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INTERVAL RELIABILITY ASSESSMENT OF REINFORCED CONCRETE BEAM PLACED ON ELASTIC FOUNDATION

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ABSTRACT: *The article proposes a formula to evaluate the reliability of the structure according to the interval model. In the case that the load effects and structural capabilities are sets of the form of interval numbers, the interval reliability evaluation formula is extended and established from the reliability in the stochastic model based on the probability geometric definition and the basic operations of interval arithmetic. Next, the paper presents a method to determine the internal force of a beam structure placed on an elastic foundation considering the uncertainty of the input parameter in the form of intervals. The finite element method combined with the interval function optimization algorithm is used to determine the interval output internal force of the beam structure. From that, the evaluation formula is applied to evaluate the reliability of reinforced concrete beam structure placed on elastic foundation with input parameters in the form of interval form: elastic modulus of material, concrete strength, and reinforcement strength, static load and Winkler's foundation coefficient.*

KEYWORDS: interval numbers, interval analysis, interval reliability of structures, reliability of structures, finite element method, elastic foundation, beam on elastic foundation.

INTRODUCTION

Evaluating the reliability of the structure is a very important content in the design and calculation of the structure. Therefore, the study of methods to evaluate the reliability of structures in general and beam structures on elastic foundations in particular is a matter of great concern. The problem of evaluating the reliability of beams on an elastic foundation is really necessary for the design and verification of the foundation structure of the building. To evaluate the reliability, it is necessary to first analyze the internal force of the beam placed on the elastic foundation. Beams placed in direct contact with an elastic foundation such as foundation beams placed on the ground, pontoon bridges, conveyer ferries lying on water, railway sleepers spread on rock, are a particularly common type of indeterminate problem In reality. Currently, the calculation of beam structure on elastic foundation determines internal force [4], [14], [18], [22..25]..deformation, stress or displacement of beam, is a quite complicated problem. Now, there are many different calculation points of view, in which, the popular model in practice is the Winkler model. This is a simple model but quite suitable for engineering problems. This model assumes that the reaction of the ground at a point is proportional to the settlement of the ground at that point. Thus, the foundation is conceived as an infinite system of independent springs, and the soil reaction strength at each point is proportional to the elastic settlement at that point through the constant elastic foundation coefficient k for each background type. If p is the ground reaction per unit area, y is the settlement at the survey point, then the Winkler ground model is represented by the relationship: $p(x) = k \times y(x)$

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However, in structural calculations, there are often structural and impact input quantities that contain random, unclear, and cannot be exacted information, these quantities are called uncertain quantities.

To describe uncertain quantities, random quantities, fuzzy numbers, interval numbers, and fuzzy-random quantities are used. Uncertainty quantities are represented as random quantities that are calculated according to the stochastic model. The State analysis and reliability structural evaluation by stochastic model had many studies [6], [7], [9],... In the case of uncertain quantities described below In fuzzy numerical form, the state analysis and evaluation must be done according to the fuzzy model. The evaluation of the reliability of the fuzzy numerical model have also been some studies [1, 2, 5, 8, 11, 17],... The calculation of structure according to the interval model with interval input parameters also have studied and published in journals [11, 15, 16, 19, 20].

In this paper, the Interval model is used to analyze the state and evaluate the reliability of the output response for the structure. In the research scope of this article, the author uses the input data with uncertainty in the form of interval numbers, the value ranges of the input variables are used according to the existing references. The calculation to determine the structural internal force should be carried out according to the operations of the interval arithmetic [3, 10] and the interval optimization algorithm.

Specifically, in the paper, we apply the interval function optimization algorithm combined with the interval finite element method to calculate the interval functions, determine the necessary internal force analysis results of the concrete beams placed on the elastic foundation in the form of intervals, as the basis for the reliability assessment of the structure. The calculation process is programmed by the authors in Maple.17 software to determine the interval output of displacement results and internal forces in beams. From there, the authors applies the proposed formula "Interval Ratio" to evaluate the reliability of the beam structure according to the interval model.

RESEARCH METHOD

Operations of interval numbers

2.1.1. Basic operations[3, 10, 13]

A real interval is a non-empty set of real numbers:

$$x = [\underline{x}, \overline{x}] = \left\{ x \in R \middle| \underline{x} \le x \le \overline{x} \right\}$$

wher $\underline{x}$ and $\overline{x}$ are lower and upper bounds of the interval $\breve{x}$, x is part of the range $\breve{x}$, R is the set of real numbers.

The four basic operations of real numbers are (+,-,×,÷), extensible for interval numbers: Any calculation o∈(+,-,×,÷) on intervals is defined as follows:

$$\breve{x}\, o\, \breve{y} = \left\{ x\, o\, y \middle| x \in \breve{x}, y \in \breve{y} \right\}$$

The set of results of operations for $x \in \breve{x}$ and $y \in \breve{y}$ forms a closed interval (if 0 is not under the denominator) with the bounds of the intervals defined as follows:

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$\breve{x} \, o \, \breve{y}$ = [*min(x o y), max(x o y)*] where $o \in (+,-,\times,\div)$.

The lower and upper bounds of the operation $\breve{x} \, o \, \breve{y}$ are determined from four pairs of numbers $\underline{x} \, o \, \underline{y}, \underline{x} \, o \, \bar{y}, \bar{x} \, o \, \underline{y}, \bar{x} \, o \, \bar{y}$.

An interval function is a function that takes the interval value of one or more interval parameters, so an interval function maps the value of one or more interval parameters onto an interval. For a function $f(x_1, \ldots, x_n)$, if the interval value function $\breve{f}(\breve{x}_1, \ldots, \breve{x}_n)$ has properties

$$\breve{f}(\breve{x}_1, \ldots, \breve{x}_n) = f(x_1, \ldots, x_n) \text{ for all arguments } x.$$

then the function $\breve{f}$ is called an interval expansion function of *f*. In particular, the natural interval expansion function of *f* can be obtained by replacing each real variable x_i with an interval variable $\breve{x}_j$ and each real operation (+,-,×,÷)with the corresponding interval operations. If the function $\breve{f}$ is an expression with a finite number of interval variables $(\breve{x}_1, \ldots, \breve{x}_n)$ and interval operations (+,-,×,÷)hen the function satisfies the basic inclusion property :

$$\text{If } \breve{x}_1 \subseteq \breve{y}_1, \ldots, \breve{x}_n \subseteq \breve{y}_n \text{ then } \breve{f}(\breve{x}_1, \ldots, \breve{x}_n) \subseteq \breve{f}(\breve{y}_1, \ldots, \breve{y}_n)$$

Where $\breve{x} \subseteq \breve{y}$ means that the interval $\breve{x}$ is a subset of the interval $\breve{y}$, only if $\underline{y} \le \underline{x}$ and $\bar{x} \le \bar{y}$.

In many cases, the boundaries determined by interval arithmetic tend to be larger than the boundaries of the true range, resulting in inaccurate results.

For example, consider the algebraic expression $f = x_1.x_2/x_3$ where $x_1 = x_2 = x_3 \in [2, 5]$, by evaluating the natural extension function, we get the value of the function *f* over the interval [2, 5] is:

$$\breve{f} = \breve{x}_1.\breve{x}_2 / \breve{x}_3 = [4, 25]/[2, 5]=[0.8, 12.5]$$

However, when considering the function $\breve{f} = \breve{x}.\breve{x} / \breve{x}$, với $x \in [2, 5]$, according to the basic operations of interval arithmetic, the function $\breve{f} = \breve{x}.\breve{x} / \breve{x}$ is computed in turn like the function $\breve{f} = \breve{x}_1.\breve{x}_2 / \breve{x}_3$ and for the output range of $\breve{f} = \breve{x}.\breve{x} / \breve{x}$ = [4, 25]/[2, 5]=[0.8, 12.5].

Meanwhile, in terms of mathematics as well as the physical meaning of the quantity $\breve{x}$, the function $\breve{f} = \breve{x}.\breve{x} / \breve{x} = \breve{x}^2 / \breve{x} = \breve{x}$ =[2, 5], So $\breve{f} = \breve{x}_1.\breve{x}_2 / \breve{x}_3$ = [0.8, 12.5] cover $\breve{f} = \breve{x}$ = [2, 5].

This unwanted expansion is known as overestimation due to the dependency problem. The reason is that in interval arithmetic, the variables appearing in the calculations are considered to be independent of each other, this is a limitation when applying interval arithmetic to solving structural problems when the parameters input or output is very tightly constrained.

Interval optimization method

This method is implemented based on the method of optimizing the output results when the input parameters contain interval parameters, now instead of using the direct calculation interval arithmetic tool to find the output result range, We perform the optimization of the objective function containing

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the interval input variables to find the maximum and minimum values with the constrained conditions that the variables of the objective function are limited in the interval of them.

$$y_j = f_j(x_1, x_2, \dots x_n) \rightarrow \min, \textbf{ constrained condition } a_j \le x_j \le b_j \quad (1)$$
$$y_j = f_j(x_1, x_2, \dots x_n) \rightarrow \max, \textbf{ constrained condition } a_j \le x_j \le b_j \quad (2)$$

Using the differential evolution algorithm (DE) [21] set up in the software Mple.17 to solve the programming problem (1) and (2) we get the ***maximum and minimum*** value of the output result. The advantage of this method is that the output is close to the analytical result because the method does not use interval arithmetic when performing calculations, so it does not suffer from natural expansion.

For example, consider the interval function $\breve{y} = \breve{x}_1^2 + \breve{x}_2^2 - 3\breve{x}_1^2.\breve{x}_2^2 + 5$,where $\breve{x}_1, \breve{x}_2$ are the interval variables $\breve{x}_1 \in$ [-2, 5]; $\breve{x}_2 \in$ [2, 7].
Sequentially performing the calculations according to the interval analysis, the results are as follows:

$\breve{x}_1^2$ =[-10, 25]; $\breve{x}_2^2$ =[4, 49]; $3\breve{x}_1.\breve{x}_2$ =[-42, 105];

Therefore $\breve{y}$ =[-10,25]+ [4,49] - [-42,105]+5 = [-111, 116]

Performing the calculations according to the interval optimization method:

the objective function: $y=f(x_1,x_2) = x_1^2 + x_2^2 - 3x_1.x_2 + 5$

constrained condition: $-2 \le x_1 \le 5$; $2 \le x_2 \le 7$

Solving the nonlinear optimization problem using Maple.17, we get the following results:

$y_{max} = 100$; $y_{min} = -26$ means $\breve{y}$ =[-26, 100].

The results according to the optimization method are narrower than those using interval arithmetic, avoiding the phenomenon of widening the output value of the basic operation in interval analysis. Below, the author will apply the interval optimization method containing the interval variable to solve the basic equation of the finite element method containing the interval parameter to determine the interval output value.

FEM for calculating beams on elastic foundation

Considering a beam on an elastic foundation according to the Winkler model, it is understood as a beam placed on a spring system with a completely independent stiffness k as shown in Figure 1. To calculates the beam on an elastic foundation follow the finite element method, firstly it is necessary to determine the stiffness matrix of the beam on an elastic foundation. The beam stiffness matrix on the elastic foundation consists of two components, the stiffness matrix of the flexural beam plus the stiffness matrix of the foundation affects the beam [4, 14, 18, 22].

Consider the eth beam element at the two ends of node i-j with displacements at the node:
q = $\{q_1, q_2, q_3, q_4\}^T$

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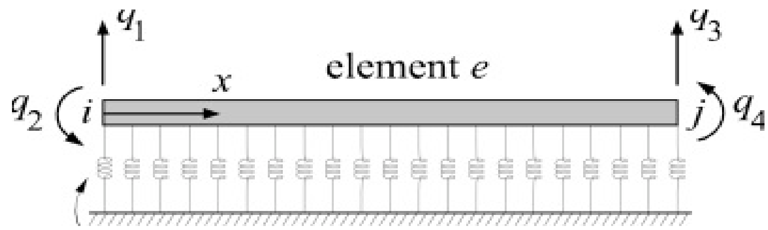

Fig.1 Diagram of beam element on elastic foundation

The eth flexural beam element stiffness matrix has the form (3).

$$[\mathbf{k}]_{eb} = \begin{bmatrix} 12EI/l^3 & 6EI/l^2 & -12EI/l^3 & 6EI/l^2 \\ 6EI/l^2 & 4EI/l & -6EI/l^2 & 2EI/l \\ -12EI/l^3 & -6EI/l^2 & 12EI/l^3 & -6EI/l^2 \\ 6EI/l^2 & 2EI/l & -6EI/l^2 & 4EI/l \end{bmatrix} \quad (3)$$

The eth beam element stiffness matrix under the influence of the foundation has the form (4).

$$[\mathbf{k}]_{ef} = \frac{k}{420}\begin{bmatrix} 156l & 22l^2 & 54l & -13l^2 \\ 22l^2 & 4l^3 & 13l^2 & -3l^2 \\ 54l & 13l^2 & 156l & -22l \\ -13l^2 & -3l^2 & -22l^2 & 4l^3 \end{bmatrix} \quad (4)$$

Where:

- E is the elastic modulus of the material of the bar element (kN/m^2)
- I is the moment of inertia of the bar section (m^4)
- l is the length of the bar (m)
- k = Kxb is the stiffness coefficient of the spring converted from the Winkler coefficient K

With:

+ K(T/m^3) is looked up according to the table in [1, 2]
+ b(m) is the width of the bar placed on the ground

So, the beam stiffness matrix on an elastic foundation has the form: $[\mathbf{k}]_e = [\mathbf{k}]_{eb} + [\mathbf{k}]_{ef}$

In the scope of the article's research, considering the uncertainty of the material $\breve{E}$ characteristics and the background coefficient $\breve{k}$ affecting the stiffness of the structure. So the stiffness matrix of the beam element on the elastic foundation has the form (5):

$$\left[\breve{k}\right]_e = \frac{\breve{k}}{420}\begin{bmatrix} (12\breve{E}I/l^3+156l) & (6\breve{E}I/l^2+22l^2) & (-12\breve{E}I/l^3+54l) & (6\breve{E}I/l^2-13l^2) \\ (6\breve{E}I/l^2+22l^2) & (4\breve{E}I/l+4l^3) & (-6\breve{E}I/l^2+13l^2 & (2\breve{E}I/l-3l^2) \\ (-12\breve{E}I/l^3+54l) & (6\breve{E}I/l^2+13l^2) & (12\breve{E}I/l^3+156l) & (-6\breve{E}I/l^2-22l) \\ (6\breve{E}I/l^2+-13l^2) & (2\breve{E}I/l-3l^2) & (-6\breve{E}I/l^2-22l^2) & (4\breve{E}I/l+4l^3) \end{bmatrix} \quad (5)$$

According to the virtual work principle, set up the basic equation of the finite element method with intervals input parameters as follows:

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$$[\breve{k}].\{\breve{q}\} = \{\breve{f}\} \qquad (6)$$

where:

- $[\breve{k}]$ is the global stiffness matrix of the structure, which is a square matrix whose size (nxn) depends on the number of degrees of freedom of all the nodes.
- $\{\breve{f}\}$ is the global nodal force vector in the form of intervals, size (nx1);
- $\{\breve{q}\} = \{\breve{q}_1\ \breve{q}_2 ... \breve{q}_n\}^T$ is the node displacement vector in the structural system.

The finite element method is used in combination with the interval function optimization algorithm presented above to determine the interval output of the beam structure as displacement and internal force.

The New formula for evaluating the reliability: "Interval Ratio"

Preamble

In practice, it is not always possible to determine the statistical law of the input quantities, but those quantities have an uncertainty of a value range, so the assessment of reliability cannot be determined according to the statistical laws of probability theory. On the basis of inheritance and development of reliability calculation methods for building structures, the author of this article proposes a way to evaluate the reliability of structures on the basis of operations in interval arithmetic and The geometric definition of probability. The new evaluation method is developed and presented below.

Evaluation is performing a comparison of two vectors, more than two sets, in general, two spaces, in this paper we use the comparison of two vectors.

Vector 1 is a standard vector, or it can be called a vector containing evaluation criteria, with dimensions equal to the number of criteria for technical, artistic, economic quality... The definite value of each criterion can be a point or interval on its axis. In the field of civil engineering, it is common for the value of each criterion to define a point or interval on the criterion axis. In other words, the boundary of the vector 1 depends on the criterion.

Vector 2 is the state vector of the object being evaluated. This vector, with the number of dimensions corresponds to vector 1. The nature of the construction work that the quality degrades over time due to use and environmental conditions, so vector 2 also changes over time and environmental conditions.

To evaluate the reliability of the construction at a certain time, we need to have full information about the state of the object, that is, we need to build vector 2. Then compare it with vector 1. In practice, it is very difficult to get a correct and accurate picture of vector 2 because it is usually not possible to collect enough information or the information is not very accurate, which we call uncertainty information. Therefore, using interval arithmetic to solve the evaluation and comparison problem in this case is a reasonable and effective approach.

Evaluation Formula

From the set containing the elements of the structural system to be evaluated, divide vector 1 and vector 2 into sub-vectors in the form of sets of simple intervals, respectively:

$$\breve{R} = \left\{ \breve{R}_1, \breve{R}_2, ..., \breve{R}_i, ..., \breve{R}_n \right\}$$

$$\breve{Q} = \left\{ \breve{Q}_1, \breve{Q}_2, ..., \breve{Q}_i, ..., \breve{Q}_n \right\}$$

From the evaluation criteria, based on analytical operations of interval arithmetic, the number of standard intervals of each element can be determined $\breve{R}_i$ (i=1,2,…,n), as figure 2.

From the results of solving the structural mechanics problem by the finite element method, the displacement state and internal force of the structural system can be determined in the form of interval numbers $\breve{Q}_i$ (i=1..n) as figure 3.

To evaluate the safety or failure level of the structural system compared with the standard, we compare the subsets of each corresponding element in pairs of 2 vectors according to the following formula presented on the basis of: Theory of interval arithmetic.

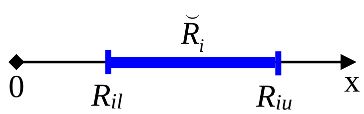

Figure 2. Interval number $\breve{R}_i$

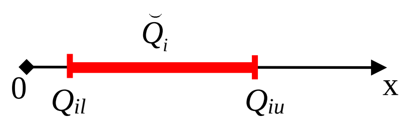

Figure 3. Interval number $\tilde{Q}_i$

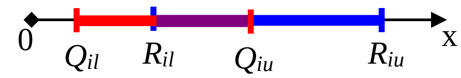

Figure 4. Interval interference model

Consider the number of intervals $\breve{Q}_i$ of the ith structural element in the set of states of the structural system to be evaluated with $\breve{Q}_i$=[Q_{il} , Q_{iu}] and interval number $\breve{R}_i$ of the ith criterion in the set of criteria used for evaluation has an interval of value $\breve{R}_i$=[R_{il} , R_{iu}] as figure 2 and figure 3. Call the set $\breve{M}_i$ = $\breve{R}_i$ - $\breve{Q}_i$ is the set of safe interval, so $\breve{R}_i$ and $\breve{Q}_i$ are sets of intervals then $\breve{M}_i$ also is sets of intervals $\breve{M}_i$ = [M_{il} , M_{iu}]. Based on operations of interval arithmetic [3, 10] to determine the interval set $\breve{M}_i$. Depending on the range of values of the interval sets $\breve{R}_i$ and $\breve{Q}_i$, from the interval interference model shown in Figure 4, three cases can occur as shown in Figure 5.

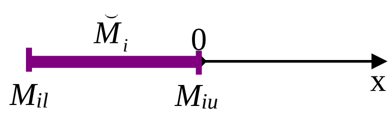

Figure 5a.

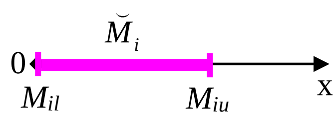

Figure 5b.

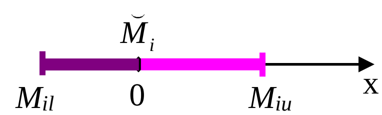

Figure 5c.

Figure 5. Cases of safe interval set $\breve{M}_i$

Where : - $M_{il} = min(R_{iu} - Q_{iu}, R_{iu} - Q_{il}, R_{il} - Q_{iu}, R_{il} - Q_{il})$.

- $M_{iu} = max(R_{iu} - Q_{iu}, R_{iu} - Q_{il}, R_{il} - Q_{iu}, R_{il} - Q_{il})$.

In Figure 5a, we see that the number of intervals of the set $\breve{M}_i$ lies completely to the left of the vertical

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axis, that is, the entire set of states $\breve{Q}_i$ of the i-th element exceeds its standard set $\breve{R}_i$. The element is in complete violation of the criterion, or the violation level against the norm set of the element is 100%. On the contrary, in Figure 5b, we see that the number of intervals of the set lies completely to the right of the vertical axis, that is, the state set $\breve{Q}_i$ of the ith element is completely non-violent compared with the possibility set or the safety level of the state set is100%.

From the above two comments, we can completely see that when the element's safety level is SP = 100%, its reliability is equivalent to Ps = 1; and in the opposite case its reliability is Ps = 0.

In the general case as shown in Figure 5c, the interval number of the set $\breve{M}_i$ has a left part and a right part of the vertical axis, that is, the state set has a part that exceeds the standard or is damaged (corresponding to the part to the left of point 0) and a standard or safe part (corresponding to the part to the right of point 0).

From the idea of a random interference model, it can be mathematically considered that the number of intervalsis a distribution of the safe interval $\breve{M}_i$, then according to the geometric definition of probability [12]: the probability of occurrence the distribution to the left of the zero point of the safe interval $\breve{M}_i$ is calculated as the ratio of that distribution over the entire distribution of $\breve{M}_i$. The probability of occurrence of the distribution to the left of the zero point of the main safety interval $\breve{M}_i$ equals the failure probability of the element (the uncertainty Pf of the element) is determined with the following formula:

$$\text{Prob}(\breve{M}_i<0) = \text{P}_\text{f} = \frac{0-M_{il}}{M_{iu}-M_{il}} = \frac{|M_{il}|}{M_{iu}-M_{il}} \tag{7}$$

By definition, the confidence Ps of element is equal to the probability of unfailure of the element calculated by the formula:

$$\text{Prob}(\breve{M}_i>0) = \text{P}_\text{s} = \frac{M_{iu}-0}{M_{iu}-M_{il}} = \frac{M_{iu}}{M_{iu}-M_{il}} \tag{8}$$

It is easy to see: Pf + Ps = 1 as in the definition of reliability under the stochastic model.
After determining the reliability of all parts of the structural system, we can completely determine the reliability of the structural system based on the definition of specific failure. Building a reliability model according to electrical diagrams or determine the reliability of the structural system according to the following formula:

$$\prod_{i=1}^{n} P_s^i \le P_s \le \min(P_s^1, P_s^2, ..., P_s^n) = P_s^i \min \tag{9}$$

Reliability assessment application for structure

3.1 Diagram of the steps of reliability assessment

The sequence of structural reliability assessment steps by the interval finite element method is briefly presented as a block diagram in Figure 6.

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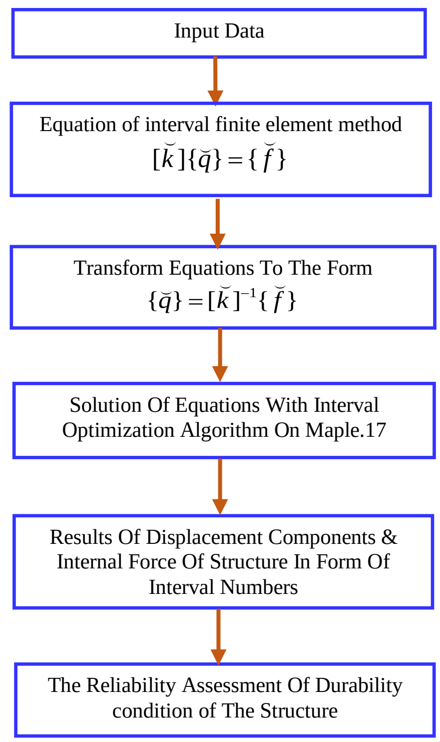


Figure 6. Block diagram of structural reliability assessment steps

3.2 Problem and input data

A two-span foundation beam structure was collected from the actual construction. The girder has dimensions bxh=70x150(cm), is placed on medium-grained sandy soil and is loaded as shown in Figure 7. The beam is arranged with reinforcement at the pillow and span with the cross-section as shown in Figure 8. The beam is used for concrete with strength grade B25, the dimensions of the area are defined numbers. As shown, elastic modulus of material $\breve{E}$, external load $\breve{N}$, $\breve{M}$, and Winkler ground coefficient $\breve{K}$ are interval numbers with lower and upper bound values given as below. The problem requires determining the internal force of the beam structure with input parameters in the form of intervals

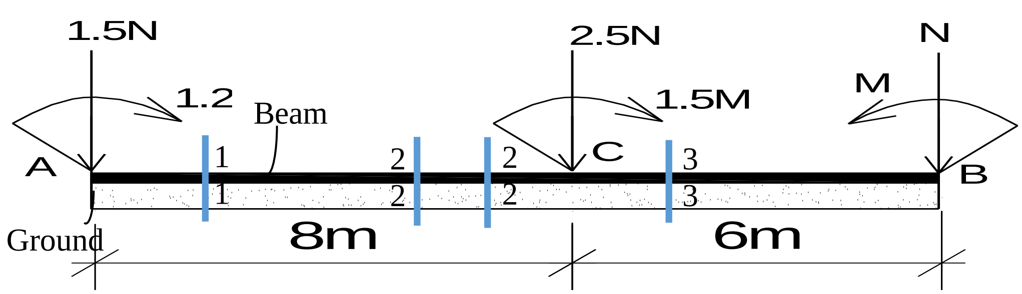
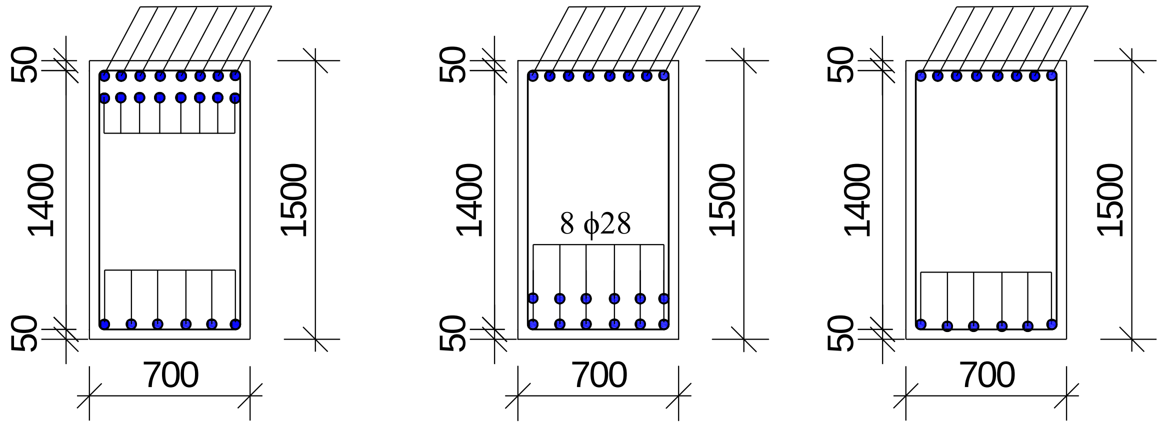


Figure 7. Structure diagram of foundation beam under load



Concrete using strength grade B25 has elastic modulus assumed to be ±5% deviation from the average value E=3100kN/cm². $\breve{E} = \left[E^L; E^U\right] = [2945; 3225]\, kN/cm^2$

The external load has a deviation of 15% from the mean value of longitudinal force N=1800kN and moment M=270kN.m

$$\breve{N} = \left[N^L; N^U\right] = [1530; 2070]\, kN$$

$$\breve{M} = \left[M^L; M^U\right] = [21250; 28750]\, kN.cm$$

The medium-grained sand base has the soil foundation coefficient: $\breve{K} = \left[K^L; K^U\right] = [30000; 50000]\, kN/m^3$

Sequence of calculation

calculation model

The beam calculation model on the elastic foundation is shown in Figure 6. Divide the beam structure into 14 elements, number the elements and the number of displacements for the beam structure according to local coordinates as shown in Figure 8.

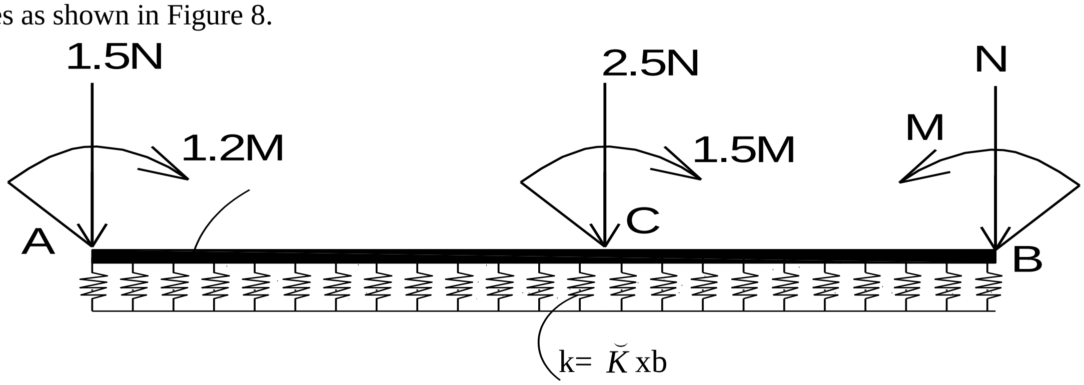


Fig. 8 Calculation model of beams on elastic foundation

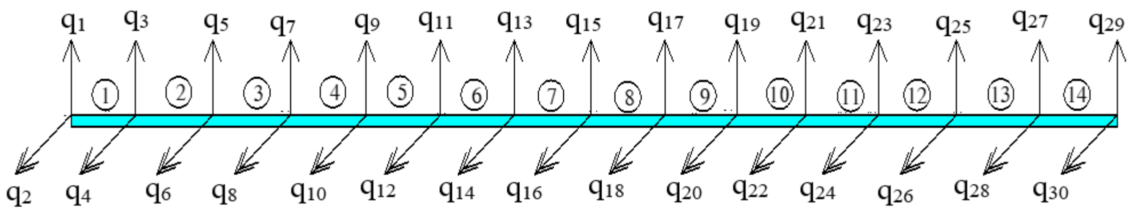


Figure 8. Element diagram of beam structure on elastic foundation

From the problem data and input data, the author conducts structural analysis according to the finite element method (FEM) according to the steps as the block diagram Figure 4. The whole calculation process is simulated by the author on Maple.17 software, integrated with the interval Optimization algorithm into the program to solve the system of equations with interval parameters to determine the internal force results as moment, numerical shear force at the nodes of beam structure. The calculation program is named FEM.BEF.

Internal force results of the structure

Using the program FEM.BEF to calculate and determine the internal force in the form of an interval for the beam structure on an elastic foundation when the parameters have the form of interval numbers which are the elastic modulus $\breve{E}$, the external load $\breve{N}$, $\breve{M}$, and the Winkler foundation coefficient $\breve{K}$. The results of the internal force calculation are shown in Table 1.

Table 1. Result of internal force of beam structure

Node order number	Interval Moment $\breve{M}$ (kN.m)
0	[255.000 ; 339.000]
1	[-2345.915 ; -1605.731]
2	[-4016.262 ; -2907.368]
3	[-486.160 ; -3614.068]
4 (Middle of span 1)	[-4966.284 ; -3769.816]
5	[-4388.819 ; -3405.839]
6	[-3161.185 ; -2540.025]
7	[-1284.096 ; -1178,723]
8 (Middle node)	[682.600 ; 1226.834] [1007.182 ; 1645.632]
9	[-742.779 ; -128.905]
10	[-1760.635 ; -1289.479]
11 (Middle of span 1)	[-2382.310 ; -1714.045]
12	[-2287.043 ; -1605.716]
13	[-1438.240 ; -946.394]
14	[212.500 ; 282.500]

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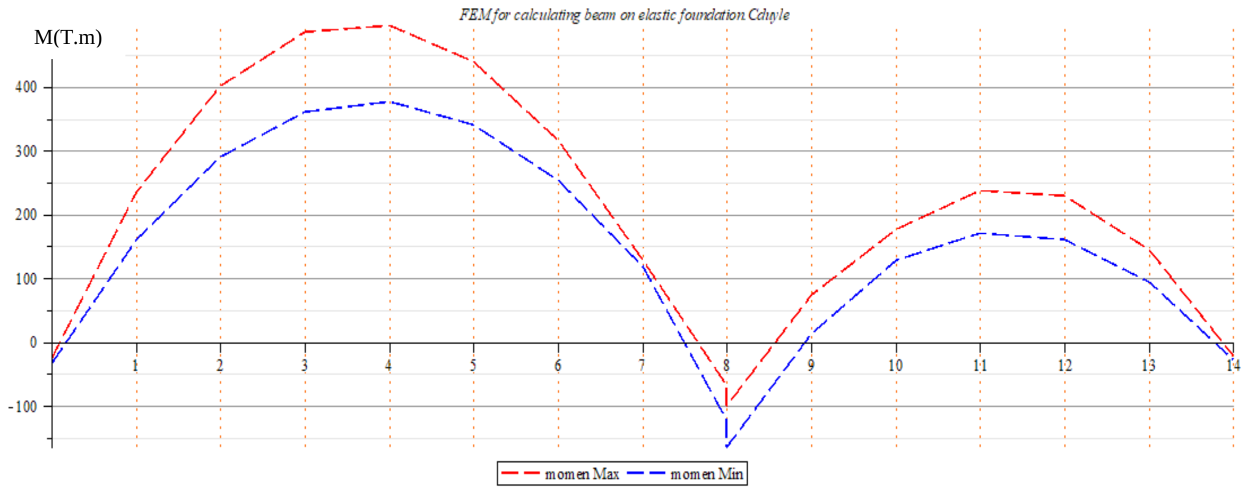

Figure 9. Interval moment graph in FEM.BEF

To check the correctness of the program FEM.BEF, the author uses the input data set corresponding to the average value of the interval numbers and recalculates it using SAP.2000 software. Moment chart, shear force graph calculated by FEM.BEF and by SAP.2000 shown as Figures below.

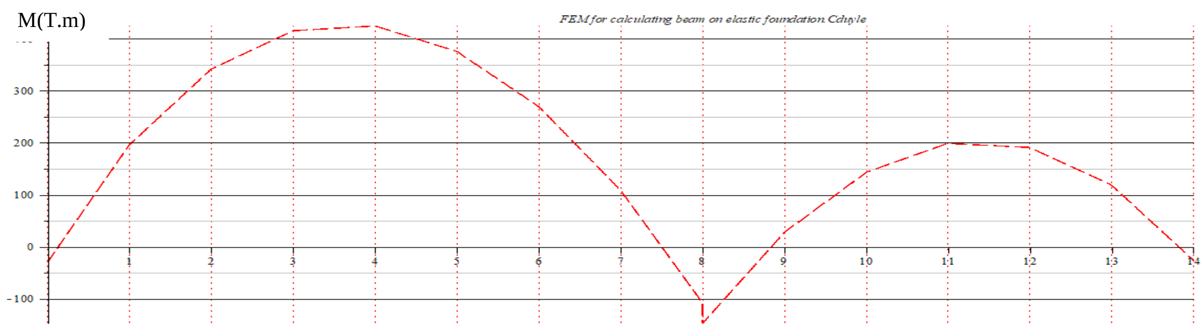

Figure 10. Moment graph in FEM.BEF

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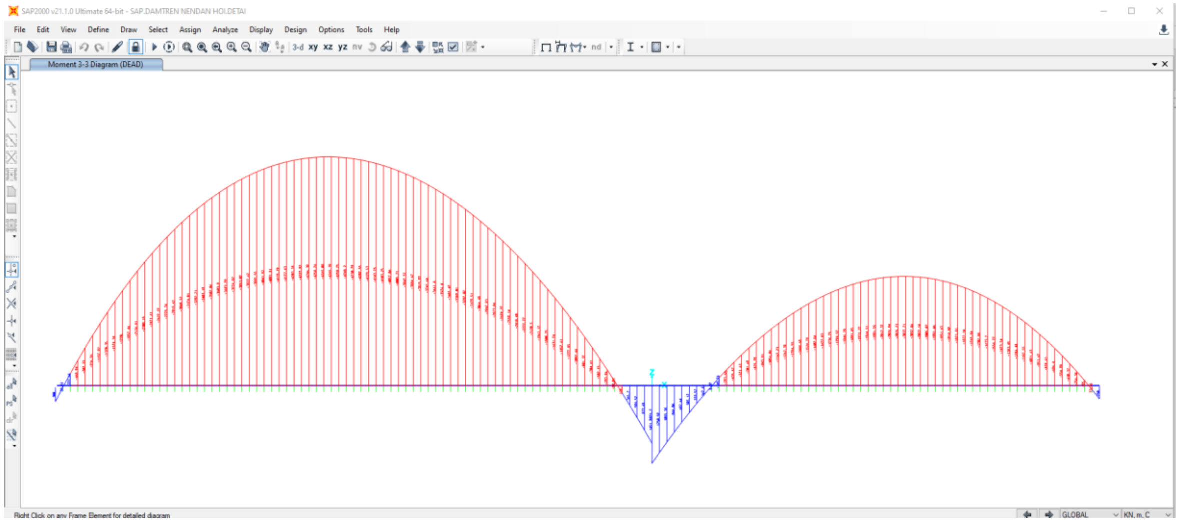

Figure 11. Moment graph in Sap.2000

Table 2. Results of calculating moment, shear force by FEM.BEF & SAP2000 for beam structure

Node order number	Moment (kN.m)		
	FEM.BEF	**SAP.2000**	**Error(%)**
0	300.00	300.00	0.00
1	-1961.30	-1942.99	0.933
2	-3413.81	-3381.21	0.954
3	-4150.93	-4111.07	0.96
4 (Middle of span 1)	-4247.14	-4265.39	0.429
5	-3753.74	-3722.73	0.826
6	-2698.22	-2680.29	0.664
7	-1087.01	-1083.35	0.336
8 (Middle node)	1087.01(left) -1462.06(right)	1076.98(left) -1451.98(right)	0.922 0.649
9	-285.49	-287.94	0.858
10	-1439.34	-1437.42	0.133
11 (Middle of span 1)	-1990.74	-1991.55	0,.046
12	-1916.22	-1906.04	0.531
13	-1183.51	-1176.55	0.588
14	250.00	250.00	0.00

The internal torque values in Tables 1 & 2 have the value (-) only showing the sign according to the convention for tensioning the upper fiber of the beam, while the moment of tensioning the lower fiber of the beam is (+). From Table 1, the internal force results show that the internal torque error between the FEM.BEF program and the SAP.2000 software is less than 1%. This error is very small and is considered insignificant in the calculation. Results of calculation allow to confirm the reliability of the program FEM.BEF that the author programmed on Maple.17 software.

Determination of bearing capacity of reinforced concrete beam

In the case of doubly reinforced beams, the ultimate moment is classified according to whether or not the compressive reinforcement has yielded. In other words, there are several cases that involve both tension reinforcement and compressive reinforcement yields. From the input data of the beam structure with the section and reinforcement arranged in the beam, the formula for calculating the bearing capacity of the beam cross section is applied. Knowing the cross-sectional dimensions and structure of the reinforcement in the beam, determine the limiting moment of the section by the formula:

$$M_{gh} = R_s.A_s.\gamma.h_o \qquad (10)$$

with $h_o = h - a$

$$\gamma = 1 - 0,5.\frac{R_s A_s}{R_b b h_o}$$

Where:
- R_s the tension strength of reinforcement
- A_s cross-sectional area of the steel on the tension side of the member
- R_b the compressive strength of concrete
- b the width of the section
- h the depth of the section
- a the distance from the tensile edge of the section to the centroid of the tensile reinforcement.

Table 3. Result of the limiting moment of the section of the beam

Section	Maximum moment in beam $\breve{M}$ (kN.m)
Section 1-1	[-4099,582 ; -4531,116]
Section 2-2	[1383,940 ; 1529,618]
Section 3-3	[-2198,551; 2429,977]

The internal torque values in Table 3 have the value (-) only showing the sign according to the convention for tensioning the upper fiber of the beam, while the moment of tensioning the lower fiber of the beam is (+). However, when calculating the reliability, the absolute value is used to compare the calculation.

Evaluation of the reliability of the beam structure

To evaluate the reliability of the structural system, it is necessary to calculate the reliability of each dangerous section on the beam structure according to the formula "Interval ratio" and then calculate the reliability for the whole structure according to the formula (9).

Reliability of beam element under tensile condition:

$$M_{max} \leq M_{gh} \qquad (11)$$

We set $Q_d = M_{max}$ and $R_d = M_{gh}$. So (11) is rewritten:

$$Q_d \leq R_d \qquad (12)$$

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When the input parameters are in the form of interval numbers, the strength condition of beam elements has the following form: $\breve{Q}_d \leq \breve{R}_d$ (113)

The structural reliability is calculated according to the formula "Interval ratio" shown in Table 4.

Table 4. Reliability calculation results for beam structure

Thứ tự mặt cắt	**Khả năng tiết diện** $\breve{R}_{di}$ **(kN.m)**	**Trạng thái tiết diện** $\breve{Q}_{di}$ **(kN.m)**	**Độ tin cậy theo"Tỷ số khoảng"**
1-1	[-4099,582 ; -4531,116]	[-3769,816 ; -4966,284]	***0,467628***
2-2	[1383,940 ; 1529,618]	[682,6004 ; 1226,834]	***1,000000***
3-3	[-2198,551; 2429,977]	[-1714,045 ; 2382,310]	***0,795752***

The reliability of the structural system is evaluated according to the formula (9):

$$\prod_{i=1}^{6} P_s^i = \leq P_s \leq min(P_s^1, P_s^2, ..., P_s^6) = P_s^i \, min$$

So the interval reliability of the structural system: $\mathbf{0,372116} \leq P_s \leq \mathbf{0,467628}$

DISCUSSION

The author of the article presented an overview of the process of developing models to evaluate the reliability of the structure, thereby proposing the formula "Interval ratio" to evaluate the reliability of the structure. The formula is established on the basis of the basic operations of interval arithmetic and on the geometrical definition of probability, the established formula is logically and mathematically rigorous enough to be used to evaluate reliability for the structure in particular and for an engineering system in general in the case of interval input data. However, in order to evaluate the reliability of the structure according to the interval model, it is necessary to analyze and determine the state of the structure when there are input parameters in the form of interval numbers. The analysis and calculation of the structure according to the interval model is done quite complicatedly on the basis of the operations of the interval arithmetic follow the analytic direction or in the numerical solution direction. This is also an issue that is currently being studied by domestic and foreign scientists.

The article has applied the theory of finite elements of the interval combined with the optimization algorithm of the interval function to analyze and determine the internal force state of the beam structure on the elastic foundation in the case of " the nature of the interval " of load parameters, material characteristics and background coefficients. The program FEM.BEF is programmed in the software language Maple.17 to calculate the output of the beam's internal force. The results of calculating the internal force of beams are appropriate,

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