MHD and Heat Generation Along a Vertical Flat Plate with Variable Viscosity and Viscous Dissipation: A Numerical Solution

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Citation: Sirazum Munira, Sree Pradip Kumer Sarker, Md. M. Alam (2022) MHD and Heat Generation Along a Vertical Flat Plate with Variable Viscosity and Viscous Dissipation: A Numerical Solution, *International Journal of Mathematics and Statistics Studies*, Vol.10, No.4, pp.78-94,

ABSTRACT: *Numerous researchers have looked into the importance of natural convection in the context of engineering, and this topic has been extensively studied. This study investigates the nature of heat generation and viscous dissipation in MHD natural convection flow with changing viscosity. Laminar flow and boundary layer equations with unstable boundaries in two dimensions are the subject of this article. The fundamental governing equation is turned into a dimensionless governing equation by using the necessary variables. The Crank Nicolson scheme is an efficient implicit finite difference approach for numerical computations of these equations. Heat generation and viscosity dissipation owing to MHD and changing viscosity were explored in this work. Several effects of various parameters are demonstrated in this study, and they are compared to the velocity and temperature profiles, skin friction, and local heat transfer coefficients of other researchers. Compare the present numerical results to the work outcomes that were previously released. It also compares the number of works available to the number of works published previously. The results are given in both figures and tables for various values of related physical parameters.*

KEYWORDS: Heat Generation, Viscous Dissipation, Variable Viscosity, Dependent Thermal Conductivity, Magneto-hydrodynamic (MHD).

INTRODUCTION

Heat is a form of energy that can be transferred from one system to another when the temperature changes. Heat transfer is the science that studies the rates of such energy transfers in a range of forms, such as spheres, various types of enclosures, and flat plates. We develop a flat plate for heat generation and MHD with changeable viscosity and dependent thermal conductivity. Because heat transport phenomena are so closely linked to hydrodynamics, convective heat transfer, and the electromagnetic field, a good background in hydrodynamics, convective heat transfer, and the electromagnetic field is required to characterize them. As the applications of Magnetohydrodynamic energy conversion proliferate, research and development for high-efficiency and low-emission electric power generation systems, MHD accelerations and/or MHD thrusters, and flow control surrounding hypersonic and re-entry vehicles are being introduced. From several perspectives, the impacts of heat generation and viscous dissipation on Magnetohydrodynamics (MHD) natural convection flow down a vertical plate with varying viscosity are crucial. For their aim, the researcher becomes interested in the technology and method. The effect of variable viscosity and dependent thermal conductivity on MHD natural convection flow along a vertical flat plate was studied by Sarker and Alam [1]. Palani and Kim [2], analyzed the effects of variable viscosity and thermal conductivity on vertical plates. In the presence of heat generation, Alam et al. [3] investigated the influence of viscous dissipation on MHD natural convection flow across a sphere. Alam et al. [4] analyzed the effect of pressure stress work and viscous dissipation in natural convection flow along with a vertical flat plate with heat conduction. On a vertical flat plate, Alim et al. [5] studied the effect of Joule heating on the coupling between conduction and MHD free convection flow. Temperature-dependent thermal conductivity was studied by Rahman et al. [6] in relation to MHD free convection flow along a vertical flat plate with heat conduction. Molla et al. [7] investigate the temperature-dependent viscosity and thermal conductivity of natural convection laminar flow along a vertical wavy surface. Temperature-dependent thermal conductivity was studied by Islam et al. [8] in conjunction with a vertical flat plate that generated heat. MHD natural convection flow along a vertical wavy surface was studied by Kabir et al. [9] for the effect of viscous dissipation. The effect of Soret and Dufour on MHD Casson fluid flow past a stretching surface under convective–diffusive conditions was investigated by Ramudu et al. [15]. Sarker and Alam [16] conducted a numerical investigation into the effects of viscosity and thermal conductivity variations in an MHD natural convection flow along a vertical flat plate under stress. Natural convection flow through a vertical flat plate with pressure work and heat conduction was studied numerically by Munira et al. [17].

Two-dimensional, laminar, unstable boundary layer equations for incompressible fluids with viscous dissipation and heat generation will be examined. Velocity and temperature profiles, as well as local and average skin friction, thermal conduction coefficient and average Nusselt's number will all be examined in this study. The present numerical results will be compared to previous investigations. Additional comparisons will be made between the current work figures and tabulated data and the previously published work. There will be a range of graphs and tables showing the results for a number of physical parameter values.

Mathematical Analysis

The flow of a viscous, incompressible fluid across a semi-infinite vertical plate is the subject of this investigation. The Y-axis is picked up perpendicular to the plate at the leading edge, as shown in Fig. 1. The X-axis is taken vertically upward along the plate. X-starting axis's point is believed to be located at the leading edge of the plate. Only the viscosity, thermal conductivity, and density variation in the body force factor in the momentum equation are considered to be constant in all fluid physical properties except for these three variables, which change exponentially with temperature and linearly with temperature, respectively.

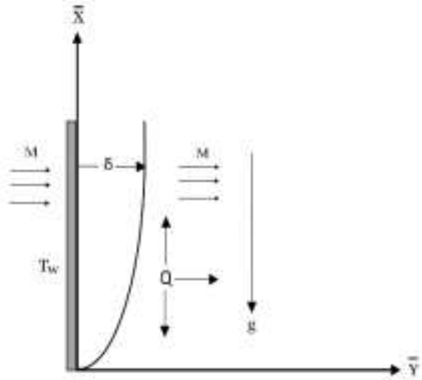

Figure 1. *Plate Configuration*

After simplifying, we arrive at the following mathematical expression of the fundamental conservation rules for mass, momentum, and energy for a steady, viscous, incompressible flow.

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \tag{1}$$

$$\frac{\partial \bar{U}}{\partial t'} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}} = \frac{1}{\rho}\frac{\partial}{\partial \bar{Y}}\left(\mu \frac{\partial \bar{U}}{\partial \bar{Y}}\right) + g\beta\left(T' - T'\infty\right) - \sigma_0 \frac{\beta_0^2 \bar{U}}{\rho} \tag{2}$$

$$\frac{\partial T'}{\partial t'} + \bar{U}\frac{\partial T'}{\partial \bar{X}} + \bar{V}\frac{\partial T'}{\partial \bar{Y}} = \frac{1}{\rho C_P}\frac{\partial}{\partial \bar{Y}}\left(k \frac{\partial T'}{\partial \bar{Y}}\right) + \frac{Q_0}{\rho C_P}\left(T' - T'_\infty\right) + \frac{\mu}{\rho C_p}\left(\frac{\partial \bar{U}}{\partial Y}\right)^2 \tag{3}$$

Where, $\bar{U}$, $\bar{V}$, $\bar{X}$, $\bar{Y}$ t', T' ,T'_∞, g, κ, ρ, C_p ,μ, $k(T)$,T' , σ_0 and β_0 are the usual notation. The amount of heat generated or absorbed per unit volume is $Q_0\left(T - T_\infty\right)$, Q_0 being a constant, which may take either positive or negative and the hydrostatic pressure $\frac{\partial P}{\partial \bar{X}} = -\rho_e g$ where, $\rho_e = \rho$. The source term represents the heat generation when $Q_0 > 0$ and the heat absorption, when $Q_0 < 0$.

The initial and boundary conditions are

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$$t' = 0: \bar{U} = 0, \bar{V} = 0, \quad T' = T'_\infty \text{ for all } Y$$
$$t' \rangle 0: \bar{U} = 0, \bar{V} = 0, \quad T' = T'_w \text{ at } Y = 0$$
$$t' \rangle 0: \bar{U} = 0, T' = T'_\infty \text{ at } X = 0 \qquad (4)$$
$$t' \rangle 0: \bar{U} \to 0, \quad T' \to T'_\infty \text{ as } Y \to \infty$$

From (1) to (4), we have

$$X = \frac{\bar{X}}{L},\ Y = \frac{\bar{Y}}{L} Gr^{1/4},\ U = \frac{L\bar{U}}{\nu} Gr^{-1/2}\quad V = \frac{\bar{V}L}{\nu} Gr^{-1/4},\ t = \frac{\nu t'}{L^2} Gr^{1/2}, \qquad (5)$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ Gr = \frac{g\beta L^3 \left(T'_w - T'_\infty\right)}{\nu^2},\ \Pr = \frac{\mu_0 C_p}{k_0},\ \nu = \frac{\mu_0}{\rho}$$

$$\frac{\mu}{\mu_0} = e^{-\lambda T} \qquad (6)$$

$$\frac{k}{k_0} = 1 + \gamma T \qquad (7)$$

Where λ, γ, μ_0, k_0 and T'_w are the usual notation.

$$\therefore \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (8)$$

Now (2) can be reduced as

$$\Rightarrow \frac{\partial U}{\partial t} + u\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \left[e^{-\lambda T}\frac{\partial^2 U}{\partial Y^2} - \lambda e^{-\lambda T}\frac{\partial T}{\partial Y}\frac{\partial U}{\partial Y} \right] + T - MU \qquad (9)$$

Again, (3) can be written as,

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{P_r}\left[\gamma \left(\frac{\partial T}{\partial Y} \right)^2 \right] + \frac{1}{P_r}\left[(1 + \gamma T)\frac{\partial^2 T}{\partial Y^2} \right] + QT + N\left(\frac{\partial U}{\partial Y} \right)^2 \qquad (10)$$

Here the condition are :

$$t = 0: U = 0, V = 0, \quad T = 0 \text{ for all } Y$$
$$t \rangle 0: U = 0, V = 0, \quad T = 1 \text{ at } Y = 0$$

$$U = 0, \quad T = 0 \text{ at } X = 0$$

$$U \to 0, \quad T \to 0 \quad at\ Y \to \infty \tag{11}$$

This free convective unstable laminar boundary layer flow with variable viscosity and thermal conductivity, as well as an isotherm semi-infinite vertical plate condition is now described in (8) to (11) where $M = \sigma_0 \frac{\beta_0^2 L^2 Gr^{-\frac{1}{2}}}{\mu}$ is the magnetic parameter, $\Pr = \mu_0 C_p / k_0$, the Prandtl's number, $Q_0 L^2 / \mu C_P Gr^{1/2} = Q$ is the heat generation parameter and $N = {g\beta L}/{C_p}$ is the viscous dissipation parameter.

The local shear stress in the plate is defined by

$$\tau_{\bar{X}} = \left(\mu \frac{\partial \bar{U}}{\partial \bar{Y}} \right)_{\bar{Y}=0} \tag{12}$$

By introducing (5)-(6) in (12), we get

$$\therefore \bar{\tau}_X = Gr^{\frac{3}{4}} e^{-\lambda} \left[\frac{\partial U}{\partial Y} \right]_{Y=0} \tag{13}$$

Now, the integration of (13) is given by

$$\bar{\tau} = e^{-\lambda} Gr^{\frac{3}{4}} \int_0^1 \left(\frac{\partial U}{\partial Y} \right)_{Y=0} d\,X \tag{14}$$

The local Nusselt number is defined by

$$\therefore \bar{N}u_X = -(1+\gamma) \left(\frac{\partial T}{\partial Y} \right)_{Y=0} \tag{15}$$

The integration of (15) is given by

$$\therefore Nu_X = -(1+\gamma) \int_0^1 \left(\frac{\partial T}{\partial Y} \right)_{Y=0} dX \tag{16}$$

Numerical Techniques

Equations (8) to (10) have a finite-difference equation, which are given by the following.

$$\frac{\left[U_{i,j}^{k+1}-U_{i-1,j}^{k+1}+U_{i,j}^{k}-U_{i-1,j}^{k}\right]}{4\Delta X}+\frac{\left[U_{i,j-1}^{k+1}-U_{i-1,j-1}^{k+1}+U_{i,j-1}^{k}-U_{i-1,j-1}^{k}\right]}{4\Delta X}+\frac{\left[V_{i,j}^{k+1}-V_{i,j-1}^{k+1}+V_{i,j}^{k}-V_{i,j-1}^{k}\right]}{2\Delta Y}=0 \quad (17)$$

$$\frac{\left[U_{i,j}^{k+1}-U_{i,j}^{k+1}\right]}{2\Delta t}+U_{i,j}^{k}\frac{\left[U_{i,j}^{k+1}-U_{i-1,j}^{k+1}\right]}{2\Delta X}+U_{i,j}^{k}\frac{\left[U_{i,j}^{k}-U_{i-1,j}^{k}\right]}{2\Delta X}$$

$$V_{i,j}^{k}\frac{\left[U_{i,j+1}^{k+1}-U_{i,j-1}^{k+1}+U_{i,j+1}^{k}-U_{i,j-1}^{k}\right]}{4\Delta Y}=\frac{1}{2}\left[T_{i,j}^{k+1}+T_{i,j}^{k}\right]+e^{-\lambda}\left[\frac{T_{i,j}^{k+1}+T_{i,j}^{k}}{2}\right]$$

$$\frac{\left[U_{i,j-1}^{k+1}-2U_{i,j}^{k+1}+U_{i,j+1}^{k}+U_{i,j-1}^{k}-2U_{i,j}^{k}+U_{i,j+1}^{k}\right]}{2(\Delta Y)^2}$$

$$-\lambda e^{-\lambda}\left[\frac{T_{i,j}^{k+1}+T_{i,j}^{k}}{2}\right]\frac{\left[T_{i,j+1}^{k+1}-T_{i,j-1}^{k+1}+T_{i,j+1}^{k}-T_{i,j-1}^{k}\right]}{4\Delta Y}\frac{\left[U_{i,j+1}^{k+1}-U_{i,j-1}^{k+1}+U_{i,j+1}^{k}-U_{i,j-1}^{k}\right]}{4\Delta Y}-\left[\frac{U_{i,j}^{k+1}+U_{i,j}^{k}}{2}\right]M \quad (18)$$

$$\left[\frac{T_{i,j}^{k+1}-T_{i,j}^{k}}{2\Delta t}\right]+U_{i,j}^{k}\frac{\left[T_{i,j}^{k+1}-T_{i-1,j}^{k+1}+T_{i,j}^{k}-T_{i-1,j}^{k}\right]}{2\Delta X}+V_{i,j}^{k}\frac{\left[T_{i,j-1}^{k+1}-T_{i,j-1}^{k+1}+T_{i,j+1}^{k}-T_{i,j-1}^{k}\right]}{4\Delta y}$$

$$=\frac{1+\gamma T_{i,j}^{k}}{\Pr}\frac{\left[T_{i,j-1}^{k+1}-2T_{i,j}^{k+1}+T_{i,j+1}^{k}+T_{i,j-1}^{k}-2T_{i,j}^{k}+T_{i,j+1}^{k}\right]}{2(\Delta Y)^2}+\frac{\gamma}{\Pr}\left[\frac{\left[T_{i,j+1}^{k+1}-T_{i,j-1}^{k+1}+T_{i,j+1}^{k}-T_{i,j-1}^{k}\right]}{4\Delta Y}\right]$$

$$+\frac{\left[U_{i,j}^{k+1}-U_{i,j-1}^{k+1}+U_{i,j}^{k}-U_{i,j-1}^{k}\right]}{2\Delta Y}\frac{\left[U_{i,j}^{k+1}-U_{i,j-1}^{k+1}+U_{i,j}^{k}-U_{i,j-1}^{k}\right]}{2\Delta Y}N+\frac{Q}{2}\left[T_{i,j}^{k+1}+T_{i,j}^{k}\right] \quad (19)$$

RESULTS AND DISCUSSIONS

The term "heat conduction" or "diffusion" refers to the microscopic process through which particles transfer kinetic energy from one system to another. Because of its high thermal conductivity and low viscosity, water is an excellent heat transfer medium. Because oil has a higher liquid temperature than water, it has been used as a solution to the problem of high pressure. Conduction, radiation, and convection all contribute to the movement of heat between the Earth's surface and the atmosphere. To move heat away from its source, convection forces a heated fluid such as air or water to flow away from the source. When hot air rises over a heated surface, it expands and becomes less dense. Molecular heat conduction is critical in the near-wall layer as well as in the flow core, even in a fully developed turbulent flow, because of the low Prandtl's number of liquid metals.

The following ranges for λ, γ and Pr are considered in the present study are:
For air: $-0.7 \le \lambda \le 0, 0 \le \gamma \le 6, \Pr = 0.733$

For water: $0 \le \lambda \le 0.6, 0 \le \gamma \le 0.12, 2 \le \Pr \le 7.00$

To check the accuracy, we compare our results with G.palani. & Kim [21] and Sarker & Alam [1]. These are plotted in Figs. 2(a), 2(b). Our results agree very well with them.

After modification of equation (10), it reduces to

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 T}{\partial Y^2}$$

Now for $\Pr = 1$, the solution of Eq. (15) is:

$$T = erfc\left(\frac{Y}{2\sqrt{t}}\right) \tag{16}$$

Figures 3(a) to 15(b) show the variation of velocity and temperature at their transient, temporal maximum and steady-state against the co-ordinate Y at the leading edge of the plate viz., $X = 1.0$ for various parameters where $0 \le Y \le 2$.

Figures 3(a) and 3(b) show the variation of transient velocity and temperature profiles. Figure 3(a) shows that as X increases, velocity U increases. At a certain distance from the wall, however, a different trend is observed. Temperature drops as increases in Fig. 3(b), as shown in the figure, the viscosity of air decreases.

From 4(a) and 4(b) figures, it is observed that the velocity and temperature distribution in the fluid increases as γ increases (thermal conductivity of air increases) for a fixed value of λ, Q and

Prandtl's number.

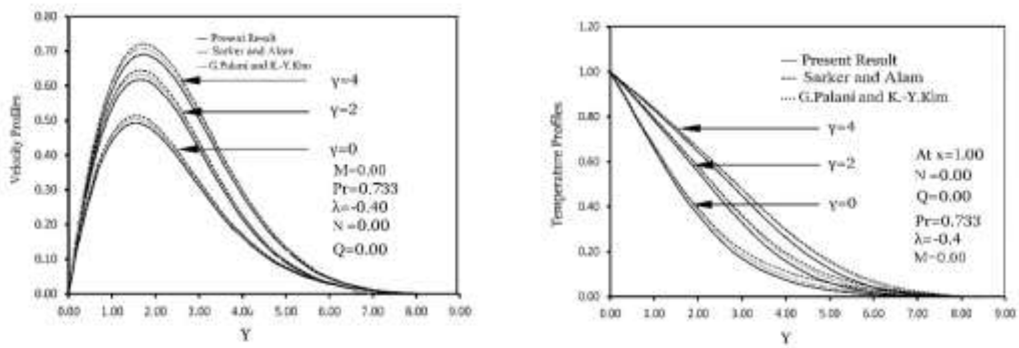

Figure 2(a) and 2(b). *Velocity profiles and temperature profiles for variation of thermal conductivity*

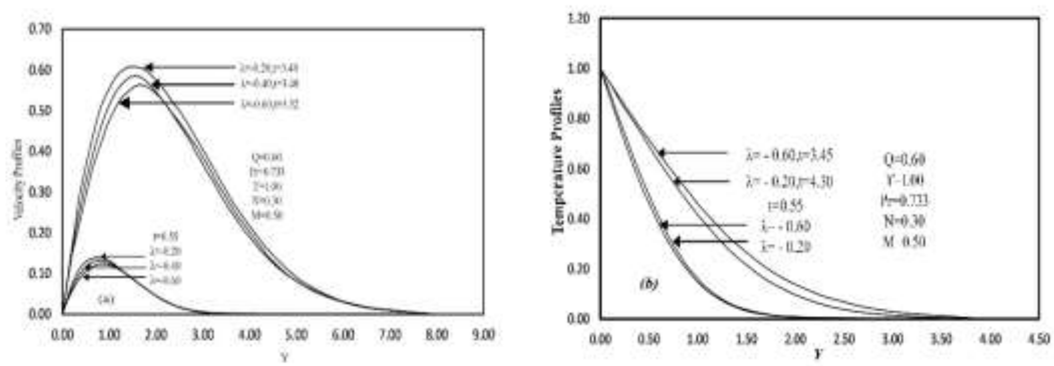

Figure 3(a) and 3(b). *Velocity profiles and temperature profiles for variable viscosity*

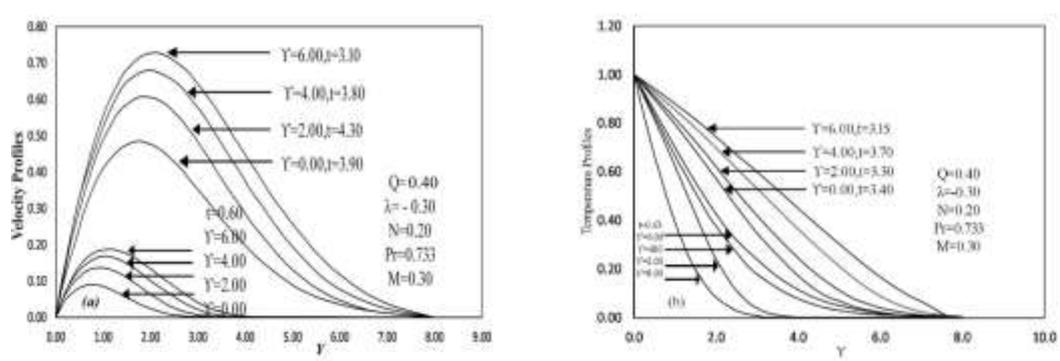

Figure 4(a) and 4(b). *Velocity profiles and temperature profiles for thermal conductivity*

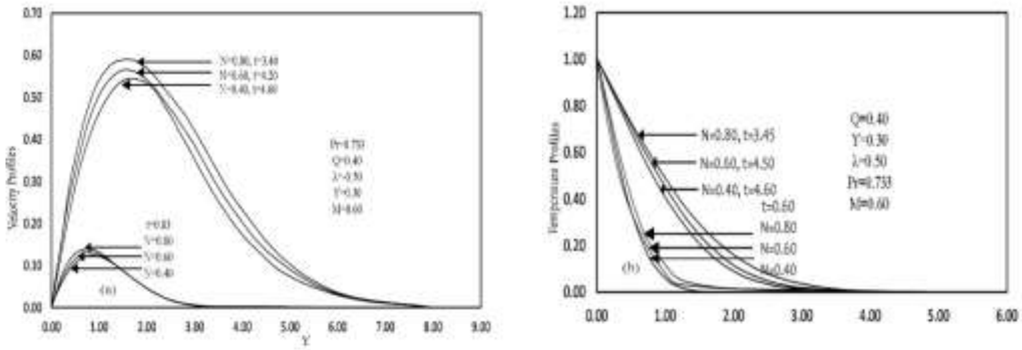

Figure 5(a) and 5(b). *Velocity profiles and temperature profiles for viscous dissipation*

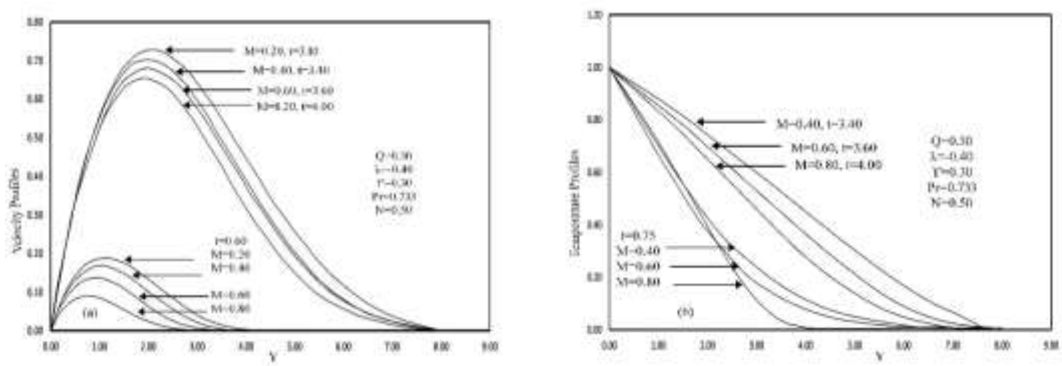

Figure 6(a) and 6(b). *Velocity profiles and temperature profiles for magnetic parameter*

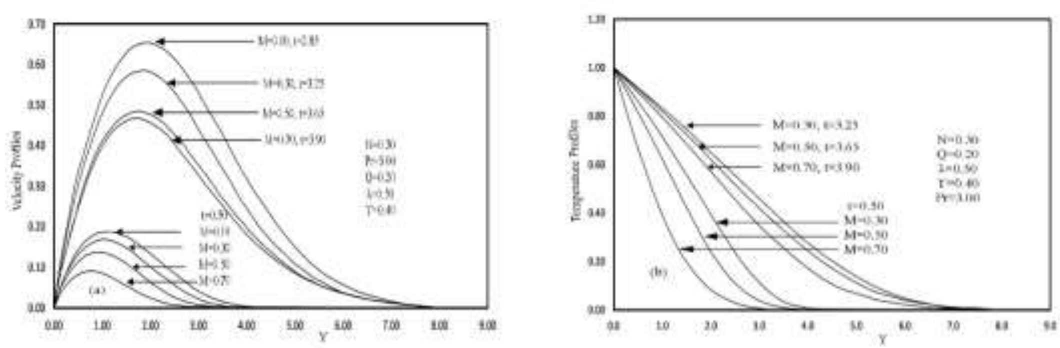

Figure 7(a) and 7(b). *Velocity profiles and temperature profiles for magnetic parameter*

Print ISSN: 2053-2229 (Print),
Online ISSN: 2053-2210 (Online)

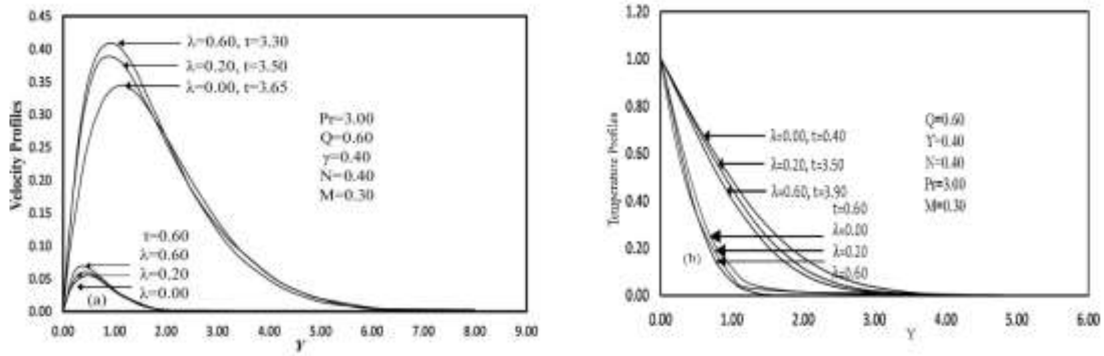

Figure 8(a) and 8(b). *Velocity profiles and temperature profiles for variable viscosity parameter*

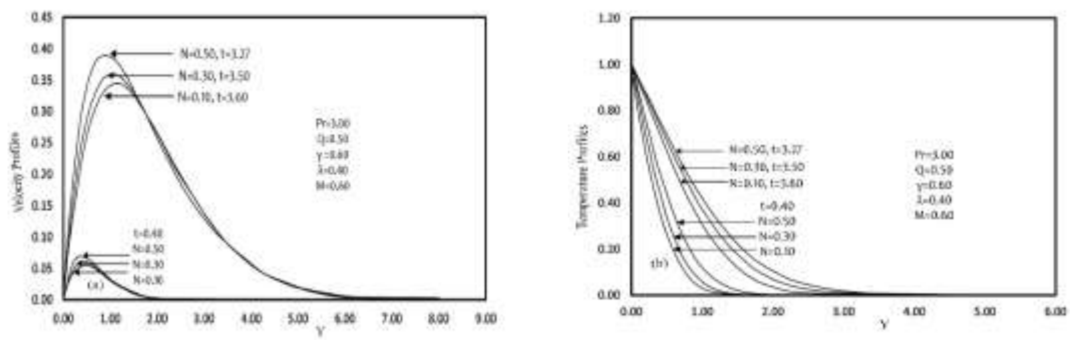

Figure 9(a) and 9(b). *Velocity profiles and temperature profiles for viscous dissipation parameter*

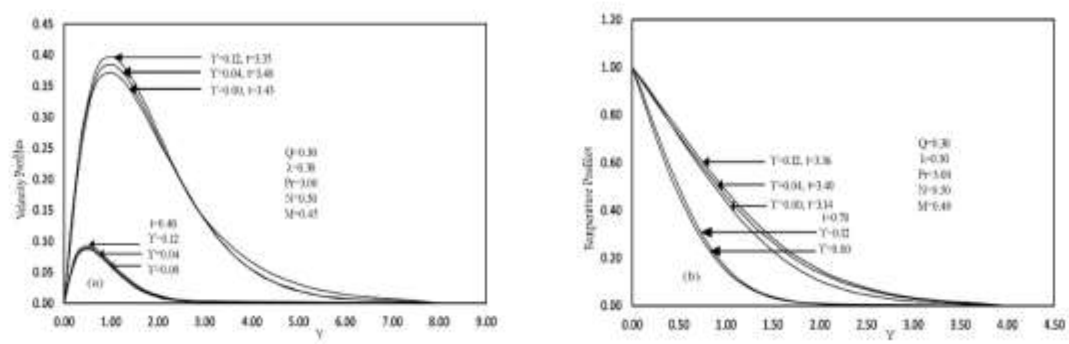

Figure 10(a) and 10(b). *Velocity profiles and temperature profiles for variable thermal conductivity parameter*

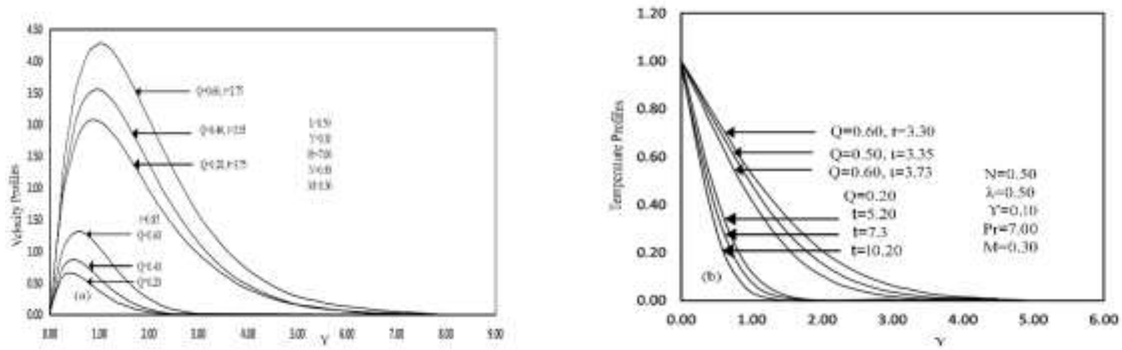

Figure 11(a) and 11(b). *Velocity profiles and temperature profiles for Heat generation parameter*

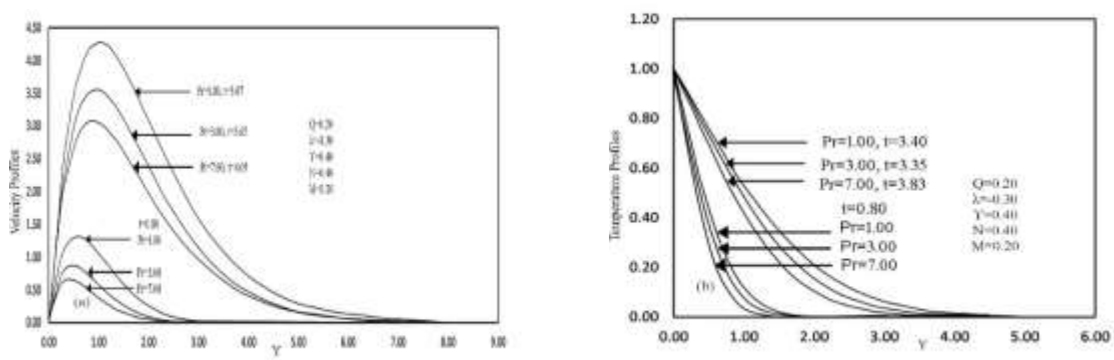

Figure 12(a) and 12(b). *Velocity profiles and temperature profiles for Prandtl number parameter*

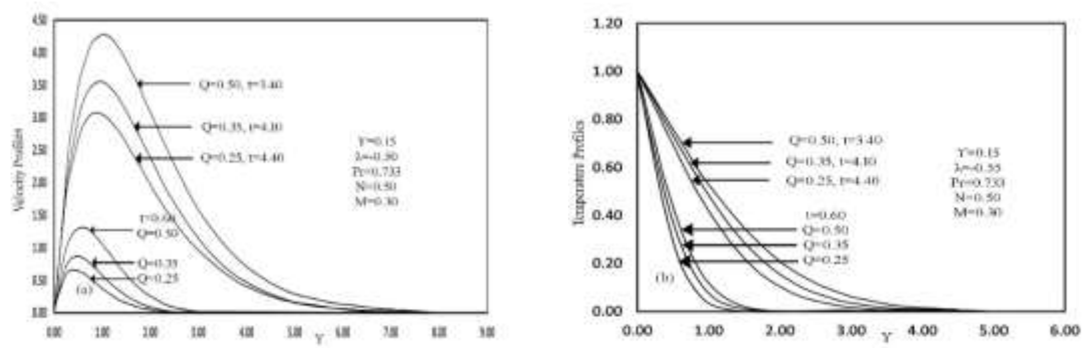

Figure 13(a) and 13(b). *Velocity profiles and temperature profiles for Heat generation parameter*

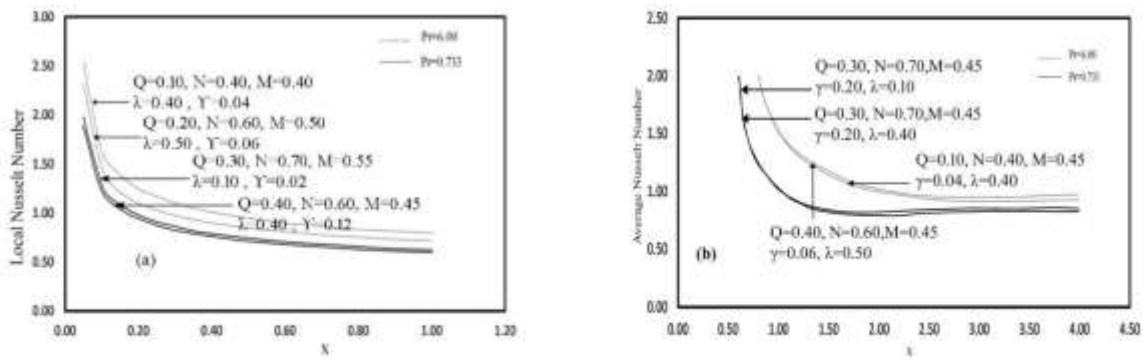

Figure 14(a) and 14(b). *Local Nusselt number Vs Average Nusselt number*

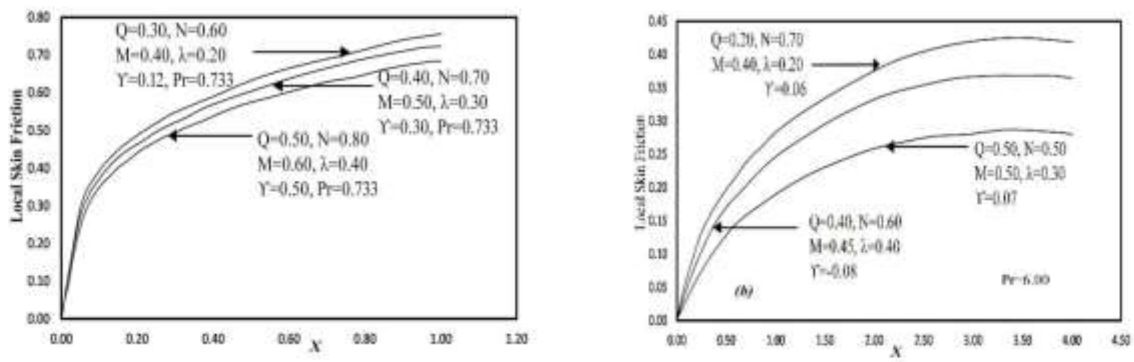

Figure 15(a) and 15(b). *Local skin friction for air and water*

The numerical values are calculated from Eqs. (13) and (14) are depicted in the graphical form in Figs. 5(a) and 5(b). When the viscous dissipation parameter N of the fluid is increased, the time it takes to reach the maximum and steady state increases. The velocity profile increases as the viscous dissipation N increases, as shown by the numerical results.

Figures 6(a) and 6(b) shows that the variation of velocity and temperature for various values of M for different fixed values. When X drops, it takes longer to get to a steady-state and λ increases towards the vertical plat, it is observed that the velocity increases. Increases in λ causes a drop in the fluid temperature.

Fig. 7(a) and 7(b) describe the velocity and temperature profiles with M for different fixed values. When x drops, it takes longer to get to a steady-state and λ increases towards the vertical plate, it is observed that the velocity also increases. Increases in λ causes fluid temperature dropped.

Vol.10, No.4, pp.78-94, 2022
Print ISSN: 2053-2229 (Print),
Online ISSN: 2053-2210 (Online)

Since the viscosity of water reduces when the viscosity variation parameter λ is increased, as shown in 8(a). Figure 8(b) shows that as λ grows, the temperature profiles decrease. Pick velocity rises in direct proportion to λ, therefore knowing this is critical. u increases in velocity as grows with increasing temperature T near the plate if the initial force is dominant. Because the second force is weaker when T is lower, then velocity u drops as λ lowers.

A rise in the viscous dissipation parameter N is connected with a considerable increase in velocity profiles, as seen in figure 9(a). Figure 9(b) shows the distribution of temperature profiles against y for several values of the viscous dissipation parameter N.

For different constant values, the numerical values of variance of velocity and temperature profiles with Q are graphically shown in figure 11(a) and 11(b). These figures show the Heat generation parameter Q lowers, which worth noticing that the velocity increases as Q approaches the vertical plate and the temperature drops as the Heat generation parameter Q lowers.

For different fixed values the figures 12(a) and 12(b) show the transient velocity and temperature with Prandtl's numbers. Here it is observed to rise as Prandtl's numbers of the fluid increase. The velocity profile reduces as Prandtl's number increases, as the numerical data show.

Figures 13(a) and 13(b) graphically depict the correlation between the variance of velocity and temperature profiles. If you look at the graphs closely, you'll see that the time it takes to reach steady state increases as the Heat generation parameter Q decreases, and that the velocity increases as the Heat generation parameter Q gets closer to the vertical plate. The fluid cools as the value of Q, a parameter of heat generation, goes down.

After a five-point approximation formula is used to evaluate the derivatives in Eqs. (12), (14), (15) and (16) are the integrals calculated using the Newton-Cotes closed integration formula.

Local Nusselt Number at Y distance for various values of Q, N, M, ϒ, λ and Pr.

Table 1. **Local Nusselt Number**

Q=0.40, N=0.30, ϒ=0.20, λ=0.10 Pr=0.733, M=0.30		Q=0.30, N=0.50, ϒ=0.10 λ=0.30, Pr=0.733, M=0.50	
Y	Local Nusselt Number	Y	Local Nusselt Number
0.61	1.9037	0.61	1.9198
0.65	1.6665	0.65	1.5928
0.71	1.4197	0.72	1.3812
0.81	1.2273	0.82	1.2048
0.93	1.0894	0.94	1.0861
1.19	0.9160	1.34	0.8486
1.31	0.8743	1.59	0.8099
1.67	0.8226	1.77	0.7969
1.95	0.8192	1.98	0.7903
2.55	0.8443	2.54	0.8154
2.85	0.8536	2.91	0.8247
3.13	0.8534	3.13	0.8309
3.44	0.8531	3.53	0.8306
3.75	0.8592	3.76	0.8271
3.99	0.8526	3.99	0.8269

CONCLUSION

In this research, we look at how changing the viscosity and thermal conductivity of a vertical plate in a laminar natural convection boundary layer impacts the heat generated by the plate. When it comes to temperature, most people believe that thermal conductivity is linear, while the viscosity of a fluid should change on an exponential scale. The dimensionless governing equations are solved using an implicit Crank-Nicolson type finite difference technique. A graphic comparison of the new numerical findings with previously published studies is drawn. It seems like the two parties have settled on a very satisfactory agreement.

The present analysis has shown that:

(i) The Fluid velocity rises as the viscosity parameter λ increases and the temperature falls. Greater velocity is found in a location near the wall when the viscosity variation parameter λ is large, resulting in a higher Nusselt number and reduced skin friction.

(ii) The fluid velocity, temperature, velocity gradient, and rate of heat transfer from the plate to the fluid all rise as the thermal conductivity parameter γ increases.

(iii) A significant amount of inaccuracy would be produced if viscosity and thermal conductivity variations were ignored. As a result, we recommend that the effects of changing viscosity and thermal conductivity be taken into account in order to better predict the results.

(iv) When the viscous dissipation parameter N is increased, the velocity profiles are slowly increased. Furthermore, when the viscous dissipation parameter rises, the temperature profile rises.

(v) When the Heat generation parameter Q causes a significant increase in velocity and temperature profiles.

(vi) For the variation of heat generation parameter Q, viscous dissipation parameter N, variable viscosity parameter λ and temperature-dependent parameter γ, there is an increase in the velocity and temperature distribution, but the local skin friction coefficient and the local Nusselt number decreases over the entire boundary layer.

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