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METHODS OF RECONSTRUCTING SIGNALS BASED ON MULTIVARIATE SPLINE

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ABSTRACT. *The article discusses the fast playback functions of several variables formed during the processing of sensor readings, combining input-output analog data and the actual arithmetic calculations on the basis of multivariate splines. In this case, the allocated basic multivariate splines, which give the Fourier transform in a clear analytic expression based on the elementary functions of frequency arguments.*

KEYWORDS. Multivariate splines basis, piecewise polynomial, piecewise-planar splines, bilinear splines, B-splines,

INTRODUCTION

Modern methods of mathematical modeling and advances in technologies of computer systems and complexes are the basis for physicists and engineers could with a high degree of efficiency to explore the complex multivariate processes and real-world objects.

Empirical multidimensional functional dependencies resulting from experimental studies, in view of measurement error, the high level of interference is largely handled by being presented surfaces of piecewise low degree, especially when the hardware implementation. Table-algorithmic methods, principles of parallelism and pipelining significantly improve performance compared to implementations on general-purpose microprocessors.

In particular, rapid tabular method based on adding an additive to the function values at the nodes of the grid so-called "corrective functions" and parallelization of selections from different memories, brings maximum performance to the asymptotic functional converters, but useful, like many other methods, only dependencies, predetermined and recorded in ROM.

Therefore becoming increasingly urgent problem of the same instant playback functions of several variables, established in the course of operative treatment of indications of many sensors, combining input-output analog data and the actual arithmetic.

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Suffice it to say that on board modern heavy aircraft is 3,000 sensors of various parameters, and in the modeling of complex aerospace systems operation on the calculation of functions of several variables occupy from 40% to 60% of CPU time.

Splines as the class of piecewise functions due to the universality of algorithms for processing samples, differential and extreme good properties, high convergence of estimates, ease of computation forms and parameters of the weak impact of rounding errors are more and more widely used in the creation of hardware and software for analyzing and recovering univariate and multivariate signals extending beyond traditional approaches [1,2,3].

THE MAIN PART.

Theory of approximation of functions of several variables by splines now has developed considerably. If we turn to multivariate polynomial spline interpolation, the entire set of definitions relating to the one-dimensional interpolation splines can be uniquely extended to the area of multivariate splines, and supplemented.

When postreniya multivariate splines on uniform rectangular grids following tasks:

- The problem of approximate representation of the domain of the function;

- The problem of approximate representation of the function.

For consideration by the two-dimensional spline as a special case of multivariate spline, denote the boundary values for each argument: and ask for the working plane (x, y) grid nodes:

$$\Delta_x : a = x_0 < x_1 < x_2 < \ldots < x_{n1} = b;$$
$$\Delta_y : c = y_0 < y_1 < y_2 < \ldots < y_{n2} = d.$$

The result is a two-dimensional grid of points on the plane dividing the area into rectangles:

$$D_{ik} \equiv \{(x, y)\}, \ x \in [x_i + x_{i+1}], \ y \in [y_k + y_{k+1}],$$

$$i = 0,1,2,\ldots,n_1 - 1; \ k = 0,1,2,\ldots,n_2 - 1.$$

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The function is called a spline of two variables of degree m defect dx = dy = 1 on the grid if it coincides with a polynomial of degree m in x and y for each rectangle Dik and belongs to the class.

The general formula of the two-dimensional representation of a spline of degree m in each argument is as follows:

$$S_{m,m}(x,y) = \sum_{l=0}^{m}\sum_{r=0}^{m} a_{lr,ik}(x-x_i)^l (y-y_k)^r. \quad (1.1)$$

For any of the D x may be formed on a two-dimensional grid spline interpolating vector values P (y). Use it to construct a two-dimensional spline reduces to finding the set of one-dimensional splines:

$$S_m(x,y) = \sum_{k=0}^{l} P_{ik,m}(x) S_{k,m}(y),$$

where - polynomials of degree m for x, - a polynomial of degree m for y, when.

The simplest multidimensional piecewise polynomial splines structures are zero and first degrees of two variables on rectangular grids. Splines of degree zero can be called piecewise pictorial space. They form a shape composed of vertical parallelepipeds, the upper face of which is a piece of the working plane arranged parallel kordinatnoy working plane (x, y).

Splines of the first degree is called a bilinear. Two-dimensional spline interpolation of the first degree is a function, which in each elementary rectangle...

$D_{ik} = [x_i, x_{i+1}] \times [y_k, y_{k+1}]$ has the form:

$$S_{1,1}(x,y) = c_{ik} + a_{ik}x + b_{ik}y + d_{ik}xy$$

and satisfies the conditions. $S_{1,1}(x_i, y_k) = f_{ik},\ i = 0,1,\ldots,n_1,\ k = 0,1,\ldots,n_2.$

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Splines of the third degree in two variables called bicubic splines. The function is called two-dimensional cubic spline interpolation function f (x), if:

- - $S_{3,3}(x,y)$ Coincides with the bicubic polynomial at

- $S_{3,3}(x,y) \in C^{(4,2)}(D)$, where $C^{(4,2)}$ Where C (4,2) - the set of functions f (x, y), defined in the region D, in which the fourth partial derivative comprising up to two differentiations with respect to each of the independent variable, and there is continuous;

-- $S_{3,3}(x_i, y_k) = f(x_i, y_k) = f_{ik}$.

set of splines is considered as a tensor product of one-dimensional splines $S_3(x)$ and $S_3(y)$.

Cubic splines of two variables defect 1 in the cell described by the formula:

$$S_{3,3}(x,y) = \sum_{l=0}^{3}\sum_{r=0}^{3} c_{ik,lr}(x-x_i)^l (y-y_k)^r. \qquad (1.3)$$

With increasing numbers of arguments to functions of several variables mathematical approximation problems are significantly increased. For example, multiple power series often characterized by slow convergence, and is primarily used as a basis for continuous analytical models that in many experimental studies, it is difficult to describe in simple formulas.

In modern computers currently allow without significant problems to obtain sufficiently comprehensive description of the piecewise polynomial function f (x, y) in three-dimensional space.

Multivariate spline is an example of a function of several variables, when given the combination of functions, each of which depends on a single independent variable. In this case can be allocated multivariate splines basis, which not only can be represented as a sum of products of functions of one variable, but give a Fourier transform in a clear analytic expression based on the elementary functions of frequency arguments.

Multivariate polynomial B-splines equal degrees m in each argument tensor defined as direct products of one-dimensional B-splines [3,4,5]:

$$B_m(x,y,...,u) = B_m(x) \otimes B_m(y) \otimes ... \otimes B_m(u) \qquad (1.4)$$

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Consequently, the recording of multidimensional polynomial S-spline becomes:

$$S_m(x_1, x_2, \ldots, x_n) = \sum_{i=-m}^{n_1+m} \sum_{k=-m}^{n_2+m} \ldots \sum_{l=-m}^{n_n+m} b_{ikl} B_{i,m}(x_1) B_{k,m}(x_2) \ldots B_{l,m}(x_n),$$

where - coefficient of multivariate approximation.

In particular, the two-dimensional spline Sm (x, y) of degree m have the formula:

$$S_m(x, y) = \sum_{i=-m}^{n_1+m} \sum_{k=-m}^{n_2+m} b_{ik} B_{m,i}(x) B_{m,k}(y), \qquad (1.5)$$

ie in the form of double sums of multiple works, where the factors are the factors and one-dimensional B-splines. Here, the domain of the non-zero values of the basic two-dimensional spline

$$B(x, y) = B_m(x) \otimes B_m(y)$$

is a rectangle obtained from the grid partitioning of the following form:

$$\Delta_x : x_0 < x_1 < x_2 < \ldots < x_{n1-1} < x_{n1};$$
$$\Delta_y : y_0 < y_1 < y_2 < \ldots < y_{n2-1} < y_{n2}.$$

Figure 1 shows a graph of a two-dimensional (bilinear) spline base first degree B1,1 (x, y) and Figure 2 shows a graph (bicubic) spline base of the third degree B3,3 (x, y).

Series-parallel algorithm for computing the values of multivariate splines can be realized if we introduce the notation for the coefficients that depend on one of the arguments [4]. In particular, in case of two independent variables, we have:

$$C_{ik}(y) = \sum_k b_{ik} B_k(y), \qquad (1.6)$$

and the expression for the spline can be written as:

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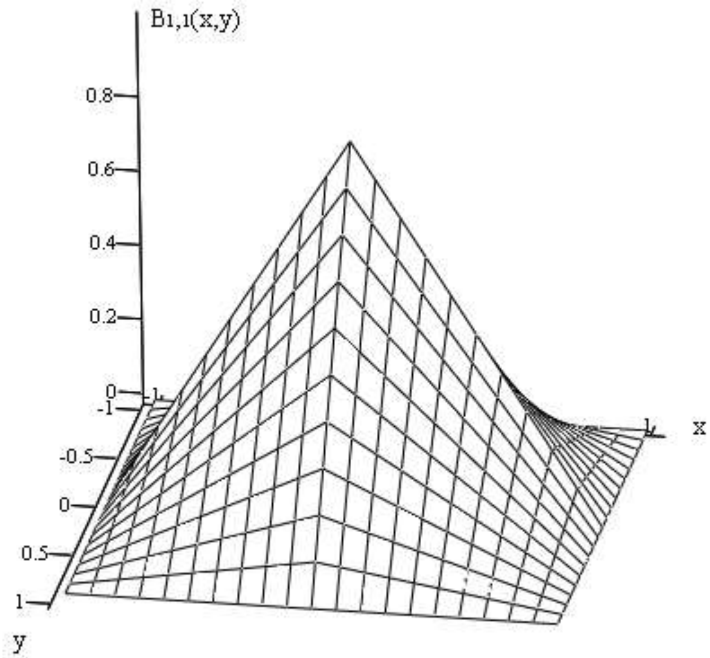
$$S_m(x,y) = \sum_k c_i(y) B_k(x), \qquad (1.7)$$

ie can be calculated by paired pieces in two stages.

By analogy dimensional spline represented as

$$S_m(x,y,z) = \sum_i \sum_k \sum_l b_{ikl} B_i(x) B_k(y) B_l(z), \qquad (1.8)$$

and its meaning can be obtained by calculating the pair of works sequences.



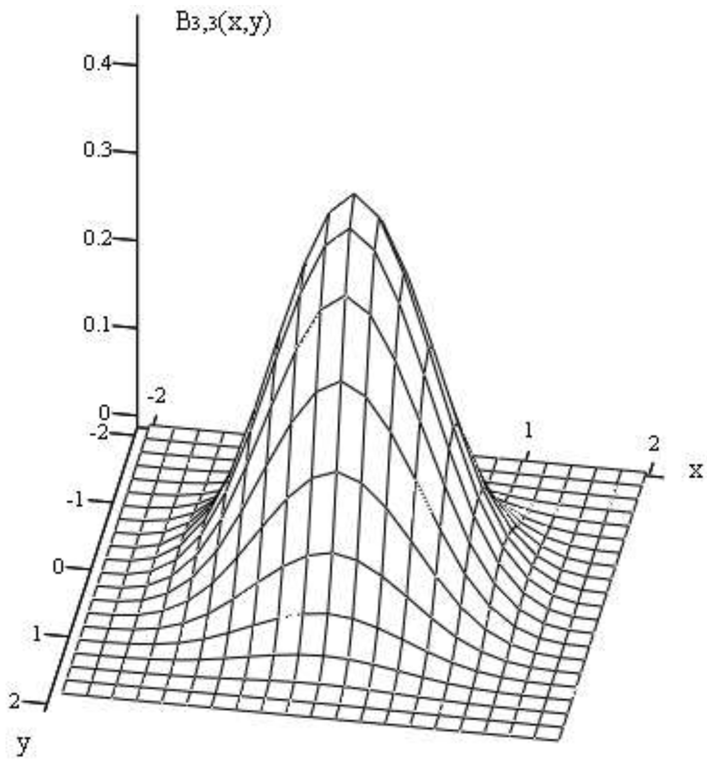


Figure 2. Two-dimensional spline basis of the third degree.

CONCLUSIONS

An analysis of the spline approximation methods revealed that the vast majority of elementary functions used in practice can be successfully approximated basic splines, or by using piecewise quadratic basis functions Haar and Hartmut. The mathematical apparatus of approximation splines basis allows us to represent the functional dependencies in the form of a sum of paired works of constant coefficients on the values of basis functions. This provides a basis for significant parallelism both analytically and table-defined functional dependencies. Local properties of basis splines defines a limited number of (N + 1, where N-degree spline) terms in the approximate amounts and minimum volume tables of values of basis functions. Combination of basis splines and table-algorithmic methods of implementation of processor means possible to obtain parallel computing structures on the basis of standard modules and BIS-based single-chip programmable DSP.

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