International Journal of Mathematics and Statistics Studies

Vol.7, No.3, pp.28-50, June 2019

Published by European Centre for Research Training and Development UK (www.eajournals.org)

NUMERICAL SOLUTION OF CURRENTS AND VOLTAGES FLOW IN ELECTRICAL CIRCUITS (RLC) USING NUMERICAL TECHNIQUES

Falade K. I[1] and **Ayodele V. I**[2]

[1]Department of Mathematics, Faculty of Computing and Mathematical Sciences.
Kano University of Science and Technology, P.M.B 3244 Wudil Kano State Nigeria.
[2]Department of Computer Science and Mathematics, Faculty of Science,
Nigeria Police Academy Wudil Kano State Nigeria.

ABSTRACT: *In this work, we present numerical solutions of electrical circuits second-order differential equations which is used as mathematical models of electrical circuits (RLC) consisting of a resistor, an inductor and a capacitor which connected in series and in parallel using numerical approaches. The Differential Transformation Method (DTM) and Exponentially Fitted Collocation Approximation Method (EFCAM) were employed to obtain numerical solutions which were compared with the analytical solutions of the electrical circuits and are found to be accurate and compatible. The graphs for voltages and currents profiles are presented to show the efficiency of numerical technique.*

KEYWORDS: Electrical circuit, RLC circuit, Differential Transformation Method (DTM), Exponentially Fitted Collocation Approximation Method (EFCAM), voltages, currents, analytical solution.

INTRODUCTION

The most important mathematical models for physical phenomena is the differential equations. Motion of objects, Electrical circuits, Fluid and heat flow, bending and cracking of materials are all modelled by both linear and non-linear differential equations. Moreover, Numerous mathematical models in science and engineering are expressed in terms of unknown quantities and their derivatives. Many applications of differential equations (DEs), particularly ODEs of different orders, can be found in the mathematical modelling of real life problems [1]. Electric circuits and electromagnetic theories are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications and instrumentation which are based on electric circuit theory [2]. Therefore, Circuits theory is also valuable in specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics and physics.

Ohm's law, Kirchhoff's Voltage and Current Laws are essential in the analysis of linear circuitry. Kirchhoff's laws deal with the voltage and current in the circuit. Ohm's law relates voltage, current and resistance to one another. These three laws apply to resistive circuits where the only elements are voltage and/or current sources and resistors. Using the three laws any resistance of current through or voltage across resistor can be found if any two are already known, Kirchhoff's Voltage Law (KVL) states that the sum of all voltages in a closed loop must be zero. A closed loop is a path in a circuit that doesn't contain any other closed loops. Loops 1 and 2 in Figure 1 are examples of closed loops [3].

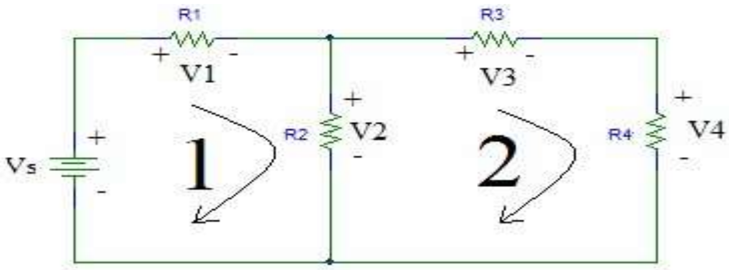

Figure 1: An example of KVL

The perimeter of the circuit is also a closed loop, but since it includes loops 1 and 2 it would be repetitive to include a KVL equation for it. If loop 1 is followed clockwise the KVL equation is

$$V_1 + V_2 - V_S = 0 \tag{1}$$

This equation holds true only if the passive sign convention is satisfied. In the case of KVL the passive sign convention states that when a positive node is encountered while following a loop the voltage across the element is positive. If a negative node is encountered the corresponding element voltage is negative. In order to simplify the KVL equations, the polarities should be assigned to satisfy the passive sign convention whenever possible [4]. The work of [5] proposed iterative method to solve resistive electrical circuit problems, in [6] presented Transient analysis of electrical circuits using Runge-Kutta method and its application, morose [7] investigated and studied of DC transients in R-L and R-C circuits and just to mention a few. It may be important to apply numerical techniques to compute numerical solutions of the electric circuits quickly. Thus, this study is to employ a very easy, fast and accurate numerical technique to obtain numerical solutions of both voltages and currents flow in electrical circuits (RLC).

ELECTRICAL CIRCUIT MODEL

Series RLC Circuit

A circuit consisting of a resistor *R*, an inductor *L*, a capacitor *C*, and a voltage source *V*(*t*) connected in series, shown in Figure 2, is called the series *RLC* circuit. Applying Kirchhoff's Voltage Law, we have

$$-V(t) + Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i dt = 0 \tag{2}$$

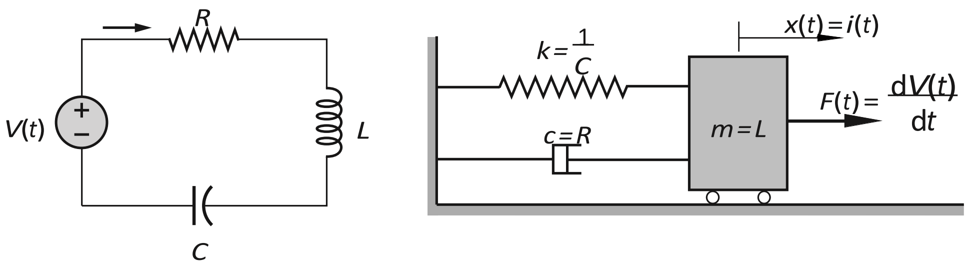

Figure 2: Serie RLC circuit.

Differentiating with respect to *t* yields

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = \frac{dV(t)}{dt} \qquad t\epsilon[a,b] \tag{3}$$

Print ISSN: 2053-2229 (Print), Online ISSN: 2053-2210 (Online)

Parallel RLC Circuit

A circuit consisting of a resistor R, an inductor L, a capacitor C, and a current source $I(t)$ connected in parallel, as shown in Figure 3, is called the parallel RLC circuit. Applying Kirchhoff's Current Law at node 1, we obtain

$$I(t) = C\frac{dv}{dt} + \frac{1}{L}\int_{-\infty}^{t} v dt + \frac{v}{R} \tag{4}$$

Differentiating with respect to t yields

$$L\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{C}v = \frac{dI(t)}{dt} \qquad t\epsilon[a, b] \tag{5}$$

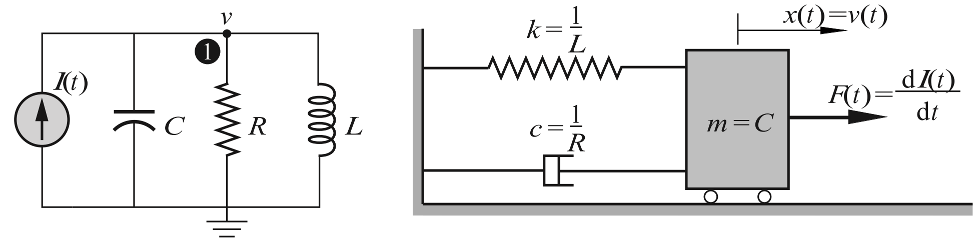

Figure 3 Parallel RLC circuit.

Where $L, R, C, E, I \; and \; V$ are measured in henrys, ohms, Farad, Volts and coulombs respectively. a and b are the domain of equations (3) and (5).

Suppose equations (3) and (5) are subject to initial conditions:

$$\begin{cases} i(t_0) = \beta_1 \\ i^{/}(t_0) = \beta_2 \\ v(t_0) = \delta_1 \\ v^{/}(t_0) = \delta_2 \end{cases} \tag{6}$$

DESCRIPTION OF TECHNIQUES:

Differential Transformation Method (DTM)

The concept of differential transformation method was first proposed by Zhou [8] and it was applied to solve linear and non-linear initial value problems in electric circuit analysis. This method constructs a semi analytical numerical technique that uses Taylor series for the solution of differential equations in the form of a polynomial. The differential transform is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. The Differential transformation method is very effective and powerful for solving various kinds of differential equations. For example, it was applied to two-point boundary value problems [9], to differential-algebraic equations [10], to the KdV and mKdV equations [11], to the Schrodinger equations [12], to fractional differential equations [13] and to the Riccati differential equation [14] and just mention a few. The main advantage of this method is that it can be applied directly to linear and nonlinear ODEs without requiring linearization, discretization or perturbation. Another important advantage is that this method is capable of greatly reducing the size of computational work while still accurately providing the series solution with fast convergence rate.

Consider an arbitrary function i(t) which can be expanded in Taylor series about a point $t = 0$ as

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$$i(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!}\left[\frac{d^k i}{dt^k}\right]_{t=0} \qquad (7)$$

The differential transformation of $I(t)$ is defined as

$$I(t) = \frac{1}{k!}\left[\frac{d^k i}{dt^k}\right]_{t=0} \qquad (8)$$

Then the inverse differential transform is

$$I(t) = \sum_{k=0}^{\infty} t^k\, I(k) \qquad (9)$$

The fundamental mathematical operations performed by differential transform method are listed in Table 1

Functional Form	**Transformed Form**
i(t) = w(t) $\pm$ v(t)	I(k) = W(k) $\pm$ V(k)
i(t) = η v(t)	I(k) = η V(k) , η is a constant
i(t) = $\frac{di(t)}{dt}$	I(k) = (k+1) I(k+1)
i(t) = $\frac{d^m i(t)}{dt^m}$	I(k) = $(k+1)\ldots\ldots(k+m)I(k+m)$
i(t)= e^t	$I(k) = \frac{1}{k!}$
i(t) = i^m	I(k) = δ(k-m) = $\begin{cases}1, & k = m\\ 0, & otherwise\end{cases}$
i(t) = w(t) v(t)	I(k) = $\sum_{r=0}^{\infty} W(r)V(k-r)$
i(t) = $i_1(t)i_2(t)i_3(t),\ldots\, i_m(t)$	I(k)= $\sum_{k_{m-1}=0}^{k} \ldots \sum_{k_1}^{k_2} I_1(k_1)I_2(k_2 - k_1)\ldots\ldots I_m(k-k_{m-1})$

Table 1: One Dimensional Differential Transformation

Exponentially Fitted Collocation Approximation Method (EFCAM)

The Exponentially Fitted Collocation Approximation Method (EFCAM) was formulated by [15] and it was applied to solve singular initial value problems and integro-differential equations. The whole idea of the method is to use power series as a basis function and its derivative substituted into a slightly perturbed equation, with perturbation term added to the right hand side of the equation. The addition of the perturbation term is to minimize the error of the problem in consideration.

Consider power series of the form:

$$i_N(t) = \sum_{k=0}^{N} p_k\, t^k \qquad (10)$$

And the Exponentially fitted approximate solution of the form:

$$i_N(t) \approx \sum_{k=0}^{N} p_k\, t^k + \tau_2 e^t \qquad (11)$$

where t represents the dependent variables in the problem, 2 represents highest derivative of the equations (3) and (5) and $p_k\ i_N(t)\ \ (k \geq 0)$ are the unknown constants to be determined, N is the length of computation and degree of chebyshev polynomials

Obtaining the second derivative of equation (3), we have

$$i^{/}(t) = \sum_{k=1}^{N} kp_k\, t^{k-1} \qquad (12)$$

$$i^{//}(t) = \sum_{k=2}^{N} k(k-1)p_k\, t^{k-2} \qquad (13)$$

Substitute equations (10), (12) and (13) into equations (3) and (5) respectively, we obtained

$$L\left[\sum_{k=2}^{N} k(k-1)p_k t^{k-2}\right] + R\left[\sum_{k=1}^{N} kp_k\, t^{k-1}\right] + \frac{1}{C}\left[\sum_{k=0}^{N} p_k\, t^k\right] = \frac{dV(t)}{dt} \qquad (14)$$

$$C\left[\sum_{k=2}^{N} k(k-1)p_k t^{k-2}\right] + \frac{1}{R}\left[\sum_{k=1}^{N} kp_k\, t^{k-1}\right] + \frac{1}{L}\left[\sum_{k=0}^{N} p_k\, t^k\right] = \frac{dI(t)}{dt} \qquad (15)$$

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Expansion of (14) and (15), we have
$L[2p_2 + 6tp_3 + 12t^2p_4 + \cdots + N(N-1)p_N t^{N-2}] + R[p_1 + 2tp_2 + 3t^2p_3 + 4t^3p_4 + \cdots + Np_N t^{N-1}] + \frac{1}{C}[p_0 + tp_1 + t^2p_2 + t^3p_3 + t^4p_4 \dots + p_N t^N] = \frac{dV(t)}{dt}$ $C[2p_2 + 6tp_3 + 12t^2p_4 + \cdots + N(N-1)p_N t^{N-2}] + \frac{1}{R}[p_1 + 2tp_2 + 3t^2p_3 + 4t^3p_4 + \cdots + Np_N t^{N-1}] + \frac{1}{L}[p_0 + tp_1 + t^2p_2 + t^3p_3 + t^4p_4 \dots + p_N t^N] = \frac{dI(t)}{dt}$

Collecting the likes terms, we have

$$\frac{1}{C}p_0 + [R+\frac{t}{C}]p_1 + \left[2L + 2tR + \frac{1}{C}t^2\right]p_2 + \left[6Lt + 3t^2R + \frac{1}{C}t^3\right]p_3 + \left[12Lt^2 + 4t^3R + \frac{1}{C}t^4\right]p_4 + \cdots + [L(N(N-1)t^{N-2} + R(N)t^{N-1} + \frac{1}{C}t^N]p_N = \frac{dV(t)}{dt} \quad (16)$$

$$\frac{1}{C}p_0 + [\frac{1}{R}+\frac{t}{C}]p_1 + \left[2C + \frac{1}{R}2t + \frac{1}{L}t^2\right]p_2 + \left[6tC + \frac{1}{R}3t^2 + \frac{1}{L}t^3\right]p_3 + \left[12t^2C + \frac{1}{R}4t^3 + \frac{1}{L}t^4\right]p_4 + \cdots + [C(N(N-1)t^{N-2} + \frac{1}{R}(N)t^{N-1} + \frac{1}{L}t^N]p_N = \frac{dI(t)}{dt} \quad (17)$$

Slightly perturbed and collocate equations (16) and (17), leads

$$\frac{1}{C}p_0 + [R+\frac{t_r}{C}]p_1 + \left[2L + 2t_rR + \frac{1}{C}t_r^2\right]p_2 + \left[6Lt_r + 3t_r^2R + \frac{1}{C}t_r^3\right]p_3 + \left[12Lt_r^2 + 4t_r^3R + \frac{1}{C}t_r^4\right]p_4 + \cdots + [L\left(N(N-1){t_r}^{N-2} + R(N){t_r}^{N-1} + \frac{1}{C}{t_r}^N\right]p_N - \tau_1 T_N(t_r) - \tau_2 T_{N-1}(t_r) = \frac{dV(t_r)}{dt} \quad (18)$$

$$\frac{1}{C}p_0 + [\frac{1}{R}+\frac{t_r}{C}]p_1 + \left[2C + \frac{1}{R}2t_r + \frac{1}{L}t_r^2\right]p_2 + \left[6Ct_r + \frac{1}{R}3t_r^2 + \frac{1}{L}t_r^3\right]p_3 + \left[12t_r^2C + \frac{1}{R}4t_r^3 + \frac{1}{L}t_r^4\right]p_4 + \cdots + [C(N(N-1){t_r}^{N-2} + \frac{1}{R}(N){t_r}^{N-1} + \frac{1}{C}t_r^N]p_N - \tau_1 T_N(t_r) - \tau_2 T_{N-1}(t_r) = \frac{dI(t_r)}{dt} \quad (19)$$

Where $t_r = a + \frac{(b-a)r}{N+2}$; $\quad r = 1,2, \dots\dots.. N+1$

Equations (18) and (19) are called Perturbed Collocation currents second order equation and Perturbed Collocation voltages second order equation respectively.
Couple with initial conditions (6) and using approximate solution (11), we have

$$\begin{cases} i(t_0) = p_0 + \tau_2 e^{t_0} = \beta_1 \\ i^/(t_0) == p_1 + \tau_2 e^{t_0} = \beta_2 \\ v(t_0) = p_0 + \tau_2 e^{t_0} = \delta_1 \\ v^/(t_0) == p_1 + \tau_2 e^{t_0} = \delta_2 \end{cases} \quad (20)$$

Altogether, we obtained (N+3) algebraic linear equations in (N+3) unknown constants. Using MAPLE 18 software to obtain (N+3) unkown constants and substitute into approximate solution equation (11).

3.3 Relative error

The relative error used in this paper can be defined as:

$$E_t = \left|\frac{\text{Analytical solution} - \text{Numerical solution}}{\text{Analytical solution}}\right| X10 \quad (21)$$

NUMERICAL EXPERIMENT

To illustrate the ability and reliability of the presented methods for the numerical solution of equations (3) and (5). We applied and present the numerical solution of currents and voltages flow in Electrical circuit (RLC) using both DTM and EFCAM techniques. Three problems were considered in which comparison shall be made between the analytical and approximate solutions The results revealed that the methods are effective and simple.

Problem 1

Consider equations (3) and (5) coupled with the following constant coefficients:

Inductor (L) =2.0, Resistor (R) =4.0, Capacitor (C) =0.05 and $\frac{dV(t)}{dt} = \frac{dI(t)}{dt} = 10e^{-t}$.

We obtained:

$$\begin{cases} 2\dfrac{d^2i}{dt^2} + 4\dfrac{di}{dt} + \dfrac{1}{0.05}i = 10e^{-t} \\ 2\dfrac{d^2v}{dt^2} + \dfrac{1}{4}\dfrac{dv}{dt} + \dfrac{1}{0.05}v = 10e^{-t} \end{cases} \qquad t\epsilon[0,1] \qquad (22)$$

Subject to initial conditions:

$$\begin{cases} i(t_0) = -1 \\ i^/(t_0) = 0 \\ v(t_0) = -1 \\ v^/(t_0) = 0 \end{cases} \qquad (23)$$

DTM technique

Taking the differential transform of (22) using table1, leads

$$2[(k+1)(k+2)I(k+2)] + 4[(k+1)I(k+1)] + \tfrac{1}{0.05}[I(k)] = -\tfrac{10}{k!}$$
$$2[(k+1)(k+2)V(k+2)] + \tfrac{1}{4}[(k+1)V(k+1)] + \tfrac{1}{0.05}[V(k)] = -\tfrac{10}{k!}$$

Taking $I(k+2)$ and $V(k+2)$ as subject of relations

$$2I(k+2) = \frac{-4[(k+1)I(k+1)] - \frac{1}{0.05}[I(k)] - \frac{10}{k!}}{(k+1)(k+2)} \qquad (24)$$

$$2V(k+2) = \frac{-\frac{1}{4}[(k+1)V(k+1)] - \frac{1}{0.05}[V(k)] - \frac{10}{k!}}{(k+1)(k+2)} \qquad (25)$$

From the initial condition given by equation (23), we have

$$\begin{cases} I(0) = -1 \\ I(1) = 0 \\ V(0) = -1 \\ V(1) = 0 \end{cases} \qquad (26)$$

Substituting equation (26) into equations (24) and (25) respectively by recursive approach when $k = 0,1,2,3,4,\dots\dots,14$, the results are listed as follow:

Print ISSN: 2053-2229 (Print), Online ISSN: 2053-2210 (Online)

International Journal of Mathematics and Statistics Studies

Vol.7, No.3, pp.28-50, June 2019

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$$\begin{cases} I(0) = -1 \ , \ I(1) = 0 \ , I(2) = \frac{15}{2} \ , I(3) = -\frac{36}{6} \ , I(4) = -\frac{25}{8} , I(5) = \frac{33}{8} \ , I(6) = -\frac{47}{144} \\ I(7) = -\frac{299}{336} \ , \ I(8) = \frac{755}{2688} \ , I(9) = \frac{4439}{72576} \ , I(10) = -\frac{3503}{80640} \ , I(11) = \frac{6221}{2661120} \\ I(12) = \frac{55589}{19160064} \ , I(13) = -\frac{19039}{31933440} \ , I(14) = -\frac{47941}{64575120} \end{cases}$$

Similarly, $$\begin{cases} V(0) = -1 \ , \ V(1) = 0 \ , V(2) = \frac{15}{2} \ , V(3) = -\frac{55}{48} \ , V(4) = -\frac{3075}{512} , V(5) = \frac{2791}{4096} \ , \\ V(6) = \frac{1176523}{589824} \ , \ V(7) = -\frac{729778}{3670016} \ , V(8) = -\frac{248714855}{704643072} \ , V(9) = \frac{4947574453}{152202903552} \\ V(10) = \frac{17503767569}{450971566080} \ , V(11) = -\frac{110278290667}{32469952757760} \ , V(12) = -\frac{59764969251697}{2057962067316736} \\ V(13) = -\frac{26544381365767}{108060002777825280} \ , V(14) = \frac{62873358856565401}{3993877026684223488 0} \end{cases}$$

Therefore, the closed form solution of currents (I) and voltages (V) in the electrical circuits can be written as

$$I(t) \approx -1 + \frac{15}{2}t^2 - \frac{36}{6}t^3 - \frac{25}{8}t^4 + \frac{33}{8}t^5 - \frac{47}{144}t^6 - \frac{299}{336}t^7 + \frac{755}{2688}t^8 + \frac{4439}{72576}t^9 - \frac{3503}{80640}t^{10} + \frac{6221}{2661120}t^{11} + \frac{55589}{19160064}t^{12} - \frac{19039}{31933440}t^{13} - \frac{47941}{64575120}t^{14} \tag{27}$$

Similarly

$$V(t) \approx -1 + \frac{15}{2}t^2 - \frac{55}{48}t^3 - \frac{3075}{512}t^4 + \frac{2791}{4096}t^5 + \frac{1176523}{589824}t^6 - \frac{729778}{3670016}t^7 - \frac{248714855}{704643072}t^8 + \frac{4947574453}{152202903552}t^9 + \frac{17503767569}{450971566080}t^{10} - \frac{110278290667}{32469952757760}t^{11} - \frac{59764969251697}{2057962067316736}t^{12} - \frac{26544381365767}{108060002777825280}t^{13} + \frac{62873358856565401}{39938770266842234880}t^{14} \tag{28}$$

EFCAM technique

Comparing equation (22) with equations (18) and (19) respectively and taking computational length (Chebyshev polynomial) $N = 14$. we have

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$$\begin{cases} 20I(0)+(4+20t_r)I(1)+(4+8t_r+20t_r^2)I(2)+(12t_r+12t_r^2+20t_r^3)I(3)+ \\ (24t_r^2+16t_r^3+20t_r^4)I(4)(40t_r^3+20t_r^4+20t_r^5)I(5)+(60t_r^4+24t_r^5+20t_r^6)I(6)+ \\ (84t_r^5+28t_r^6+20t_r^7)I(7)+(112t_r^6+32t_r^7+20t_r^8)I(8)+(114t_r^7+36t_r^8+20t_r^9)I(9)+ \\ +(180t_r^8+40t_r^9+20t_r^{10})I(10)+(220t_r^9+44t_r^{10}+20t_r^{11})I(11)+(264t_r^{10}+48t_r^{11}+20t_r^{12})I(12) \\ +(312t_r^{11}+52t_r^{12}+20t_r^{13})I(13)+(364t_r^{12}+56t_r^{13}+20t_r^{14})I(14) \\ -\begin{Bmatrix} 134217728t_r^{14}-939524096t_r^{13}+2936012800t_r^{12}-5402263552t_r^{11}+ \\ 6499598336t_r^{10}-5369233408t_r^9+3111714816t_r^8-1270087680t_r^7+ \\ 3611811184t_r^6-69701632t_r^5+8712704t_r^4-652288t_r^3+25480t_r^2- \\ 372t_r+1 \end{Bmatrix}\tau_1 \\ -\begin{Bmatrix} 33554432t_r^{13}-218103808t_r^{12}+5402263552t_r^{11}-1049624576t_r^{10}+ \\ 1133117440t_r^9-825556992t_r^8+412778496t_r^7-1413213696t_r^6+ \\ 32361471t_r^5-4759040t_r^4+416416t_r^3-18928t_r^2+338t-1 \end{Bmatrix}\tau_2=10e^{-t_r} \end{cases} \tag{29}$$

Similarly

$$\begin{cases} 20V(0)+\left(\frac{1}{4}+20t_r\right)V(1)+\left(4+\frac{1}{2}t_r+20t_r^2\right)V(2)+\left(12t_r+\frac{3}{4}t_r^2+20t_r^3\right)V(3)+ \\ (24t_r^2+t_r^3+20t_r^4)V(4)+\left(40t_r^3+\frac{5}{4}t_r^4+20t_r^5\right)V(5)+\left(60t_r^4+\frac{3}{2}t_r^5+20t_r^6\right)V(6)+ \\ +\left(84t_r^5+\frac{7}{4}t_r^6+20t_r^7\right)V(7)+(112t_r^6+2t_r^7+20t_r^8)V(8) \\ +\left(114t_r^7+\frac{9}{4}t_r^8+20t_r^9\right)V(9)+\left(180t_r^8+\frac{5}{2}t_r^9+20t_r^{10}\right)V(10) \\ +\left(220t_r^9+\frac{11}{4}t_r^{10}+20t_r^{11}\right)V(11)+(264t_r^{10}+3t_r^{11}+20t_r^{12})V(12) \\ +\left(312t_r^{11}+\frac{13}{4}t_r^{12}+20t_r^{13}\right)V(13)+\left(364t_r^{12}+\frac{7}{2}t_r^{13}+20t_r^{14}\right)V(14) \\ -\begin{Bmatrix} 134217728t_r^{14}-939524096t_r^{13}+2936012800t_r^{12}-5402263552t_r^{11}+ \\ 6499598336t_r^{10}-5369233408t_r^9+3111714816t_r^8-1270087680t_r^7+ \\ 3611811184t_r^6-69701632t_r^5+8712704t_r^4-652288t_r^3+25480t_r^2- \\ 372t_r+1 \end{Bmatrix}\tau_1 \\ -\begin{Bmatrix} 33554432t_r^{13}-218103808t_r^{12}+5402263552t_r^{11}-1049624576t_r^{10}+ \\ 1133117440t_r^9-825556992t_r^8+412778496t_r^7-1413213696t_r^6+ \\ 32361471t_r^5-4759040t_r^4+416416t_r^3-18928t_r^2+338t-1 \end{Bmatrix}\tau_2=10e^{-t_r} \end{cases} \tag{30}$$

Collocate equation (29) and (30) as follows:

$t_r = a + \frac{(b-a)r}{N+2}$; $r = 1,2,3 \ldots\ldots N+1$ where $a = 0$, $b = 1$, $N = 14$

$$t_1 = \frac{1}{16}, t_2 = \frac{2}{16}, t_3 = \frac{3}{16}, t_4 = \frac{4}{16}, t_5 = \frac{5}{16}, t_6 = \frac{6}{16}, t_7 = \frac{7}{16}, t_8 = \frac{8}{16}, t_9 = \frac{9}{16}$$
$$t_{10} = \frac{10}{16}, t_{11} = \frac{11}{16}, t_{12} = \frac{12}{16}, t_{13} = \frac{13}{16}, t_{14} = \frac{14}{16}, t_{15} = \frac{15}{16}$$

Consider initial conditions (22) and using MAPLE 18 software to obtain seventeen (17) unkown constants of equations (29) and (30), we obtained the following constants

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$$\begin{cases} I(0) = -1.0000001160000000 \\ I(1) = -0.0000001158246719 \\ I(2) = 7.4999648490000000 \\ I(3) = -5.8327085980000000 \\ I(4) = -3.1314962740000000 \\ I(5) = 4.1695703950000000 \\ I(6) = -0.5362652712000000 \\ I(7) = -0.1946550513000000 \\ I(8) = -1.3613488700000000 \\ I(9) = 2.8359199730000000 \\ I(10) = -3.367929853000000 \\ I(11) = 2.7580963510000000 \\ I(12) = -1.499035609000000 \\ I(13) = 0.4833205534000000 \\ 1(14) = -0.069829910730000 \\ \tau_1 = 0.000000554929090000 \\ \tau_2 = 0,000000115824671900 \end{cases} \quad \begin{cases} V(0) = -1.0000000760000000 \\ V(1) = -0.0000000756372912 \\ V(2) = 7.4999858830000000 \\ V(3) = -1.145589808000000 \\ V(4) = -6.008371471000000 \\ V(5) = 0.6985832133000000 \\ V(6) = 1.9138332130000000 \\ V(7) = 0.0692240279400000 \\ V(8) = -0.987270961300000 \\ V(9) = 1.1067339970000000 \\ V(10) = 1.2517337780000000 \\ V(11) = 1.0696018710000000 \\ V(12) = -0.589694498000000 \\ V(13) = 0.1901078941000000 \\ V(14) = -0.02741564170000 \\ \tau_1 = 0.0000002050255278000 \\ \tau_2 = 0,000000075637291200 \end{cases}$$

Substitute the above values into approximation solution (11). Hence, the approximate solution of currents (I) and voltages (V) in the electrical circuits equation (22) can be written as,

$$\begin{aligned} I(t) \approx\ & I(0) + I(1)t + I(2)t^2 + I(3)t^3 + I(4)t^4 + I(5)t^5 + I(6)t^6 + I(7)t^7 + I(8)t^8 + I(9)t^9 \\ & + I(10)t^{10} + I(11)t^{11} \\ & + I(12)t^{12} + I(13)t^{13} + i(14)t^{14} + \tau_2 e^t \end{aligned}$$

Similarly,

$$\begin{aligned} V(t) \approx\ & V(0) + V(1)t + V(2)t^2 + V(3)t^3 + V(4)t^4 + V(5)t^5 + V(6)t^6 + \\ & V(7)t^7 + V(8)t^8 + V(9)t^9 + V(10)t^{10} + V(11)t^{11} + \\ & V(12)t^{12} + V(13)t^{13} + V(14)t^{14} + \tau_2 e^t \end{aligned}$$

$$\begin{aligned} I(t) \approx\ & -1.0000001160000000 - 0.0000001158246719t + \\ & 7.4999648490000000t^2 - 5.8327085980000000t^3 - \\ & 3.1314962740000000t^4 + 4.1695703950000000t^5 - \\ & 0.5362652712000000t^6 - 0.1946550513000000t^7 - \\ & 1.3613488700000000\ t^8 + 2.8359199730000000t^9 - \\ & 3.367929853000000t^{10} + 2.7580963510000000t^{11} - \\ & 1.499035609000000t^{12} + 0.48332055340000003t^{13} - \\ & 0.069829910730000t^{14} + 0,000000115824671900e^t \end{aligned} \tag{31}$$

$$\begin{aligned} V(t) \approx\ & -1.0000000760000000 - 0.0000000756372912t + \\ & 7.4999858830000000t^2 - 1.1455898080000000t^3 - \\ & 6.008371471000000t^4 + 0.6985832133000000t^5 + \\ & 1.9138332130000000t^6 + 0.0692240279400000t^7 - \\ & 0.987270961300000\ t^8 + 1.1067339970000000t^9 + \\ & 1.2517337780000000t^{10} + 1.0696018710000000t^{11} - \\ & 0.589694498000000t^{12} + 0.1901078941000000t^{13} - \\ & 0.02741564170000t^{14} + 0,000000075637291200e^t \end{aligned} \tag{32}$$

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Problem 2

Consider equations (3) and (5) coupled with the following constant coefficients:

Inductor (L) =4.0, Resistor (R) =8.0, Capacitor (C) =0.1 and $\frac{dV(t)}{dt} = \frac{dI(t)}{dt} = 10e^{-t}$.

We obtained:

$$\begin{cases} 4\frac{d^2i}{dt^2} + 8\frac{di}{dt} + \frac{1}{0.1}i = 10e^{-t} \\ 4\frac{d^2v}{dt^2} + \frac{1}{8}\frac{dv}{dt} + \frac{1}{0.1}v = 10e^{-t} \end{cases} \tag{33}$$

Subject to initial conditions:

$$\begin{cases} i(t_0) = -1 \\ i^{/}(t_0) = 0 \\ v(t_0) = -1 \\ v^{/}(t_0) = 0 \end{cases} \tag{34}$$

DTM technique

Taking the differential transform of (33) using table1, leads

$$4[(k+1)(k+2)I(k+2)] + 8[(k+1)I(k+1)] + \frac{1}{0.1}[I(k)] = -\frac{10}{k!}$$

$$4[(k+1)(k+2)V(k+2)] + \frac{1}{8}[(k+1)V(k+1)] + \frac{1}{0.1}[V(k)] = -\frac{10}{k!}$$

Taking $I(k+2)$ and $V(k+2)$ as subject of relations

$$4I(k+2) = \frac{-8[(k+1)I(k+1)] - \frac{1}{0.1}[I(k)] - \frac{10}{k!}}{(k+1)(k+2)} \tag{35}$$

$$4V(k+2) = \frac{-\frac{1}{8}[(k+1)V(k+1)] - \frac{1}{0.1}[V(k)] - \frac{10}{k!}}{(k+1)(k+2)} \tag{36}$$

From the initial condition given by equation (23), we have

$$\begin{cases} I(0) = -1 \\ I(1) = 0 \\ V(0) = -1 \\ V(1) = 0 \end{cases} \tag{37}$$

Substituting equation (37) into equations (35) and (36) respectively by recursive approach when $k = 0,1,2,3,4 \ldots \ldots \ldots \ldots 14$, the results are listed as follow:

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$$\begin{cases} I(0) = -1 \ , \ I(1) = 0 \ , I(2) = \frac{5}{2}, I(3) = -\frac{25}{12} \ , I(4) = \frac{5}{8}, I(5) = -\frac{1}{96} \ , I(6) = -\frac{13}{288} \\ I(7) = \frac{5}{384} \ , \ I(8) = -\frac{19}{16128} \ , I(9) = -\frac{229}{1161216} \ , I(10) = \frac{47}{645120} \ , I(11) = -\frac{41}{46444864} \\ I(12) = \frac{37}{383201280} \ , I(13) = \frac{479}{3795517440} \ , I(14) = -\frac{10783}{557941063680} \end{cases}$$

Similarly,

$$\begin{cases} V(0) = -1 \ , \ V(1) = 0 \ , V(2) = \frac{5}{2}, V(3) = -\frac{85}{192} \ , V(4) = -\frac{3385}{8192}, V(5) = \frac{29167}{786432}, \\ V(6) = \frac{5694481}{150994944} \ , \ V(7) = -\frac{4625677}{1619612736} \ , V(8) = -\frac{13943861231}{8658654068736}, \\ V(9) = \frac{245440387567}{2493692371795968} \ , V(10) = \frac{4000066686409}{886646176638566640} \ , V(11) = -\frac{681920178393617}{2808895087590978355520} \\ , V(12) = -\frac{90916666323048431}{10786157136349356883968 0} \ , V(13) = -\frac{866008535476774 27}{2136686366957777363681280} \\ V(14) = \frac{23150450861480 6180881}{2010194533187156943751348 2240} \end{cases}$$

Therefore, the closed form solution of currents (I) and voltages (V) in the electrical circuits can be written as

$$I(t) \approx -1 + \frac{5}{2}t^2 - \frac{25}{12}t^3 + \frac{5}{8}t^4 - \frac{1}{96}\ t^5 - \frac{13}{288}t^6 + \frac{5}{384}t^7 - \frac{19}{16128}t^8 - \frac{229}{1161216}t^9 + \frac{47}{645120}\ t^{10} - \frac{41}{46444864}t^{11} + \frac{37}{383201280}\ t^{12} + \frac{479}{3795517440}t^{13} - \frac{10783}{557941063680}t^{14} \tag{38}$$

Moreso,

$$V(t) \approx -1 + \frac{5}{2}t^2 - \frac{85}{192}t^3 - \frac{3385}{8192}t^4 + \frac{29167}{786432}t^5 + \frac{5694481}{150994944}t^6 - \frac{4625677}{1619612736}t^7 - \frac{13943861231}{8658654068736}t^8 + \frac{245440387567}{2493692371795968}t^9 + \frac{4000066686409}{886646176638566640}t^{10} - \frac{681920178393617}{2808895087590978355520}t^{11} - \frac{90916666323048431}{107861571363493568839680}\ t^{12} - \frac{86600853547677427}{2136686366957777363681280}t^{13} + \frac{231504508614806180881}{20101945331871569437513482240}t^{14} \tag{39}$$

EFCAM technique

Comparing equation (33) with equation (18) and (19) respectively and taking computational length (Chebyshev polynomial) $N = 14$. we have

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$$\begin{cases} 10\mathrm{I}(0)+(8+10t_r)\mathrm{I}(1)+(4+16t_r+10t_r^2)\mathrm{I}(2)+(24t_r+24t_r^2+10t_r^3)\mathrm{I}(3)+ \\ (48t_r^2+32t_r^3+10t_r^4)\mathrm{I}(4)+(80t_r^3+40t_r^4+10t_r^5)\mathrm{I}(5)+(120t_r^4+48t_r^5+10t_r^6)\mathrm{I}(6) \\ +(168t_r^5+52t_r^6+10t_r^7)\mathrm{I}(7)+(224t_r^6+64t_r^7+10t_r^8)\mathrm{I}(8)+(288t_r^7+72t_r^8+10t_r^9)\mathrm{I}(9) \\ +(360t_r^8+80t_r^9+10t_r^{10})\mathrm{I}(10)+(440t_r^9+88t_r^{10}+10t_r^{11})\mathrm{I}(11)+(528t_r^{10}+96t_r^{11}+10t_r^{12})\mathrm{I}(12) \\ +(624t_r^{11}+104t_r^{12}+10t_r^{13})\mathrm{I}(13)+\left(728t_r^{12}+112t_r^{13}+20t_r^{14}\right)\mathrm{I}(14) \\ -\begin{Bmatrix} 134217728t_r^{14}-939524096t_r^{13}+2936012800t_r^{12}-5402263552t_r^{11}+ \\ 6499598336t_r^{10}-5369233408t_r^9+3111714816t_r^8-1270087680t_r^7+ \\ 3611811184t_r^6-69701632t_r^5+8712704t_r^4-652288t_r^3+25480t_r^2- \\ 372t_r+1 \end{Bmatrix}\tau_1 \\ -\begin{Bmatrix} 33554432t_r^{13}-218103808t_r^{12}+5402263552t_r^{11}-1049624576t_r^{10}+ \\ 1133117440t_r^9-825556992t_r^8+412778496t_r^7-1413213696t_r^6+ \\ 32361471t_r^5-4759040t_r^4+416416t_r^3-18928t_r^2+338t-1 \end{Bmatrix}\tau_2=10e^{-t_r} \end{cases} \quad (40)$$

Similarly

$$\begin{cases} 10V(0)+\left(\frac{1}{8}+10t_r\right)V(1)+\left(8+\frac{1}{4}t_r+10t_r^2\right)V(2)+\left(24t_r+\frac{3}{8}t_r^2+10t_r^3\right)V(3)+ \\ \left(48t_r^2+\frac{1}{2}t_r^3+10t_r^4\right)V(4)+\left(80t_r^3+\frac{5}{8}t_r^4+10t_r^5\right)V(5)+\left(120t_r^4+\frac{3}{8}t_r^5+10t_r^6\right)V(6) \\ +\left(168t_r^5+\frac{7}{8}t_r^6+10t_r^7\right)V(7)+(224t_r^6+t_r^7+10t_r^8)V(8)+\left(288t_r^7+\frac{9}{8}t_r^8+10t_r^9\right)V(9) \\ +\left(360t_r^8+\frac{5}{4}t_r^9+10t_r^{10}\right)V(10)+\left(440t_r^9+\frac{11}{8}t_r^{10}+10t_r^{11}\right)V(11)+\left(528t_r^{10}+\frac{3}{2}t_r^{11}+10t_r^{12}\right)V(12) \\ +\left(623t_r^{11}+\frac{13}{8}t_r^{12}+10t_r^{13}\right)V(13)+\left(728t_r^{12}+\frac{7}{4}t_r^{13}+10t_r^{14}\right)V(14) \\ -\begin{Bmatrix} 134217728t_r^{14}-939524096t_r^{13}+2936012800t_r^{12}-5402263552t_r^{11}+ \\ 6499598336t_r^{10}-5369233408t_r^9+3111714816t_r^8-1270087680t_r^7+ \\ 3611811184t_r^6-69701632t_r^5+8712704t_r^4-652288t_r^3+25480t_r^2- \\ 372t_r+1 \end{Bmatrix}\tau_1 \\ -\begin{Bmatrix} 33554432t_r^{13}-218103808t_r^{12}+5402263552t_r^{11}-1049624576t_r^{10}+ \\ 1133117440t_r^9-825556992t_r^8+412778496t_r^7-1413213696t_r^6+ \\ 32361471t_r^5-4759040t_r^4+416416t_r^3-18928t_r^2+338t-1 \end{Bmatrix}\tau_2=10e^{-t_r} \end{cases} \quad (41)$$

Collocate equation (29) and (30) as follows:

$t_r = a + \frac{(b-a)r}{N+2}$; $r = 1,2,3 \ldots \ldots . N+1$ where $a = 0$, $b = 1$, $N = 14$

$$t_1 = \frac{1}{16}, t_2 = \frac{2}{16}, t_3 = \frac{3}{16}, t_4 = \frac{4}{16}, t_5 = \frac{5}{16}, t_6 = \frac{6}{16}, t_7 = \frac{7}{16}, t_8 = \frac{8}{16}, t_9 = \frac{9}{16}$$
$$t_{10} = \frac{10}{16}, t_{11} = \frac{11}{16}, t_{12} = \frac{12}{16}, t_{13} = \frac{13}{16}, t_{14} = \frac{14}{16}, t_{15} = \frac{15}{16}$$

Consider initial conditions (34) and using MAPLE 18 software to obtain seventeen (17) unkown constants of equations (40) and (41), we obtained the following constants

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$$\begin{cases} I(0) = -0.99999987600000 \\ I(1) = 0.0000001239546647 \\ I(2) = 2.4999954490000000 \\ I(3) = -2.083261593000000 \\ I(4) = 0.6242938667000000 \\ I(5) = -0.005881303353000 \\ I(6) = -0.064977933870000 \\ I(7) = 0.0738002110600000 \\ I(8) = -0.133650912600000 \\ I(9) = 0.2061266745000000 \\ I(10) = -0.22775426850000 \\ I(11) = 0.174099691800000 \\ I(12) = -0.08754080575000 \\ I(13) = 0.026045097910000 \\ 1(14) = -0.0034734825490 \\ \tau_1 = 0.00000005052650570 \\ \tau_2 = -0.0000012395466400 \end{cases}, \quad \begin{cases} V(0) = -0.999999987700000 \\ V(1) = 0.0000000123445447 \\ V(2) = 2.4999965700000000 \\ V(3) = -0.442651782000000 \\ V(4) = -0.413782974400000 \\ V(5) = 0.0409372053100000 \\ V(6) = 0.0200350612100000 \\ V(7) = 0.0542752049200000 \\ V(8) = -0.13350345230000 \\ V(9) = 0.2181513885000000 \\ V(10) = -0.25597630310000 \\ V(11) = 0.208289971500000 \\ V(12) = -0.11159054110000 \\ V(13) = 0.035397619950000 \\ V(14) = -0.00503565605300 \\ \tau_1 = 0.00000008000316770 \\ \tau_2 = -0,0000001234454471 \end{cases}$$

Substitute the above values into approximation solution (11). Hence, the approximate solution of currents (I) and voltages (V) in the electrical circuits equation (33) can be written as,

$$I(t) \approx I(0) + I(1)t + I(2)t^2 + I(3)t^3 + I(4)t^4 + I(5)t^5 + I(6)t^6 + I(7)t^7 + I(8)t^8 + I(9)t^9 + I(10)t^{10} + I(11)t^{11} + I(12)t^{12} + I(13)t^{13} + i(14)t^{14} + \tau_2 e^t$$

Similarly,

$$V(t) \approx V(0) + V(1)t + V(2)t^2 + V(3)t^3 + V(4)t^4 + V(5)t^5 + V(6)t^6 + V(7)t^7 + V(8)t^8 + V(9)t^9 + V(10)t^{10} + V(11)t^{11} + V(12)t^{12} + V(13)t^{13} + V(14)t^{14} + \tau_2 e^t$$

$$\begin{aligned} I(t) \approx\ & -0.99999987600000 - 0.0000001239546647t + \\ & 2.4999954490000000t^2 - 2.083261593000000t^3 - \\ & 0.6242938667000000t^4 - 0.005881303353000t^5 - \\ & 0.0649779338700000t^6 + 0.0738002110600000t^7 - \\ & 0.133650912600000\, t^8 + 0.2061266745000000t^9 - \\ & 0.22775426850000t^{10} + 0.174099691800000t^{11} - \\ & 0.08754080575000t^{12} + 0.026045097910000t^{13} - \\ & 0.0034734825490t^{14} - 0.0000012395466400e^t \end{aligned} \tag{42}$$

$$\begin{aligned} V(t) \approx\ & -0.999999987700000 + 0.0000000123445447t + \\ & 2.4999965700000000t^2 - 0.442651782000000t^3 - \\ & 0.413782974400000t^4 + 0.0409372053100000t^5 + \\ & 0.0200350612100000t^6 + 0.0542752049200000t^7 - \\ & 0.13350345230000\, t^8 + 0.2181513885000000t^9 - \end{aligned}$$

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$$0.25597630310000t^{10} + 0.208289971500000t^{11} - 0.11159054110000t^{12} + 0.035397619950000t^{13} - 0.00503565605300t^{14} - 0,0000001234454471e^{t} \quad (43)$$

Problem 3

Consider equations (3) and (5) coupled with the following constant coefficients:

Inductor (L) =8.0, Resistor (R) =16.0, Capacitor (C) =0.2 and $\frac{dV(t)}{dt} = \frac{dI(t)}{dt} = 10e^{-t}$.

We obtained:

$$\begin{cases} 8\frac{d^2i}{dt^2} + 16\frac{di}{dt} + \frac{1}{0.2}i = 10e^{-t} \\ 8\frac{d^2v}{dt^2} + \frac{1}{16}\frac{dv}{dt} + \frac{1}{0.2}v = 10e^{-t} \end{cases} \quad (44)$$

Subject to initial conditions:

$$\begin{cases} i(t_0) = -1 \\ i^{/}(t_0) = 0 \\ v(t_0) = -1 \\ v^{/}(t_0) = 0 \end{cases} \quad (45)$$

DTM technique

Taking the differential transform of (44) using table1, leads

$$8[(k+1)(k+2)I(k+2)] + 16[(k+1)I(k+1)] + \frac{1}{0.2}[I(k)] = -\frac{10}{k!}$$
$$8[(k+1)(k+2)V(k+2)] + \frac{1}{16}[(k+1)V(k+1)] + \frac{1}{0.2}[V(k)] = -\frac{10}{k!}$$

Taking $I(k+2)$ and $V(k+2)$ as subject of relations

$$8I(k+2) = \frac{-16[(k+1)I(k+1)] - \frac{1}{0.2}[I(k)] - \frac{10}{k!}}{(k+1)(k+2)} \quad (46)$$

$$8V(k+2) = \frac{-\frac{1}{16}[(k+1)V(k+1)] - \frac{1}{0.2}[V(k)] - \frac{10}{k!}}{(k+1)(k+2)} \quad (47)$$

From the initial condition given by equation (23), we have

$$\begin{cases} I(0) = -1 \\ I(1) = 0 \\ V(0) = -1 \\ V(1) = 0 \end{cases} \quad (48)$$

Substituting equation (48) into equations (46) and (47) respectively by recursive approach when $k = 0,1,2,3,4 \dots \dots \dots \dots 14$, the results are listed as follow:

$$\begin{cases} \mathrm{I}(0) = -1 \ , \ \mathrm{I}(1) = 0 \ , \mathrm{I}(2) = \dfrac{15}{16} \ , \mathrm{I}(3) = -\dfrac{5}{6} \ , \mathrm{I}(4) = \dfrac{215}{512} , \mathrm{I}(5) = -\dfrac{39}{256} \ , \\ \mathrm{I}(6) = \dfrac{3227}{73728} , \mathrm{I}(7) = -\dfrac{451}{43008} \ , \ \mathrm{I}(8) = \dfrac{23827}{11010048} \ , \mathrm{I}(9) = -\dfrac{58463}{148635648} \\ \mathrm{I}(10) = \dfrac{169067}{264211520} \ , \mathrm{I}(11) = -\dfrac{51391}{5449973760} , \mathrm{I}(12) = \dfrac{31925809}{25113479086080} \\ \mathrm{I}(13) = -\dfrac{8597219}{54412538019840} \ , \mathrm{I}(14) = \dfrac{24675121}{1354267612938240} \end{cases}$$

Moreso,

$$\begin{cases} \mathrm{V}(0) = -1 \ , \ \mathrm{V}(1) = 0 \ , \mathrm{V}(2) = \dfrac{15}{16} \ , \mathrm{V}(3) = -\dfrac{1295}{6144} \ , \mathrm{V}(4) = \dfrac{3845}{1048576} , \mathrm{V}(5) = -\dfrac{514817}{134217728} \\ \mathrm{V}(6) = \dfrac{514791683}{309237645312} \ , \ \mathrm{V}(7) = -\dfrac{2543766199}{13194139533312} \ , \mathrm{V}(8) = \dfrac{1192681759489}{945792174780416} \\ \mathrm{V}(9) = -\dfrac{582466468688131}{3268532465560411177} \ , \mathrm{V}(10) = \dfrac{12006496765893547}{46485795065748070072320} \\ \mathrm{V}(11) = -\dfrac{381525633492004003}{178505453052472589077708 80} , \mathrm{V}(12) = \dfrac{1267254845309447427}{90480844043237305951 7} \\ \mathrm{V}(13) = -\dfrac{4476245325113707959749}{3860516012511458387272918 8925440} \ , \mathrm{V}(14) = \dfrac{8627836726166533153125121}{8993458102746693458990185207040} \end{cases}$$

Therefore, the closed form solution of currents (I) and voltages (V) in the electrical circuits can be written as

$$\begin{aligned} I(t) \approx -1 + \frac{15}{16}t^2 - \frac{5}{6}t^3 + \frac{215}{512}t^4 - \frac{39}{256}\,t^5 + \frac{3227}{73728}t^6 - \frac{451}{43008} + \frac{23827}{11010048} \\ - \frac{58463}{148635648}\,t^9 + \frac{169067}{264211520}\,t^{10} - \frac{51391}{5449973760}t^{11} + \\ \frac{31925809}{25113479086080}\,t^{12} - \frac{8597219}{54412538019840}t^{13} + \frac{24675121}{1354267612938240}t^{14} \end{aligned} \qquad (49)$$

Moreso,

$$\begin{aligned} V(t) \approx -1 + \frac{15}{16}t^2 - \frac{1295}{6144}t^3 + \frac{3845}{1048576}t^4 - \frac{514817}{134217728}t^5 + \frac{514791683}{309237645312}t^6 \\ - \frac{2543766199}{13194139533312}t^7 + \frac{1192681759489}{945792174780416}t^8 - \frac{582466468688131}{3268532465560411177}t^9 \\ + \frac{12006496765893547}{46485795065748070072320}t^{10} - \frac{381525633492004003}{17850545305247258907770880}t^{11} \\ + \frac{1267254845309447427}{904808440432373059517}\,t^{12} - \frac{4476245325113707959749}{38605160125114583872729188925440}t^{13} \\ + \frac{8627836726166533153125121}{8993458102746693458990185207050240}t^{14} \end{aligned} \qquad (50)$$

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EFCAM technique

Comparing equation (44) with equation (18) and (19) respectively and taking computational length (Chebyshev polynomial) $N = 14$. we have

$$\begin{cases} 5I(0) + (16 + 5t_r)I(1) + (16 + 32t_r + 5t_r{}^2)I(2) + (48t_r + 48t_r{}^2 + 5t_r{}^3)I(3) + \\ (96t_r{}^2 + 64t_r{}^3 + 5t_r{}^4)I(4) + (1600t_r{}^3 + 80t_r{}^4 + 5t_r{}^5)I(5) + (240t_r{}^4 + 96t_r{}^5 + 5t_r{}^6)I(6) \\ +(336t_r{}^5 + 112t_r{}^6 + 5t_r{}^7)I(7) + (448t_r{}^6 + 128t_r{}^7 + 5t_r{}^8)I(8) + (576t_r{}^7 + 144t_r{}^8 + 5t_r{}^9)I(9) \\ +(720t_r{}^8 + 160t_r{}^9 + 5t_r{}^{10})I(10) + (880t_r{}^9 176t_r{}^{10} + 5t_r{}^{11})I(11) + (1056t_r{}^{10} + 192t_r{}^{11} + 5t_r{}^{12})I(12) \\ +(1248t_r{}^{11} + 208t_r{}^{12} + 5t_r{}^{13})I(13) + \left(1456t_r{}^{12} 224t_r{}^{13} + 5t_r{}^{14}\right)I(14) \\ -\begin{Bmatrix} 134217728t_r{}^{14} - 939524096t_r{}^{13} + 2936012800t_r{}^{12} - 5402263552t_r{}^{11} + \\ 6499598336t_r{}^{10} - 5369233408t_r{}^9 + 3111714816t_r{}^8 - 1270087680t_r{}^7 + \\ 3611811184t_r{}^6 - 69701632t_r{}^5 + 8712704t_r{}^4 - 652288t_r{}^3 + 25480t_r{}^2 - \\ 372t_r + 1 \end{Bmatrix}\tau_1 \\ -\begin{Bmatrix} 33554432t_r{}^{13} - 218103808t_r{}^{12} + 5402263552t_r{}^{11} - 1049624576t_r{}^{10} + \\ 1133117440t_r{}^9 - 825556992t_r{}^8 + 412778496t_r{}^7 - 1413213696t_r{}^6 + \\ 32361471t_r{}^5 - 4759040t_r{}^4 + 416416t_r{}^3 - 18928t_r{}^2 + 338t - 1 \end{Bmatrix}\tau_2 = 10e^{-t_r} \end{cases} \quad (51)$$

Similarly,

$$\begin{cases} 5V(0) + \left(\frac{1}{16} + 5t_r\right)V(1) + \left(16 + \frac{1}{8}t_r + 10t_r{}^2\right)V(2) + \left(48t_r + \frac{3}{16}t_r{}^2 + 5t_r{}^3\right)V(3) + \\ \left(96t_r{}^2 + \frac{1}{4}t_r{}^3 + 5t_r{}^4\right)V(4)\left(160t_r{}^3 + \frac{5}{16}t_r{}^4 + 5t_r{}^5\right)V(5) + \left(240t_r{}^4 + \frac{3}{8}t_r{}^5 + 5t_r{}^6\right)V(6) \\ +\left(336t_r{}^5 + \frac{7}{16}t_r{}^6 + 5t_r{}^7\right)V(7) + \left(448t_r{}^6 + \frac{1}{2}t_r{}^7 + 5t_r{}^8\right)V(8) + \left(576t_r{}^7 + \frac{9}{16}t_r{}^8 + 5t_r{}^9\right)V(9) \\ +\left(720t_r{}^8 + \frac{5}{8}t_r{}^9 + 5t_r{}^{10}\right)V(10) + \left(880t_r{}^9 + \frac{11}{16}t_r{}^{10} + 5t_r{}^{11}\right)V(11) + \left(1056t_r{}^{10} + \frac{3}{4}t_r{}^{11} + 5t_r{}^{12}\right)V(12) \\ +\left(1248t_r{}^{11} + \frac{13}{16}t_r{}^{12} + 5t_r{}^{13}\right)V(13) + \left(1456t_r{}^{12} + \frac{7}{8}t_r{}^{13} + 5t_r{}^{14}\right)V(14) \\ -\begin{Bmatrix} 134217728t_r{}^{14} - 939524096t_r{}^{13} + 2936012800t_r{}^{12} - 5402263552t_r{}^{11} + \\ 6499598336t_r{}^{10} - 5369233408t_r{}^9 + 3111714816t_r{}^8 - 1270087680t_r{}^7 + \\ 3611811184t_r{}^6 - 69701632t_r{}^5 + 8712704t_r{}^4 - 652288t_r{}^3 + 25480t_r{}^2 - \\ 372t_r + 1 \end{Bmatrix}\tau_1 \\ -\begin{Bmatrix} 33554432t_r{}^{13} - 218103808t_r{}^{12} + 5402263552t_r{}^{11} - 1049624576t_r{}^{10} + \\ 1133117440t_r{}^9 - 825556992t_r{}^8 + 412778496t_r{}^7 - 1413213696t_r{}^6 + \\ 32361471t_r{}^5 - 4759040t_r{}^4 + 416416t_r{}^3 - 18928t_r{}^2 + 338t - 1 \end{Bmatrix}\tau_2 = 10e^{-t_r} \end{cases} \quad (52)$$

Collocate equation (51) and (52) as follows:

$$t_r = a + \frac{(b-a)r}{N+2}; \quad r = 1,2,3 \ldots\ldots.. N+1 \text{ where } a = 0 \ , \ b = 1 \ , \ N = 14$$

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$$t_1 = \frac{1}{16}, t_2 = \frac{2}{16}, t_3 = \frac{3}{16}, t_4 = \frac{4}{16}, t_5 = \frac{5}{16}, t_6 = \frac{6}{16}, \ t_7 = \frac{7}{16}, t_8 = \frac{8}{16}, t_9 = \frac{9}{16}$$
$$t_{10} = \frac{10}{16}, t_{11} = \frac{11}{16}, \ t_{12} = \frac{12}{16}, t_{13} = \frac{13}{16}, \ t_{14} = \frac{14}{16}, t_{15} = \frac{15}{16}$$

Consider initial conditions (48) and using MAPLE 18 software to obtain seventeen (17) unkown constants of equations (51) and (52), we obtained the following constants

$$\begin{cases} I(0) = -0.999999892700000 \\ I(1) = 0.0000001072866583 \\ I(2) = 0.9374975700000000 \\ I(3) = -0.833294772000000 \\ I(4) = 0.4195400350000000 \\ I(5) = -0.149871704700000 \\ I(6) = 0.0328478018700000 \\ I(7) = 0.0233595164700000 \\ I(8) = -0.072557880960000 \\ I(9) = 0.1176179575000000 \\ I(10) = -0.13220368840000 \\ I(11) = 0.102680042300000 \\ I(12) = -0.05249560506000 \\ I(13) = 0.015892862110000 \\ 1(14) = -0.00215842210100 \\ \tau_1 = 0.000000066086035000 \\ \tau_2 = -0.0000001072866580 \end{cases} , \quad \begin{cases} V(0) = -1.00000000600000000 \\ V(1) = 0.000000005670923487 \\ V(2) = 0.937499410300000000 \\ V(3) = -0.21076485180000000 \\ V(4) = 0.003564974435000000 \\ V(5) = -0.00314296369000000 \\ V(6) = -0.00156842875000000 \\ V(7) = 0.010430511760000000 \\ V(8) = -0.02489717008000000 \\ V(9) = 0.041807682410000000 \\ V(10) = -0.0497932500100000 \\ V(11) = 0.04105354240000000 \\ V(12) = -0.0222679838900000 \\ V(13) = 0.00714532374800000 \\ V(14) = -0.0010273751960000 \\ \tau_1 = 0.0000003309564203000 \\ \tau_2 = 0.0000000056709234800 \end{cases}$$

Substitute the above values into approximation solution (11). Hence, the approximate solution of currents (I) and voltages (V) in the electrical circuits equation (44) can be written as

$$\mathrm{I(t) \approx I(0) + I(1)t + I(2)t^2 + I(3)t^3 + I(4)t^4 + I(5)t^5 + I(6)t^6 + I(7)t^7 + I(8)t^8 + I(9)t^9 + I(10)t^{10} + I(11)t^{11} + I(12)t^{12} + I(13)t^{13} + i(14)t^{14} + \tau_2 e^t}$$

Similarly,

$$V(t) \approx V(0) + V(1)t + V(2)t^2 + V(3)t^3 + V(4)t^4 + V(5)t^5 + V(6)t^6 + V(7)t^7 + V(8)t^8 + V(9)t^9 + V(10)t^{10} + V(11)t^{11} + V(12)t^{12} + V(13)t^{13} + V(14)t^{14} + \tau_2 e^t$$

$$\begin{aligned} I(t) \approx {} & -0.999999892700000 + 0.0000001072866583t + \\ & 0.9374975700000000t^2 - 0.833294772000000t^3 - \\ & 0.4195400350000000t^4 - 0.149871704700000t^5 - \\ & 0.0328478018700000t^6 + 0.0233595164700000t^7 - \\ & 0.072557880960000\, t^8 + 0.1176179575000000t^9 - \\ & 0.13220368840000t^{10} + 0.102680042300000t^{11} - \\ & 0.05249560506000t^{12} + 0.015892862110000t^{13} - \\ & 0.00215842210100t^{14} - 0.0000001072866580e^t \end{aligned} \tag{53}$$

$$\begin{aligned} V(t) \approx {} & -1.00000000600000000 + 0.000000005670923487t + \\ & 0.937499410300000000t^2 - 0.21076485180000000t^3 - \\ & 0.003564974435000000t^4 - 0.00314296369000000t^5 - \end{aligned}$$

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$$0.00156842875000000t^6 + 0.0104305117600000000t^7 - 0.02489717008000000\,t^8 - 0.0418076824100000000t^9 - 0.0497932500100000t^{10} + 0.04105354240000000t^{11} - 0.0222679838900000t^{12} + 0.00714532374800000t^{13} - 0.0010273751960000t^{14} + 0.0000000056709234800e^t \quad (54)$$

Problem 1 Current I(t)				E_t (%)	
t	**Analytical Solution**	**DTM**	**EFCAM**	**DTM**	**EFCAM**
0.0	-1.0000000000	-1.0000000000	-1.0000000000	0.00000000000	0.00000000000
0.1	-0.9311049955	-0.9311049958	-0.9311050809	0.00000000322	0.00000009171
0.2	-0.7503782006	-0.7503782002	-0.7503783436	0.00000000053	0.00000019057
0.3	-0.4982019233	-0.4982019232	-0.4982020855	0.00000000020	0.00000032557
0.4	-0.2136925059	-0.2136925058	-0.2136926505	0.00000000046	0.00000067667
0.5	0.06855105330	0.06855105210	0.06855095584	0.00000001779	0.00000142171
0.6	0.32070639920	0.32070638180	0.32070637260	0.00000005425	0.00000008294
0.7	0.52297149450	0.52297132110	0.52297155180	0.00000331566	0.00000010956
0.8	0.66386469650	0.66386342020	0.66386484360	0.00000192253	0.00000021581
0.9	0.73972505180	0.73971765330	0.73972529290	0.00001000168	0.00000032593
1.0	0.75360246310	0.75356687620	0.75360276750	0.00004722237	0.00000040392

Table 2: Numerical Results: Inductor (L) = 2.0, Resistor (R) = 4.0, Capacitor (C) = 0.05 , $F(t) = 10e^{-t}$

Problem I Voltages V(t)				E_t (%)	
t	**Analytical Solution**	**DTM**	**EFCAM**	**DTM**	**EFCAM**
0.0	-1.0000000000	-1.0000000000	-1.0000000000	0.00000000000	0.0000000000
0.1	-0.9267376344	-0.9267376339	-0.9267376700	0.00000000053	0.0000000393
0.2	-0.7184337616	-0.7184337623	-0.7184338275	0.00000000097	0.0000000917
0.3	-0.4015408145	-0.4015408147	-0.4015408922	0.00000000049	0.0000001935
0.4	-0.0124802464	-0.0124802464	-0.0124803169	0.00000000160	0.0000056465
0.5	0.40603226290	0.40603226250	0.40603221840	0.00000000098	0.0000001095
0.6	0.80923992690	0.80923992630	0.80923992590	0.00000000074	0.0000000012
0.7	1.15474960700	1.15474960500	1.15474966500	0.00000000173	0.0000000502
0.8	1.40665190000	1.40665188200	1.40665202400	0.00000001279	0.0000000881
0.9	1.53895377400	1.53895365700	1.53895397500	0.00000007602	0.0000001306
1.0	1.53799373100	1.53799302000	1.53799399500	0.00000046229	0.0000001716

Table 3: Numerical Results: Inductor (L) = 2.0, Resistor (R) = 4.0, Capacitor (C) = 0.05 . $F(t) = 10e^{-t}$

Problem 2 Current I(t)				E_t (%)	
t	**Analytical Solution**	**DTM**	**EFCAM**	**DTM**	**EFCAM**
0.0	-1.0000000000	-1.0000000000	-1.0000000000	0.0000000000	0.0000000000
0.1	-0.9770209810	-0.9770209813	-0.9770209960	0.00000000030	0.0000000151
0.2	-0.9156727250	-0.91567272530	-0.9156727560	0.00000000032	0.0000000380
0.3	-0.8262429520	-0.8262429529	-0.8262430094	0.00000000012	0.0000000689
0.4	-0.7176043720	-0.7176043722	-0.7176044556	0.00000000027	0.0000001164
0.5	-0.5972881770	-0.5972881760	-0.5972882898	0.00000000100	0.0000001888
0.6	-0.4715728657	-0.4715728656	-0.4715730090	0.00000000212	0.0000003038
0.7	-0.3455837718	-0.3455837718	-0.3455839486	0.00000000000	0.0000005139
0.8	-0.2233992615	-0.2233992623	-0.2233994755	0.00000000381	0.0000009792

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0.9	-0.1081601058	-0.1081601088	-0.1081603564	0.00000000739	0.0000023169
1.0	-0.0021791053	-0.0021791053	-0.00217939763	0.00000005750	0.0001341518

Table 4: Numerical Results: Inductor (L) = 4.0, Resistor (R) = 8.0, Capacitor (C) = 0.1 $F(t) = 10e^{-t}$

Problem 2 Voltages V(t)				E_t (%)	
t	**Analytical Solution**	**DTM**	**EFCAM**	**DTM**	**EFCAM**
0.0	-1.0000000000	-1.0000000000	-1.00000000	0.0000000000	0.0000000000
0.1	-0.9754836220	-0.9754836208	-0.9754836308	0.0000000012	0.0000000090
0.2	-0.9041885596	-0.9041885586	-0.9041885803	0.0000000011	0.0000000228
0.3	-0.7901832261	-0.7901832254	-0.7901832583	0.0000000088	0.0000000407
0.4	-0.6383829383	-0.6383829375	-0.6383829817	0.0000000125	0.0000000679
0.5	-0.4544442771	-0.4544442765	-0.4544443316	0.0000000013	0.0000001199
0.6	-0.2446394667	-0.2446394652	-0.2446395302	0.0000000061	0.0000002595
0.7	-0.0157141305	-0.0157141285	-0.0157142031	0.0000000127	0.0000046194
0.8	0.2252678086	0.2252678111	0.22526772860	0.0000000111	0.0000003551
0.9	0.4710880495	0.4710880513	0.47108796140	0.0000000038	0.0000000187
1.0	0.7145423982	0.7145424014	0.71454230370	0.0000000044	0.0000001322

Table 4: Numerical Results: Inductor (L) = 4.0, Resistor (R) = 8.0, Capacitor (C) = 0.1 . $F(t) = 10e^{-t}$

Problem 3 Current I(t)				E_t (%)	
t	**Analytical Solution**	**DTM**	**EFCAM**	**DTM**	**EFCAM**
0.0	-1.0000000000	-1.0000000000	-1.00000000	0.0000000000	0.0000000000
0.1	-0.9914178200	-0.9914178220	-0.991417823	0.0000000017	0.0000000099
0.2	-0.9685408690	-0.9685408694	-0.968540888	0.0000000004	0.0000000198
0.3	-0.9350640790	-0.9350640792	-0.935064111	0.0000000002	0.0000000349
0.4	-0.8939799140	-0.8939799150	-0.893979963	0.0000000011	0.0000000550
0.5	-0.8476975820	-0.8476975829	-0.847697649	0.0000000011	0.0000000785
0.6	-0.7981431010	-0.7981431016	-0.798143187	0.0000000007	0.0000001078
0.7	-0.7468432073	-0.7468432074	-0.746843316	0.0000000005	0.0000000145
0.8	-0.6949956253	-0.6949956252	-0.694995758	0.0000000001	0.0000000190
0.9	-0.6435278373	-0.6435278375	-0.643527998	0.0000000003	0.0000000249
1.0	-0.5931461752	-0.5931461735	-0.593146366	0.0000000028	0.0000000311

Table 6: Numerical Results: Inductor (L) = 8.0, Resistor (R) = 16.0, Capacitor (C) = 0.2 and $F(t) = 10e^{-t}$

Problem 3 Voltages V(t)				E_t (%)	
t	**Analytical Solution**	**DTM**	**EFCAM**	**DTM**	**EFCAM**
0.0	-1.0000000000	-1.0000000000	-1.00000000	0.00000000000	0.0000000000
0.1	-0.9908354450	-0.9908354447	-0.990835447	0.00000000030	0.0000000003
0.2	-0.9641814543	-0.9641814543	-0.964181458	0.00000000000	0.0000000002
0.3	-0.9212943646	-0.9212943648	-0.921294370	0.00000000021	0.0000000058
0.4	-0.8634284783	-0.8634284780	-0.863428485	0.00000000034	0.0000000073
0.5	-0.7918379772	-0.7918379767	-0.791837985	0.00000000063	0.0000000099
0.6	-0.7077779132	-0.7077779132	-0.707777921	0.00000000141	0.0000000111
0.7	-0.6125043462	-0.6125043459	-0.612504356	0.00000000048	0.0000000159
0.8	-0.5072737265	-0.5072737262	-0.507273737	0.00000000059	0.0000000021
0.9	-0.3933415757	-0.3933415757	-0.393341587	0.00000000000	0.0000000270
1.0	-0.2719605636	-0.2719605634	0.2719605751	0.00000000073	0.0000000422

Table 7: Numerical Results: Inductor (L) = 8.0, Resistor (R) = 16.0, Capacitor (C) = 0.2 and $F(t) = 10e^{-t}$

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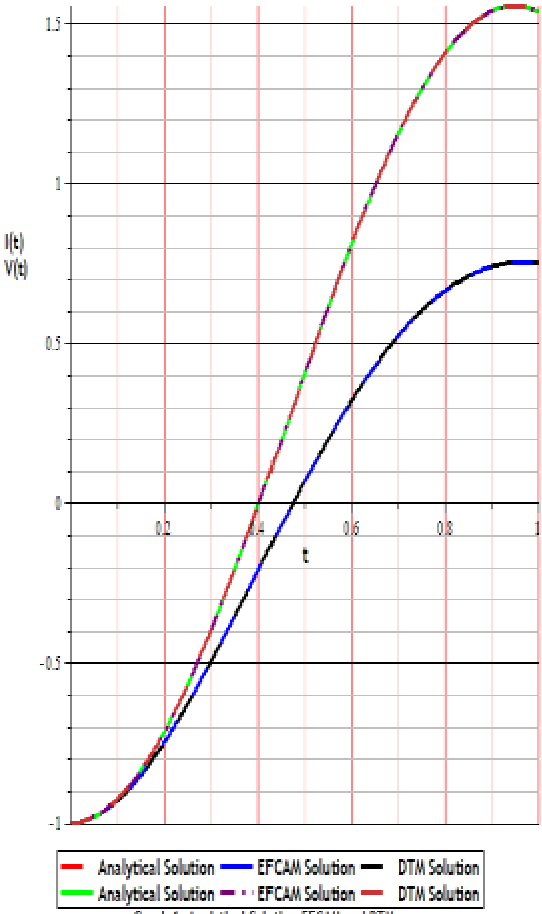
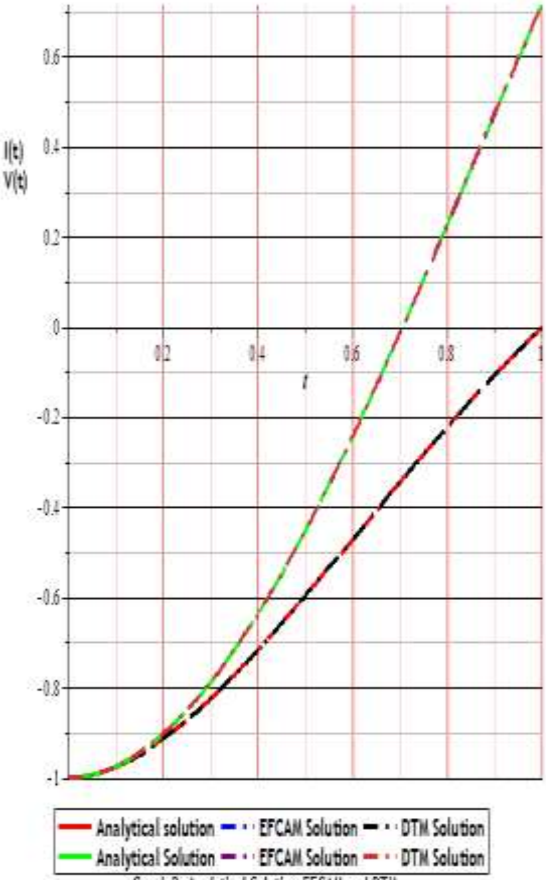


Graph 1. Analytical Solution,EFCAM and DTM
Problem 1: Current I(t) and Voltages V (t) L=2 , R=4 , C=0.05



Graph 2. Analytical Solution,EFCAM and DTM
Problem 2: Current I(t) and Voltages V (t) L=4 , R=8 , C=0.1

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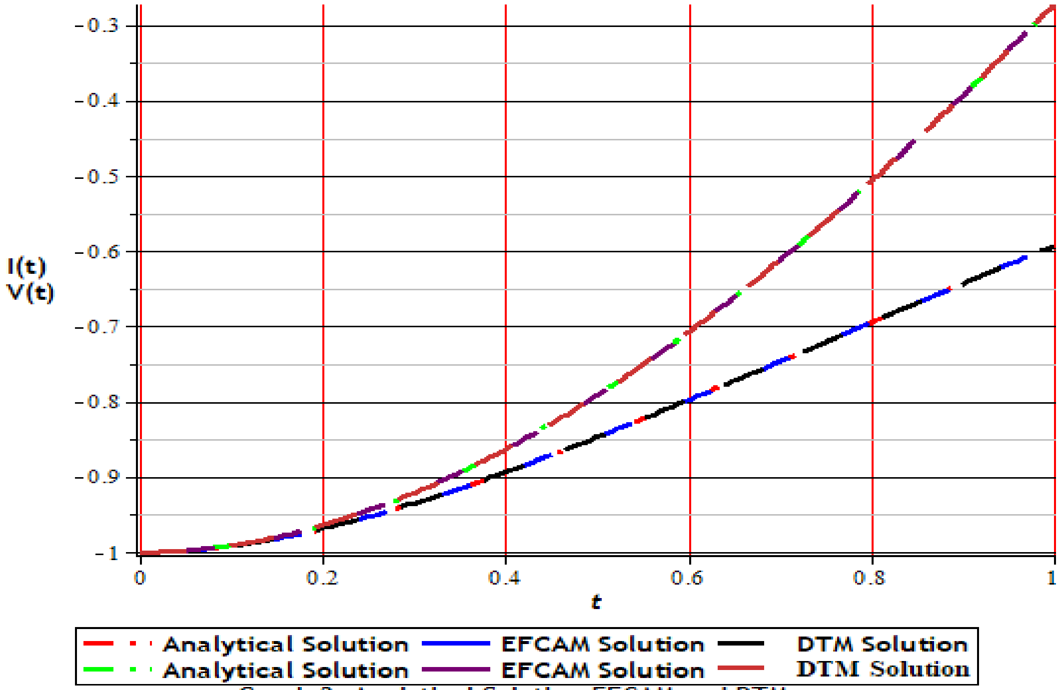
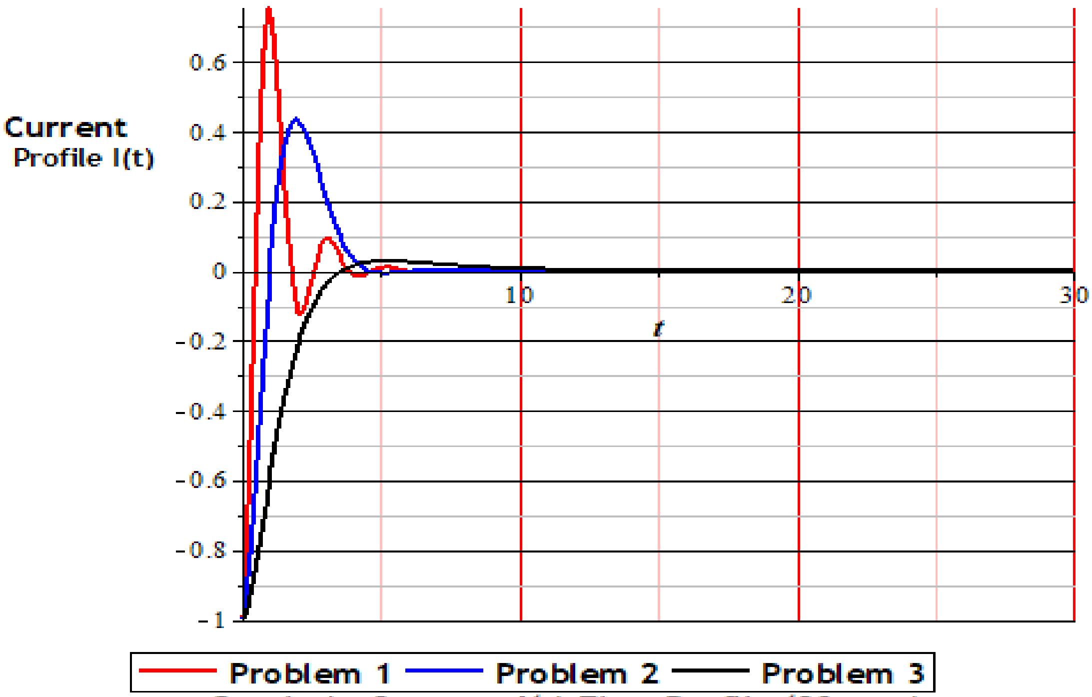


Graph 3. Analytical Solution,EFCAM and DTM
Problem 3: Current I(t) and Voltages V (t) L=8 , R=16 , C=0.2



Graph 4. Currents I(t) Flow Profile (30 sec.)

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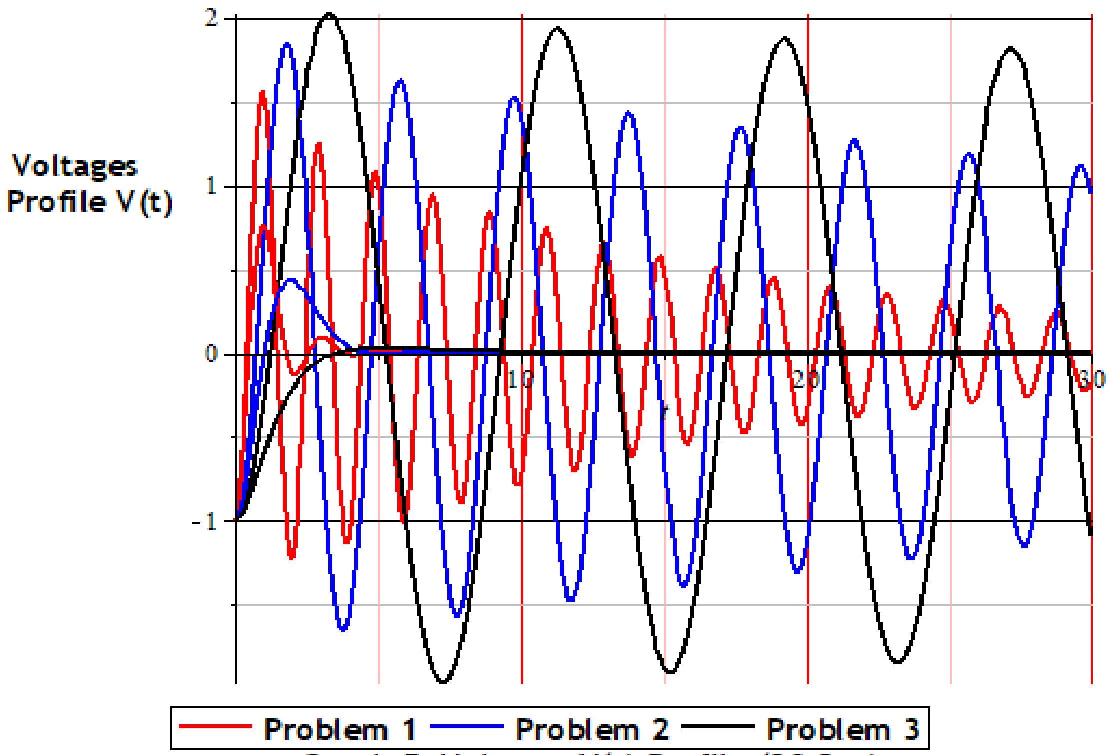

Graph 5. Voltages V(t) Profile (30 Sec)

IMPLICATION TO RESEARCH AND PRACTICE

Three problems were considered using proposed numerical techniques in which the computed results were compared with the analytical solutions. It was observed that multiplying resistor (R), inductor (L) and capacitor (C) by two (2) gives decreases in currents I(t) and voltages V(t) see tables (2,3,4,5,6,7) and graphs (1,2,3). Moreso, graphs (4,5) showed the flow profile of currents and voltages within a given seconds (0-30 seconds).

CONCLUSION

This study showed that Differential Transformation Method (DTM) and Exponentially Fitted Collocation Approximate Method (EFCAM) are very efficient in solving and obtaining numerical solutions of second-order differential equation of voltages and currents in the RLC electrical circuits model. The general conclusion is that this study will serve as a good alternative to that of laboratory experiment instrumental measurements. Finally, the numerical solutions obtained was compared to that analytical solutions which was in good agreement with less relative errors obtained.

Print ISSN: 2053-2229 (Print), Online ISSN: 2053-2210 (Online)

FUTURE RESEARCH

Base on this study, it will serve as basis or prototype for numerical solution of second order electrical circuit (RCL) model equation and focused solely on external force acting on the circuit (Non-homogenous) in which was exponential function $\frac{dV(t)}{dt} = \frac{dI(t)}{dt} = \mathbf{10e^{-t},}$ Thus, we hereby suggest further study when the functions are trigonometric and logarithm. Moreso, the proposed method (EFCAM) is equally recommended in finding numerical solution to Higher Order Ordinary Differential Equations.

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Print ISSN: 2053-2229 (Print), Online ISSN: 2053-2210 (Online)