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ON THE EXISTENCE AND UNIQUENESS OF A STEADY HYDRO MAGNETIC FLOW OF A RADIATING VISCOUS FLUID IN A POROUS MEDIUM

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ABSTRACT: *This study examines the existence and uniqueness of a steady hydro magnetic flow of a radiating viscous fluid in a porous medium. The combine effects of a radioactive heat transfer and transverse magnetic field on a steady flow of an electrically conducting optically thin fluid through a horizontal channel with porous medium in non-uniform temperatures at the wall were considered. The transformed non-linear coupled partial differential equations were solved numerically using shooting method via Runge-Kutta method of order four using Maple software. Numerical results for the effects of the parameters such as magnetic, radiation and porosity on velocity and temperature distributions were discussed graphically.*

KEYWORDS: hydro magnetic flow, optically thin, porous medium, electrically conducting fluid, radiating viscous fluid, convective flow

INTRODUCTION

Convective flow in a porous media has been widely studied in the recent years due to its wide applications in engineering as post accidental heat removal in nuclear reactors, drying processes, heat exchangers, geothermal and oil recovery , building constructions see (Nield and Bejan 2005; Ingham and Pop 2005; Vafai 2005).

Many industrial processes involve the transfer of heat by means of a flowing fluid in either the laminar or turbulent regime as well as flowing or stagnant boiling points. Furthermore, many processes in industrial areas occur at high temperature and the knowledge of radiation heat transfer in the system can perhaps lead to a desired product with a desired characteristic hence many researchers focused attention on Magntohydrodynamics (MHD) applications where the operating temperatures are high.

Studies on MHD was carried out by (Crammer and Pal 1973; Moreau 1990; Raptis *et al.* 1982) in which they examined the problems on hyromagnetic free convection flow through a porous medium between two parallel plates while Kearsley (1994) considered the problem of steady state coquette flow with viscous heating.

Consideration of a MHD steady flow in a channel with slip at permeable boundaries was done by (Makinde and Osalusi (2006).

Grief *et.al* (1976) considered the exact solution for the problem of laminar convection flow in a vertical heated channel within the optically thin limit of Cogley *et al.* (1968).

Makinde and Mhone (2005) studied the effect of thermal radiation on MHD oscillatory flow in a channel filled with saturated porous medium and in non-uniform wall temperatures.

Kumar et al. (2010) considered the problem of unsteady MHD periodic flow of viscous fluid through a planar channel in a porous medium using perturbation method.

Equally, Narahari (2010) examined the effects of thermal radiation and free convection currents on the unsteady coquette flow between two vertical parallel plates with constant heat flux at one boundary.

In like manner, Isreal –Cookey and Nwaigwe (2010) examined unsteady MHD flow of a radiating fluid over a vertical moving heated porous plate with time- dependent suction.

Isreal- Cookey *et. al*. (2010) examined the combined effects of thermal radiation and transverse magnetic field on steady flow of an electrically optically thin fluid and gave the close form.

The main focus of this paper is to establish the existence and uniqueness of solution to the model and to equally enact a theorem to support the existence and uniqueness of solution, then to come up with a numerical solution which is an approximate solution to the given model.

MODEL FORMULATION

A steady flow of an electrically conducting fluid bounded by two horizontal plates which was filled with saturated porous medium and a transverse uniform magnetic field B_0 was considered. At the lower plate, the temperature was maintained at $T = T_0$, while at the upper plate, $y = h$, the temperature was maintained at $T = T_1$.. Using the radiative heat flux and a Boussinesq incompressible fluid model invoke, the momentum and energy equations governing the flow are given by

$$\nu\frac{\partial^2 U}{\partial y^2} + g\beta_T(T - T_0) - \frac{\nu}{\kappa}U - \frac{\sigma_c {B_0}^2}{\rho_0}U = 0 \qquad (1)$$

And

$$\frac{K_T}{\rho_0 c_P}\left(\frac{\partial^2 T}{\partial y^2} - \frac{1}{K_T}\frac{\partial q}{\partial y}\right) = 0 \qquad (2)$$

Following Cogley *et al.* (1968), and assuming the fluid is optically thin with relative low density where

$$\frac{\partial q}{\partial y} = 4(T - T_0)\int_0^\infty \left(\propto_\lambda \frac{\partial \beta_\lambda}{\partial T}\right) d\lambda \qquad (3)$$

Where α_λ is the absorption coefficient, β_λ is the Planck's function , q is the component of radiative flux U is the axial velocity, T is the temperature, g is the gravitational

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acceleration, β_T is the coefficient of thermal expansion,ν is the kinematic viscosity, σ_c is the electric conductivity , ρ_0 is the fluid density, c_p is the specific heat capacity at constant pressure, κ is the permeability of the porous medium and κ_T is the thermal conductivity .

The corresponding boundary conditions are:

$$\left.\begin{array}{l} U = 0,\ T = T_0 \quad \text{on} \quad y = 0 \\ U = 0 \ \ , T = T_1 \quad \text{on} \quad y = h \end{array}\right\} \tag{4}$$

To simplify the problem the investigation is limited to the so called optically thin non-grey gas. On non-dimensionalize the momentum and energy equations $(1), (2)$ and (3) subject to the boundary conditions in (4) , by using the following non-dimensional variables and parameters

$$y = h\eta\ , \quad U = U_0 u\ , \qquad \theta = \frac{T - T_1}{T_1 - T_0}$$

$$k = h^2\chi^2\ , M^2 = \frac{\sigma_{cB_0}{}^2 h^2}{\rho_0 \nu}, Gr = \frac{g\beta_1 h^2 (T_0 - T_1)}{\nu U_0}\ ,\ F^2 = \frac{4\alpha^2 h^2}{\kappa_T} \tag{5}$$

Where U_0 represents the mean velocity.

After taken the advantages of the dimensionless quantities and parameters in (5) the momentum and energy equations (1) and (2) $and\ using$ (3) in (2) respectively become:

$$\frac{\partial^2 u}{\partial \eta^2} + Gr\theta - \lambda^2 u = 0 \tag{6}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} - F^2\theta = 0 \tag{7}$$

Where $\lambda^2 = M^2 + \chi^2$, M is the magnetic parameter, χ is the porosity parameter, Gr is the Grashof number and F is the radiation parameter.

The momentum equation (6) and energy equation (7) are to be solved subject to the following boundary conditions

$$u = 0\ , \theta = 1 \text{ on } \eta = 0 \qquad u = 0\ , \theta = 1 \text{ on } \eta = 1 \tag{8}$$

METHOD OF SOLUTION

In the steady case of the problem the second order boundary value problem is transformed into a system of first order equations by using Shooting method and finally these system of equations is solved numerically using Runge-Kutta technique of order four.

Existence and Uniqueness of Solution.

Preliminary Remarks

Consider the initial value system [3]

$$\left.\begin{aligned} X_1^I &= f_1(t, x_1, x_2, \ldots, x_n),\ X_1(t_0) = x_{10} \\ X_2^I &= f_2(t, x_1, x_2, \ldots, x_n),\ X_2(t_0) = x_{20} \\ &\cdot \\ &\cdot \\ &\cdot \\ X_n^I &= f_n(t, x_1, x_2, \ldots, x_n),\ X_n(t_0) = x_{no} \end{aligned}\right\} \tag{9}$$

Which in vector form we have yields

$$X' = \begin{pmatrix} x_1 \\ \cdot \\ x_n \end{pmatrix}$$

$$\bar{f}(t,x) = \begin{cases} f_1(t\,, x_1\,, x_2\,, \ldots\ldots, x_n) \\ \cdot \\ f_n(t\,, x_1\,, x_2\,, \ldots\ldots\ldots x_n) \end{cases}$$

$$\underline{X_0'} = \begin{pmatrix} x_{10} \\ \cdot \\ x_{n0} \end{pmatrix}$$

The above can be written in compact form as

$$\underline{X'} = \overline{f}(t\,, X)\,,\ \ \underline{X}\,(t_0) = X_0 \tag{10}$$

In order to generalize the result, it is necessary to generalize the notion of the absolute value of a number to the absolute value or norm of a vector $\underline{X}$ or matrix A . If $\underline{X} = (x_1,\ .\ .\ .\ ,x_n)$ is a two valued-vector, then the distance from $(x_1,\ .\ .\ ,x_2)$ to the origin is given by the norm

$$\|\ \underline{X}\ \| = \sqrt{{x_1}^2 + {x_2}^2} \tag{11}$$

Therefore, it is natural to define the length or norm of an n −vector

$$\|\ \underline{X}\ \| = (x_1 + x_2\ \ .\ .\ .\ + x_n\ )^{\frac{1}{2}} \tag{12}$$

If A is an $n \times n$ matrix, there are several ways to define its norm. For our purposes, the simplest choice for the norm of A is $\|\ A\ \| = \sum_{i=`}^{n} \sum_{j=1}^{n} |a_{ij}|$

Now using this notation and norm $\|.\|$, the existence and uniqueness of a local vector solution $\underline{X}(t)$ of the initial value problem is hereby obtained.

According to [3] , Let D denote the region in $(n\,(n+1))$ dimensional space, one dimension for t and n dimensions for the vector $\underline{X}$.

$$|t\text{-}t_0| \le a\,,\quad \|\ X\text{-}X_0\ \| \le b \tag{13}$$

And suppose that $\underline{f}\ (t\,,\underline{X})$ satisfies the Lipschitz condition

$$\|\ \underline{f}\ (t\,,\underline{X_1}) - \underline{f}\ (t\,,\underline{X_2})\ \| \le k\ \|\ X_1 - X_2\ \| \tag{14}$$

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Whenever the pairs $(t, \underline{X_1})$ and $(t, \underline{X_2})$ belong to D, where k is a positive constant. Then there is a constant $\delta > 0$ such that there exist a unique continuous vector solution $\underline{X}(t)$ of the system (14) in the interval $|t - t_0| \leq \delta$.

It is easy to see that (14) is implied by the inequality

$$|f_i(t, x_{11}, \ldots, x_{1n}) - f_i(t, x_{21}, \ldots . x_{2n})| \leq k_1 \sum_{j=1}^{n} |x_{1j} - x_{2j}| \tag{15}$$

For some number k_1. This fact follows from the double inequality

$$\frac{1}{n} \sum_{j=1}^{n} |x_j| \leq \| x \| \leq \sum_{j=1}^{n} |x_j|$$

Which is an immediate consequence of the definition of $\| X \|$.

Another condition emanating from equation (14) is

$$|f_i(t, x_{11,\ldots,} x_{1n}) - f_i(t, x_{21}, \ldots, x_{2n})\Big| \leq k_2 \max_j |x_{1j} - x_{2j}\Big| \tag{16}$$

for $i = 1, 2, \ldots, m$

The two inequalities (13) and (14) are useful since it is often very difficult to verify inequalities (15) directly. Finally, if the partial derivatives

$$\frac{\partial f_i}{\partial f_j} \quad i, j = 1, 2, \ldots . n \tag{17}$$

are continuous in D, then they are bounded on D the conditions (15) and (16) both follows from the mean value theorem of differential calculus.

The proof of existence and uniqueness of solution of the problem formulated is hereby presented.

Now consider the problem

$$u(y) + Grt\theta(y) - \lambda^2 u(y) = 0$$

$$\frac{d^2}{dy^2}\theta(y) - F^2\theta(y) = 0 \tag{18}$$

Subject to

$$\left.\begin{array}{l} u(0) = 0, \ u(1) = 0 \\ \theta(0) = 0, \ \theta(1) = 1 \end{array}\right\} \tag{19}$$

THEOREM:

Let $\Phi = \{(X, y): |X_i| \leq b_0, 0 \leq y, a_0\}$, where $X = (u(y), \ \theta(y)), F, Gr$ and λ are real positive constants and $F \neq \lambda \neq 0, a_0, b_0 < \infty$.Then the system of equation (16) satisfying (17) has a unique solution.

Proof:

Let; $u = X_1, \ u' = X_2, X_2' = \lambda X_1 - GrtX_3, \theta = X_3, \theta' = X_4, X_4' = FX_3$, then

$$\left.\begin{aligned} u' &= X_1' = X_2 \\ u'' &= X_2' = \lambda X_1 - GrtX_3 \\ \theta' &= X_3' = X_4 \\ \theta'' &= X_4' = FX_3 \end{aligned}\right\} \tag{20}$$

The system of equation (20) can be written in vector form as thus

$$\frac{d}{dt}\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix}' = \begin{pmatrix} f_{1(X_1,X_2,X_3,X_4)} \\ f_{2(X_1,X_2,X_3,X_4)} \\ f_{3(X_1,X_2,X_3,X_4)} \\ f_{4(X_1,X_2,X_3,X_4)} \end{pmatrix} = \begin{pmatrix} X_2 \\ \lambda X_1 - GrtX_3 \\ X_4 \\ FX_3 \end{pmatrix}$$

With the initial condition

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} = (0) = \begin{pmatrix} 0 \\ \alpha_1 \\ 1 \\ \alpha_2 \end{pmatrix}$$

Let $f_{i(X_1,X_2,X_3,X_4)}$ be defined as follows

$$f_1(X_1, X_2, X_3, X_4) = X_2$$

$$f_2(X_1, X_2, X_3, X_4) = \lambda X_1 - GrtX_3$$

$$f_3(X_1, X_2, X_3, X_4) = X_4$$

$$f_4(X_1, X_2, X_3, X_4) = FX_3$$

Since X_i is bounded, then $f_j(X, t)$, $j = 1(1)4$ are defined and continuous for all points $(X.t)$, $j = 1(1)4$ in Φ in which they take their maximum in Φ.

Let this maximum be defined by

$$M_i' = \sup_{(t,X)\epsilon\Phi} |f_j\,(X, t)|\,, i, j = 1, \ldots .4$$

$$M_i' = \sup_{(t,X)\epsilon\Phi} |f_j\,(t, \underline{X})|\,, i, j = 1,2,3,4$$

Thus $f_j(X, t)$ are continuous over Φ.

Then there exist at most an M' such that $|f_i(X, t)| \leq M'$ and $\delta = min\left(a_0, \frac{b_0}{M'}\right)$ which imply $f_i(X, t)$ are continuous and bounded in Φ, then the system of equations (15) and(16) has a solution in the interval $|t| < \delta$.

Now $\left|\frac{\partial f_1}{\partial X_j}\right| = 0 \quad, j = 1,3,4 \qquad \left|\frac{\partial f_1}{\partial X_2}\right| = 1$

$$\left|\frac{\partial f_2}{\partial X_j}\right| = 0 \quad, j = 2,4 \qquad \left|\frac{\partial f_2}{\partial X_1}\right| = \lambda \quad \left|\frac{\partial f_2}{\partial X_3}\right| = Grt$$

$$\left|\frac{\partial f_3}{\partial X_j}\right| = 0 \quad, j = 1,2,3 \qquad \left|\frac{\partial f_3}{\partial X_4}\right| = 1$$

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$$\left|\frac{\partial f_4}{\partial X_j}\right| = 0 \ , j = 1,2,4 \qquad \left|\frac{\partial f_4}{\partial X_3}\right| = F$$

By the condition of the theorem $\lambda, F,$ and given that Grt are real and continuous in Φ, then $\left|\frac{\partial f_i}{\partial X_j}\right|, i = 1(1)4$ are continuous and bounded.

Hence, the system of equations (6) and (7) subject to (8) has a unique solution.

NUMERICAL SIMULATION

The transformed non-linear equations (18) under the boundary conditions (19) are solved numerically by shooting method alongside with fourth order Runge- Kutta method algorithm. The corresponding velocity and temperature profiles are shown in figures (1) to(4)

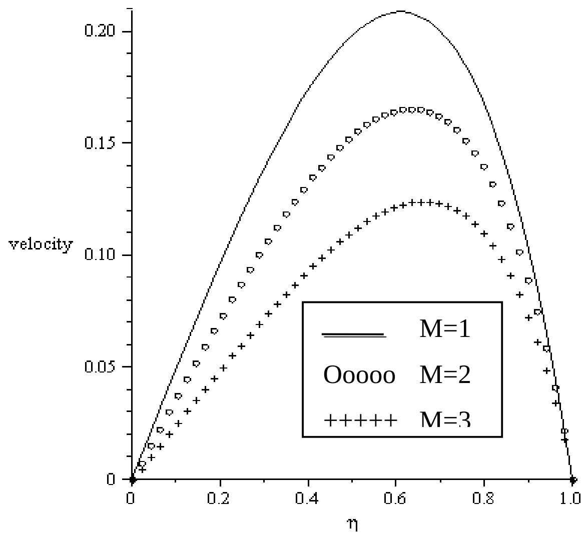

Figure 1: Graph of velocity profile for $\chi = 0.2, F = 2, Gr = 5$ with various values of M

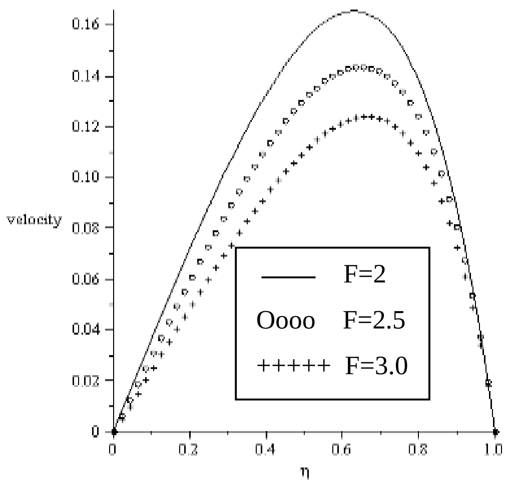

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Figure 2: Graph of velocity profile for $\chi = 0.2, M = 2, Gr = 5$ with various values of F

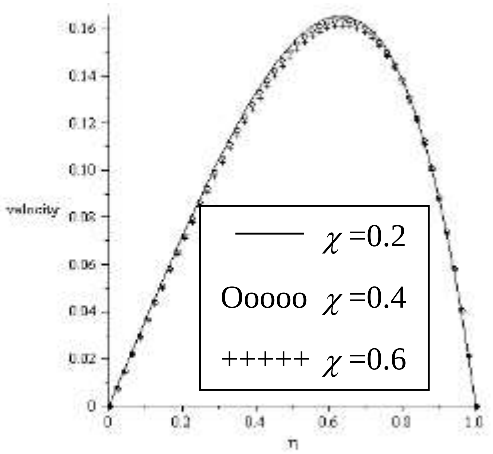

Figure 3: Graph of velocity profile for F=2, M=2, Gr=5 with various values of χ

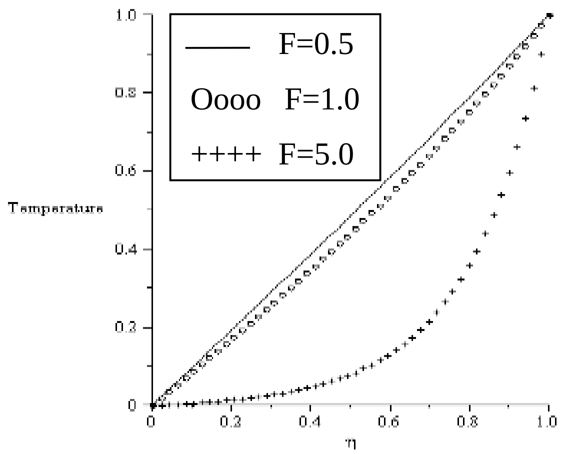

Figure 4: Graph of temperature profile for $\chi = 0.2, M = 2, Gr = 5$ with various values of F

DISCUSSION OF RESULTS

The coupled non-linear ordinary differential equations (18) subject to the boundary conditions (19) were solved numerically using Maple.

This software uses a fourth order Runge-Kutta method to solve the boundary value problems numerically. In the numerical analysis, some parameters like the magnetic, radiation and porosity were varied to simulate physically realistic situations.

We considered in details the influence of physical parameters like magnetic parameter, radiation parameter, porosity parameter on the velocity and the effect of radiation parameter on temperature profile.

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In figures (1) to (3) , it was observed that the velocity profile was parabolic in nature and the fluid velocity boundary layer thickness decreases across the channel as magnetic (M) , radiation (F) and porosity (χ) parameters increases.

Figure (4) , shows the temperature profile for various values of the radiation parameter(F). It was observed that the temperature increases and is minimum at the lower plate and maximum at the upper plate. Generally, there is a decrease in the fluid temperature profile within the channel when the radiation parameter increases.

CONCLUSION

The communication deals with the existence and uniqueness of a steady hydro magnetic flow of a radiating viscous fluid in a porous medium where a theorem was formulated to support the existence and uniqueness of the model .The transformed non-linear equations are solved numerically by shooting method alongside with fourth order Runge-Kutta iteration scheme.

The results are sketched and discussed for the fluid and flow parameters variations.

The major results from this study can be summarized as follows:

Existence and uniqueness of solution for the model was established.

Temperature increases as the radiation parameter increases.

The velocity flow rate decreases as the magnetic and radiation increases.

(4)Also observed that increase in porosity parameter bring a slight decrease in temperature boundary layer.

In conclusion therefore, the problem of a steady hydro magnetic flow of a radiating viscous fluid in a porous medium with an optically thin case is affected by the magnetic parameter, radiation parameter and porosity parameter.

It was clearly seen that the influence of radiation (F) cannot be underestimated in engineering applications (electronic cooling).

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