INFLUENCE OF RADIATIVE HEAT AND MASS TRANSFER IN CHEMICAL REACTIVE ROTATING FLUID ON A STRATIFIED STEADY STATE IN A POROUS MEDIUM.

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ABSTRACT: *An analysis of radiative heat and mass transfer on the onset chemical reactive rotating fluid on a stratified steady state in a porous medium has been carried out .In addition the influence on rotation, radiative heat transfer and chemical reaction where investigated by imposes a time dependent perturbation on concentration, temperature and velocity. Their involvements are assumed to be large so that heat radiation, chemical reaction and heat transfer is significant. This renders the problem inhomogeneous even on assumption of differential approximation for the radiative flux with the chemical reaction. When the perturbation is small, the transient flow is tackled by laplace transform technique with the involvement of modified Bessel function of first and modified second order given solution for stable steady state, temperature solute concentration and velocity. Consequence of the stable steady state Analysis and numerical solution where obtained by the use of the ratio of marginal state and asymptotic state on the concentration and temperature are presented graphically display. Their profile on the chemical reaction parameters, concentration decreases due to the variations of the chemical reaction parameter, causing a corresponding asymptotic change in the porous medium. Concentration profile on the Schmidt number, concentration decreases as a result of the variation of the Schmidt number parameter causing a corresponding asymptotic change in the porous medium. Temperature profiles on the radiation parameter, temperature decrease due to the variation of radiation parameter resulting to a corresponding asymptotic change in the porous medium. Temperature profiles on the Prandtl number, temperature decrease following the variation of the Prandtl number, resulting to a corresponding asymptotic change in the porous medium.*

KEYWORDS: Radiative Heat, Mass Transfer, Chemical Reactive Rotating Fluid, Porous Medium.

INTRODUCTION

The problem of influence on radiative heat and mass transfer in chemical reactive rotating fluid on stratified stable steady state in a porous medium are prevalent through everyday life and the study of such fluid flow is gaining increasing application in the study of meteorology, Geophysics Engineering, Global Climate, and Astrophysics (William and John ,1999), more

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so the atmosphere and the weather are in porous medium that are governed by the dynamic of fluids

In 1916 Lord Raleigh investigated the instability of Bernard cells by considering the buoyancy-driven instability in a homogenous medium. His study shows that the parameter that determines the instability is the product of the Prandtl and Grashof numbers (i.e. ratio of buoyancy forces to viscous dissipation). Presently the product is called Raleigh number (R).

The buoyancy-driven instability has also been studied by other workers based on the theory of infinitesimal disturbances. Among these are Likhovskii and Ludovich (1963), Eckhaus (1965), Chandrasekhar (1961), Matkowisk (1970) and Sattinger (1973). Also, Bestman (1983) studied the case of instability due to mass concentration gradient in a porous medium. He established that as the permeability of the porous medium decreases, the Raleigh number for the onset of instability rises linearly over a wide range of values of the permeability. Thus although porosity increases stability, the critical Raleigh number (R_e) which determine the onset of instability was found to be 920 for the value of porosity $\chi = 6$. Consequently, turbulence is likely to form the low value of R_e.

The study of thermal stability analysis in compressible fluid flow through a porous medium abound in nature, engineering and in scientific applications. A number of workers have studied such flow problems; Ahmadi and Manvi (1971) have derived an equation of motion for fluid flow. Bestman [(1989), (1990)], Varshney (1979) and Raptis and Perdikis (1988), also studied the steady state problem associated with flow in the porous media. Similarly, unsteady flow has engaged the attention of other workers such as Gulab and Mishra (1977) who investigated the unsteady hydrodynamic flow in a porous medium. Kumar et al (1985) studied the unsteady magneto hydrodynamic flow through a porous medium in a channel, while Singh and Soundalgekar (1990) considered the transient free convection of water at $4^0 C$ past an infinite vertical porous plate with time-dependent suction. Thermal stability of an incompressible fluid in a porous medium generally considers fluid in a basic state of steady motion when a small disturbance is made in the fluid, possibly controlled or uncontrolled possibilities occurs. The first is that, the disturbance may generate waves in the fluid which propagate through it but do not pick up energy from the basic state; an example is a wave seen on the surface of water. However, the buoyancy-driven thermal stability of a radiating non-grey gas between two infinitely long vertical plates has been studied by Arpaci and Bayazitoglu (1973). In their study, the instability of natural convection in a slot involving two infinitely long vertical plates at different isothermal temperatures appears in two regimes (the Conduction and Convection regimes) which are distinguished by the temperature of the initial state; the initial temperature of the conduction regime is independent of and that of the convection regime depends linearly on the vertical direction. In this study, each regime is unstable, setting in form of stationary cells or travelling waves.

In the past decades, several papers dealing with this problem have been published [Opara et al ,(2001)]. Similarly, the study of the effect of combined thermal and mass concentration gradient on the stability of a chemically reacting fluid in a porous medium has been studied by Opara et al (1996). Their study revealed that in the absence of chemical reaction, instability sets in a stationary convection at the critical Raleigh number $R_e = 500$ with the corresponding wave number $a_c = 0.3$. Although, the extension of the problem of thermal stability of a incompressible fluid in a porous medium including the effect of rotation or that

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of non-Newtonian behaviour have recently been considered, Opara et al (1997), the combined influence on radiative heat and mass transfer in chemical reactive rotating fluid has apparently been left untreated especially areas on stratified steady state is the concern of this study. Key word steady state, porous medium, asymptotic state, marginal state.

MATHEMATICAL ANALYSIS

We consider a fluid rotating in $X-Y$ plane about the y-axis in a porous medium with combined effects of radiation, chemical reaction, temperature and concentration gradient respectively. In this consideration, the flow pattern in the plane is the same as that in all other parallel plane with the fluid and the fluid medium bounded by a horizontal free surface with constant pressure, density and velocity. The geometrical description of the model and the co-ordinates of the fluid is rectilinear Cartesian system (X,Y,Z) rotating steadily with angular velocity with the y-axis being vertical upward in the positive direction and the X,Z axis mutually perpendicular to Y to allow a columnof the fluid flow compresses a horizontal layer of fluid of thickness $|r_2 - r_1| = d$. Batchelor (2000).

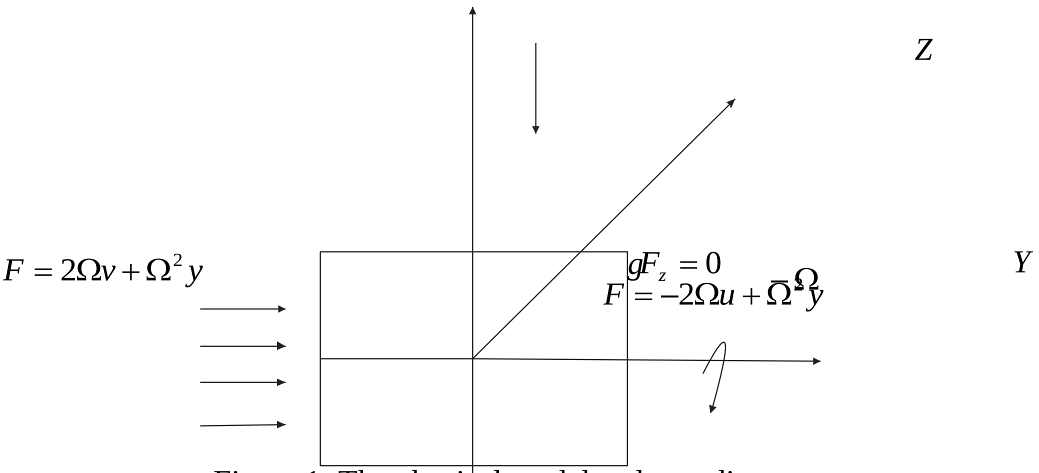

Figure 1: The physical model and coordinate system

The above flow description is bounded by plane $y = 0$ and $y' = |r_2 - r_1| = d$ with temperature T_0 and T_1 and concentration C_0 and C_1 respectively. Here the fluid is assumed to be at temperature T_0 and concentration C_0 at the lower plane and temperature T_1 and concentration C_1 rotating at the upper plane. Rotation, Ω and angular velocity, ω about y-axis is sustained mainly by the action of a fictitious body force per unit mass of the fluid lying in the X,Z plane with components

$$F_x = 2\Omega v,\ F_z = 0,\ F_y = 2\Omega u \qquad (2.0.1)$$

In consideration of the above fluid, the entire layer is acted upon by a uniform gravitational field and heated from below such that a uniform temperature gradient $\alpha_T = \left|\frac{dT}{dy}\right|$ and concentration gradient $\alpha_C = \left|\frac{dC}{dy}\right|$ are maintained across it. The lower plane is assumed to be in a state represented by the velocity,

$$U = \frac{v}{d} y \qquad (2.0.2)$$

Where v is the characteristic velocity and d the unit length; the velocity component u, v, w are in Cartesian co-ordinate system with axis X, Y, Z.

Generally the porous medium considered is one whose structure is statistically isotropic so that pressure gradient applied in different directions produce the same flux and is given by,

$$\nabla p = -\frac{\mu v}{\xi} \qquad (2.0.3)$$

Where ξ is the permeability, μ is the viscosity, v is the characteristic velocity, p is pressure and ∇ is the laplace equation.

In this research, the problem of the Reynolds parameter is small which is being ignored. Unfortunately, high temperature phenomena bound in the medium and assume the medium as optically thin body that can transfer radiative heat and chemical reaction into the medium. A primary difficulty in thermal radiative heat and chemical reaction study stems from the fact that the radiative flux and chemical reaction is governed by an integral expression and one has to handle a non-linear integro-differential equation. However, under fairly realistic assumption, the integral expression is replaced by a differential approximation for radiation and chemical reaction respectively. Thus in one space, co-ordinate y, the flux q satisfy the non-linear differential equation as state by ,Bestman *et al* ,(1988);Alabraba et *al*,(2007). Pekene and Ekpe,(2015)

$$\frac{\partial^2 q}{\partial y^2} - 3\alpha^2 q - 16\sigma\alpha T^3 \frac{\partial T}{\partial y} = 0 \qquad (2.0.4)$$

Where T, the temperature of heat transferred, σ is the Stefan-Boltzmann constant and α is the absorption coefficient which will be assumed constant in the model. Take into account the medium permit finite transparent for diffusing particle $\alpha << 1$ and equation 2.0.4 is approximated by,

$$\frac{\partial q}{\partial y} = 4\sigma\alpha\left(T^4 - T_\infty^4\right) \qquad (2.0.5)$$

In which subscript "∞" will be used to denote condition in the undisturbed porous medium.

Mathematical Formulation

The system incorporates a steady motion with the following designed conditions: an incompressible viscous fluid flowing in a porous medium, heated below in a horizontal plane and generating a sufficient thermal flux with combined radiation, chemical reaction with angular rotation through a layer of diffusing particle with concomitant variation in

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temperature and solute concentration. In the resulting motion, the fluid layer column assumes a uniform horizontal velocity v rotating about y-axis with angular velocity Ω (See figure 1 above); thus the fluid layer is maintained at transient temperature $T = T_\infty(1-\varepsilon f(t))$ and transient concentration $C = C_\infty(1-\varepsilon f(t))$ in which $T_\infty >> 1 f(t), C_\infty >> 1 f(t)$ and in which an arbitrary function of time and Σ are parameters. The temperature is high enough to sustain an adverse temperature and concentration gradient in the fluid for radiative heat transfer. C_∞ is the concentration undisturbed constant; $f(t)$ is an arbitrary function relating to time in the continuum but for this problem $f(t)$ will be approximated to the Heavy side unit function. However, we have neglected all effects of electromagnetic potential and we assumed that the hydrodynamic state is influenced entirely by adverse temperature gradient. This generate radiative heat flux which spreads free convection, in addition, concentration gradient generates molecular migration and chemical reaction that interact in the flow. On the above conditions, the resulting system admits the Boussenessq approximation for the thermal fluid with finite transparency of the porous medium with the governing equations of particle motion of horizontal momentum transfer as follows: Following the argument of Opara et al (1990), we employed equations 2.0.1, 2.0.3 and 2.0.5 the governing equation for transparency medium was modelled.

$$\frac{\partial u'}{\partial t} - 2\Omega v' = \frac{v\partial^2 u}{\partial y^2} - \frac{\mu u}{\xi} + g\beta_T(T - T_\infty) + g\lambda_C(C - C_\infty) \qquad (2.1.01)$$

The above equation is a two-dimensional form in horizontal motion in a rotating plane that is specified in terms of two components along u with radial adverse temperature and concentration variation along v with transverse component ξ in the porosity. Medium permeability was introduced followed Brinkman (1947) in Darcy laws (1956).

$$\frac{\partial v}{\partial t} + 2\Omega u = \frac{v\partial^2 v}{\partial y^2} - \frac{\mu v}{\xi} \qquad (2.1.02)$$

Equation (2.1.02) above establish the energy equation in a horizontal motion rotating frame in terms of two components along v with permeability of the medium (Transverse component) along v under rotational symmetry with swirl.

$$\rho C_p \frac{\partial T}{\partial t} = k_r \frac{\partial^2 T}{\partial y^2} - 4\sigma\alpha(T^4 - T_\infty^4) + \frac{D_m k \partial^2 C}{\xi \rho C_P \partial y^2} \qquad (2.1.03)$$

Equation 2.1.03 is the distribution of adverse temperature under the action of radiative heat flux and concentration with effective diffusion which migrate on the action of the radiative flux and mass diffusion due to the time independent temperature variation.

$$\frac{\partial C}{\partial t} = \frac{D_m \partial^2 C}{\partial y^2} - k_r T 4\sigma\alpha(C^4 - C_\infty^4) + \frac{D_m k_r \partial^2 T}{\xi \partial y^2} \qquad (2.1.04)$$

Equation 2.1.04 gives account of chemical reaction to varying strength, arbitrary during the process of chemical reaction with convectional mass diffusion induced by an applied concentration gradient over the average temperature variation.

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Equations 2.1.01-2.1.04 is subject to the boundary conditions.

Non-Dimensional Variable

$$\left.\begin{array}{l} C, C_\infty / C_\infty, \tau = t\dfrac{u_0^2}{v}, (u,v) = (u,v)/u_0 \\ V(u,v,w), (\theta,\theta_\infty)/\theta_\infty = (T, T_C)/T_\infty, (T,T_\infty) = T_\infty(\theta,\theta_\infty) \\ (C,C_\infty) = C_\infty(C,C_\infty) \\ t = \dfrac{v}{u_0^2}\tau, (u', v') = U_\infty(u', v'), y = \dfrac{v_y}{u_0}, q = u + iv, i = \sqrt{-1} \\ \text{The boundary conditions governing the problem are;} \\ T = T_w(1+\varepsilon f(t)), C = C_w[1+\varepsilon f(t)], \text{ when } y = 0, v = 0 \\ u = u_0, u = v = 0, T = T_\infty, y \to \infty, C \to 0, C = C_0 \end{array}\right\}, \quad (2.1.05)$$

Under suitable non-dimensional, equations 2.1.01-2.1.04 Where subjected to the Boussinesq approximation were after modification, reduced to the equations below:

$$\frac{\partial q}{\partial t} + 2i\varepsilon q = \frac{\partial^2 q}{\partial y^2} - \chi^2 q + G_r(\theta - 1) + G_C(C-1) \quad (2.1.06)$$

$$P_r\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial y^2} - P_r R(\theta^4 - 1) + D_C\frac{\partial^2 C}{\partial y^2} \quad (2.1.07)$$

$$S_C\frac{\partial C}{\partial \tau} = \frac{\partial^2 C}{\partial y^2} - k_r S_C(C^4 - 1) + S_t\frac{\partial^2 \theta}{\partial y^2} \quad (2.1.08)$$

The boundary conditions to be imposed are:

$$\left.\begin{array}{l} u' = u_0, T = T_W[1+\varepsilon f(t)], C = C_W[1+\varepsilon f(t)], y = 0 \\ u' = 0, v' = 0, T = T_\infty, C = C_\infty \to y \to \infty, q = 0, q = 1, C = 1, C' = C_\infty \\ q' = [G_r T' + G_C C], \theta = 1, \theta \to 0 \end{array}\right\} \quad (2.1.09)(a,b)$$

Use is made of the following non-dimensional equation in the above non-dimensional procedure.

$$\left.\begin{array}{l} (U,V) = (u',v'), t = (t,u_0), T, T_\infty = T, T_\infty / T_\infty \\ \theta = \theta_W[1+\varepsilon f(t)], C = C_W[1+\varepsilon f(t)], u = 0, v = 0, C = 1 \\ q = u + iv, \dfrac{\partial q}{\partial y} = \dfrac{\partial u}{\partial y} + i\dfrac{\partial v}{\partial y}, y = y'\dfrac{U_0}{V}, \dfrac{\mu u}{\xi} = \chi^2 q, i = \sqrt{-1} \end{array}\right\} \quad 2.1.10$$

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Where

$$E = \frac{\upsilon\Omega}{u_0} = \text{Rotational parameter}$$

$$S_C = \frac{\mu}{D_m} = \text{Schmidt number}$$

$$\chi^2 = \frac{\mu\upsilon}{\xi u_0^2} = \text{Prosity parameter}$$

$$G_r = \frac{g\beta_T T v}{u_0^3} = \text{Grashof number due temperature}$$

$$G_C = \frac{g\lambda_C T v}{u_0^3} = \text{Grashof parameter due to concentration}$$

$$\frac{\rho C_P}{K} = P_r = \text{Prandtl number}$$

$$R = \frac{4v\alpha k_\infty^3 v^2}{\rho K u^2 C_P} = \text{Radiation parameter}$$

$$D_C = \frac{D_m k_r C_\infty^2}{k T_\infty \xi C_P} = \text{Diffusion due to concentration}$$

$$S_t = \frac{K_T T_\infty D_m}{D_m \xi u} = \text{Diffusion due to temperature}$$

$$K = \frac{k4C_\infty^3 \alpha v^2}{u_0} = \text{Chemical reaction}$$

$\Omega = (0, \Omega, 0) =$ The whole configuration rotates about the y-axis with angular velocity.

The statement of the problem therefore is to solve equation 2.1.06-2.1.08 based on the boundary conditions of equation 2.1.09 (a,b). Following Opara et al (1994), Equation 2.1.06-2.1.08 are solved by invoking to equation 2.1.09 (a,b) with out loss of generality which involves step by step numerical integration by using the explicit finite differential scheme.

However, in order to analyse the solution, it could be possible to adopt regular perturbation scheme for the independent variable of the type and this is the problem of this research; and to solve this we follow the example of Opara et al (1997) for which,

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$$\left.\begin{array}{l}\theta_{y,t} = \theta_y^0 + \varepsilon\theta_{y,t}^{(')} + \ldots \\ C_{y,t} = C_y^0 + \varepsilon C_{y,t}^{(')} + \ldots \\ q_{y,t} = q_y^0 + \varepsilon q_{y,t}^{(')} + \ldots\end{array}\right\} \quad 2.1.11(a,b,c)$$

Where,

q_y^0 = Velocity field steady state component
θ_y^0 = Temperature field steady state component
C_y^0 = Concentration field steady state component
$q_y^{(')}$ = Velocity field unsteady state component
$\theta_y^{(')}$ = Temperature field unsteady state component
$C_y^{(')}$ = Concentration field unsteady state component

Substituting equation 2.1.11 (a,b,c) into equations 2.1.06-2.1.08 respectively, the above equations reduce the problem into a set of zero order equations as shown in equations 2.1.12-2.1.14, which are characterized by unsteady state flow with eigen values peculiar with boundary conditions presented in equation 2.1.15 (a,b) below,

$$\frac{\partial q^0}{\partial t} + \left(2i\mathrm{E} + \chi^2\right)q^0 = \frac{\partial^2 q^0}{\partial y^2} + G_r\left(\theta^0 - 1\right) + G_C\left(C^0 - 1\right) \quad 2.1.12$$

$$\frac{\partial^2 \theta^0}{\partial y^2} - P_r R\left(\theta^{(0)4} - 1\right) + D_C \frac{\partial^2 C^{(0)}}{\partial y^2} = 0 \quad 2.1.13$$

$$\frac{\partial^2 C^0}{\partial y^2} - k_r S_C\left(\theta^{(0)4} - 1\right) + S_t \frac{\partial^2 \theta^{(0)}}{\partial y^2} = 0 \quad 2.1.14$$

Boundary conditions:

$$\left.\begin{array}{l} q^0 = 1, \theta = \theta_W\left[1 + \varepsilon H(t)\right]\varepsilon^0, q = \theta_W, C^0 = C_W, q^0 = 1, y \to 0 \\ \theta^0 + \theta = \theta_W\left[1 + \varepsilon H(t)\right], \theta^0 = 1, C^0 = 1, q^{(0)} = 0 \text{ as } y \to \infty \\ C^0 + C = C_W\left[1 + \varepsilon H(t)\right], C = C_W\left[1 + \varepsilon H(t)\right]\theta^0, \varepsilon, << 1, \text{Signifying low} \\ \qquad \text{speed incompressible flow} \end{array}\right\} \quad 2.1.15\ (a,b)$$

In another development, we substitute equation 2.1.11 (a,b,c) into equations 2.1.06-2.1.08 to obtained first order equation characteristics of unsteady state.

$$\frac{\partial q^{(1)}}{\partial y} + 2i\mathrm{E}q^{(1)} = \frac{\partial^2 q^{(1)}}{\partial y^2} - \chi^2 q^{(1)} + G_r\theta^{(1)} + G_C C^{(1)} \quad 2.1.16$$

$$\frac{\partial^2 \theta}{\partial y^2} - 4P_r R\left(\theta^{(1)3} - 1\right) + D_C \frac{\partial^2 C^{(1)}}{\partial y^2} \quad 2.1.17$$

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$$\frac{\partial^2 C}{\partial y^2} - 4k_r S_C \left(C^{(1)3} - 1\right) + S_t \frac{\partial^2 \theta^{(1)}}{\partial y^2} \qquad 2.1.18$$

The boundary conditions for equation 2.1.16-2.1.18 are as follows:

$$\left.\begin{array}{l} \theta > 0, y > 0, \theta^1 = \theta_W, C^1 = C_W, q^1 = 0 \text{ for } y \to 0 \\ y = 0, q_y^{'} = P_r \theta^{'} + S_C C^{(1)}, \theta^{'} = 0, C^{'} = 0, q = 0 \text{ as } y \to \infty \\ q_y^{'} = G_r \theta + G_C C \end{array}\right\} \quad 2.1.19 \text{ (a,b)}$$

However, in this research work we shall assume the chemical reaction is greater than zero (homogenous mixture). Similarly, the medium in this research is porous and both the radiation and chemical reaction are in combination. Hence, equations 2.1.13 and 2.1.14 were modified, transformed and reduce to 2.1.20 and 2.1.21 below.

$$\frac{\partial^2 C}{\partial y^2} = S_t \frac{\partial^2 \theta}{\partial y^2} \qquad 2.1.20$$

$$\frac{\partial^2 \theta}{\partial y^2} = D_C \frac{\partial^2 C}{\partial y^2} \qquad 2.1.21$$

On rearranging we substitute equation 2.1.20 into 2.1.13 to obtain equation 2.1.22 below.

$$\frac{\partial^2 \theta}{\partial y^2} + D_C S_t \frac{\partial^2 \theta}{\partial y^2} - P_r R\left(\theta^{04} - 1\right) \qquad 2.1.22$$

We substitute equation 2.1.21 into equation 2.1.14 to get

$$\frac{\partial^2 C}{\partial y^2} + D_C S_t \frac{\partial^2 C}{\partial y^2} - K_r S_C \left(C^{04} - 1\right) \qquad 2.1.23$$

In general, the complete statement of the problem is a solution of equations 2.1.12, 2.1.13, 2.1.14, 2.1.17, 2.1.18, 2.1.20, 2.1.21, 2.1.22, and 2.1.23 respectively, without great loss in generality and subject to boundary conditions presented in equations 2.1.15 (a,b) and 2.1.19 (a,b).

Nevertheless, this study shall be restricted to the following: temperature field, concentration field and velocity field; the ratio of marginal solution to the asymptotic solution will be employed to solve annalistically the steady state for the temperature and concentration. Numerical analysis will be used graphically to obtain the result of Prandtl (P_r), Schmidt (S_C), Chemical reaction (K) and Radiative flux, R through the rotating medium.

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METHOD OF SOLUTIONS

To determine the thermal stability state components of temperature profile θ, concentration profile C, Schmidt profile S_C and Prandtl profile P_r on the effect of chemical reaction, radiative heat flux combined with rotation of the fluid in a porous medium, equations 2.1.13, 2.1.14, 2.1.22, 2.1.23 and 2.1.07, 2.1.08 where rearranged to obtain the following equations:

$$\frac{d^2\theta}{dy^2} = \left(\frac{RP_r}{1 - S_t D_C}\right)\theta^{(0)4} - 1 \quad 3.1.01(a)$$

$$\frac{d^2C}{dy^2} = \left(\frac{k_r S_C}{1 - S_t D_C}\right)C^{(0)4} - 1 \quad 3.1.01(b)$$

$$\frac{d^2\theta}{dy^2} = \left(\frac{4RP_r}{1 - S_t D_C}\right)\theta^{(0)3} - 1 \quad 3.1.01(c)$$

$$\frac{d^2C}{dy^2} = \left(\frac{4k_r S_C}{1 - S_t D_C}\right)C^{(0)3} - 1 \quad 3.1.01(d)$$

Equations 3.1.01 (a,b,c,d) are non-linear and non-homogenous. Generally, it involves a heuristic approach by using numerical integration of the explicit differential scheme. It is assumed that the above equations are integrated together and are operating in the same medium; they are not operating independently as the fluid rotates.

Equations 3.1.01 (a) and 3.1.01 (c) are multiplied by $\frac{d\theta}{dy}$, while equations 3.1.01 (b) and 3.1.01 (d) are multiplied by $\frac{dC}{dy}$ on both sides and expand without loss of generality. Furthermore, we employ another integration scheme in equation 3.1.01 (a,b,c,d) with respect to y from θ_0 to θ_∞ and C_0 to C_∞ respectively. In order to find the function which satisfy the given differential equation and particular condition, equation 2.1.19 (a,b) was subjected to the differential and integral scheme and equation 3.1.01 (a,b,c,d) was reduced to the following:

$$y = \sqrt{\frac{5}{2}\left(\frac{1 - S_t D_C}{RP_r}\right)}\int_{\theta_0}^{\theta_\infty} \frac{d\xi}{\left(\xi^5 - 5\xi + 4\right)^{1/2}} \quad 3.1.02$$

$$y = \sqrt{\frac{5}{2}\left(\frac{1 - S_t D_C}{k_r S_C}\right)}\int_{C_0}^{C_\infty} \frac{d\xi}{\left(\xi^5 - 5\xi + 4\right)^{1/2}} \quad 3.1.03$$

$$y = \sqrt{\frac{1}{2}\left(\frac{1 - S_t D_C}{RP_r}\right)}\int_{\theta_0}^{\theta_\infty} \frac{d\xi}{\left(\xi^5 - 5\xi + 1\right)^{1/2}} \quad 3.1.04$$

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$$y=\sqrt{\frac{1}{2}\left(\frac{1-S_tD_C}{k_rS_C}\right)}\int_{C_0}^{C_\infty}\frac{d\xi}{\left(\xi^5-5\xi+1\right)^{1/2}} \qquad 3.1.05$$

Without any loss of generality, equations 3.1.02 and 3.1.03 are the same; similarly, equations 3.1.04 and 3.1.05 are the same and are in the same medium. We further assumed S_t and D_C are constant; equations 3.1.02, 3.1.03, 3.1.04 and 3.1.05 are reduced to obtain the following equations:

$$y=\left(\frac{5}{2RP_r}\right)^{1/2}\int_{\theta_0}^{\theta_\infty}\frac{d\xi}{\left(\xi^5-5\xi+4\right)^{1/2}} \qquad 3.1.06$$

$$y=\left(\frac{5}{2k_rS_C}\right)^{1/2}\int_{C_0}^{C_\infty}\frac{d\xi}{\left(\xi^5-5\xi+4\right)^{1/2}} \qquad 3.1.07$$

$$y=\left(\frac{1}{2RP_r}\right)^{1/2}\int_{\theta_0}^{\theta_\infty}\frac{d\xi}{\left(\xi^5-5\xi+1\right)^{1/2}} \qquad 3.1.08$$

$$y=\left(\frac{1}{2k_rS_C}\right)^{1/2}\int_{C_0}^{C_\infty}\frac{d\xi}{\left(\xi^5-5\xi+1\right)^{1/2}} \qquad 3.1.09$$

Equation 2.1.20, 2.1.21 were solved without any loss of generality

$$C^{(0)}=-S_t\theta+A_1+A_2 \qquad 3.1.10(a)$$

$$C^{(0)}=-D_CC+B_1+B_2 \qquad 3.1.10(b)$$

Where equations 3.1.10a and 3.1.10b reduce to

$$\infty=\frac{C^0+S_t\theta-\left(C_W+S_t\theta_W\right)}{A_1} \qquad 3.1.11(a)$$

$$\infty=\frac{\theta^0+D_CC-\left(\theta_W+D_CC_W\right)}{B_1} \qquad 3.1.11(b)$$

Equations 3.1.11a, 3.1.11b are satisfied if and only if $A_1=0, B_1=0$ respectively. The following equations were obtained:

$$C^{(0)}=-S_t\theta^{(0)}+\left(C_W+S_t\theta_W\right) \qquad 3.1.12(a)$$

$$\theta^{(0)}=-D_CC^{(0)}+\left(\theta_W+D_CC_W\right) \qquad 3.1.12(b)$$

Substituting equations 3.1.12(a,b) into equation 2.1.12 respectively, we get a non-homogenous second order differential equation in q^0.

$$\left.\begin{array}{l}\frac{d^2q}{dy^2}-\left(\chi^2+2i\mathrm{E}\right)q^0-G_r\left(-D_C C+\left(\theta_W+D_C C_W\right)-1\right)\\+G_C\left(-S_t\theta^0+\left(C_W+S_t\theta_W\right)-1\right)\end{array}\right\}\qquad 3.1.13$$

The homogenous equation is,

$$\frac{d^2q}{dy^2}-\left(\chi^2+2i\mathrm{E}\right)q^0=0\qquad 3.1.14$$

$$D^2-\left(\chi^2+2i\mathrm{E}\right)q=0\qquad 3.1.15$$

$$D^2=\left(\chi^2+2i\mathrm{E}\right)q\qquad 3.1.16$$

The complementary function q_C is,

$$D^0=\left(\chi^2+2i\mathrm{E}\right)^{1/2}=\alpha\qquad 3.1.17$$

$$\left.\begin{array}{l}q^0_{(y)}=D_1e^{m_1y}+D_2e^{-m_2y}\\q_{(y)}=D_1e^{\alpha y}+D_2e^{-\alpha y}\end{array}\right\}\qquad 3.1.18(a,b)$$

Consequently, upon bounded equation 3.1.18(b) above, we obtained,

$$q^{(0)}_{(y)}=e^{-m_2y}=\exp\left(\chi^2+2i\mathrm{E}\right)^{1/2}y\qquad 3.1.19$$

From equations 3.1.06, 3.1.07, 3.1.08 and 3.1.09 the particular integral q^0_p is given as follows:

$$q^0_{(p)}=\left(\frac{5}{2RP_r}\right)^{1/2}\int_{\theta_0}^{\theta_\infty}\sinh\left(\chi^2+2i\mathrm{E}\right)^{1/2}\left[y\left(\theta^0\right)-y(\xi)\right]\left[\left(G_r-G_CS_t\right)(\xi-1)\right]d\xi\qquad 3.1.20$$

$$q^0_{(p)}=\left(\frac{5}{2k_rS_C}\right)^{1/2}\int_{C_0}^{C_\infty}\sinh\left(\chi^2+2i\mathrm{E}\right)^{1/2}\left[y\left(C^0\right)-y(\xi)\right]\left[\left(G_C-G_rD_C\right)(\xi-1)\right]d\xi\qquad 3.1.21$$

$$q^0_{(p)}=\left(\frac{1}{2RP_r}\right)^{1/2}\int_{\theta_0}^{\theta_\infty}\sinh\left(\chi^2+2i\mathrm{E}\right)^{1/2}\left[y(\theta)-y(\xi)\right]\left[\left(G_r-G_CS_t\right)(\xi-1)\right]d\xi\qquad 3.1.22$$

$$q^0_{(p)}=\left(\frac{1}{2k_rS_C}\right)^{1/2}\int_{C_0}^{C_\infty}\sinh\left(\chi^2+2i\mathrm{E}\right)^{1/2}\left[y(C)-y(\xi)\right]\left[\left(G_C-G_rD_C\right)(\xi-1)\right]d\xi\qquad 3.1.23$$

Adding equations 3.1.07, 3.1.09 to 3.1.19, 3.1.20, 3.1.21, 3.1.22 and 3.1.23 respectively we obtain the following:

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$$\left.\begin{aligned} q^0_{(y)} &= \exp\left[\left(-\chi^2+2i\mathrm{E}\right)^{1/2} y\right]-\left(\frac{5}{2RP_r}\right)^{1/2}\int_{\theta_0}^{\theta_W}\int_{C_0}^{C_W}\left(\frac{5}{2k_rS_C}\right)^{1/2}\sinh\left(\chi^2+2i\mathrm{E}\right)\\ &\frac{\left[y\left(\theta,C^0\right)-y(\xi)\right](\xi-1)d\xi}{\left(\xi^5-5\xi+4\right)^{1/2}}\end{aligned}\right\} \quad 3.1.24$$

$$\left.\begin{aligned} q^0_{(y)} &= \exp\left[\left(-\chi^2+2i\mathrm{E}\right)^{1/2} y\right]-\left(\frac{1}{2RP_r}\right)^{1/2}\int_{\theta_0}^{\theta_W}\int_{C_0}^{C_W}\left(\frac{1}{2k_rS_C}\right)^{1/2}\sinh\left(\chi^2+2i\mathrm{E}\right)\\ &\frac{\left[y\left(\theta,C^0\right)-y(\xi)\right](\xi-1)d\xi}{\left(\xi^5-5\xi+1\right)^{1/2}}\end{aligned}\right\} \quad 3.1.25$$

In order to solve equations 2.1.16, 2.1.17 and 2.1.18, we employ a function that will satisfy the given particular differential equation and the particular boundary conditions in equation 2.1.19(a,b). We obtain the laplace transform with respect to time and denoting the transformed variable by ξ, placing the tilde over the transformed function the equation satisfied by $(q^{(1)}),\theta^{(1)},C^{(1)}$, in equations 2.1.16, 2.1.17 and 2.1.18 reduce to,

$$\frac{d^2q^{'}}{dy^2}-\left(\chi^2+2i\mathrm{E}+\xi\right)q^{()}=-G_r\theta^{()}+G_rC^{()} \quad 3.1.26$$

$$\frac{d^2\theta}{dy^2}-\left(4RP_r\theta^{(0)3}+\xi\right)\theta^{()}=0 \quad 3.1.27$$

$$\frac{d^2C^{()}}{dy^2}-\left(4k_rS_CC^{(0)3}+\xi\right)C^{()}=0 \quad 3.1.28$$

With the boundary conditions, substituting equation 2.1.19(a,b) into 3.1.27 and 3.1.28 we get the solutions for $\theta^{()}$ and $C^{()}$ as follows:

$$\theta^{()}=\exp\frac{\left(-4RP_r+\xi\right)^{1/2}y}{\xi} \quad 3.1.29$$

$$C^{()}=\exp\frac{\left(-4k_rS_C+\xi\right)^{1/2}y}{\xi} \quad 3.1.30$$

Employing the shifting theorem and taking the inverse laplace transform on equations 3.1.29 and 3.1.30 we obtain the following equations:

$$\theta^{()}=\frac{1}{2}\left\{\begin{aligned}&e^{-(4RP_r)^{1/2}y}\beta_y\operatorname{erfc}\left[\frac{Y}{(2t)^{1/2}}-\left(4RP_rt\right)^{1/2}\right]\\&+e^{(4RP_r)^{1/2}}\beta_y\operatorname{erfc}\left[\frac{Y}{(2t)^{1/2}}+\left(4RP_rt\right)^{1/2}\right]\end{aligned}\right\} \quad 3.1.31$$

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$$C^{()} = \frac{1}{2}\left\{\begin{array}{l} e^{-(4k_r S_C)^{1/2} y}\gamma_y \operatorname{erfc}\left[\frac{Y}{(2t)^{1/2}} - (4k_r S_C)^{1/2}\right] \\ + e^{(4k_r S_C)^{1/2}}\gamma_y \operatorname{erfc}\left[\frac{Y}{(2t)^{1/2}} + (4k_r S_C t)^{1/2}\right] \end{array}\right\} \quad 3.1.32$$

When we consider θ_W, C_W, β and γ arbitrary, and approximate θ^0 and $C^{(0)}$ by,

$$\theta^{(0)3} = \left(\theta_W{}^{(0)3} - 1\right)e^{-2\beta y} + 1 \quad 3.1.33$$

$$C^{(0)3} = \left(C_W{}^{(0)3} - 1\right)e^{-2\gamma \cdot y} + 1 \quad 3.1.34$$

From equations 3.1.33 and 3.1.34 the solution for $\theta^{()}$ and $C^{()}$ were obtain and are reduced to get the marginal state solution.

$$\frac{\theta^0}{\theta_W} = \frac{J_n(4RP_r + \xi)^{1/2}\left(i\eta e^{-\beta_y}\right)}{\xi J_n(4RP_r + \xi)^{1/2} in} = \frac{I_n(RP_r + \xi)^{1/2}\left(\eta e^{-\beta_y}\right)}{\xi I_n(RP_r + \xi)(\eta)} \quad 3.1.35$$

$$\frac{C^0}{C_W} = \frac{J_n(4k_r S_C + \xi)^{1/2}\left(i\eta e^{-\gamma_y}\right)}{\xi J_n(4k_r S_C + \xi)^{1/2} in} = \frac{I_n(k_r S_C + \xi)^{1/2}\left(\eta e^{-\gamma_y}\right)}{\xi I_n(k_r S_C + \xi)^{1/2}(\eta)} \quad 3.1.36$$

Equations 3.1.35 and 3.1.36 are the marginal state solution of the temperature and concentration respectively, were $\eta = 4RP_r\left(\theta^{(0)3} - 1\right), 4k_r S_C\left(C^{(0)3} - 1\right)$ and $J_n(x), I_n(x)$ are the Bessel and modified Bessel function of the first kind. Equations 3.1.35 and 3.1.36 have simple pole at $\xi = 0$ and another at $\xi = 4RP_r$ and $4k_r S_C$. However, without any lose of generality equation 3.1.35 and 3.1.36 could be inverted by using the Bromwich contour with a suitable branch cut and the result obtain as follows:

$$\left.\begin{array}{l} \frac{\theta^{()}}{\theta_W} = \frac{I(4RP_r)^{1/2}\left(\eta e^{-\beta_y}\right)}{I(4RP_r)^{1/2}(\eta)} + \\ \frac{e^{-4RP_r t}}{2\pi i}\left\{\int_0^\infty\left[\frac{e^{-x} I_{ix}\left(\eta e^{-\beta_y}\right)}{(x + 4RP_r)I_{(ix)^{1/2}}(\eta)} - \int_0^\infty \frac{e^{-ix} I_{ix}\left(\eta e^{\beta_y}\right)}{(x + 4RP_r)I_{-ix}(\eta)}\right]dy\right\} \end{array}\right\} \quad 3.1.37$$

$$\left.\begin{array}{l} \frac{C^{()}}{C_W} = \frac{I(4k_r S_C)^{1/2}\left(\eta e^{-\gamma_y}\right)}{I(4k_r S_C)^{1/2}(\eta)} + \\ \frac{e^{-4KS_C t}}{2\pi i}\left\{\int_0^\infty\left[\frac{e^{-x} I_{ix}\left(\eta e^{-\gamma_y}\right)}{(x + 4k_r S_C)I_{(ix)^{1/2}}(\eta)} - \int_0^\infty \frac{e^{-ix} I_{ix}\left(\eta e^{\gamma_y}\right)}{(x + 4k_r S_C)I_{-ix}(\eta)}\right]dy\right\} \end{array}\right\} \quad 3.1.38$$

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Equations 3.1.37 and 3.1.38 are highly complex is expedient to take limiting value with,

$$J_n(x) \approx \frac{1}{(2\pi\eta)^{1/2}}\left(\frac{e^x}{2n}\right)^n, n \to \infty \qquad 3.1.39$$

$$\frac{\theta^{()}}{\theta_W} \approx \exp\left[\frac{-\beta_y(4RP_r+\xi)^{1/2}}{\xi}\right] \qquad 3.1.40$$

$$\frac{C^{()}}{C_W} \approx \exp\left[\frac{-\gamma_y(4k_rS_C+\xi)^{1/2}}{\xi}\right] \qquad 3.1.41$$

With the condition that $\xi \to \infty$ in the form of equation 3.1.29 and 3.1.30 we obtain the following:

$$\frac{\theta^{()}}{\theta_W} \approx \frac{1}{2}\left\{e^{-2\beta(RP_r)^{1/2}}\operatorname{erfc}\left[\frac{\beta_y}{(2t)^{1/2}}-(4RP_rt)^{1/2}\right]\right\}+e^{-\beta(RP_r)^{1/2}}\operatorname{erfc}\left[\frac{\beta_y}{(2t)^{1/2}}+(4RP_rt)^{1/2}\right]$$

3.1.42

$$\frac{C^{()}}{C_W} \approx \frac{1}{2}\left\{e^{-2\gamma(KS_C)^{1/2}}\operatorname{erfc}\left[\frac{\gamma_y}{(2t)^{1/2}}-(4k_rS_Ct)^{1/2}\right]\right\}+e^{-\gamma(KS_C)^{1/2}}\operatorname{erfc}\left[\frac{\gamma_y}{(2t)^{1/2}}+(4KS_Ct)^{1/2}\right]$$

3.1.43

When $4RP_r$ is large and of order 0.1, k_rS_C is also large and of order 0.1, then

$$\frac{\theta^{()}}{\theta_W} \approx \operatorname{erfc}\left[\frac{\beta_y}{(2t)^{1/2}}\right] \qquad 3.1.44$$

$$\frac{C^{()}}{C_W} \approx \operatorname{erfc}\left[\frac{\gamma_y}{(2t)^{1/2}}\right] \text{as } t \to \infty \qquad 3.1.45$$

Also, as $\eta \to 0$

$$I_n(x) = I_0(x) \to nK_0(x),$$

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Where $K_0(x)$ is the modified Bessel function of the second kind of order zero, we obtain

$$\frac{\theta^{()}}{\theta_W} \approx \frac{1}{I_0(\eta)}\left\{I_0(\eta e^{-\beta_y}).\frac{1}{\xi} + \left[\frac{I_0(\eta e^{-\beta_y})}{I_0(\eta)}K_0(\eta) - K_0\left(\eta e^{-\beta_y}\right)\right](\xi + 4RP_r)^{1/2}\right\} \qquad 3.1.46$$

Inverting equation 3.1.46 we get

$$\frac{\theta^{()}}{\theta_W} \approx \frac{1}{I_0(\eta)}\left\{\begin{array}{l} I_0\left(\eta e^{-\beta_y}\right) + \left[K_0(\eta)I_0(\eta e^{-\beta_y}) - K_0(\eta e^{-\beta_y})\right] \\ .e^{-4RP_r}\left[\frac{1}{\pi} + (4RP_r)^{1/2}.e^{-4RP_r t}\operatorname{erfc}(4RP_r)^{1/2}\right] \end{array}\right\} \text{as } t \to \infty \qquad 3.1.47$$

$$\frac{C^{()}}{C_W} \approx \frac{1}{I_0(\eta)}\left\{I_0(\eta e^{-\gamma_y}).\frac{1}{\xi} + \left[\frac{I_0(\eta e^{-\gamma_y})}{I_0(\eta)}K_0(\eta) - K_0\left(\eta e^{-\gamma_y}\right)\right](\xi + 4KS_C)^{1/2}\right\} \qquad 3.1.48$$

Which on inverting equation 3.1.48, we obtain

$$\frac{C^{()}}{C_W} \approx \frac{1}{I_0(\eta)}\left\{\begin{array}{l} I_0\left(\eta e^{-\gamma_y}\right) + \left[K_0(\eta)I_0(\eta e^{-\gamma_y}) - K_0(\eta e^{-\gamma_y})\right] \\ .e^{-4KS_C}\left[\frac{1}{\pi t} + (4k_r S_C)^{1/2}.e^{-4KS_C t}\operatorname{erfc}(4k_r S_C)^{1/2}\right] \end{array}\right\} \text{as } t \to \infty \qquad 3.1.49$$

Equations 3.147 and 3.1.49 are the asymptotic state of the solution.

For the velocity profile, $q^{()}$ in equation 2.1.16 when solved by putting $\varsigma^2 = \varsigma + \chi^2 + 2i\mathrm{E}$, we obtain

$$q^{(1)} = \frac{1}{2}K\int_0^\infty \frac{e^{-\varsigma(y-\bar{y})}}{\varsigma}\theta(y)dy$$

Applying Laplace transform of inverse, we have

$$L^{-1}F(s) = f(t) = \frac{1}{2\pi j}\int_{C_0 - j\infty}^{C + j\infty} F(S)e^{ts}ds$$

$$\left.\begin{array}{l} L\theta^{-1}\left[e\frac{^{-\varsigma(y-\bar{y})}}{\varsigma}\right] = e^{-(\chi^2+2i\mathrm{E})t}L\theta^{-1}\left[e\frac{^{-\xi^{1/2}}(y-\bar{y})}{\xi^{1/2}}\right] = \\ e^{-(\chi^2+2i\mathrm{E})t}\frac{1}{(\pi t)^{1/2}} - e\frac{-(y-\bar{y})}{t} \end{array}\right\} \qquad 3.1.50$$

Employing convolution theorem on equation 3.1.50 we obtain,

$$q^{(1)} = \frac{1}{(2\pi)^{1/2}}\int_0^y dy\int_0^t \frac{1}{\tau^{1/2}}e^{-(\chi^2+2i\mathrm{E})\tau}e\frac{-(y-\bar{y})}{4\tau}.\left[\theta^{(1)}(t-\tau)d\tau + C^{(1)}(t-\tau)d\tau\right] \qquad 3.1.51$$

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Where, $L\left[\int_0^t F_1(x) f_2(t-x)dx\right] = F_1(s) * F_2(s)$

$$f * g = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x-\xi : x-\beta) g(\xi,\beta) d\xi d\beta$$

This is the completion of the solution.

CONSEQUENCE OF THE STEADY STATE.

If we recall, equations 3.1.35 and 3.1.36 are the marginal state solution for the temperature and concentration. Similarly, equations 3.1.47 and 3.1.49 are the asymptotic state solution for the temperature and concentration respectively.

Where,

$$\eta = 4RP_r(\theta_W - 1)$$

$$\varsigma = 4k_r S_C(C_W - 1)$$

$J_n(x) =$ Bessel function of the first kind

$I_n(x) =$ Modified Bessel function for the first kind

$K_0(x) =$ The modified Bessel function of the second kind

Taking the ratio of equation 3.1.35 to 3.1.47 as $\frac{\theta^1}{\theta_W}$ and the ratio of equation 3.1.36 to 3.1.49 as $\frac{C^1}{C_W}$, we establish the stable steady state conditions which states that if the ratio of the marginal state of the solution to the ratio of the asymptotic state solution for the temperature and concentration is less than or equal to one (1) then the system is assumed stable either with respect to temperature profile or with respect to the concentration profile. Rainville E.D. (1960).

Equation 3.1.35 divided by equation 3.1.47

$$\frac{3.1.35}{3.1.47} \leq 1 \qquad 3.2.01$$

Similarly, equation 3.1.36 divided by equation 3.1.49

$$\frac{3.1.36}{3.1.49} \leq 1 \qquad 3.2.02$$

Solving equation 3.2.01 we obtain the value of θ_0 as follows:

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$$\theta_0 = \frac{1}{\xi}\left(4RP_r + \xi\right)^{\frac{1}{2}(1-\eta)} \leq 1 \qquad 3.2.03$$

In a similar approach, we solve for equation 3.2.02 and have C_0 as follows:

$$C_0 = \frac{1}{\xi}\left(4KS_C + \xi\right)^{\frac{1}{2}(1-\eta)} \leq 1 \qquad 3.2.04$$

The results of equations 3.2.03 and 3.2.04 above confirm the stable steady state of the system if $\eta \leq 1$, i.e. $\eta = 0.5$ for the radiation and chemical reaction.

This is the complete solution of the problem of the stable steady state analysis in a porous medium on the influence of rotation, radiation and chemical reaction. From assumption, rotation may likely alter the condition of stable steady state.

RESULT AND DISCUSSIONS.

The formulation of the problem of two-dimensional stable steady state analysis in an incompressible fluid flow in a porous medium with the combined effects of rotation, radiation and chemical reaction were presented. By invoking the differential approximation for the chemical reaction, radiation and rotation in optically thin medium, the non-linear problem was tackled by asymptotic approximation resulting to a stable steady state on which is superimposed a first order and zero order transient flow.

To comprehend the totality on the effect of the dependent parameter on the flow state parameter, use is made of the following numerical computation for concentration C_0 for various values of chemical reaction parameter $K = 0.2, 0.8, 1.20.$ $S_C = 0.24, C_0 = 1,$ for concentration for various values of (S_C) Schmidt, $S_C = 0.24, 0.40, 0.60, K = 0.20.$ For temperature, θ_0 for the various values of the radiation parameter, $R = 0.20, 0.8, 1.20,$ Prandtl $(P_r) = 0.24$ and for the temperature $\theta_0 = 1,$ for various values of Prandtl number , $P_r = 0.24, 0.40, 0.60, R = 0.20.$ Following the evaluation of the marginal state solution on the ratio of the asymptotic state solution on the temperature and concentration profile respectively, equations 3.2.03, 3.2.04 and equation 2.1.11(a,b,c) gives the solution for the temperature and concentration field where evaluated by numerical integration.

Fig. 2, 3, 4 and 5 below shows the graphical display representation of the solution on equation , 3.2 01,3.2.02, 3.2 03, 3.2.04 respectively the problems.

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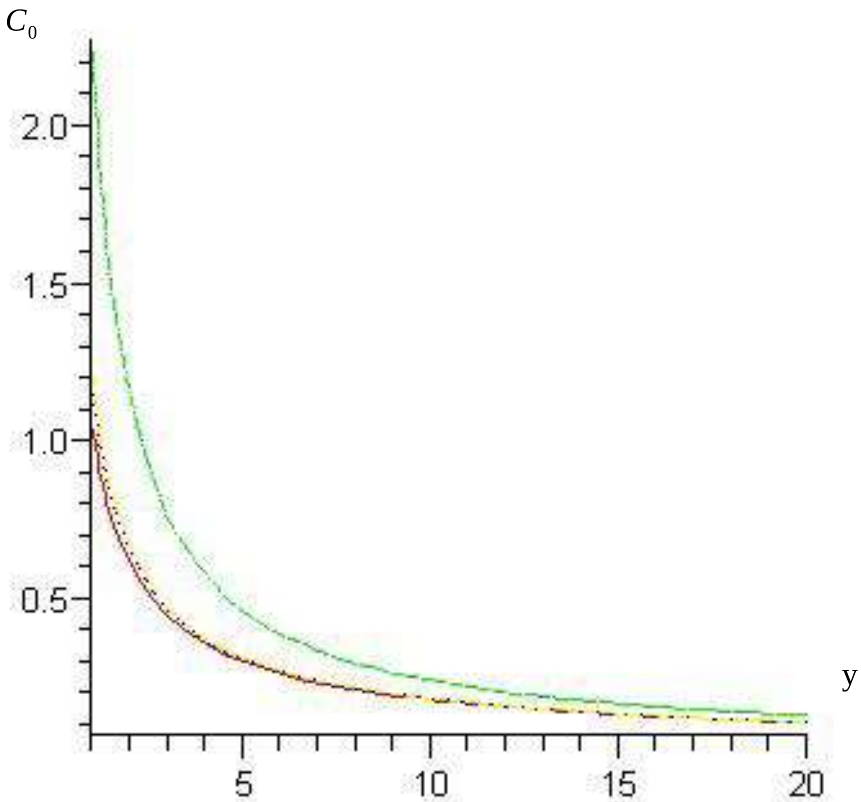

Fig.2: Concentration profile for various values of the chemical reaction parameter k_r

$$k_r = 0.20, 0.80, 1.20$$
$$S_C = 0.24$$

The concentration profile, C_0 shows the plot of the effect of chemical reaction parameter, k_r. The graph shows that the concentration, C_0 decreases as the chemical reaction parameter increases.

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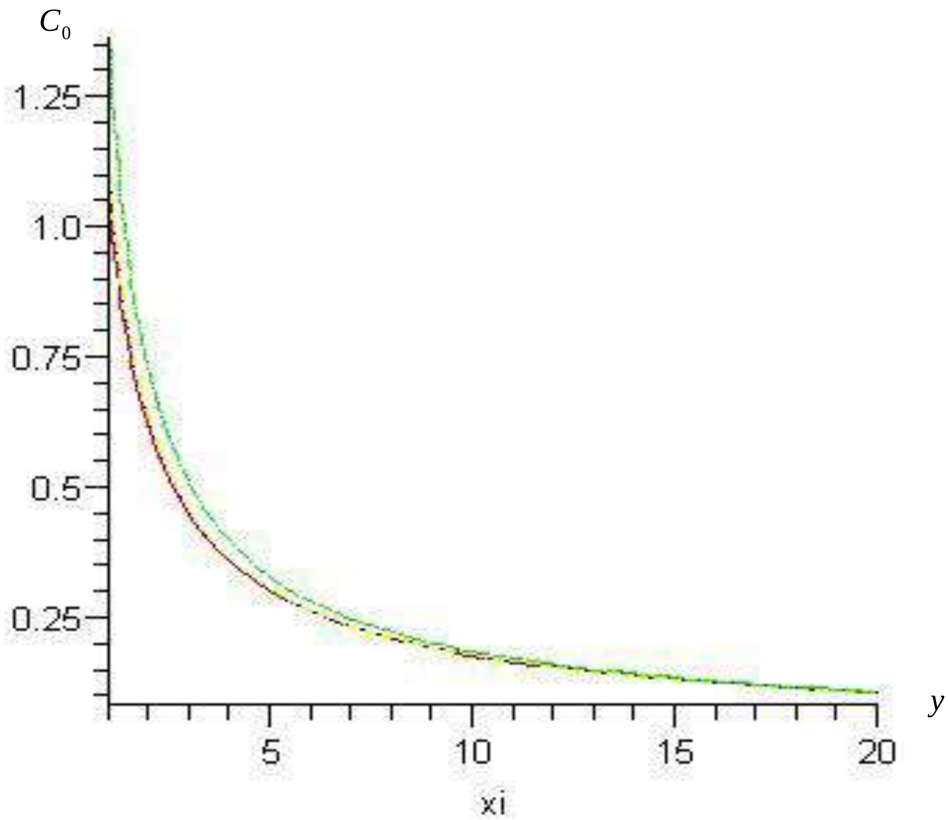

Fig. 3: Concentration profile for various values of the Schmidt number S_C

$$S_C = 0.24, 0.40, 0.60,$$
$$K = 0.20$$

The concentration profile C_0 showing the effect of Schmidt number in the medium has been plotted in figure 3 above; the graph shows that the concentration C_0 decreases as the Schmidt number increases.

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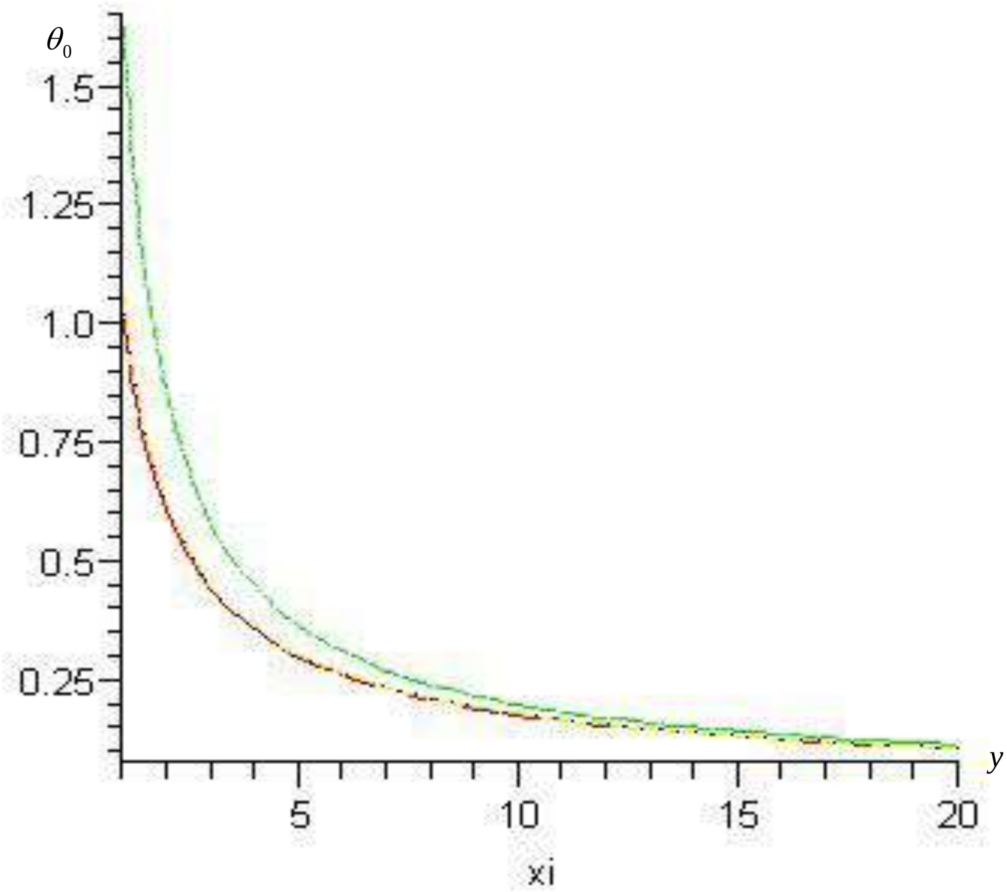

Fig. 4: Temperature profile for various values of radiation parameters R.

$$R = 0.20, 0.80, 1.20,$$
$$P_r = 0.24$$

The temperature profile, θ_0 have been plotted for various values of radiation parameter, R in figure 4 and the graph shows that the temperature, θ_0 decreases as the radiation parameter, R increases.

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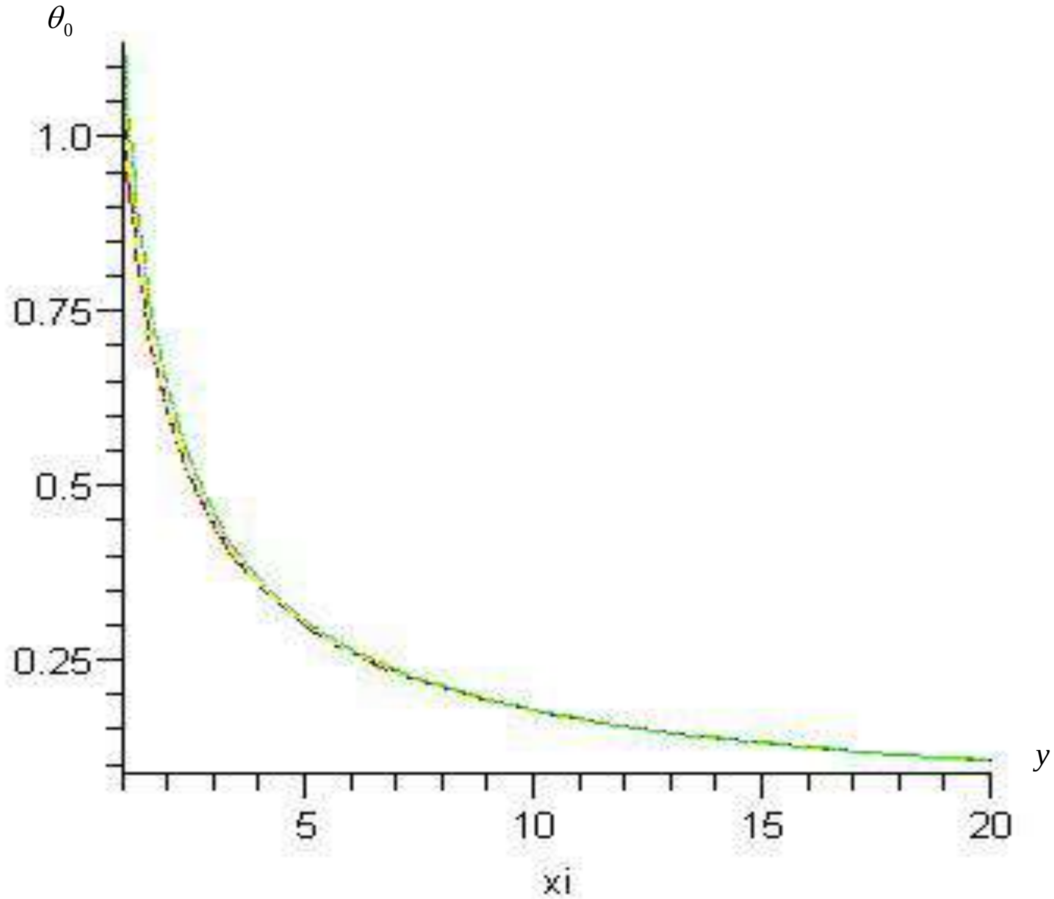

Fig.5: Temperature profile for various values of Prandtl number P_r

$$P_r = 0.24, 0.40, 0.60,$$
$$R = 0.20$$

The temperature, θ_0 showing the effect of the Prandtl number, (P_r) is plotted in figure 5; it is observed that the temperature θ_0 decreases as the Prandtl parameter, P_r increases.

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APPENDIX 1

Integrate Equation 2.0.4

$$\frac{\partial^2 q}{\partial y^2} - 3\alpha^2 q - 16\sigma\alpha T^3 \frac{dT}{dy} = 0 \qquad 2.0.4$$

or $\alpha <<< 1$ for a thin transparent layer

Reduce $\frac{\partial^2 q}{\partial y^2} = 3\alpha^2 q - 16\sigma\alpha T^3 \frac{dT}{dy}$

$$\frac{\partial q}{\partial y} = 4\delta\alpha\left(T^4 - T_\infty\right) \qquad 2.0.5$$

$$\frac{\partial u^{'}}{\partial t^{'}} - 2\Omega\upsilon = \frac{\upsilon\partial^2 u^{'}}{\partial y^2} - \frac{\mu u}{\xi} + g\beta_t\left(T - T_\infty\right) + g\lambda_C\left(C - C_\infty\right) \qquad 1.0$$

Use non-dimensional variable $\left(\theta, \theta_K\right) = \left(T, T_e\right)\Big| T_\infty - C, C_\infty = C, C_\infty \Big| C_\infty$

$$\left.\begin{aligned}
&\left(C, C_\infty\right)\Big| C_\infty \tau = \frac{tu_0^2}{\upsilon} = \left(u, \upsilon\right) = \left(u, \upsilon\right)\Big| u_0 \\
&T^4 - T_\infty^4 = \left(\theta - \theta_1\right)C^4 - C_\infty^4 = \left(C^4 - 1\right) \\
&\left(T^4, T_\infty\right) = T_\infty\left(\theta_1, \theta_2\right), \left(C, C_\infty\right) = C_\infty\left(C, C_\infty\right) \\
&t = \frac{\upsilon}{u_0^2}\tau\left(u^{'}, \upsilon^{'}\right) = u_\infty\left(u^{'}, \upsilon^{'}\right), y^{'} = \frac{\upsilon y}{u_0}
\end{aligned}\right\} \qquad 2.0$$

Equation 1

$$\left.\begin{aligned}
&\frac{\partial v^{'}}{\partial t} = \frac{u_0}{\frac{\upsilon}{u_0^2}}\frac{\partial u}{\partial \tau} - 2ui\Omega v = \frac{\upsilon u_0}{\frac{v^2}{u_0^2}}\frac{\partial^2 u}{\partial y^2} - \frac{\mu u_0}{\xi} + g\beta_\infty TT_\infty\left(\theta - 1\right) \\
&+ g\lambda CC_\infty\left(C - 1\right) + g\lambda_C CC_\infty\left(C - 1\right)
\end{aligned}\right\}$$

$$\frac{u_0^3}{\upsilon}\frac{\partial u}{\partial t} - 2u_0 i\Omega\upsilon = \frac{u_0^2}{\upsilon}\frac{\partial^2 u}{\partial y^2} - \frac{\mu u_0}{\xi}u_0 + g\beta_T T_\infty\left(\theta - 1\right) + g\lambda_C C_\infty\left(C - 1\right) \qquad 3.0$$

Divide through by $\frac{u_0^3}{\upsilon}$ we obtain

$$\frac{\partial u}{\partial \tau} - \frac{2\upsilon\Omega}{u^2} = \frac{\partial^2 u}{\partial y^2} - \frac{\mu\upsilon u}{\xi u_0^2} + \frac{g\beta_C T_\infty \upsilon}{u_0^3}\left(\theta - 1\right) + g\lambda\frac{C_\infty \upsilon u_0}{u_0^3}\left(C - 1\right) \qquad 4.0$$

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$$q = u + iv, \frac{\partial q}{\partial y} = \frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}, \frac{\partial^2 q}{\partial y^2} = \frac{\partial^2 y}{\partial y^2} + i\frac{\partial^2 v}{\partial y^2}$$

The complex number system is therefore a natural extension of the real number system $(i)^2 = -1$

$u =$ Primary axial

$i\upsilon =$ Secondary axial

$$\frac{\partial v}{\partial t} + 2\Omega u = \upsilon\frac{\partial^2 v}{\partial y^2} - \frac{\mu v}{\xi} \qquad 5.0$$

By putting the non-dimensional variable I have

$$\frac{\partial v}{\partial t} - \frac{2\Omega\upsilon u}{u_0^2} = \frac{\partial^2 u}{\partial y^2} - \frac{u\upsilon v}{\xi u_0^2} \qquad 6.0$$

Combining equations 4 and 6 by putting $q = u + iv$

Multiplying equation through 6 by i I have

$$i\frac{\partial v}{\partial \tau} - \frac{2\upsilon\Omega iu}{u_0^2} = i\frac{\partial^2 u}{\partial y^2} - \frac{\mu\upsilon}{\xi u^2}i\upsilon \qquad 7.0$$

Adding equations 4 and 7 I obtained

$$\frac{\partial q}{\partial \tau} + \frac{2\upsilon\Omega iq}{u_0^2} = \frac{\mu\upsilon q}{\xi u_0^3} + \frac{g\beta_C T_\infty \upsilon}{u_0^3}(\theta - 1) + \frac{g\lambda_C C_\infty}{u_0^3}(C - 1)$$

$$\frac{\partial q}{\partial t} + i2\varepsilon q = \frac{\partial^2 q}{\partial y^2} - \chi^2 q + G_r(\theta - 1) + G_C(C - 1) \qquad 2.1.06$$

Energy equation: $\frac{\partial q}{\partial y} = 4\sigma\alpha\left(T^4 - T_0^4\right)$

$$\rho C_P \frac{\partial T}{\partial t} = \frac{k\partial^2 T}{\partial y^2} - \frac{\partial q}{\partial y} + \frac{D_m K_t C_\infty}{\xi C_P}\frac{\partial^2 C}{\partial y^2}$$

$$\rho C_P \frac{\partial T}{\partial t} = \frac{k\partial^2 T}{\partial y^2} - +4\sigma\alpha\left(T^4 - T_\infty^4\right) + \frac{D_m K_t C_\infty}{\xi C_P}\frac{\partial^2 C}{\partial y^2} \qquad 2.1.03$$

Using non-dimensional analysis equation above

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$$\frac{\rho C_P T_\infty}{\frac{\upsilon^2}{u_0^2}}\frac{\partial\theta}{\partial\tau}=\frac{k_r T_\alpha}{\frac{\upsilon^2}{u_0^2}}\frac{\partial^2\theta}{\partial y^2}-4\sigma\alpha T_\alpha^4\left(\theta^4-1\right)+\frac{D_m K_t C_\alpha^2}{\xi\rho C_P\frac{\upsilon^2}{u_0^2}}u_0^2\frac{\partial^2 C}{\partial y^2}$$

$$=\frac{\rho C_P T_\alpha}{\upsilon_0^2}u_0^2\frac{\partial\theta}{\partial T}=\frac{k_r T_\alpha}{\upsilon^2}u_0^2\frac{\partial^2\theta}{\partial y^2}-4\sigma\alpha T_\alpha^4\left(\theta^4-1\right)+\frac{D_m K_t C_\infty^2}{\xi\rho C_P\frac{\upsilon^2}{u_0^2}}\frac{\partial^2 C}{\partial y^2}$$

Divide through by $\frac{k_r T_\infty u_0^2}{\upsilon^2}$

$$\frac{\rho C_P\upsilon}{k}\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial y^2}-\frac{4\sigma\alpha T_\infty^3\upsilon^2}{ku_0^2}\left(\theta^4-1\right)+\frac{D_m k_i C_\alpha^2}{kT_\infty\xi\rho C_P}\frac{\partial^2 C}{\partial y^2}$$

$$P_r\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}-\frac{4\sigma\alpha T_\infty^3\upsilon^2}{ku_0^2}\left(\theta^4-1\right)+\frac{D_m k_i C_i^2}{kT_\infty\xi\rho C_P}\frac{\partial^2 C}{\partial y^2}$$

$$\frac{4\sigma\alpha T_\infty\upsilon^2}{ku_0^2}\frac{\rho C_P\upsilon}{\rho C_P\sigma}=\frac{4\sigma\alpha T_\infty^3\upsilon}{\rho C_P u_0^2}\frac{\rho C_P\upsilon}{k}=P_r R$$

$$P_r\frac{\partial\theta}{\partial\tau}=0$$

$$P_r\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial y^2}-P_r R\left(\theta^4-1\right)+D_C\frac{\partial^2 C}{\partial y^2}$$

$$\frac{\partial^2\theta}{\partial y^2}-P_r R\left(\theta^4-1\right)+D_C\frac{\partial^2 C}{\partial y^2} \qquad 2.1.07$$

Concentration equation

$$\frac{\partial C}{\partial t}=D_m\frac{\partial^2 C}{\partial y^2}-k_r T\sigma\alpha\left(C^4-C_\infty^4\right)+D_m k_r\frac{\partial^2 T}{\partial y^2} \qquad 2.1.04$$

Non-dimensional

$$\frac{C_\infty}{\frac{\upsilon^2}{u_0^2}}\frac{\partial C}{\partial\tau}=\frac{D_m D_\infty}{\frac{\upsilon^2}{u_0^2}}\frac{\partial^2 C}{\partial y^2}-\frac{v4k_r\sigma\alpha T_\infty}{\frac{\upsilon^2}{u_0^2}}C_\infty^4\left(C^4-1\right)+\frac{D_m k_r T_\infty}{\frac{\upsilon^2}{u_0^2}}\frac{\partial^2\theta}{\partial y^2}$$

$$\Rightarrow\left(\frac{C_\infty u_0^2}{\frac{\upsilon^2}{u_0^2}}\frac{\partial C}{\partial\tau}=\frac{D_m C_\infty u_0^2}{\upsilon^2}\frac{\partial^2 C}{\partial y^2}-\frac{4k_r\alpha T_\infty C_\infty^4 u_0^2}{\upsilon^2}\left(C^4-1\right)+\frac{D_m k_r T_\alpha u_0^2}{\upsilon^2}\frac{\partial^2\theta}{\partial y^2}\right)\frac{\upsilon^2}{\xi D_m C_\infty u^2}$$

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Multiply the equation above by $\frac{\upsilon^2}{D_m C_\infty u_0^2}$

$$\frac{\upsilon}{D_m}\frac{\partial C}{\partial \tau}=\frac{\partial^2 C}{\partial y^2}-\frac{4k_r\alpha T_\infty C_\infty^4\upsilon^2 u}{D_m u_0^2 C_\infty \upsilon^2}\left(C^4-1\right)+\frac{D_m k_t T_\infty \upsilon^2 u^2}{D_m C_\infty u_0^2}\frac{\partial^2\theta}{\partial y^2}$$

$$\frac{k_r T_\infty}{C_\infty}=S_C$$

$$S_C\frac{\partial C}{\partial \tau}=\frac{\partial^2 C}{\partial y^2}-k_r S_C C\left(C^4-1\right)+S_t\frac{\partial^2\theta}{\partial y^2} \qquad 2.1.08$$

$$\left.\begin{aligned} q(y,\tau)&=q^0(y)+\xi q^{'}(y,\tau)\\ \theta(y,\tau)&=\theta^0(y,t)+\xi\theta^{'}(y,\tau)\\ C(y,\tau)&=C^0(y,t)+\xi C^{'}(y,\tau)\end{aligned}\right\} \qquad 8$$

The above perturbation is used to substitute into equation 2.1.06

$$\frac{\partial q}{\partial \tau}+i2\mathrm{E}q+\chi^2 q=\frac{\partial^2 q}{\partial y^2}+G_r(\theta-1)+G_C(C-1) \qquad 2.1.07$$

$$\left.\begin{aligned}&\frac{\partial q}{\partial \tau}\left(q^0+Cq^{'}\right)+i2\mathrm{E}\left(q^0+\varepsilon q\right)+\chi^2\left(q^0\right)+\varepsilon q^{'}=\\ &\frac{\partial^2}{\partial t}\left(q^0+\varepsilon q^{'}\right)+G_r\left(\theta^0+\varepsilon\theta^{'}-1\right)+G_C\left(C^0+\varepsilon C^{'}-1\right)\end{aligned}\right\}$$

$$\frac{\partial q}{\partial t}+\left(2\mathrm{E}+X^2\right)q^0=\frac{\partial^2 q^0}{\partial y^2}+G_r\left(\theta^0-1\right)+G_C(C-1)$$

$$\frac{\partial q^{'}}{\partial t}+\left(i2\mathrm{E}+\chi^2\right)q^{'}=\frac{\partial^2 q}{\partial y^2}+G_r\theta^{'}+G_C C^{'}$$

Using the above equation and substituting into equations 2.1.07, 2.1.08

$$\frac{\partial^2\theta}{\partial y^2}-P_r R\left(\theta^{04}-1\right)+D_C\frac{\partial^2 C}{\partial y^2} \qquad 2.1.08$$

$$\frac{\partial^2 C}{\partial y^2}-k_r S_C\left(C^{04}-1\right)+S_t\frac{\partial^2\theta}{\partial y^2}$$

$$\frac{\partial^2}{\partial y^2}\left(\theta^0+\xi\theta^{'}\right)-P_r R\left[\left(\theta^0+\xi\theta\right)^4-1\right)+D_C\frac{\partial^2}{\partial y^2}\left(C^0+\xi C^{'}\right)\right]$$

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$$\frac{\partial^2\theta}{\partial y^2}-P_rR\left(^{04}-1\right)+D_C\frac{\partial^2 C^0}{\partial y^2}$$

If we substitute equation 2.1.07 – 2.1.09 and expand q,θ,C in the power of ε the Eckert number under the assumption $\varepsilon<<1$. This justified in the lower speed of incompressible flow. If we substitute in respective equation 2.1.07-2.1.09 and equate the coefficient of different powers of ε and neglect those of $\varepsilon, C^{04}, \theta^{04}$.

$$P_r\frac{\partial}{\partial t}\left\{\theta^0 y+\mathrm{E}\theta'(y,t)\right\}=\frac{\partial^2}{\partial y^2}\left\{\theta^0 y+\mathrm{E}\theta'(y,t)\right\}$$

$$P_rR\left\{\left[\theta^0+\xi\theta'(y,t)-1\right]\right\}+D_C\frac{\partial^2}{\partial y^2}\left\}C^0 y+\xi C'(y,t)\right\}$$

$$\theta^0(y)+\xi\theta^{(1)}(y,t)\xi^4=\theta^{04}+4\xi\theta^{03}\theta'+6\xi^2\theta^{(0)2}\theta^{12}+4\xi^3\theta^{(0)}\theta^{(0)}\theta^{13}+\xi^4\theta^{(1)4}$$

$$\Rightarrow\theta^{04}+4\xi\theta^{03}\theta^{(1)}$$

So equation 2.1.07, 2.1.08 becomes

$$\frac{\partial}{\partial t}\left\{\theta^{(0)}+\xi\theta^{(1)}\right\}=\frac{\partial^2}{\partial y^2}\left\{\theta^0+\xi\theta^{(1)}\right\}-P_rR\left\{\theta^{(0)4}+4\xi\theta^{02}\theta^{(1)}-1\right\}+D_C\frac{\partial^2}{\partial y^2}\left\{C^{(0)}+\xi C^{(1)}\right\}$$

Similarly, equation 2.1.08

$$\frac{\partial}{\partial t}\left\{C^{(0)}+\xi C^{(1)}\right\}=\frac{\partial^2}{\partial y^2}\left\{C^0+\xi C^{(1)}\right\}-k_rS_C\left\{C^{(0)4}+4\xi C^{03}C^{(1)}-1\right\}+S_t\frac{\partial^2}{\partial y^2}\left\{\theta^{(0)}+\xi\theta^{(1)}\right\}$$

Zero order

$$\frac{d^2q}{dy^2}+\left(2\mathrm{E}+\chi^2\right)q^0=G_r(\theta-1)+G_C(C-1)$$

$$\frac{d^2\theta}{dy^2}-RP_r\left(\theta^4-1\right)+D_C\frac{d^2C^0}{dy^2} \qquad 2.1.22$$

$$\frac{d^2C^0}{dy^2}-k_rS_C\left(C^{04}-1\right)+S_t\frac{d^2\theta}{dy^2} \qquad 2.1.23$$

1st order

$$\frac{dq}{dy}+2i\mathrm{E}q^{(1)}=\frac{d^2q^{(1)}}{dy^2}-\chi^2q^{(1)}+G_r\theta+G_CC^{(1)}$$

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$$P_r \frac{d\theta^{(1)}}{dt} = \frac{d^2\theta}{dy^2} - 4R.P_r \theta^{(0)3}\theta' + D_C \frac{d^2C}{dy^2} \qquad 2.1.17$$

$$S_C \frac{dC^{(1)}}{dt} = \frac{d^2C'}{dy^2} - 4k_r S_C C^{(0)3}C' + S_C \frac{d^2\theta}{dy^2} \qquad 2.1.18$$

From the zero order

$$\frac{d^2C^0}{dy^2} = -S_t \frac{d^2\theta^{(0)}}{dy^2}$$

Substitute above equation into the zero order

$$0 = \frac{d^2\theta^0}{dy^2} - RP_r\left(\theta^{04} - 1\right) - D_C S_C \frac{d^2\theta}{dy^2} \qquad 3.1.01(a,b,c,d)$$

$$0 = \left(1 - D_C S_t\right)\frac{d^2\theta^0}{dy^2} - R.P_r\left\{\theta^{04} - 1\right\}$$

$$\frac{d^2\theta^0}{dy^2} = \frac{R.P_r}{1 - D_C S_t}\left\{\theta^{04} - 1\right\} \text{Where } \frac{R.P_r}{1 - D_C S_t} = P_1$$

$$\frac{d^2\theta}{dy^2} = P_1\left\{\theta^{(0)4} - 1\right\}\frac{d\theta}{dy}$$

$$\frac{d\theta^0}{dy}\frac{d^2\theta^0}{dy^2} = P\frac{d\theta^0}{dy}\theta^{04} - P_1\frac{d\theta^0}{dy}$$

$$\frac{1}{2}\frac{d}{dy}\left(\frac{d\theta}{dy}\right)^2 = \frac{P_1}{5}\frac{d\theta^{05}}{dy} - P_1\frac{d\theta}{dy}$$

$$\frac{d}{dy}\left(\frac{d\theta}{dy}\right)^2 = \frac{2P_1}{5}\frac{d\theta^{05}}{dy} - 2P\frac{d\theta}{dy}$$

$$\left(\frac{d\theta}{dy}\right)^2 = \frac{2}{5}P_1\theta^{05} - 2P\theta^0 + A_1$$

$$\frac{d\theta}{dy} = \sqrt{\frac{2}{5}P_1\theta^{05} - 2P\theta^0 + A_1}$$

$$dy = \frac{d\theta}{\sqrt{\frac{2}{5}P_1\theta^{05} - 2P\theta^0 + A_1}}$$

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$$y = \int \frac{d\theta^0}{\sqrt{\frac{2}{5}P_1\theta^{05} - 2P\theta^0 + A_1 + A_2}}$$

$y = 0, \theta^0 = \theta_w$ Also $y \to \infty, \theta^0 = 1; \theta = 0 + A_2 = A_2 = 0$

$$\infty = \int_1^{\theta_W} \frac{d\theta}{\sqrt{\frac{2}{5}P_1\theta^{05} - 2P\theta^0 + A_1}}$$

Since the integral is infinite this condition will be satisfied only when

$$\frac{2}{5}P_1\theta^{05} - 2P_1\theta^0 + A_1 = 0$$

Substitute the value of $\theta^{(0)} = 1$

$$\frac{2}{5}P_1\theta^{05} - 2P_1\theta^0 + A_1 = 0$$

$$A_1 = 2P_1 - \frac{2P_1}{5} = 2P_1\left(1 - \frac{1}{5}\right) = 2P_1\frac{4}{5} = \frac{8P_1}{5}$$

$$\therefore\ y = \left\{\frac{5}{2}\left(\frac{1 - D_C S_t}{R.P_r}\right)^{1/2} \int_0^{\theta_W} \frac{d\xi}{\left(\xi^5 - 5\xi + 4\right)^{1/2}}\right\} \qquad 3.1.02\text{-}3.1.05$$

Similarly for the zero order in the 3rd equation

$$\frac{d^2\theta}{dy^2} = D_C \frac{d^2C}{dy^2}$$

If we substitute into the 3rd equation and solve as above we have

$$y = \left\{\frac{5}{2}\left(\frac{1 - S_t D_C}{k_r S_C}\right)^{1/2} \int_0^{\theta_W} \frac{d\xi}{\left(\xi^5 - 5\xi + 4\right)^{1/2}}\right\} \qquad 3.1.03$$

From 1st order

$$\frac{R.P_r}{1 - D_C S_t} = P_1$$

$$\frac{d^2\theta}{dy^2} - 4R.P_r\left(\theta^{03} - 1\right) - D_C S_t \frac{d^2\theta}{dy^2}$$

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$$\frac{d^2\theta}{dy^2} - D_C S_t \frac{d^2\theta}{dy^2} = 4R.P_r\left(\theta^{03} - 1\right)$$

$$\frac{d^2\theta}{dy^2}\left(1 - D_C S_t\right) = 4R.P_r\left(\theta^{03} - 1\right)$$

$$\frac{d^2\theta}{dy^2} = \left(\frac{4RP_r\left(\theta^{03} - 1\right)}{1 - D_C S_t}\right)$$

$$\frac{d^2\theta}{dy^2} = \frac{4R.P_r\theta^{03}}{1 - D_C S_t} - 4R.P_r$$

$$\frac{d^2\theta}{dy^2}\frac{d\theta}{dy} = \frac{4R.P_r\theta^{03}}{1 - D_C S_t}\frac{d\theta}{dy} - \frac{4R.P_r}{1 - D_C S_t}\frac{d\theta}{dy}$$

$$\frac{d^2\theta}{dy^2}\frac{d\theta}{dy} = 4P_r\frac{d\theta^{04}}{dy} - 4P_r\frac{d\theta^{(0)}}{dy}$$

$$\frac{1}{2}\frac{d}{dy}\left(\frac{d\theta}{dy}\right)^2 = \frac{8}{4}P_r\theta^{04} - 8P_r\theta^0$$

$$\left(\frac{d\theta}{dy}\right)^2 = \frac{8}{4}P_r\theta^{04} - 8P_r\theta^0 + A_1$$

$$\frac{d\theta}{dy} = \sqrt{\frac{8}{4}P_r\theta^{04} - 8P_r\theta^0 + A_1}$$

$$dy = \frac{d\theta}{\sqrt{\frac{8}{4}P_r\theta^{04} - 8P_r\theta^0 + A_1}}$$

$$dy = \frac{d\theta}{\sqrt{2P_r\theta^{04} - 2P_r\theta^0 + A_1}}$$

$$dy = \int_0^{\theta_W}\frac{d\theta}{\sqrt{2P_r\theta^{04} - 2P_r\theta^0 + A_1 + A_2}}$$

Since the integrals are infinite the conditions will be satisfied only were

$$\frac{8}{1}P_r\theta^{04} - 2P_r\theta^0 + A_1 = 0$$

Substituting the value $\theta^0 = 1$

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$$A_1 = 2P_r - 2P_r = 2P_r\left(1 - \frac{1}{1}\right), A = 0$$

$$y = \left\{\frac{1}{2}\left(\frac{1 - D_C S_t}{R.P_r}\right)^{1/2}\right\}\int_0^{\theta_W} \frac{d\xi}{\left(\xi^5 - 5\xi + 1\right)^{1/2}} \quad 3.1.04$$

Similarly, for the first order in equation 5

$$\frac{d^2\theta}{dy^2} = -D_C \frac{d^2 C}{dy^2}$$

If we substitute the above equation into 3rd equation in the 1st order equation and solve similarly above we arrived at

$$y = \left(\frac{1}{2}\left(\frac{1 - S_t D_C}{k_r S_C}\right)^{1/2}\right)\int_0^{\theta_W} \frac{d\xi}{\sqrt{\xi^5 - 5\xi + 1}} \quad 3.1.05$$

Equations 3.1.02, 3.1.03, 3.1.04, 3.1.05 respectively reduce equation 3.1.06, 3.1.07, 3.1.08 and 3.1.09. I assumed S_t, D_C are constants. The equation for $q^{(0)}$ therefore becomes,

$$0 = \frac{d^2 q^{(0)}}{dy^2} - \left(\chi^2 + 2i\mathrm{E}\right)q^0 + G_r\left(\theta^{(0)} - 1\right) + G_C\left(C^{(0)} - 1\right)$$

From the relationship

$$\frac{d^2 C^{(0)}}{dy^2} = -S_t \frac{d^2\theta^{(0)}}{dy^2} \quad 3.1.10(a,b)$$

$$C^{(0)} = -S_t\theta^{(0)} + B_1 y + B_2$$

When $y = 0, C^{(0)} = C_W, \theta^0 = \theta_\infty$

$$C_W = -S_C\theta_\infty + B_2$$

$$B_2 = C_W + S_t\theta_W$$

When $y \to \infty$

$$y = \frac{\left\{C^{(0)} + S_t\theta^{(0)} - \left(C_W + S_t\theta_W\right)\right\}}{B_1}$$

$$\infty = \frac{\left\{C^{(0)} + S_t\theta^{(0)} - \left(C_W + S_t\theta_W\right)\right\}}{B_1}$$

This is satisfied when $B_1 = 0$

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$$C^{(0)} = -S_t\theta^{(0)} + \left(C_W + S_t\theta_W\right)$$

Similarly from the relationship

$$\frac{d^2\theta^{(0)}}{dy^2} = -D_C\frac{d^2C^{(0)}}{dy^2}$$

$$\theta^{(0)} = -D_C C^{(0)} + B_1 y + B_2$$

When $y = 0, C^{(0)} = C_W, \theta^0 = \theta_W$

$$\theta_W = -D_C C_W + B_2$$

$$B_2 = \theta_w + D_C C_W$$

When $y \to \infty$

$$y = \frac{\left\{\theta^{(0)} + D_C C^{(0)} - \left(\theta_W + D_C C_W\right)\right\}}{B_1}$$

$$\infty = \frac{\left\{\theta^{(0)} + D_C C^{(0)} - \left(\theta_W + D_C C_W\right)\right\}}{B_1}$$

This is satisfied when $B_1 = 0$

$$\theta^0 = -D_C C^{(0)} + \left(\theta_W + D_C C_W\right)$$

Therefore the equation for $q^{(0)}$ becomes

$$\frac{d^2q}{dy^2} - \left(\chi^2 + 2i\mathrm{E}\right)q^{(0)} + G_r\left(\theta - 1\right) + G_C\left(C^{(0)} - 1\right)$$

$$\left.\begin{array}{l}\frac{d^2q}{dy^2} - \left(\chi^2 + 2i\mathrm{E}\right)q^{(0)} + G_r\left(-D_C C^0 + \theta_W + D_C\theta_W - 1\right) \\ + G_C\left(-S_t\theta^0 + \left(C_W + S_t\theta_W\right) - 1\right)\end{array}\right\} \qquad 3.1.13\text{-}3.1.19$$

The homogenous equation is

$$\frac{d^2q^{(0)}}{dy^2} - \left(\chi^2 + 2i\mathrm{E}\right)q^{(0)} = 0$$

$$D^2 - \left(\chi^2 + 2i\mathrm{E}\right)q^{(0)} = 0$$

$$M^2 - \left(\chi^2 + 2i\mathrm{E}\right)q^{(0)} = 0$$

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$M_{1,2} = \left(\chi^2 + 2i\mathrm{E}\right)^{1/2} = \pm\alpha$

$q^{(0)} = D_1 \mathrm{e}^{m_1 y} + D_2 e^{-m_2 y}$

$q^{(0)} = D_1 e^{\alpha y} + D_2 e^{-\alpha y}$

Subject to boundary condition

$y = 0, q^{(0)} = 1, 1 = D_1 + D_2$

$y \to \infty : q^{(0)} = 0$

The term $D_1 e^{\alpha y}$ is unbounded and so I neglected it, i.e. $D = 0$, and $D_2 = 1$

$q^{(0)} = 0$ and $\exp\left\{- X^2 + 2i\mathrm{E}\right\}^{1/2} y$

Expression for C_W

Assuming $\theta_W = 10$

When $y = 0, \theta^{(0)} = \theta_W, C^{(0)} = C_W$

Since $C_W = -S_t \theta^0 + C_W + S_t \theta_W$

I have $C_W = -S_t \theta_W + C_W + S_t \theta_W$

$C_W = C_W$

When $y \to \infty, \theta^0 = 1, C^0 = 1$

$1 = -S_t + C_W + 10 S_t = 1 - 9 S_t = C_\infty$

$C_W = 1 - 9 S_t$

Similarly, expression for θ_W

Assuming $C_W = 10$

When $y = 0, \theta^0 = \theta_W, C^{(0)} = C_W$

$\theta_W = -D_C C_W + \theta_W + C_W$

I have

$\theta_W = -D_C C_W + \theta_W + D_C C_W$

$\theta_W = \theta_W$

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$$y \to \infty, \theta^0 = 1, C_W = 1$$

$$1 = -D_C + \theta_W + 10D_C$$

$$1 - 9D_C = \theta_W$$

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APPENDIX 2

$$t^2 \frac{d^2 p}{dt^2} + t\frac{dp}{dt} - \left(t^2 + n^2\right)p = 0$$

Where $p = p_n(it)$ called the modified Bessel function of the first kind and denoted by $\ln(\xi)$. They are given by

$$\ln(\xi) = i^{-n} J_n(i\xi) = \sum_{1=0}^{\infty} \frac{(\xi/2)^{2\lambda+n}}{\lambda!(\lambda+n)!}$$

When n is a non integer $\ln(\xi)$ and $1-n$ are independent solution of the modified Bessel equation. When n is an integer $\ln(\xi) = 1-n(\xi)$. The modified Bessel function of the second kind $k_n(\xi)$ are defined by

$$k_n(\xi) = \frac{\pi}{2}\left|\frac{1-n(\xi)-\ln(\xi)}{\sin n\pi}\right|$$

$$I_n(\xi) = \sqrt{\frac{\pi}{2\xi}\ln + \frac{1}{2}(\xi)},\ k_n(\xi) = \sqrt{\frac{2}{\pi\xi}k_n + \frac{1}{2}(\xi)}$$

$$I_n(\xi) = \frac{1}{\sqrt{2\pi\xi}}e^{\xi}, k_n(\xi) = \sqrt{\frac{\pi}{2\xi}}e^{-\xi}$$

Bessel function of the 1st kind

$$I_n(x) = i^{-n} J_n(ix) e^{-n\pi \frac{1}{2}} J_n(x)$$

$$I_v(z) = e^{-i/2 v\pi} J_v\left(ze^{i/2\pi}\right), \left(-\pi < \arg z \le \frac{1}{2}\pi\right)$$

$$I_v(z) = e^{3v\pi\, i/2} J_v\left(ze^{-3\pi\, i/2}\right), \left(\frac{1}{2}\pi \arg z \le \pi\right)$$

$$I_{-n} = \ln(x), I_{-n}(z), k_{-v}(z) = k_v(z)$$

$$\sqrt{\frac{1}{2}\frac{\pi}{z}} I_n + \frac{1}{2}(z) = e^{-n\pi\, i/2} J_n\left(ze^{\pi\, t/2}\right)\left(-\pi < \arg z \le \frac{1}{2}\pi\right)$$

$$e^{-x} I_v(x), e^{-x} I_0,(ix), e^{x} k_0(ix), e^{x} k_1(ix)$$

$$\gamma_v\left(ze^{\frac{1}{2}\pi i}\right) = e^{\frac{1}{2}(v+1)\pi i} I_v(z) - \left(\frac{2}{\pi}\right) e^{-\frac{1}{2}v\pi i} K_v(z)\left(-\pi < \arg z \le \frac{1}{2}\pi\right)$$

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Bessel function of the 2nd kind

$$K_n(x) = \frac{\pi}{2}\left|\frac{I_{-n}(x) - I_n(x)}{\sin nx}\right| \quad n \neq 0,1,2,3,...$$

$$K_n(x) = \lim \frac{\pi}{2}\left|\frac{I_{-P}(x) - I_P(x)}{\sin p\pi}\right|$$

$$Y(x) = C_1 J_v(ix) + C_2 J_v(\lambda_x)$$

$$y(x) = C_1 J_v(ix) + C_2 J_v(ix)$$

$$J_v(ix) = \sum_{k=0}^{\infty} \frac{(-1)^k (ix)^{2k+v}}{2^{2k+v} K_i T(v+k+i)} = i^v I_v(x)$$

$$I_v(x) = \sum_{k=0}^{\infty} \frac{X^{2k+v}}{2^{2k+v} KT(v+k+i)}$$

$$L^{-1}\frac{J_0(i\delta)^{\frac{1}{2}}}{J_0(i\delta)^{\frac{1}{2}}} = e^{-k^2 t}\sum 2\delta_m e^{\delta m^2}\ \frac{J_0(\delta - r)}{J_0 \delta_m}$$

$$L^{-1}\frac{I_0(k^2+m)^{\frac{1}{2}} r}{I_0(k^2+m)^{\frac{1}{2}}} = e^{-k^2 t}, L^{-1}\frac{J_0}{I_0}\frac{(\delta)^{\frac{1}{2}}}{\delta^{\frac{1}{2}}} - e^{-k^2}$$

$$L^{-1}\frac{(k+m)^{\frac{1}{2}}}{m} = L^{-1}\frac{(k^2+m)^{\frac{1}{2}}}{(k^2+m)-k^2} = e^{-k^2 r} \qquad \text{3.1.35-3.1.38}$$

$$L^{-1}\frac{(m)^{\frac{1}{2}}}{m-k^2 r} = \frac{e^{-k\ r}}{\sqrt{\pi t}} + k_r \operatorname{erf}\left(k_r \sqrt{t}\right)$$

$$L^{-1}\frac{I_0}{I_0} = \left(\frac{\delta^{\frac{1}{2}} r}{\delta^{\frac{1}{2}}}\right) = e^{-k^2 r}$$

$$\left(\frac{d}{ydy}\right)^m \left[y^v J_v(y)\right] = y^{v-m} J_v - m(y)$$

$$\left(\frac{d}{ydy}\right)^m \left[y^{-v} J_v(y)\right] = y^{-v-m} J_{v+m}(y)$$

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$$J_v - I_0(y) + J_v + I_0(y) = \frac{2}{y} J_v(y), J_v - I_0(y) - J_v + I_0(y) = 2J_0(y)$$

$$\left.\begin{aligned} \text{erfc} = \frac{\gamma_y}{2\pi t}\int_0^{C_W} e^{-t^2} dt, \text{erf}(x) = 1 - \text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_r^{\infty} e^{-\xi^2} d\xi \\ \text{erfc} = \frac{\beta_y}{2\pi t}\int_0^{\theta_W} e^{-t^2} d\xi, \text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_r^{\infty} e^{-\xi^2} d\xi, J_n(x) + I_0(x) = m_0(x) \end{aligned}\right\} \quad 3.1.43\text{-}3.1.50$$

$$L^{-1}\frac{(m)^{\frac{1}{2}}}{m - k^2 r} = \frac{\mathrm{e}^{-k\ r}}{\sqrt{\pi t}} + k_r \text{erf}\left(k_r \sqrt{t}\right)$$

$$\sum_{n=1}^{\infty}\frac{\delta_n J_0(\delta_n r)}{J_0(\delta_n)} = \int_0^1 \left[1 - \frac{e^{-k^2 r}}{\sqrt{\pi t}} + k_r \text{erf}\left(k_r \sqrt{t}\right)\right] \mathrm{e}^{-(kr^2 + \delta n)(t-r)dr}$$

$$L\theta^{-1}\left[e^{-S(y-y)}\right]$$

$$q' = \frac{1}{2}\int_0^y e\frac{S(y-y)}{S} dy$$

$$\left.\begin{aligned} L^{-1}F(S) = f(t) = \frac{1}{2\pi t}\int_{C-C\infty}^{C+C\infty} F(S) e^{St} dS = \\ f(t) = \frac{1}{2\pi i}\int_{C-C\infty}^{C+C\infty} e\frac{St - a\sqrt{S}}{S} dS \end{aligned}\right\}$$

$$f(t) = \sum \text{ Reduces of } e^{\delta t}F(S) \text{at all poles of } F(S)$$

$$L\theta^{-1}\left[\frac{e^{-S\frac{1}{2}(y-y)}}{S^{\frac{1}{2}}}\right] = e^{-(\chi^2 + i2\mathrm{E})^{\frac{1}{2}}} \frac{1}{\pi t} e^{-S(y-y)^{\frac{1}{2}}} dt$$

$$L\{\{u(t-a)\} = \frac{e^{-aS}}{S} S > 0$$

$$\left\{L^{-1}\left\{\frac{e^{-aS}}{S}\right\} = u(t-a)\right\}$$

$$\{\{e^{-at}\} = \int_0^{\infty} e^{st} e^{-at} dt = \int_0^{\infty} e^{-(S-a)} dt = \left.\frac{e^{-(S-a)t}}{S-a}\right|_0^{\infty}$$

$\frac{1}{S-\alpha}$ Provides $S - a > 0$ that is $S > a$

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$$L\left\{\{u(t-a)\} = \frac{e^{-aS}}{S} \text{ if } S > 0\right.$$

$$u(t-a) = \begin{cases} 0 & t < a \\ 1 & t < a \end{cases} \text{ so that}$$

$$L\{\ u(t-a) = \int_0^a e^{-St} 0dt + \int_0^\infty e^{-St} dt = 0 + \left.\frac{e^{-St}}{S}\right|_0^\infty = \frac{e^{-St}}{S} \text{ if } S > 0$$

$$L[f_1(t) = f_1(\delta), L[f_2(t) = f_2(t)$$

$$L\left\{\int_0^T f_1(x) f_2(t-x)dx\right\} = f(\delta), f_2(\delta)$$

$$f_1(\delta), f_2(\delta) = L^{-1}\int_0^t f_1(x), f_2(t-x)dt$$

$$L\left\{\int_0^a f_1(x) f_2(t-x)dx = \int_0^a e^{-\delta t}\int_0^t f_1(x) f_2(t-x)dx\right.$$

$$= \int_0^a \int_0^t e^{-St} f_1(y) f_2(t-x)dxdt$$

Where the double integrals is taken over the finite region in the first quadrant lying between the line $y = 0$ and $y = t$. Changing the order of integration, the above equation becomes,

$$\int_0^\alpha \int_y^y e^{St} f_1(y) f_2(t+y)dtdy$$

$$= \int_0^a e^{-St} f_1(y)dy \int_y^a e^{-S(y-y)} f_2(y-y)dy$$

$$= \int e^{-Sy} f_1(y)dy \int_0^a e^{-Sy} f_2(y)dy$$

$$= \left[\int e^{-Sy} f_1(y) f_2(S)dy = \int_0^a e^{-Sy} f_1(y)dy\right] f_2(S) f_1(S)$$

$$L\theta^{-1}\left[e^{-}\frac{S(y-y)}{S}, q = \frac{1}{2}\int_0^y e^{-S(y-y)dydy}\right. \qquad 3.1.50$$

$$L\theta^{-1}\left[e^{-}\frac{S\frac{1}{2}(y-y)}{S^{\frac{1}{2}}} = e^{-(x+2i\text{E})\frac{1}{2}}\frac{1}{\pi t}e^{-S(y-y)\frac{1}{2}}dt\right. \qquad 3.1.51$$

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Shifting Rule

$$L^{-1}\left(\frac{k^2+m}{m}\right)^{\frac{1}{2}} = L^{-1}\left(\frac{k^2+m}{k^2+m}\right)^{\frac{1}{2}}_{-kr^2} = e^{-kr^2}$$

$$L^{-1}\frac{(m)^{\frac{1}{2}}}{m-kr^2} = \frac{e^{-k^2r}}{\sqrt{\pi t}} + k_r \operatorname{erfc}\left(k_r\sqrt{t}\right)$$

$$\theta = \theta_W \frac{I_0}{I_0}\frac{k_r r}{(k_r)}\left[e^{-}\frac{k^2 r}{(\pi t)^{\frac{1}{2}}} + k_r \operatorname{erfc}\left(k_r\sqrt{t}\right)\right] +$$

$$= 2\theta_m \sum_{n-1}^{\alpha}\delta_m \frac{J_0}{J_v}\frac{(\delta_m .r)}{(\delta_m)}\int_0^1\left[1-\frac{e^{-k^2r}}{\sqrt{\pi t}} + k_r \operatorname{erfc}\left(k_r\sqrt{t}\right)\right]e^{-\left(k^2+\delta_m\right)(t-r)dr}$$

Stability Analysis

$$\frac{\theta^{'}}{\theta_\alpha} = \frac{I_n\left(4RP_r+\xi\right)^{\frac{1}{2}}\left(\eta e^{-\beta y}\right)}{\xi I_n\left(4RP_r+\xi\right)^{\frac{1}{2}}\eta} = \frac{I_n\left(RP_r+\xi\right)^{\frac{1}{2}}\left(\eta e^{-\beta y}\right)}{\xi I_n\left(RP_r+\xi\right)^{\frac{1}{2}}\eta} \qquad 3.1.35$$

$$\frac{C^{'}}{C_\alpha} = \frac{I_n\left(4k_r S_C+\xi\right)^{\frac{1}{2}}\left(i\eta e^{-\gamma y}\right)}{\xi I_n\left(4k_r S_C+\xi\right)^{\frac{1}{2}}\eta} = \frac{I_n\left(k_r S_C+\xi\right)^{\frac{1}{2}}\left(\eta e^{-\gamma y}\right)}{\xi I_n\left(k_r S_C+\xi\right)^{\frac{1}{2}}\eta} \qquad 3.1.36$$

Where

$\eta = 4RP_r\left(\theta_\infty - 1\right) = RP_r\left(\theta_\infty - 1\right)$

$\xi = 4k_r S_C\left(C_\infty - 1\right) = k_r S_C\left(C_\infty - 1\right)$

$I_n(\eta)$ = Modified Bessel function of the first kind

$K_0(x)$ Modified Bessel function of the second kind

Compare with equations 3.1.35, 3.1.47, 3.1.36 and 3.1.49

For $\frac{\theta^{'}}{\theta_\infty}$ and $\frac{C^{'}}{C_\alpha}$ respectively

The problem of stability analysis

The stability condition

If the ratio of the marginal solution to the asymptotic solution is less than or equal to 1 then I claim that the system is stable either with respect to temperature or with respect to concentration. That is,

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$$\frac{(\theta'/\theta_\infty)\text{Marginal Solution}}{(\theta^{(1)}/\theta_\infty)\text{AsymptoticSolution}} \le 1 \qquad 3.2.01$$

$$\frac{3.1.35}{3.1.47} \le 1 \qquad 3.2.01$$

And,

$$\frac{(C'/C_\infty)\text{Marginal Solution}}{(C^{(1)}/C_\infty)\text{AsymptoticSolution}} \le 1 \qquad 3.2.02$$

$$\frac{3.1.36}{3.1.49} \le 1 \qquad 3.2.02$$

Taking $\frac{3.1.35}{3.1.47} \le 1$

$$\frac{\theta^0}{\theta_\infty} = \frac{\dfrac{I_n(4RP_r+\xi)^{1/2}(\eta e^{-\beta_y})}{\xi I_n(4RP_r+\xi)^{1/2}\eta}}{\dfrac{1}{I_0(\eta)}\left\{\begin{matrix} I_0(\eta e^{-\beta y}) + [K_0(\eta e^{-\beta y}) - K_0(\eta e^{-\beta y})] \\ .e^{-4RP_r}\left[+\dfrac{1}{\pi}(4RP_r)^{1/2}.e^{-RP_r t}\operatorname{erfc}(RP_r)^{1/2}\right]\end{matrix}\right\}} \le 1 \quad \text{at } K=0, t\to\infty$$

$$\frac{\theta^0}{\theta_\infty} = \frac{\dfrac{I_n(4RP_r+\xi)^{1/2}(i\eta e^{-\beta_y})}{\xi I_n(4RP_r+\xi)^{1/2}\eta}}{\dfrac{I}{I_0(\eta)}\left\{I_0\left(\eta e^{-\beta y}+\dfrac{1}{\pi}(4RP_r)\right)^{1/2} e^{-4RP_r t}\operatorname{erfc}(4RP_r t)^{1/2}\right\}} \le 1$$

$$\frac{I_n(4RP_r+\xi)^{1/2}(\eta e^{-\beta y})I_0(\eta)}{\xi I_n(4RP_r+\xi)^{1/2}\left\{I_0(\eta e^{-\beta y}) + \dfrac{1}{\pi}(4RP_r)^{1/2} e^{-4RP_r t}\operatorname{erfc}(4RP_r)^{1/2}\right\}} \quad \text{as } t\to\infty$$

$$\frac{I_n I_0(\eta)(4RP_r+\xi)^{1/2}(\eta e^{-\beta y})}{\xi I_n(4RP_r+\xi)^{1/2\eta}(I_0(\eta e^{-\beta y}))} \le 1$$

$$= \frac{I_n.I_0\eta(4RP_r+\xi)^{1/2}(\eta e^{-\beta y})}{\xi I_n.(4RP_r+\xi)^{1/2\eta}.I_0.\eta e^{-\beta y}} \le 1$$

$$\theta_0 = \frac{(4RP_r+\xi)^{1/2}}{\xi(4RP_r+\xi)^{1/2\eta}} \le 1$$

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$$\theta_0 = \frac{(4RP_r + \xi)^{1/2}.(4RP_r + \xi)^{-1/2\eta}}{\xi} \le 1$$

$$\theta_0 = \frac{(4RP_r + \xi)^{1/2 - 1/2\eta}}{\xi} \le 1$$

$$\theta_0 = \frac{1}{\xi}(4RP_r + \xi)^{1/2(1-\eta)} \le 1 \qquad 3.2.03$$

We see that the system is thermally stable iff $\eta \ge 1$

Similarly, the system is chemically stable if

$$C_0 = \frac{(4k_r S_C + \xi)^{1/2}}{\xi(4k_r S_C + \xi)^{1/2\eta}} \le 1$$

$$C_0 = \frac{1}{\xi}(4k_r S_C + \xi)^{1/2(1-\eta)} \le 1 \qquad 3.2.04$$

And so the system will be chemically stable iff $\eta \ge 1$

CONDITIONS FOR THERMAL STABILITY AND ASYMPTOTIC EXPANSIONS OF FUNCTIONS ABOUT THE ORIGIN.

Let $a_j(z)$ and $b_j(z)$ be functions admitting the asymptotic expansions as:

$$a_j(z) = \sum_{j=0}^{\infty} a_j z^{-j}$$

$$b_j(z) = \sum_{j=0}^{\infty} b_j z^{-j}$$

Now let $U(z)$ be functions of $a_j(z)$ and $b_j(z)$ symmetric about the origin (i.e. the equilibrium position or value) the two neighbouring values of $U(z)$ about the origin can be expressed in cluster terms as:

$$U_0(z) = \sum_{n=0}^{\infty} a_n(z) e^{-i(\omega_0 t - k_0 x)}$$

and

$$U_1(z)=\sum_{n=0}^{\infty} a_n(z)e^{-i(\omega_1 t-k_1 x)}$$

Thus an auxiliary value of $U(z)$ between $U_0(z)$ and $U_1(z)$ is given by,

$$\frac{U_0(z)}{U_1(z)}=\sum_{n=0}^{\infty}\frac{a_n(z)}{a_b(z)}e^{i(\omega_1 t-k_1 x)}e^{i(\omega_0 t-k_0 x)}$$

$$\frac{U_0(z)}{U_1(z)}=\sum_{n=0}^{\infty} C_n(z)e^{i[(\omega_1-\omega_0)t+(k_1-k_0)x]}$$

$\therefore$ as $n\to\infty$

$\omega_1\to\omega_0$ and $k_1\to k_0$

$\therefore \frac{\omega_1}{\omega_0}\to 1$ and $\frac{k_1}{k_0}\to 1$

Hence $\frac{U_0(z)}{U_0(z)}\to 1$

Or $U_1(z)\approx U_0(z)$

So that the functions $U(z), U_0(z)$ or $U_1(z)$ are asymptotically stable if and only if:

$$\frac{U_0(z)}{U_1(z)}\le 1$$

Hence, under this conditions each of $U_0(z)$ or $U_1(z)$ can be expanded as:

$$U_0(z)=\sum_{n=0}^{\infty} a_n(z)e^{i(\omega t-kx)}$$

And $$U_1(z)=\sum_{n=0}^{\infty} b_n(z)e^{i(\omega t-kx)}$$

Were $\omega=\omega_0=\omega_1$ and $k=k_0=k_1$

Also, their sum and product can be expressed as:

$$U_0(z)+U_1(z)\cong\sum_{n=0}^{\infty}(a_n(z)+b_n(z))e^{i(\omega t-kx)}$$

And $$U_0(z).U_1(z)\cong\sum_{n=0}^{\infty}\sum_{n=r}^{n} a_r b_{n-r}e^{-i(\omega t-kx)}$$

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