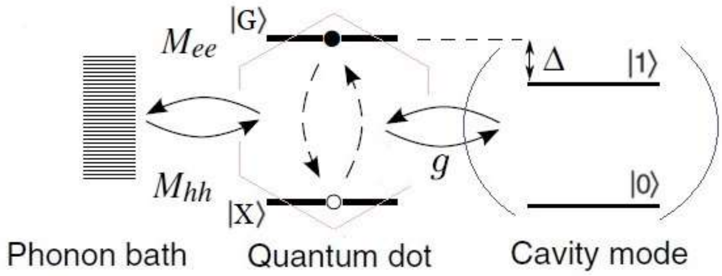

Figure 1. Schematic of the solid-state cavity QED system and the phonon bath

In a solid-state cavity QED system as shown in Figure 1, g is the coupling strength between the two-level quantum dot and the cavity and Δ is the detuning for them. Main part of the Hamiltonian for the cavity system reads as

$$H_s = \sum_{i=X,G} \hbar\omega_i c_i^\dagger c_i + \hbar\omega_{cav} a^\dagger a + \hbar g(a^\dagger c_g^\dagger c_e + c_e^\dagger c_g a)\,, \quad (1)$$

Where $\hbar$ is Plank constant and $\hbar\omega_G$ and $\hbar\omega_X$ are the energy for quantum dot when it is in ground state or excited respectively. $c_i^\dagger$ and c_i are corresponding fermionic operators. The energy of the cavity photon is $\hbar\omega_{cav}$, and the corresponding bosonic operators are $a^\dagger$ and a. The dissipative items from both quantum dot and the microcavity are not included in H_s.

The Hamiltonian for a phonon bath in the solid-state environment reads as

$$H_{ph} = \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} b^\dagger_{-\mathbf{k}} b_{\mathbf{k}}\,, \quad (2)$$

where $\hbar\omega_{\mathbf{k}}$ is the engergy for phonon with wave vector $\mathbf{k}$. The bosonic operators for the phonons are $b^\dagger_{-\mathbf{k}}$ and $b_{\mathbf{k}}$. And the Hamiltonian for the interaction of electrons and phonons is given by

$$H_{eph} = \sum_{\mathbf{k}} (M_{ee}^{\mathbf{k}} c_G^\dagger c_G + M_{hh}^{\mathbf{k}} c_X^\dagger c_X)(b^\dagger_{-\mathbf{k}} + b_{\mathbf{k}})\,, \quad (3)$$

where $M_{ee}^{\mathbf{k}}$ ($M_{hh}^{\mathbf{k}}$) are the matrix elements in the electron-phonon interaction.

We only consider a single electron (hole), the effective matrix element $M^{\mathbf{k}}$ can be introduced as

$$M^{\mathbf{k}} = M_{hh}^{\mathbf{k}} - M_{ee}^{\mathbf{k}}\ . \quad (4)$$

Phonon Spectrum And Dynamic Parameters

In order to compute the phonon spectrum

$$J(\omega) = \sum_{\mathbf{k}} |M^{\mathbf{k}}|^2 \delta(\omega - \omega_{\mathbf{k}}), \quad (5)$$

we need to model the wave function as for the electron (hole) in quatnum dot. We consider harmonic confinement in all three dimensions. So wave functions for both the ground state comes as the following form

$$\phi_v(\mathbf{r}) = (\frac{1}{\sqrt{\pi} l_v})^{\frac{3}{2}} \exp(-\frac{r^2}{2l_v^2}) \quad (6)$$

where $v \in \{e, h\}$, and l_v represents the confinement length for electrons (hole) in the quantum dot. In the theoretical treatment of polaron transformation [8], the light-matter coupling strength g will be renormalized by a factor of $\langle X \rangle$ due to phonon bath. We will refer $\langle X \rangle$ as DW factor later.

$$\langle X \rangle = \exp(-\sum_{\mathbf{k}} |\frac{M^{\mathbf{k}}}{\hbar\omega_{\mathbf{k}}}|^2 [n_{\mathbf{k}} + \frac{1}{2}]) \quad (7)$$

where $\mathbf{k}$ is the phonon vector and $n_{\mathbf{k}} = 1/[\exp(\hbar\omega_{\mathbf{k}}/k_B T) - 1]$ is the occupation number of the phonon mode $\mathbf{k}$ in thermal equilibrium with temperature T.

The effective phonon spectrum $D(\omega)$ is defindead as the following

$$D(\omega) = \pi \sum_{\mathbf{k}} |M^{\mathbf{k}}|^2 [n_{\mathbf{k}} \delta(\omega + \omega_{\mathbf{k}}) + (n_{\mathbf{k}} + 1)\delta(\omega - \omega_{\mathbf{k}})]. \quad (8)$$

$D(\omega)$ is calculated as the real part of the Fourier transform of the phonon reservoir correlation function $D(t)$ which describes the scattering process when the phonon bath interacts with the quantum dot. The scattering rate $\Gamma_{ph}(\Delta)$ for a phonon-mediated transition from the excitation to the cavity mode is given by

$$\Gamma_{ph}(\Delta) = 2(\frac{g}{\Delta})^2 D(\omega)_{\omega=\Delta} \quad (9)$$

The phonon reservoir correlation function $D(t)$ determines pure phonon depasing rate $\gamma_{ph}(t)$ and the spectral lineshape in semiconductor quantum dots.

$$\begin{aligned} D_{\pm}(t) &= \sum_{\mathbf{k}} |M^{\mathbf{k}}|^2 [n_{\mathbf{k}} \exp(\pm i\omega_{\mathbf{k}} t) + (n_{\mathbf{k}} + 1)\exp(\mp i\omega_{\mathbf{k}} t)] \\ &= \sum_{\mathbf{k}} |M^{\mathbf{k}}|^2 [(2n_{\mathbf{k}} + 1)\cos(\omega_{\mathbf{k}} t) \mp \sin(\omega_{\mathbf{k}} t)] \end{aligned} \quad (10)$$

The pure depasing rate $\gamma_{ph}(t)$ induced by phonon can be calculated from $D(t)$.

$$\gamma_{ph}(t) = Re \int_0^t \frac{1}{\hbar^2} [(1-\xi)D(t') + \xi \cos(t'\sqrt{\Delta^2 + 4g^2}) D(t')] dt' \quad (11)$$

$$\text{where } \xi = \frac{4\lambda_+}{(1+\lambda_+^2)^2} \quad \text{with} \quad \lambda_+ = \frac{\Delta}{(2g)} + \sqrt{\Delta^2/(2g)^2 + 1}$$

Next we present analytical spectral function expressions with different phonon modes (longitudinal acoustic, LA and transverse acoustic, TA) by two different mechanisms (deformation potential, or piezoelectric). The coupling matrix $M^{\mathbf{k}}$ can be calculated with the electronic form factor $F_v(\mathbf{k})$ which is the overlap between the electron (hole) and the phonon wave function.

$$M_{vv}^{\mathbf{k}} = D_v \sqrt{\frac{\hbar k}{2dc_l V}} F_v(\mathbf{k}) \text{ with } F_v(\mathbf{k}) = \int d\mathbf{r} |\phi_v(\mathbf{r})|^2 e^{-i\mathbf{k}\cdot\mathbf{r}} \quad (12)$$

The bulk phonon has a linear dispersion relation and $\omega_{\mathbf{k}} = c_l|\mathbf{k}|$ is assumed. D_e (D_h) is the deformation potential constant of a conduction band electron (valence hole). c_l is the velocity of LA phonons. d is the mass density and V is the phonon quantization volume.

Deformation Potential Coupling And Piezoelectric Coupling

For Gaussian-spherical wave function shown in Eq. (6), we get the phonon spectrum $J_{LA}(\omega)$ due to deformation potential (DF) coupling of LA phonons as the following analytical expression

$$J_{LA}^{DF}(\omega) = \frac{\hbar\omega^3}{4\pi^2 dc_l^5}[D_e e^{-\omega^2 l_e^2/(4c_l^2)} - D_h e^{-\omega^2 l_h^2/(4c_l^2)}]^2 . \quad (13)$$

Both TA and LA phonons contribute to piezoelectric (PZ) coupling which is calculated from three parts, one LA mode and two TA modes. We performed angular averages over the longitudinal and transverse modes separately.

$$M_{vv}^{\mathbf{k}} = \sqrt{\frac{\hbar}{2d\omega_{\mathbf{k}} V}} \overline{M}_j(\mathbf{k}) F_v(\mathbf{k}) \quad (13)$$

where the subscript j of $\overline{M}_j(\mathbf{k})$ refer to LA or TA mode. For LA mode,

$$\overline{M}_{LA}(\mathbf{k}) = \frac{24\pi e e_{14}}{\epsilon k^3} k_x k_y k_z \quad (14)$$

where e is the charge of an electron and e_{14} the piezoelectric constant, ϵ dielectric constant. For the two TA modes,

$$\overline{M}_{TA1}(\mathbf{k}) = \frac{8\pi e e_{14}}{\epsilon k^2} \frac{(k_y^2 - k_x^2)k_z}{\sqrt{k_x^2 + k_y^2}} \quad (15)$$

$$\overline{M}_{TA2}(\mathbf{k}) = \frac{8\pi e e_{14}}{\epsilon k^2} \frac{k_x k_y (k_x^2 + k_y^2 - 2k_z^2)}{k\sqrt{k_x^2 + k_y^2}} \quad (16)$$

The phonon spectra corresponding to piezoelectric (PZ) coupling are

$$J_{LA}^{PZ}(\omega) = \frac{3(ee_{14})^2 \hbar\omega}{35(\pi\epsilon)^2 dc_l^3}[e^{-\omega^2 l_e^2/(4c_l^2)} - e^{-\omega^2 l_h^2/(4c_l^2)}]^2 \quad (17)$$

$$J_{TA}^{PZ}(\omega) = \frac{4(ee_{14})^2 \hbar\omega}{35(\pi\epsilon)^2 dc_t^3}[e^{-\omega^2 l_e^2/(4c_t^2)} - e^{-\omega^2 l_h^2/(4c_t^2)}]^2 \quad (18)$$

where c_t is the velocity of TA phonons.

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RESULTS AND DISCUSSION

Since the analytical expressions for phonon spectra have been calculated from spherical-Gaussian wave function, we compute the coupling renormalization factor $\langle X\rangle$, the off-resonance assisted feeding rate $\Gamma_{ph}(\Delta)$ and the pure dephasing rate $\gamma_{ph}(t)$ from phonon spectra. InGaAs system material parameters [4] used in the calculation are given in Table 1.

Parameters	Value	Parameters	Value
Mass density d	5370 kg/m^3	DF Constant for electron D_e	-14.6 eV
Velocity, LA c_l	5110 m/s	DF Constant for hole D_h	-4.8 eV
Velocity, TA c_t	3340 m/s	Coupling strength of cavity $\hbar g$	150 μeV
PZ constant e_{14}	0.16 C/m^2	Dielectric constant ϵ	12.53 ϵ_0

Table 1. InGaAs system material parameters used in the calculation

The Effective Phonon Spectrum

In the spherical-Gaussian wave function Eq. (6), l_e (l_h) represents the confinement length for electrons (hole) in the quantum dot. In order to simulate different sizes of quantum dot, we set l_e (l_h) to be 4 nm, 8 nm, 12nm and 16 nm. At first, we don't consider the difference between l_e and l_h. Then piezoelectric coupling has no effect. Figure 2 shows the effective phonon spectra $D(\omega)$ dependence on the confinement length of quantum dot at temperature $T = 4K$. While $D(\omega)$ dependence on the temperature with $l_e = 8nm$ is shown in Figure 3 and the inset shows renormalization factor $\langle X\rangle$ wherein.

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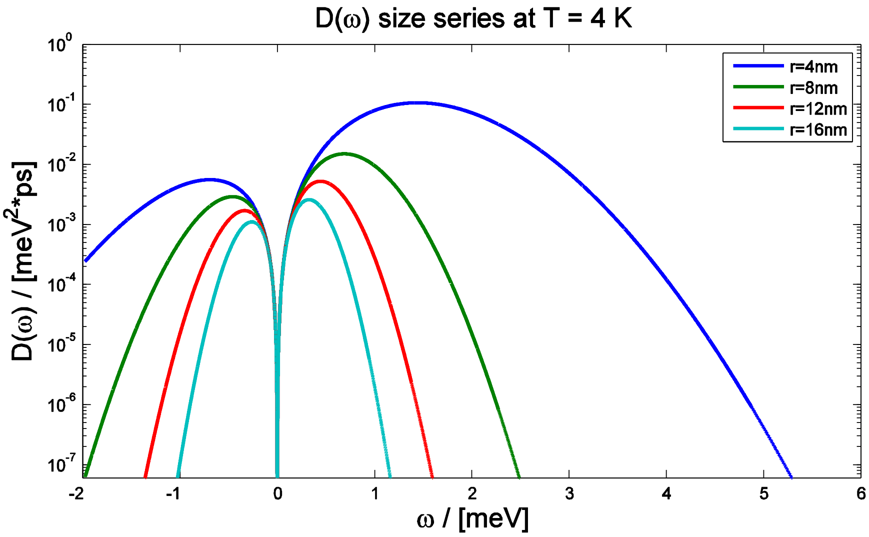

Figure 2. (Color online) $D(\omega)$ dependence on the confinement length of quantum dot

The effective phonon spectrum has a large amplitude and frequency range in the reciprocal space for small confinement length. Because the electronic form factor $F_v(\mathbf{k})$ in Eq. (12) is the Fourier transform of the wave function squared. Small volume of confinement leads to large frequency range of interaction with phonon.

We plot $D(\omega)$ dependence on the temperature in Figure 3. A positive frequency means a phonon with energy $\hbar\omega$ being emitted into the solid environment, while a negative frequency means a phonon with energy $\hbar|\omega|$ being absorbed from the environment. $D(\omega)$ has smaller amplitude when $\omega < 0$ because a phonon is esier to be emitted than be absorbed at the same temperature and frequency. However, amplitude of $D(\omega)$ increases and tends to be symmetric when the temperature goes up. The average thermal occupation number $n_{\mathbf{k}}$ is responsible for this symmetry.

Renormalization factor $\langle X \rangle$ is shown in the inset of Figure 3. At low temperature, renormalization of coupling strength g due to phonon bath is not significant. $\langle X \rangle$ seems decreases as temperature increases linearly.

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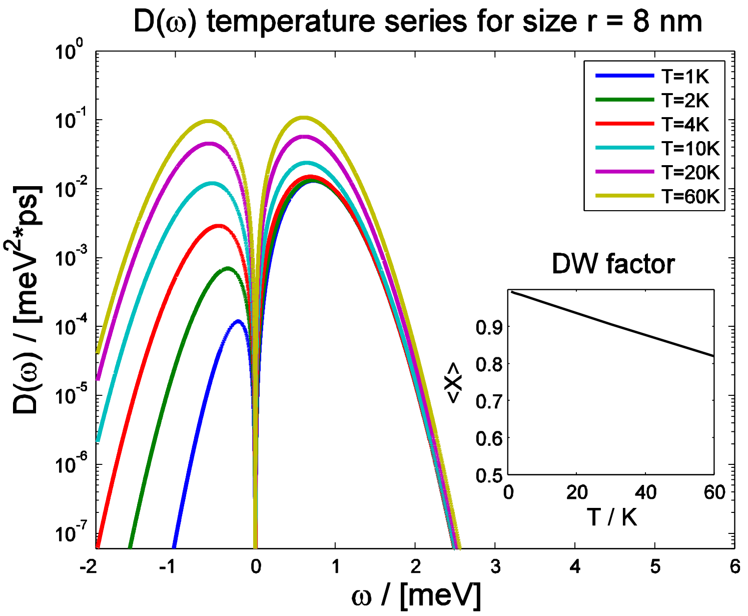

Figure 3. (Color online) $D(\omega)$ dependence on the temperature

In fact, electron and hole in the quantum dot have different effective mass. If we consider this difference, l_e doesn't equal l_h. We assume $l_h = 0.8 l_e$, then piezoelectric coupling has effect to the phonon spectra. At temperature $T = 4K$, we contrasted the piezoelectric (PZ) coupling with deformation potential (DF) coupling for two confinement lengths l_e 4 nm and 16 nm in Figure 4.

The amplitude of $D(\omega)$ due to piezoelectric coupling is one order at least smaller than deformation potential. That's why it is usually omitted in theory analysis. However, piezoelectric coupling might be significant if there is a considerable difference in the confinement lengths of electron and hole in relative large quantum dot. Difference in the confinement lengths also causes the dip in $D(\omega)$ curves due to deformation potential (DF) coupling.

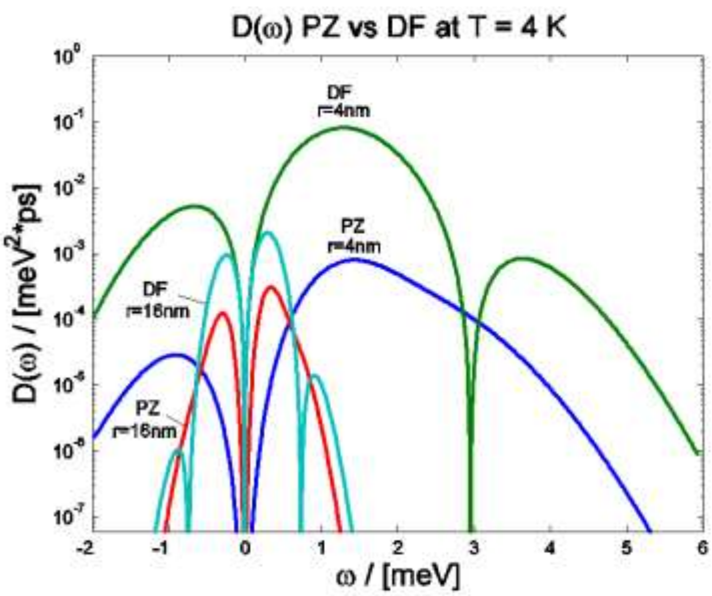

Figure 4. (Color online) $D(\omega)$ due to difference in the confinement length of electron and hole

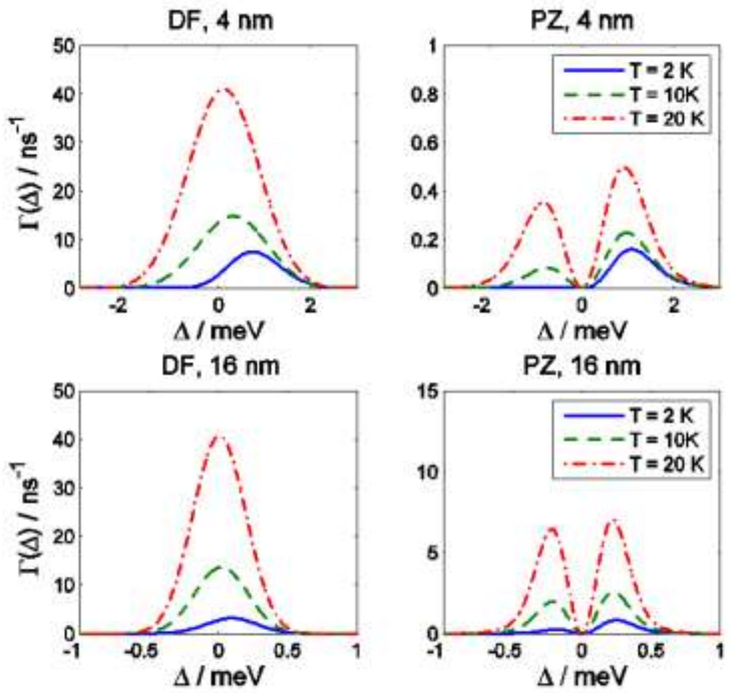

Figure 5. (Color online) Cavity feeding rate $\Gamma_{ph}(\Delta)$ for the phonon-assited transition

Cavity feeding rate $\Gamma_{ph}(\Delta)$ for the phonon-assited transition from the excitation to the cavity is shown in Figure 5. We considered the two confinement length and three temperature points 2K, 10K and 20K. Piezoelectric (PZ) coupling contribution to the feeding rate has been taken into consideration although it is negligible for small size of quantum dot. At low temperatures, the scattering is efficient for positive detunings. The corresponding is associated with phonon emission. At higher temperatures, it turns to be more symmetric because the phonon absorption becomes important. For quantum dot with large confinement length of 16 nm, the detuning that phonon can feed is limited to a narrower range compared to the confinement length of 4 nm.

In Figure 6, we plot the pure phonon dephasing rate $\gamma_{ph}(t)$ at the limit $t \to \infty$ for different confinement lengths and temperature from 1K to 16K. When the detuning Δ is zero, $\gamma_{ph}(\infty)$ has a large value for small confinement length. $\gamma_{ph}(\infty)$ decreases with Δ inreasing. $\gamma_{ph}(\infty)$ increases with temperature increasing when Δ is kept constant. $\gamma_{ph}(\infty)$ implies to a Fourier transform of $D(t)$ and depends only on $D(\omega = 0)$ and $D(\omega)$ when $\omega = \sqrt{\Delta^2 + 4g^2}$, corresponding to sampling the spectrum at the polaritonic eigenenergies [7].

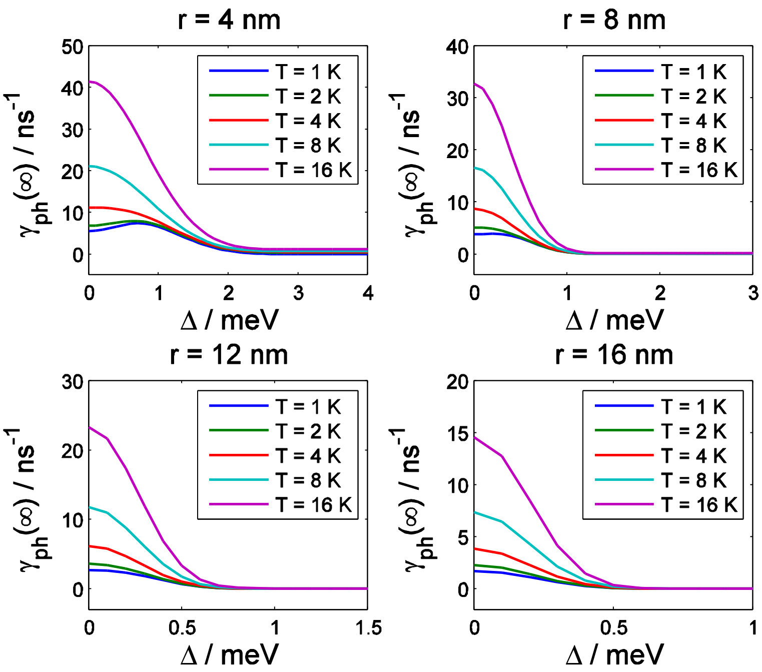

Figure 6. (Color online) Pure phonon dephasing rate $\gamma_{ph}(\infty)$ dependence on size and temperature

CONCLUSION

We have derived the analytic expressions of phonon spectrum in the quantum dot cavity QED system. Spherical-Gaussian wave function has been used to model the electron (hole) in the quantum dot. Difference in the confinement lengths of electron and hole has been taken into consideration. Piezoelectric coupling contributes much less than deformation potential coupling to phonon spectrum. The effective phonon spectrum, renormalizaton factor, feeding rate and pure phonon dephasing rate was analyzed. The results show that both the amplitude and extending range of the phonon spectrum increases as the confinement lengths of quantum dot decreases. As a consequence, the interaction between the electron-hole pair and the acoustic phonon strengthens the renormalization of the coupling factor between the quantum dot and the cavity. Pure dephasing rate increases as the confinement lengths of the quantum dot decreases, and increases as the temperature increases. The confinement length of quantum dot dominates the coupling between the electron-hole and the phonon.

We hope this analysis is helpful to control the interaction between the quantum dot and phonon bath in solid-state cavity QED system which has many applications in quantum information technology.

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