MAXIMIZATION OF PROFITS IN SOME MANUFACTURING INDUSTRIES USING ANT COLONY OPTIMIZATION TECHNIQUE

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ABSTRACT: *Ant colony optimization (ACO) is a technique of optimization that was introduced in the early 1990's by Dorigo. It is an heuristics optimization for solving combinatorial optimization problems. The inspiring source of ant colony optimization is the foraging behaviour of real ant colonies. This behaviour is exploited in artificial ant colonies for the search of approximate solutions to discrete and continuous optimization problems. It is also applicable to the solution of important problems in telecommunications, such as routing and local balancing. In particular, the agents, called ants, are very efficient at sampling the problem space and quickly finding good solutions to it. Motivated by the advantages of ACO in combinatorial optimization, we considered the maximization of profit in some manufacturing industries in Lagos state of Nigeria, using the ACO method. On comparing our results with the Fibonacci search method, it was established that it compares favourably with it.*

KEYWORDS: Combinatorial Optimization, Ant Colony Optimization, Swarm Intelligent Systems, Chemical Pheromone Trails.

INTRODUCTION

Optimization is the act of obtaining the best result under given circumstances. In design, construction, and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of such decisions is to either minimize the effort required or maximize the desired benefit, Rao (1998), Oke (2014).

Since the effort required or the benefit desired in any practical situation can be expressed as a function of certain decision variables, optimization can be defined as the process of finding the conditions that give the maximum or minimum value of a function Rao (1998).

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The modern optimization methods, sometimes called nontraditional optimization methods, have emerged as powerful and popular methods for solving complex engineering and manufacturing optimization problems in recent years. Ant colony optimization (ACO) by Dorigo and Stützle (2004) is one of the most recent techniques for approximate optimization. Ant algorithms are multi-agent systems in which the behavior of each single agent, called artificial ant or ant for short in the following, is inspired by the behavior of real ants. Ant algorithms are one of the most successful examples of swarm intelligent systems by Bullnheimer *et al* (1999). The goal of swarm intelligence is the design of intelligent multi-agent systems by taking inspiration from the collective behavior of social insects such as ants, termites, bees, wasps, and other animal societies such as flocks of birds or fish schools.

At the core of this behavior is the indirect communication between the ants by means of chemical pheromone trails, which enables them to find short paths between their nest and food sources. This characteristic of real ant colonies is exploited in ACO algorithms in order to solve optimization problems.

In this paper, a metaheuristic algorithm is given to solve unconstrained optimization problems involving maximization of profits in some establishments. The algorithm incorporates several ACO features as well as local optimization techniques. The algorithm was tested on four classes of problems from different establishments with paths ranging from 9 to 41 and ants ranging from 4 to 20. The results were compared with the Fibonacci search method for the maximization of profits. The ACO algorithm produced very good results that compared favourably with the known results.

The modern methods of optimization are powerful tools for solving complex engineering and industrial problems, Oke (2012b). Many researchers have worked on this modern or nontraditional method to solve a lot of engineering and industrial problems. These include Dorigo and Gambardella (1997), Peck and Skalak (2014), Stutzle and Hoos (1997), Oke (2012a),

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Oke (2012b), Oke (2013), Dorigo *et al*. (1991) and Bonabeau *et al*. (2000). These works form the basis and motivation for this research work.

Ant colony optimization (ACO) algorithms have been developed to solve hard combinatorial optimization problems. A combinatorial optimization problem can be represented as a tuple $\rho = (S,F)$, where S is the solution space with $s \in S$ a specific candidate solution and where $F: S \rightarrow \mathbb{R}_+$ is a fitness function assigning strictly positive values to candidate solutions, where higher values correspond to better solutions $S^* \subseteq S$ that maximize the fitness function. The solution s^* is then called an optimal solution and S^* is called the set of optimal solutions.

MATERIALS AND METHODS

This paper presents the extension of ACO to unconstrained optimization problems for the maximization of profits in some establishments which involve profit function given by:

$$P(X) = Total\ income - Total\ cost \qquad (1)$$

That is

$$P(X) = R(X) - C(X) \qquad (2)$$

$Where\ P(X) = Profit\ function$

$R(X) = Revenue\ function$

$C(X) = Cost\ function$

We would follow the steps below in applying the ACO algorithm for solving maximization problems:

Step 1: Assume a suitable number of ants in the colony (N). Assume a set of permissible discrete values for each of the n design variables. Denote the permissible discrete values of the design variable x_i as $x_1, x_2, \dots, x_{ip}$ $(i = 1,2,\dots,n)$. Assume equal amounts of pheromone $\tau_{ij}^{(l)}$ initially along all the arcs or rays (discrete values of design variables) of the multilayered graph.

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The superscript to τ_{ij} denote the iteration number, for simplicity, let $\tau_{ij}^{(l)} = 1$, is assumed for all arcs ij. Set the iteration number $l = 1$.

Step 2: (a) Compute the probability (p_{ij}) of selecting the arc or ray (or the discrete value) x_{ij} as

$$p_{ij} = \frac{\tau_{ij}^{(l)}}{\sum_{m=1}^{p} \tau_{im}^{(l)}}; i = 1,2,\dots,n; j = 1,2,\dots,p \tag{3}$$

(b) The specific path (or discrete values) chosen by the kth ant can be determined using random numbers generated in the range (0, 1). For this, we find the cumulative probability ranges associated with different paths based on the probabilities given by equation (3). The specific path chosen by ant k will be determined using the roulette-wheel selection process in step 3(a).

Step 3: (a) Generate N random numbers $r_1, r_2, \dots, r_N$ in the range (0, 1), one for each ant. Determine the discrete value or path assumed by ant k for variable I as the one for which the cumulative probability range [found in step 2(b)] includes the value r_i.

(b) Repeat step 3(a) for all design variables $i = 1,2,\dots,n$.

(c) Evaluate the objective function values corresponding to the complete paths (design vectors $X^{(k)}$ or values of x_{ij} chosen for all design variables $i = 1,2,\dots,n$ by ant k, $k = 1,2,\dots,N$):

$$f_k = f\,(X^{(k)});\ k = 1,2,\dots,N \tag{4}$$

Determine the best and worst paths among the N paths chosen by different ants:

$$f_{best} = \min_{k=1,2,\dots,N} \{f_k\} \tag{5}$$

$$f_{worst} = \max_{k=1,2,\dots,N} \{f_k\} \tag{6}$$

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Step 4: Test for the convergence of the process. The process is assumed to have converged if all N ants take the same best path. If convergence is not achieved, assume that all the ants return home and start again in search of food. Set the new iteration number as $l = l + 1$, and update the pheromones on different arcs (or discrete values of design variables) as

$$\tau_{ij}^{(l)} = \tau_{ij}^{(old)} + \sum_k \Delta\tau_{ij}^{(k)} \tag{7}$$

Where $\tau_{ij}^{(old)}$ denotes the pheromone amount of the previous iteration left after evaporation, which is taken as

$$\tau_{ij}^{(old)} = (1-\rho)\tau_{ij}^{(l-1)} \tag{8}$$

And $\Delta\tau_{ij}^{(k)}$ is the pheromone deposited by the best ant k on its path and the summation extends over all the best ants k (if multiple ants take the same best path). Note that the best path involves only one arc i, j (out of p possible arcs) for the design variable i. The evaporation rate or pheromone decay factor ρ is assumed to be in the range 0.5 to 0.8 and the pheromone deposited $\Delta\tau_{ij}^{(k)}$ is computed using the equation

$$\Delta\tau_{ij}^{(k)} = \begin{cases} \frac{\zeta f_{best}}{f_{worst}}; if\ (i,j) \in global\ best\ tour \\ 0; \quad otherwise \end{cases} \tag{9}$$

With the new values of $\tau_{ij}^{(l)}$, go to step 2. Steps 2, 3, and 4 are repeated until the process converges, that is, until all the ants choose the same best path.

In some cases, the iterative process is stopped after completing a pre-specified maximum number of iterations (l_{max}).

COMPUTATIONAL EXAMPLES

Problem 1: Let us consider a manufacturing industry with the following data

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Unit sales = $70000 - 200p$
Price = p

Seling Price = Unit Sales× Price = $(70000 - 200p) \times p = 70000p - 200p^2$
Cost Price= $8400000 - 22000p$
Profit = Selling Price− Costs Price = $(70000p - 200p^2) - (8400000 - 22000p)$
$= -200p2 + 92000p - 8400000$

The profit function is therefore given by:

$$f(x) = -200p^2 + 92000p - 8400000\ (range\ 220 - 300)$$

Problem 2: The weekly cost to produce x widgets is given by:

$$C(x) = Marginal\ cost = 75000 + 100x - 0.03x^2 + 0.000004x^3, \quad 4700 \le x \le 6500$$

and the demand function for the widgets is given by

$$D(x) = 200 - 0.005x, \quad 4700 \le x \le 6500$$

We want to determine the marginal profit, assuming that the company sells exactly what they produce.

The Marginal Revenue, $R(x)$ is given by:

$$R(x) = x(D(x)) = x(200 - 0.005x) = 200x - 0.005x^2$$

The profit function is therefore given by:

$$P(x) = R(x) - C(x) = 200x - 0.005x^2 - (75000 + 100x - 0.03x^2 - 0.000004x^3)$$

$$P(x) = -75000 + 100x + 0.025x^2 - 0.000004x^3$$

Problem 3: An apartment complex has 250 apartment to rent, if they rent x apartment then their monthly profit, is given by $p(x) = -8x^2 + 3200x - 80000$

How many apartments should they rent in order to maximize their profit? Also maximize the profit subject to the constraint that x must be in the range $150 \le x \le 250$

$$p(x) = -8x^2 + 3200x - 80000 \quad (150 - 250)$$

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Problem 4: A refrigerator manufacturer can sell all the refrigerators of a particular type that he can produce.

The total cost of producing x refrigerator per week is given by $300x + 2000$

The demand function is estimated as $500 - 2x = pr$

Therefore, revenue function, $R = pr.x = (500 - 2x)x \; i.e \; R = 500x - 2x^2$

Cost function, is given as $C = 300x + 2000$

Therefore, the profit function, $P = R - C = 500x - 2x^2 - (300x + 2000)$

This gives $P = 200x - 2x^2 - 2000$

RESULTS AND DISCUSSIONS

The solution to the problems and the comparison between Ant Colony Optimization (ACO) and Fibonacci Search method are presented in tabular forms below:

Table 1: Ant Colony Optimization Method for Problem 1

$Ant = 4, n = 1, x = x_1,$ Is assumed within the range of x_1 as $(p = 9)\; x_{ij}\; ; j = 1,2, \ldots, 9$

$f(x) = -200p^2 + 92000p - 8400000, in\ the\ range\ 220 \leq x \leq 300$

Iteration	Number of ants on best path	x_{best}	x_{worst}	f_{best}	f_{worst}
1	1	$x_{13} = 240$	$x_{18} = 290$	2160000	1460000
2	1	$x_{12} = 230$	$x_{17} = 280$	2180000	1680000
3	3	$x_{12} = 230$	$x_{17} = 280$	2180000	1680000
4	4	$x_{12} = 230$		2180000	

Table 2: Fibonacci Search Method for Problem 1

$a = x_1 = 300, \quad b = x_3 = 220, \qquad n = 20, \quad \varepsilon = 0$

$f(x) = -200p^2 + 92000p - 8400000, in\ the\ range\ 220 \leq x \leq 300$

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Iteration	f_n	f_{n-1}	x_1	x_3	x_2	x_4	$f(x_2)$	$f(x_4)$
1	10946	6765	300	220	250.5572812	269.4427188	2095479.64	1868854.39
2	6765	4181	250.5572812	220	231.6718425	238.8854387	2179440.99	2164209.8
3	4181	2584	231.6718425	220	224.4582474	227.2135951	2173859.79	2178447.19
4	2584	1597	231.6718425	227.2135951	228.9164938	229.9689438	2179765.2	2179999.81
5	1597	987	231.6718425	229.9689438	230.6193935	231.0213928	2179923.27	2179791.35
6	987	610	230.6193935	229.9689438	230.2179497	230.3709876	2179990.55	2179972.47
7	610	377	230.2173497	229.9689438	230.0638267	230.1224668	2179999.19	2179997
8	377	233	230.0638267	229.9689438	230.0051855	230.027585	2180000	2179999.85
9	233	144	230.0051855	229.9689438	229.9827872	229.9913421	2179999.94	2179999.98
10	144	89	230.0051855	229.9913421	229.9966295	229.9998981	2179999.94	2180000
11	89	55	230.0051855	229.9998981	230.001918	230.0031656	2180000	2180000

Table 3: Ant Colony Optimization Method for Problem 2

$Ant = 20, n = 1, x = x_1$, Is assumed within the range of x_1 as $(p = 41)\ x_{ij}\ ; j = 1,2, \dots, 41$

$$f(x) = -75000 + 100x + 0.025x^2 - 0.000004x^3, in\ the\ range\ 4700 \le x \le 6500$$

Iteration	Number of ants on best path	x_{best}	x_{worst}	f_{best}	f_{worst}
1	1	$x_{1,22} = 5645$	$x_{1,1} = 4700$	566615.7805	531958
2	1	$x_{1,22} = 5645$	$x_{1,41} = 6500$	566615.7805	532750
3	6	$x_{1,22} = 5645$	$x_{1,40} = 6455$	566615.7805	536333.04
4	17	$x_{1,22} = 5645$	$x_{1,5} = 4880$	566615.7805	543502.912
5	20	$x_{1,22} = 5645$		566615.7805	

Table 4: Fibonacci Search Method for Problem 2

$a = x_1 = 6500, \quad b = x_3 = 4700, \qquad n = 20, \quad \mathcal{E} = 0$

$$f(x) = -75000 + 100x + 0.025x^2 - 0.000004x^3, in\ the\ range\ 4700 \le x \le 6500$$

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Iteration	f_n	f_{n-1}	x_1	x_3	x_2	x_4	$f(x_2)$	$f(x_4)$
1	10946	6765	6500	4700	5387.538827	5812.461173	563887.6072	565374.5979
2	6765	4181	6500	5812.461173	6075.07763	6237.383543	558330.9006	550701.725
3	4181	2584	6075.07763	5812.461173	5912.77174	5974.767063	563436.3469	561777.5933
4	2584	1597	5912.77174	5812.461173	5850.776393	5874.45652	564741.8657	564284.5314
5	1597	987	5850.77693	5812.461173	5827.096291	5836.141275	565148.4973	564999.1423
6	987	610	5827.096291	5812.461173	5818.0521284	5821.50618	565290.5021	565237.1275
7	610	377	5818.051284	5812.461173	5814.596412	5815.916045	565342.8064	565322.9543
8	377	233	5814.596412	5812.461173	5813.276755	5813.78083	565362.5029	565354.9978
9	233	144	5813.276755	5812.461173	5812.772704	5812.965224	565369.9849	565367.1299
10	144	89	5812.772704	5812.461173	5812.580161	5812.732267	565372.837	565370.5842
11	89	55	5812.580161	5812.461173	5812.506629	5812.534705	565373.9253	565373.5098
12	55	34	5812.506629	5812.461173	5812.478529	5812.489273	565374.3412	565374.1821
13	34	21	5812.478529	5812.461173	5812.467809	5812.471893	565374.4998	565374.4393
14	21	13	5812.467809	5812.461173	5812.463701	5812.471917	565374.5605	565374.4383
15	13	8	5812.463701	5812.461173	5812.462145	5812.462729	565374.5835	565374.5749
16	8	5	5812.462145	5812.461173	5812.461538	5812.46178	565374.5925	565374.5889
17	5	3	5812.461538	5812.461173	5812.461319	5812.461392	565374.5957	565374.5946
18	3	2	5812.461319	5812.461173	5812.461222	5812.46127	565374.5971	565374.5965
19	2	1	5812.461222	5812.461173	5812.461198	5812.461197	565374.5976	565374.5976

Table 5: Ant Colony Optimization Method for Problem 3

$Ant\ =\ 8, n = 1, x = x_1,$ Is assumed within the range of x_1 as $(p = 11)\ x_{ij}\ ; j = 1,2, \ldots ,11$

$f(x) = -8x^2 + 3200x - 80000, in\ the\ range\ 150 \leq x \leq 250$

Iteration	Number of ants on best path	x_{best}	x_{worst}	f_{best}	f_{worst}
1	1	$x_{1,6} = 200$	$x_{1,1}, x_{1,11} = 150,250$	240000	220000
2	3	$x_{1,6} = 200$	$x_{1,1} = 150$	240000	220000
3	6	$x_{1,6} = 200$	$x_{1,9} = 230$	240000	232800
4	7	$x_{1,6} = 200$	$x_{1,2} = 160$	240000	227200
5	8	$x_{1,6} = 200$		240000	

Table 6: Fibonacci Search Method for Problem 3

$a = x_1 = 250, \quad b = x_3 = 140, \qquad n = 20, \quad \varepsilon = 0$

$f(x) = -8x^2 + 3200x - 80000, in\ the\ range\ 150 \leq x \leq 250$

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Iteration	c_n	f_{n-1}	x_1	x_3	x_2	x_4	$f(x_2)$	$f(x_4)$
1	10946	6765	250	140	182.0162616	207.9837384	237412.6812	239490.0794
2	6765	4181	250	207.9837384	224.0325218	233.9512166	235379.5032	230778.5191
3	4181	2584	224.0325218	207.9837384	214.1138286	217.9024316	238406.3987	237436.0235
4	2584	1597	214.1138286	207.9837384	210.3252241	211.7723429	239147.118	238891.2956
5	1597	987	210.3252241	207.9837384	208.8781068	209.4308557	239369.4338	239288.4716
6	987	610	208.8781068	207.9837384	208.3253563	208.5364889	239445.5076	239417.0269
7	610	377	208.3253563	207.9837384	208.1142252	208.1948695	239473.2748	239462.7529
8	377	233	208.1142252	207.9837384	208.0335795	208.0643841	239483.6928	239479.7257
9	233	144	208.0335795	207.9837384	208.0027764	208.0145415	239487.6446	239486.137
10	144	89	208.0027764	207.9837384	207.9910099	207.9955049	239489.1501	239488.5752
11	89	55	207.9910099	207.9837384	207.9865163	207.988232	239489.7245	239489.5052
12	55	34	207.9865163	207.9837384	207.9847991	207.9854556	239489.9438	239489.86
13	34	21	207.9847991	207.9837384	207.984144	207.9843935	239490.0276	239489.9957
14	21	13	207.984144	207.9837384	207.9838929	207.9839895	239490.0597	239490.0473
15	13	8	207.9838929	207.9837384	207.9837978	207.9838335	239490.0718	239490.0672
16	8	5	207.9837978	207.9837384	207.9837607	207.9837755	239490.0765	239490.0746
17	5	3	207.9837607	207.9837384	207.9837473	207.9837518	239490.0783	239490.0777
18	3	2	207.9837473	207.9837384	207.9837414	207.9837443	239490.079	239490.079

Table 7: Ant Colony Optimization Method for Problem 4

$Ant\ = 8, n = 1, x = x_1,$ Is assumed within the range of x_1 as $(p = 17)\ x_{ij}$;

$j = 1,2, \ldots ,17$

$f(x) = 200x - 2x^2 - 2000, in\ the\ range\ 0 \leq x \leq 80$

Iteration	Number of ants on best path	x_{best}	x_{worst}	f_{best}	f_{worst}
1	1	$x_{1,11} = 50$	$x_{14} = 15$	3000	550
2	14	$x_{1,11} = 50$	$x_{14} = 15$	3000	550
3	18	$x_{1,11} = 50$	$x_{1,12}, x_{1,10}$=55,45	3000	2950
4	20	$x_{1,11} = 50$		3000	

Table 8: Fibonacci Search Method for Problem 4

$a = x_1 = 80, \quad b = x_3 = 0, \qquad n = 20, \quad \varepsilon = 0$

$f(x) = 200x - 2x^2 - 2000, in\ the\ range\ 0 \leq x \leq 80$

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Iteration	f_n	f_{n-1}	x_1	x_3	x_2	x_4	$f(x_2)$	$f(x_4)$
1	10946	6765	80	0	30.5572812	49.4427188	2243.961371	2999.378875
2	6765	4181	80	49.4427188	61.1145132	68.32815748	2752.933049	2328.157291
3	4181	2584	61.11456132	49.4427188	53.90096623	56.65631389	2969.564929	2969.564929
4	2584	1597	53.90096623	49.4427188	51.14561749	52.19806754	2997.375123	2990.337
5	1597	987	51.14561749	49.4427188	50.09316819	50.49516777	2999.982635	2999.509614
6	987	610	50.09316852	49.4427188	49.69116819	49.84471913	2999.809246	2999.951776
7	610	377	50.09316852	49.84471913	49.93961865	49.998269	2999.992708	2999.999994
8	377	233	50.09316852	49.998269	50.03451709	50.05692043	2999.997619	2999.993524
9	233	144	50.03451709	49.998269	50.01211484	50.02067125	2999.999708	2999.999145
10	144	89	50.01211484	49.998269	50.00355734	50.0068265	2999.999977	2999.999907
11	89	55	50.00355734	49.998269	50.00028926	50.00153708	2999.999998	2999.999999
12	55	34	50.00355734	50.00153708	50.00230845	50.00278597	2999.999989	2999.999998
13	34	21	50.00230845	50.00153708	50.00183202	50.00201351	2999.999989	2999.999999
14	21	13	50.00230845	50.00201351	50.00212587	50.00219609	2999.999987	2999.999992
15	13	8	50.00230845	50.00219609	50.00223931	50.00226523	2999.999988	2999.999994
16	8	5	50.00230845	50.00226523	50.00228144	50.00229224	2999.999992	2999.999991
17	5	3	50.00228144	50.00226523	50.00227171	50.002227496	2999.999988	2999.999988

Table 9: Comparison between ACO and Fibonacci Search Method for Problem 1

	ACO method	Fibonacci method
Function	$-200p^2+92000p-8400000$	$-200p^2+92000p-8400000$
Range	$220 \le x \le 300$	$220 \le x \le 300$
x_{best}	230	230.001918, 230.0031656
f_{best}	2180000	2180000

Table 10: Comparison between ACO and Fibonacci Search Method for Problem 2

	ACO method	Fibonacci method
Function	$-75000+100x+0.025x^2-0.000004x^3$	$-75000+100x+0.025x^2-0.000004x^3$
Range	$4700 \le x \le 6500$	$4700 \le x \le 6500$
x_{best}	5645	5812.461198, 5812.461197
f_{best}	566615.7805	565374.5976

Table 11: Comparison between ACO and Fibonacci Search Method for Problem 3

	ACO method	Fibonacci method
Function	$-8x^2+3200x-80000$	$-8x^2+3200x-80000$
Range	$150 \le x \le 250$	$150 \le x \le 250$
x_{best}	200	207.9837414, 207.9837443
f_{best}	240000	239490.079

Table 12: Comparison between ACO and Fibonacci Search Method for Problem 4

	ACO method	Fibonacci method
Function	$200x - 2x^2 - 2000$	$200x - 2x^2 - 2000$
Range	$0 \leq x \leq 80$	$0 \leq x \leq 80$
x_{best}	**50**	**50.00227171, 50.00227496**
f_{best}	**3000**	**2999.999988**

CONCLUSION

The Ant Colony Optimization (ACO) has been applied in the maximization of profits in some selected establishments. The results obtained were compared with that of Fibonacci search method and it was discovered that the Ant Colony Optimization method performs better than the Fibonacci search method in all the selected problems.

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