FUNDAMENTAL THEOREMS TO IND OPTMAL PATH LENGTH IN ROUTING OF MESSAGES IN OCN

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ABSTRACT: This paper presents the most fundamental theorems in octagon-cell interconnected network (OCN), which find optimal path length between the sources to destinations in routing of messages. Also it presents the theorems to find number of nodes present in OCN of any depth along horizontal and vertical direction. The theorems of OCN have been proved by taking sufficient test cases. The optimal routing algorithm in OCN has been presented in our past research work. OCN is represented by an undirected graph and it is assumed a type of integrated circuit with an array of hundreds or thousands of central processing units (CPUs) and random-access memory banks. OCN is expandable due to its recursive structure and it has the interesting features such as node degree, bisection width and small network diameter.

KEYWORDS: Routing of messages, Octagon-Cell, Parallel System, Routing Algorithm

INTRODUCTION

An interconnection network can be represented as an undirected graph, in which vertices correspond to processors and edges correspond to the bidirectional communication links between processing elements [3]. In parallel system, the efficiency of communication system depends on the routing algorithm adopted in the system. Parallel processing system is used in many areas such as image processing and scientific computing and many problems can be solved in these areas using parallel processing elements [1-7]. The degree of a node in an interconnection network is related to the cost of hardware and message passing time is related to diameter of the network [8]. In an interconnection network, throughput is increased if the degree of network is decreased. Some features in the network of parallel machines are highly desirable such as minimal communication cost, efficient routing and the capability of topological structures [3]. In parallel system information can be transmitted over a network from a source to a destination through a sequence of intermediate switching / buffering stations or nodes through routing. The optimal routing algorithm of OCN and its important features have already been described in our past research work [1]. Many interconnected network topological structures have been designed and presented by researchers for parallel system. OCN is one of the interesting interconnected networks, which has very attractive properties and also can be useful in massively parallel system. This paper explores the fundamental theorems of OCN, which find the optimal path length in routing of information. Alternatively the theorems are the correctness of optimal routing algorithm described in [1].
LITERATURE SURVEY

Different topological structures for interconnected networks with routing algorithms have been presented by many researchers. In [3] a new class of interconnected network has been introduced called hex-cell. Efficient routing algorithm and its properties have been presented.

In [9] a new interconnection topology called the Hierarchical Cross Connected Recursive network (HCCR) with a shortest path routing algorithm for the HCCR has been presented. Proposed topology offers a high degree of regularity, scalability and symmetry with a reduced number of links and node degree. A unique address encoding scheme has been proposed for hierarchical graphical representation of HCCR networks, and based on this scheme a shortest path routing algorithm has been devised.

In [10] a new scalable interconnection network topology, called Double-Loop Hypercube (DLH), has been presented. It has been shown that the DLH network combines the positive features of the hypercube topology, such as small diameter, high connectivity, symmetry and simple routing, and the scalability and constant node degree of a new double-loop topology. Both unicasting and broadcasting routing algorithms have been designed for the DLH network, and it is based on the hybrid coding scheme. In [11], an optimal data routing scheme has been presented for mesh embedded hypercube interconnection network with faulty nodes. A fault tolerant communication scheme has been described that facilitates optimal routing in mesh embedded hypercube interconnection networks subject to node failures in parallel computing.

DESCRIPTION OF OCTAGON-CELL NETWORK

OCN has already been explained in our past research work [1]. An interconnection network can be viewed as an undirected graph, in which vertices correspond to processors and edges correspond to the bidirectional communication links between processing elements [3].

An octagon-cell has eight nodes. It has d levels numbered from 1 to d with depth d. Level 1 represents one octagon-cell. Level 2 represents eight octagon-cells surrounding the octagon-cell at level 1. Level 3 represents 16 octagon-cells surrounding the 8 octagon-cells at level 2 and so on.

Each level i has $N_i$ nodes, representing processing elements and interconnected in a ring structure. In an octagon-cell network, the number of nodes at level i is:

$$N_i = 8(4i-3)$$
Now at level 1, \( N_1 = 8 \), since there is a single octagon-cell with 8 vertices. Level 2 introduces 8 octagon-cells. Therefore at level 2 and level 3 the number of nodes

\[
N_2 = 8(4\times2-3) = 8\times5 = 40 \quad \text{and} \quad N_3 = 8(4\times3-3) = 8\times9 = 72
\]

respectively.

In octagon-cell the level \((i+1)\) has 32 nodes in addition to corresponding nodes to those at level \(i\).

Therefore \( N_i = 8 + (i-1)\times32 = 8+32\times i-32 = 32\times i-24 = 8(4\times i-3) \)

The total number of nodes in an octagon-cell network is,

\[
N = \sum_{i=1}^{d} 8(4i - 3) = 32\sum_{i=1}^{d} i - 24\sum_{i=1}^{d} = 32\sum_{i=1}^{d} (d+1)/2-24d = 16d^2 - 8d = 8d(2d-1)
\]

or \( N = 8i(2i-1) \)

Now \( N = 16d^2 - 8d \) or \( 16d^2 = N+8d \) or \( d^2 = N+8d/16 \) or \( d = 1/4 \sqrt{(N + 8d)} \)

Therefore the total no of nodes at level 1 is \( N = 8(2^1-1) = 8 \)

At level 2, \( N = 8(2^2-2) = 48 \)

At level 3, \( N = 8(2^3-3) = 120 \) and so on.

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**Figure 2: Addressing Nodes in OCN with Level 2**

In OCN when the message is received by an intermediate node, it will consider itself as a new node. Due to its recursive structure routing is done easily. In routing algorithm the level number scheme has been used. That is each node in OCN is identified by a pair \((X,Y)\), where \(X\) denotes the line number in which the node exists and \(Y\) denotes the serial number of the node in that line. A node with address \((1,1)\) is the 1\(^{st}\) node in 1\(^{st}\) line. A node with address \((1,2)\) is the 2\(^{nd}\) node in 1\(^{st}\) line and so on. Also the address of the node \((1,2)\) represents the horizontal line number 1 and vertical line number 2. \(X_s\) represents the address of source node in the horizontal direction/Line number of source node in horizontal direction. \(Y_s\) represents the address of source node in the vertical direction/Serial number of the source node in \(X_s\) line. \(X_d\) represents...
The address of destination node in the horizontal direction/Line number of destination node in horizontal direction. \( Y_d \) represents the address of destination node in the vertical direction/Serial number of destination node in \( X_d \) line.

The following theorems are devised for OCN of any depth.

**Theorem 1**

The total number of horizontal lines or the maximum line numbers of a node in OCN of depth \( d \).

\[
[(d \times 5) + (d - 2)].
\]

Remarks: For \( d = 1, 2, 3, \) and \( 4 \), Total horizontal lines = 4, 10, 16 and 22 respectively.

**Theorem 2**

The total number of vertical lines or the maximum serial number of a node at which the line numbers of the nodes mod 3 = 1 is given by \([ (3 \times d) + (d - 2) ]\) in the OCN of depth \( d \).

Remarks: For \( d = 1, 2, 3, \) and \( 4 \), Total vertical lines = 2, 6, 8 and 14 respectively.

**Theorem 3**

The total number of vertical lines or the maximum serial number of a node at which the line numbers of the nodes mod 3 \( \neq 1 \) is given by \((2 \times d)\) in the OCN of depth \( d \).

Remarks: For \( d = 1, 2, 3, \) and \( 4 \), Total vertical lines = 2, 4, 6 and 8 respectively.

**FUNDAMENTAL THEOREMS OF ROUTING ALGORITHM TO FIND THE OPTIMAL PATH LENGTH FROM SOURCE TO DESTINATION NODE**

**Theorem 1 (Horizontal Move)**

If \([(X_s \text{ mod } 3 = 1 \&\& X_s = X_d)]\) Then the optimal path length can be calculated as:

\[
\text{OPL} = |Y_d - Y_s| + \left\lceil \frac{|Y_d - Y_s|}{2} \right\rceil, \text{ If } \{(Y_s \text{ is even } \&\& Y_s < Y_d) \| (Y_s \text{ is odd } \&\& Y_s > Y_d)\}
\]

And

\[
\text{OPL} = |Y_d - Y_s| + \left\lfloor \frac{|Y_d - Y_s|}{2} \right\rfloor, \text{ If } \{(Y_s \text{ is even } \&\& Y_s > Y_d) \| (Y_s \text{ is odd } \&\& Y_s < Y_d)\}
\]

**Proof**

Let \( M \) represents the line number in horizontal direction such that \( M \text{ mod } 3 = 1 \) and \( N \) represents the line number in the same direction such that \( N \text{ mod } 3 \neq 1 \).

If \( Y_s \text{ is even and } Y_s < Y_d \), then the signal moves to right of OCN. In horizontal move the signal crosses the nodes of two horizontal lines to reach at the destination. It is well known that the number of nodes in the line \( M \) is more than that in \( N \). So the signal crosses more nodes in line \( M \) than \( N \).
In our formula we have found $|Y_d - Y_s|$ which gives the number of nodes decreased by one present in line $M$ in between $Y_s$ and $Y_d$. Next we have added it by $\left\lceil \frac{|Y_d - Y_s|}{2} \right\rceil$. In this case we have found the number of nodes crossed by the signal in line $N$. By the structure of OCN, if there are $n$ nodes along the optimal path, the shortest path length is $n-1$. So the formula counts the total number of edges crossed by the signal.

The proof of second part also follows the above concept. Test cases are shown in table 1.

**Theorem 2 (Horizontal Move)**

If $(X_s \mod 3 \neq 1 \&\& X_s = X_d)$, then the OPL is $|Y_d - Y_s| \times 3$

**Proof**

Let $M$ represents the line number in horizontal direction such that $M \mod 3 \neq 1$ and $N$ represents the line number in the same direction such that $N \mod 3 = 1$. If the source node is on the line $M$, then the signal can go to the next node of same line through 3 links. So the total number of nodes including the source and destination must be four, if no other intermediate node is present on the line $M$ except source and destination. The number of link is $n-1$ for $n$ nodes, which is given by the formula $|Y_d - Y_s|$. Therefore $|Y_d - Y_s| \times 3$ is the required formula. Test cases are shown in table 1.

**Theorem 3 (Vertical Move)**

If $(X_s \mod 3 = 1 \&\& X_d \mod 3 = 1 \&\& Y_s = Y_d)$, then the OPL = $|X_d - X_s|$

**Proof**

Let $M$ represents the line number in horizontal direction such that $M \mod 3 = 1$ and $N$ represents the line number in the same direction such that $N \mod 3 \neq 1$. If $Y_s = Y_d$, then this is vertical move. In this case the signal crosses two intermediate nodes along the optimal path with line numbers belong to $N$ in between $M$ and $M+3$. The total number of nodes including source and destination is four, if no other intermediate nodes (excluding the two nodes) are there on $M$. By addressing design of OCN the formula $|X_d - X_s|$ gives the desired result. Test cases are shown in table 1.

**Theorem 4 (Vertical Move)**

If $(X_s \mod 3 \neq 1 \&\& X_d \mod 3 \neq 1 \&\& Y_s = Y_d)$, then the OPL = $|X_d - X_s|$

**Proof**

Proof of this theorem follows the above theorem.

**Theorem 5 (Vertical Move)**

If $(X_s \mod 3 = 1 \&\& X_d \mod 3 \neq 1 \&\& (X_s < X_d || X_s > X_d) \&\& Y_s = Y_d)$, then the optimal path length is given by

OPL =
\[ \text{OPL}((X_s, Y_s) \text{ to } (X_d, Y_d)) = \text{OPL}((X_s, Y_s) \text{ to } (X_d, Y'_d)) \]

1. If \( \text{HOZL move exists along the optimal path} \)
2. \( \text{OPL}((X_s, Y_s) \text{ to } (X_d, Y'_s)) + \text{OPL}((X_d, Y'_s) \text{ to } (X_d, Y_d)) \)

\[ (n + 1) + (p \times 2) \quad \text{If } Y_s \text{ is odd} \]
\[ n + (p \times 2) \quad \text{If } Y_s \text{ is even} \]

Where \( n \) is the number of node with line number belongs to \( N \) (\( N \text{ mod } 3 \neq 1 \)) and \( n = |X_d - X_s| - p \)

\( P \) is the number of node with line number belongs to \( M \) (\( M \text{ mod } 3 = 1 \), (Excluding the line of source node) and \( p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor \)

**Proof**

The theorem is proved for the case \( X_s < X_d \).

Here \( Y_s = Y_d \), so this is vertical move. \( M \) represents the line number in horizontal direction and \( M \text{ mod } 3 = 1 \). The source node belongs to this line. \( N \) represents the line number in horizontal direction and \( N \text{ mod } 3 \neq 1 \). The destination node belongs to this line. It is known that the number of nodes on \( M \) greater that in \( N \) for a given depth \( d \) of OCN. In this case the signal always moves towards right of OCN to reach at the destination. The number of horizontal lines present in optimal path which belong to \( M \) can be calculated by the formula \( \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1 \). In between two lines \( M \) and \( M + 3 \), there are two lines \( N \) and \( N + 1 \) along the optimal path.

First \( X_s + 1 \) should be calculated and matched with \( X_d \). If match occurs, then this is considered as horizontal move exist at first step.

Otherwise \( Y_s \) shall be found, if \( Y_s \) is even and find \( Y_s + 2 \), if \( Y_s \) is odd. Once it is found, the serial numbers of nodes (i.e. \( Y_s \)) can be determined for all lines belong to \( N \). For two consecutive lines belong to \( N \), the serial number of nodes remains constant. There are two lines belong to \( N \) in between \( M \) and \( M + 3 \). It is not required to calculate the addresses of all nodes along the path. Continuing this process there must be one of the three cases: i) There may be a node whose line number matches with \( X_d \), which means horizontal move exists. ii) There may be a node whose column/serial number matches with \( Y_d \), which means vertical move exists. iii) The above two cases may not occur, means direct move exists.

The OPL \( \{(X_s, Y_s) \text{ to } (X'_s, Y_d)\} \) or \( \{(X_s, Y_s) \text{ to } (X_d, Y'_s)\} \) or \( \{(X_s, Y_s) \text{ to } (X_d, Y_d)\} = \)

\[ (n + 1) + (p \times 2) \quad \text{If } Y_s \text{ is odd} \]
\[ n + (p \times 2) \quad \text{If } Y_s \text{ is even} \]
If \( Y_s \) is odd, the number nodes are increased by one because the signal moves to right. Here \( n \) is the number of nodes with line number belongs to \( N \) and \( p \) is the number of horizontal lines (belong to \( M \)) along optimal path (Excluding the line of source node).

The case for \((X_s > X_d)\) is similar to above case.

This theorem can be very clear from the following test case:

**Test Case** \((X_s, Y_s) = (1, 11), (X_d, Y_d) = (9, 11)\)

Now \( \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1 = 3 \) gives the number of lines belong to \( M \). They are 1, 4 and 7. The serial number of nodes in line 4 and 7 will never match with \( Y_d \). In this way there will be a node, whose line number is 9 (belongs to \( N \)), whose serial number is 9. So \( X_d \) matches and a horizontal move exists.

Therefore \( \text{OPL} = \text{OPL}\{(1, 11) \text{ to } (9, 9)\} + \text{OPL}\{(9, 9) \text{ to } (9,11)\} \)

Here \( n = 6, p = 2 \)

So \( \text{OPL} = 17 \)

**Theorem 6**

If \((X_s \mod 3 \neq 1 && X_d \mod 3 = 1 && (X_s < X_d || X_s > X_d) \&\& Y_s = Y_d)\), then the optimal path length is given by

\[
\text{OPL} = \left\{ \begin{array}{l}
1. \text{OPL}\{(X_s, Y_s) \text{ to } (X_s^* Y_d)\} + \text{OPL}\{(X_s^* Y_d) \text{ to } (X_d, Y_d)\} \\
\quad \text{[If HOZL move exist on the optimal path]} \\
2. \text{OPL}\{(X_s, Y_s) \text{ to } (X_d, Y_s^*)\} + \text{OPL}\{(X_d, Y_s^*) \text{ to } (X_d, Y_d)\} \\
\quad \text{[If VERT move exist on the optimal path]} \\
3. \text{Direct Move from } (X_s, Y_s) \text{ to } (X_d, Y_d) \\
\quad \text{[If intermediate node does match with either } X_d \text{ or } Y_d]\end{array} \right. 
\]

The \( \text{OPL}\{(X_s, Y_s) \text{ to } (X_s^* Y_d)\} \text{ or } \{(X_s, Y_s) \text{ to } (X_d, Y_s^*)\} \text{ or } \{(X_s, Y_s) \text{ to } (X_d, Y_d)\} = \left\{ \begin{array}{l}
(n + 1) + (p \times 2) \quad \text{If } (Y_s^* \text{ or } Y_d) \text{ is odd} \\
(n + (p \times 2)) \quad \text{If } (Y_s^* \text{ or } Y_d) \text{ is even}\end{array} \right. 
\]

Where \( n \) is the number of node with line number belongs to \( N \) (\( N \mod 3 \neq 1 \)) and \( n = |X_d - X_s| - p \)

\( P \) is the number of horizontal lines with line number belongs to \( M \) (\( M \mod 3 = 1 \), (Excluding the line of destination node)) and \( p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor \)

**Proof**

This theorem is proved for \((X_s < X_d)\).
Theorem 7

Here $Y_s = Y_d$, so this is vertical move. $N$ represents the line number in horizontal direction and $N \mod 3 \neq 1$. The source node belongs to this line. $M$ represents the line number in horizontal direction and $M \mod 3 = 1$. The destination node belongs to this line. It is known that the number of nodes on $M$ greater than that in $N$ for a given depth $d$ of OCN. In this case the signal always moves towards left of OCN to reach at the destination. The number of lines (belong to $M$) present along the optimal path can be calculated by the formula $\frac{|X_d - X_s|}{3} + 1$. In between two lines $M$ and $M + 3$, there are two lines $N$ and $N + 1$ for any values of $M$ and $N$.

First $(X_s + 2, 2Y_s - 2)$ and $(X_s + 1, 2Y_s - 2)$ should be calculated, if $X_s \mod 3 = 2$ and if $X_s \mod 3 = 0$ respectively. When matching the line numbers with $X_d$ and if match occurs, then this is considered as horizontal move exists at first step.

Otherwise $2Y_s - 2$ will get found, which is serial number of the node and whose line number belongs to $M$. Also this is $1^{st}$ line belongs to $M$, which is present along optimal path. Once it is found, the addresses of nodes along optimal path of all lines belong to $M$ can be calculated. As the signal moves to left, so the serial number of nodes in optimal path will be decreased by 2 repeatedly, if we move down (whose line number belong to $M$). Calculating $X_s + 3$ repeatedly $(X_s$ belongs to $M)$ till destination reached, the line numbers belong to $M$ can be found. The nodes can be matched with $X_d$ and $Y_d$. Continuing this process there must be one of the three cases: i) There may be a node whose line number matches with $X_d$, which means horizontal move exists. ii) There may be a node whose column number matches with $Y_d$, which means vertical move exists. iii) The above two cases may not occur, means direct move exists.

The OPL $\{(X_s, Y_s) \text{ to } (X_{s*}, Y_{d})\}$ or $\{(X_s, Y_s) \text{ to } (X_{d*}, Y_{s})\}$ or $\{(X_{s*}, Y_{s}) \text{ to } (X_{d}, Y_{d})\}$

$$= \begin{cases} 
(n + 1) + (p \times 2) & \text{if } (Y_{s*} \text{ or } Y_d) \text{ is odd} \\
(n + (p \times 2)) & \text{if } (Y_{s*} \text{ or } Y_d) \text{ is even}
\end{cases}$$

If $(Y_{s*} \text{ or } Y_d)$ is odd, the number nodes are increased by one because the signal moves to left. Here $n$ is the number of nodes with line number belongs to $N$ and $p$ is the horizontal lines belong to $M$ (Excluding the line of destination).

The proof for the case $(X_s > X_d)$ is similar to above case.

This theorem can be very clear from the following test case:

Test Case $(X_s, Y_s) = (5, 6), (X_d, Y_d) = (10, 6)$

Now $\frac{|X_d - X_s|}{3} + 1 = 2$ gives the number of horizontal lines belong to $M$ including the line of destination. They are 7 and 10. So $2Y_s - 2 = 10$. Some of the required nodes along the optimal path are $(7, 10), (10, 8)$ and $X_d$ matches. So a horizontal move exists in optimal path.

Therefore $OPL = OPL\{(5, 6) \text{ to } (10, 8)\} + OPL\{(10, 8) \text{ to } (10, 6)\}$

Here $n = 4, p = 1$

So $OPL = 9$

Theorem 7
If \((X_s \mod 3 = 1 \&\& X_d \mod 3 = 1 \&\& (X_s < X_d | X_s > X_d) \&\& Y_s < Y_d)\), then the optimal path length is given by

\[
OPL = \begin{cases} 
1. OPL\{(X_s, Y_s) \to (X_s, Y_d)\} + OPL\{(X_s, Y_d) \to (X_d, Y_d)\} \\
2. OPL\{(X_s, Y_s) \to (X_s^*, Y_d)\} + OPL\{(X_s^*, Y_d) \to (X_d, Y_d)\} \\
[\text{If HOZL move exists along optimal path}] \\
3. OPL\{(X_s, Y_s) \to (X_d, Y_s^*)\} + OPL\{(X_d, Y_s^*) \to (X_d, Y_d)\} \\
[\text{If VERT move exists along optimal path}] \\
4. Direct Move from \((X_s, Y_s)\) to \((X_d, Y_d)\) \\
[\text{If intermediate node doesn’t match with either } X_d \text{ or } Y_d] 
\end{cases}
\]

The \(OPL\) {\{(X_s, Y_s) \to (X_s^*, Y_d)\}} or \{(X_s, Y_s) \to (X_d, Y_s^*)\} or \{(X_s, Y_s) \to (X_d, Y_d)\} =

\[
\begin{align*}
&= \begin{cases} 
(n + 1) + (p \times 2) & \text{If } (Y_s \text{ is odd } \text{and } (Y_s^* \text{ or } Y_d \text{ is even})) \\
n + (p \times 2) & \text{If } \left((Y_s \text{ is odd and } (Y_s^* \text{ or } Y_d \text{ is odd})) \right) \\
(n - 1) + (p \times 2) & \text{If } (Y_s \text{ is even } \text{and } (Y_s^* \text{ or } Y_d \text{ is odd})) 
\end{cases} \\
&= \left\lceil \frac{|X_d - X_s|}{3} \right\rceil + 1
\end{align*}
\]

Where \(n\) is the number of node with line number belongs to \(N\) \((N \mod 3 \neq 1)\) and \(n = |X_d - X_s| - p\).

\(P\) is the number of horizontal lines belong to \(M\) along the optimal path \((M \mod 3 = 1, \text{ and excluding the line of source node})\) and \(p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor \)

**Proof**

This theorem is proved for \((X_s < X_d)\).

This is neither vertical nor horizontal move. Here \(Y_s < Y_d\). The signal always moves to right. \(N\) represents the line number in horizontal direction and \(M \mod 3 \neq 1\). \(M\) represents the line number in horizontal direction and \(M \mod 3 = 1\). The source node and the destination node both belong to this line. The number of horizontal lines present which belongs to \(M\) can be calculated by the formula \(\left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1\).

First \((X_s, Y_s + 1)\) is to be calculated and the column number of this node may match with \(Y_d\). If match occurs, then both vertical and horizontal moves exist. First case of this theorem is proved. Otherwise the line numbers in optimal path belonging to \(M\) can be found by repeatedly calculating \(X_s + 3\). Starting with the line and column numbers of source node we can have the addresses of nodes with line numbers belonging to \(M\) by repeatedly decreasing 2 in column number and increasing 1 in line number in the optimal path. When a signal is on the line (belongs to \(M\)), then it must pass two nodes on that line before jumping to next line. So we can have the addresses of two nodes (consecutive) on this line.

Continuing this process there must be one of the three cases: i) There may be a node whose line number matches with \(X_d\), which means horizontal move exists. ii) There may be a node whose column number matches with \(Y_d\), which means vertical move exists. iii) The above two cases may not occur, means direct move exists.
The OPL $\{ (X_s, Y_s) \to (X_s', Y_s') \}$ or $\{ (X_s, Y_s) \to (X_d, Y_d) \}$ or $\{ (X_s, Y_s) \to (X_d, Y_d) \}$

$$OPL = \begin{cases} 
(n + 1) + (p \times 2) & \text{if } (Y_s \text{ is odd and } (Y_s' \text{ or } Y_d \text{ is even})) \\
n + (p \times 2) & \text{if } [(Y_s \text{ is odd and } (Y_s' \text{ or } Y_d \text{ is odd})) \\
\text{or } (Y_s \text{ is even and } (Y_s' \text{ or } Y_d \text{ is even}))]) \\
(n - 1) + (p \times 2) & \text{if } (Y_s \text{ is even and } (Y_s' \text{ or } Y_d \text{ is odd})) 
\end{cases}$$

Here $n$ is the number of nodes, whose line number belongs to $N$ and $p$ is the number of nodes(excluding source node), whose line number belongs to $M$.

The proof for the case $(X_s > X_d)$ is similar to above case.

This theorem can be very clear from the following test case:

**Test Case** $(X_s, Y_s) = (4, 10)$, $(X_d, Y_d) = (13, 16)$

Now $\left\lceil \frac{|X_d - X_s|}{3} \right\rceil + 1 = 4$ gives the number of lines belongs to $M$ including source and destination. They are 4, 7, 10, 13. Some of the required nodes on the optimal path are $(4, 10)$, $(7, 12)$, $(10, 14)$, and $(13, 16)$. Before the node $(13, 16)$, there is $(13, 15)$. Now $X_d$ matches. So a horizontal move exists in optimal path.

Therefore $OPL = OPL\{(4, 10) \to (13, 15)\} + OPL\{(13, 15) \to (13, 16)\}$

Here $n = 6$, $p = 3$

So $OPL = 12$

Similarly for $(X_s > X_d)$, the test cases are shown in table 1.

**Theorem 8**

If $(X_s \text{ mod } 3 = 1 \& \& X_d \text{ mod } 3 \neq 1 \& \& (X_s < X_d || X_s > X_d) \& \& Y_s < Y_d$, then the optimal path length is given by

$$OPL = \begin{cases} 
1. OPL \{ (X_s, Y_s) \to (X_s', Y_d) \} + OPL\{ (X_s, Y_d) \to (X_d, Y_d) \} & \text{if HOZL move exists along optimal path} \\
2. OPL\{ (X_s, Y_s) \to (X_d, Y_s') \} + OPL\{ (X_d, Y_s') \to (X_d, Y_d) \} & \text{if VERT move exists along optimal path} \\
3. Direct Move from $(X_s, Y_s)$ to $(X_d, Y_d)$ & \text{if intermediate node doesn't match with either $X_d$ or $Y_d$} 
\end{cases}$$

The OPL $\{ (X_s, Y_s) \to (X_s', Y_d) \}$ or $\{ (X_s, Y_s) \to (X_d, Y_s') \}$ or $\{ (X_s, Y_s) \to (X_d, Y_d) \} = 
\begin{cases} 
(n + 1) + (p \times 2) & \text{if } (Y_s \text{ is odd}) \\
n + (p \times 2) & \text{if } (Y_s \text{ is even}) 
\end{cases}$

Where $n$ is the number of nodes with line number belongs to $N$ $(N \text{ mod } 3 \neq 1)$ and $n = |X_d - X_s| - p$
p is the number of horizontal lines belong to M along the optimal path (M mod 3 = 1 and excluding the line of source node) and

\[ p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1 \]

**Proof**

This theorem is proved for \( X_s < X_d \). This is neither vertical nor horizontal move. Here \( Y_s < Y_d \).

The signal always moves to right. \( N \) represents the line number in horizontal direction and \( N \mod 3 \neq 1 \). \( M \) represents the line number in horizontal direction and \( M \mod 3 = 1 \). The source node and the destination node belong to \( M \) and \( N \) respectively.

The number of lines along \( OP \) belongs to \( M \) can be calculated by the formula

\[ \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1 \]

By the structure and the addressing scheme of OCN [1], calculating \( (Y_s/2 + 2) \) and \( (Y_s/2 + 1) \), if \( Y_s \) is odd and \( Y_s \) is even respectively, this gives the addresses of the nodes, whose line numbers belong to \( N \) in the optimal path. As it is well known that there are two lines \( N \) and \( N + 1 \), in between \( M \) and \( M + 3 \), we can find the addresses of nodes, whose line numbers belong to \( N \) by repeatedly increasing the line number by 1, but increasing column number by 1 only once in between \( M \) and \( M + 3 \).

Continuing this process there must be one of the three cases: i) There may be a node whose line number matches with \( X_d \), which means horizontal move exists. ii) There may be a node whose column number matches with \( Y_d \), which means vertical move exists. iii) The above two cases may not occur, means direct move exists.

The OPL \( \{(X_s, Y_s) \to (X_d, Y_d)\} \) or \( \{(X_s, Y_s) \to (X_d, Y_s)\} \) or \( \{(X_s, Y_s) \to (X_d, Y_d)\} \)

\[ \text{OPL} = \begin{cases} (n + 1) + (p \times 2) & \text{If } Y_s \text{ is odd} \\ n + (p \times 2) & \text{If } Y_s \text{ is even} \end{cases} \]

Here \( n \) is the number of nodes, whose line number belongs to \( N \) and \( p \) is the number of horizontal lines belongs to \( M \) along \( OP \) (excluding the line of source node).

This theorem can be very clear from the following test case:

**Test Case** \( (X_s, Y_s) = (7, 3), (X_d, Y_d) = (12, 7) \)

Now \[ \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1 = 2 \] gives the number of lines belongs to \( M \) including source node. They are 7 and 10.

Since \( Y_s \) is odd, so \( Y_s/2 + 2 = 3 \). This is the column number of the first node, whose line number belongs to \( N \) along \( OP \). So some of the required nodes along the optimal path are (7, 3), (9, 3), (11, 4) and (12, 4). Now \( X_d \) matches. So a horizontal move exists in optimal path.

Therefore \( \text{OPL} = \text{OPL} \{(7, 3) \to (12, 4)\} + \text{OPL} \{(12, 4) \to (12, 7)\} \)

Here \( n = 4, p = 1 \)

So \( \text{OPL} = 16 \)

Similarly for the case \( X_s > X_d \) test cases are shown in table 1.

**Theorem 9**

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If \((X_s \mod 3 \neq 1 \&\& X_d \mod 3 = 1 \&\& (X_s < X_d \| X_s > X_d) \&\& Y_s < Y_d\), then the optimal path length is given by

\[
\text{OPL} = \begin{cases} 
\text{OPL} \{ (X_s, Y_s) \text{ to } (X_s^*, Y_d) \} + \text{OPL} \{ (X_s^*, Y_d) \text{ to } (X_d, Y_d) \} & \text{[If HOZL move exists along OP]} \\
\text{OPL} \{ (X_s, Y_s) \text{ to } (X_s^*, Y_s^*) \} + \text{OPL} \{ (X_s^*, Y_s^*) \text{ to } (X_d, Y_d) \} & \text{[If VERT move exists along OP]} \\
\text{Direct Move from } (X_s, Y_s) \text{ to } (X_d, Y_d) & \text{[If intermediate node doesn’t match with either } X_d \text{ or } Y_d]\end{cases}
\]

The OPL \{ (X_s, Y_s) \text{ to } (X_s^*, Y_d) \} or \{ (X_s, Y_s) \text{ to } (X_d, Y_s^*) \} or \{ (X_s, Y_s) \text{ to } (X_d, Y_d) \}

\[
= \begin{cases} 
(n + 1) + (p \times 2) & (n + (p \times 2)) \\
\end{cases}
\]

The 1\textsuperscript{st} one works, if \{[(2Y_s - 1) \leq Y_d \&\& (Y_s^* \text{ or } Y_d) \text{ is even}] || [(2Y_s - 1) > Y_d \&\& (Y_s^* \text{ or } Y_d) \text{ is odd}]\}

The 2\textsuperscript{nd} one works, if \{[(2Y_s - 1) \leq Y_d \&\& Y_s^* \text{ is odd}] || [(2Y_s - 2) > Y_d \&\& Y_s^* \text{ is even}]\}

Here \(n\) is the number of node on line \(N\) (\(N \mod 3 \neq 1\)) and \(n = |X_d - X_s| - p\) and \(p\) is the number of horizontal lines excluding the line of destination node belong to \(M\) (\(M \mod 3 = 1\)) and \(p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1\).

**Proof**

The Theorem is proved for \(X_s < X_d\). This is neither vertical nor horizontal move. Here \(Y_s < Y_d\). The signal may move to right or left. \(N\) represents the line number of the node in horizontal direction and \(N \mod 3 \neq 1\). \(M\) represents the line number of the node in horizontal direction and \(M \mod 3 = 1\). The source node and the destination node belong to \(N\) and \(M\) respectively. The number of lines present which belongs to \(M\) along OP can be calculated by the formula \(\left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1\).

By the structure and the addressing scheme of OCN \([1]\), if \((2Y_s - 1) > Y_d\) the signal moves to left of OCN and if \((2Y_s - 1) \leq Y_d\) the signal moves to right of OCN. As \(X_d \mod 3 = 1\), So moving towards down the addresses of the nodes, whose line numbers belong to \(M\) can be calculated. Keeping in mind whether the signal moves to left or right, we can calculate \(2Y_s - 2\) or \(2Y_s - 1\) respectively. So once the address of 1\textsuperscript{st} node in optimal path whose line number belongs to \(M\) is found, then the addresses of all nodes of this character can be found along that path. The addresses of these nodes can be compared with the destination node. Continuing this process there must be one of the three cases: i) There may be a node whose line number matches with \(X_d\), which means horizontal move exists. ii) There may be a node whose column number matches with \(Y_d\), which means vertical move exists. iii) The above two cases may not occur, means direct move exists.

The OPL \{ (X_s, Y_s) \text{ to } (X_s^*, Y_d) \} or \{ (X_s, Y_s) \text{ to } (X_d, Y_s^*) \} or \{ (X_s, Y_s) \text{ to } (X_d, Y_d) \}
\[
OPL = \begin{cases} 
(n + 1) + (p \times 2) \\
(n + (p \times 2))
\end{cases}
\]

The 1\textsuperscript{st} one works, if \([(2Y_s - 1) \leq Y_d \&\& (Y_s^* \text{ or } Y_d) \text{ is even}] \implies [(2Y_s - 1) > Y_d \&\& (Y_s^* \text{ or } Y_d) \text{ is odd}]\]

The 2\textsuperscript{nd} one works, if \([(2Y_s - 1) \leq Y_d \&\& (Y_s^* \text{ or } Y_d) \text{ is odd}] \implies [(2Y_s - 1) > Y_d \&\& (Y_s^* \text{ or } Y_d) \text{ is even}]\]

Here \(n\) is the number of nodes, whose line number belongs to \(N\) and \(p\) is the number of horizontal lines along \(OP\) (excluding the line of destination node), with line number belongs to \(M\).

This theorem can be very clear from the following test case:

Test Case (\(X_s, Y_s\) = (2, 3), \(X_d, Y_d\) = (7, 5))

Now \(\left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1 = 2\) gives the number of lines belongs to \(M\) including destination node. They are 4 and 7.

\(2Y_s - 1 \leq Y_d\). So the signal moves to right. \(2Y_s - 1 = 5\). The address of 1\textsuperscript{st} node along the optimal path, whose line number belongs to \(M\) is \((4, 5)\). Now \(Y_d\) matches. So a vertical move exists in optimal path.

Therefore \(OPL = OPL \{(2, 3) \text{ to } (4, 5)\} + OPL \{(4, 5) \text{ to } (7, 5)\}\)

Here \(n = 2\), \(p = 0\). So \(OPL = 5\)

For the case \((X_s > X_d)\), the prove is simlilar and the test cases are shown in table 1.

**Theorem 10**

If \((X_s \text{ mod } 3 \neq 1 \&\& X_d \text{ mod } 3 \neq 1 \&\& X_s < X_d \&\& (Y_s < Y_d \| Y_s > Y_d)\), then the optimal path length is given by

\[
OPL = \begin{cases} 
1. OPL \{(X_s, Y_s) \text{ to } (X_s^*, Y_d)\} + OPL\{(X_s^*, Y_d) \text{ to } (X_d, Y_d)\} \\
\quad \text{[If HOZL move exists along } OP]\} \\
2. OPL\{(X_s, Y_s) \text{ to } (X_d, Y_s^*)\} + OPL\{(X_d, Y_s^*) \text{ to } (X_d, Y_d)\} \\
\quad \text{[If VERT move exists along } OP]\} \\
3. \text{Direct Move from } (X_s, Y_s) \text{ to } (X_d, Y_d) \\
\quad \text{[If inyermidate node doesn’t match with either } X_d \text{ or } Y_d]\}
\end{cases}
\]

The OPL \ \{(X_s, Y_s) \text{ to } (X_s^*, Y_d)\} \text{ or } \{(X_s, Y_s) \text{ to } (X_d, Y_s^*)\} \text{ or } \{(X_s, Y_s) \text{ to } (X_d, Y_d)\}

\[= n + (2 \times p)\]

Here \(n\) is the total number of nodes decreased by one with line number belongs to \(N\) (\(N \text{ mod } 3 \neq 1\)) and \(n = |X_d - X_s| - p\). \(p\) is the total number of horizontal lines with line number belongs to \(M\) along \(OP\) (\(M \text{ mod } 3 = 1\)).
\[ p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor \text{ if } X_s \neq 3t, \quad (t \text{ is any natural number}) \]

\[ p = \left\lfloor \frac{|(X_s - 1) - X_d|}{3} \right\rfloor \text{ if } X_s = 3t, \quad (t \text{ is any natural number}) \]

**Proof**

The theorem is proved for \( Y_s < Y_d \). This is neither vertical nor horizontal move. The signal always moves to right. \( N \) represents the line number of the node in horizontal direction and \( N \mod 3 \neq 1 \). \( M \) represents the line number of the node in horizontal direction and \( M \mod 3 = 1 \). The source and destination node both belong to \( N \). The number of horizontal lines with line numbers belong to \( M \) along \( OP \) can be calculated by the formula:

\[ p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor \text{ if } X_s \neq 3t, \quad (t \text{ is any natural number}) \]

\[ p = \left\lfloor \frac{|(X_s - 1) - X_d|}{3} \right\rfloor \text{ if } X_s = 3t, \quad (t \text{ is any natural number}) \]

By the structure and the addressing scheme of OCN [1], calculating \( (2Y_s - 1) \), the column number of the \( 1^{st} \) node will get found whose line number belongs to \( M \). Also line number of this node can be calculated from \( X_s \). It is well known that there are two horizontal lines belong to \( N \) in between \( M \) and \( M + 3 \), for any values of \( M \) and \( N \) according to the depth of OCN. In between \( M \) and \( M + 3 \), the column numbers of the two nodes aligned vertically are same. As the signal moves to right bottom like a ladder, so the column number of nodes is increased by once in between \( N \) and \( N + 4 \) along the \( OP \) repeatedly till destination reached. The aim is to check whether the node matches with the destination node. Clearly it is to find the address of intermediate node which matches with either \( X_d \) or \( Y_d \), with line number belongs to \( N \). Continuing this process there must be one of the three cases: i) There may be a node whose line number matches with \( X_d \), which means horizontal move exists. ii) There may be a node with column number matches with \( Y_d \), which means vertical move exists. iii) The above two cases may not occur, means direct move exists.

The OPL \( \{(X_s, Y_s) \text{ to } (X_d, Y_d)\} \) or \( \{(X_s, Y_s) \text{ to } (X_d, Y'_s)\} \) or \( \{(X_s, Y_s) \text{ to } (X_d, Y_d)\} \)

\[ = n + (2 \times p) \]

Where \( n \) is the number of nodes decreased by one with line number belongs to \( N \) (\( N \mod 3 \neq 1 \)) along the optimal path and \( n = |X_d - X_s| - p \). \( p \) is the number of horizontal lines with line number belongs to \( M \) (\( M \mod 3 = 1 \)) along the optimal path.

\[ p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor \text{ if } X_s \neq 3t, \quad (t \text{ is any natural number}) \]
\[ p = \left\lfloor \frac{|X_s - 1| - X_d}{3} \right\rfloor \text{ if } X_s = 3t, (t \text{ is any natural number}) \]

This theorem can be very clear from the following test case:

**Test Case** \((X_s, Y_s) = (2, 4), (X_d, Y_d) = (8, 8)\)

Now \(p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor = 2\)

This gives the number of lines belongs to \(M\). They are 4 and 7. Some required nodes on the optimal path are: \((2, 4)\), \((6, 5)\), \((8, 6)\)

Now \(X_d\) matches. So a horizontal move exists in optimal path.

Therefore \(OPL = OPL \{ (2, 4) \text{ to } (8, 6) \} + OPL \{ (8, 6) \text{ to } (8, 8) \} \)

\(P = 2, n = 4, OPL = 14\)

For the case \((Y_s > Y_d)\), the proof is similar to above and test cases for this case are shown in table 1.

**NOTE** The following theorem are not proved as these can be proved in similar way as above.

**Theorem 11**

If \((X_s \mod 3 = 1 \land X_d \mod 3 = 1 \land (X_s < X_d) \land Y_s > Y_d)\), then the optimal path length is given by

\[
OPL =\]

\[
\begin{align*}
1. & \quad OPL\{(X_s, Y_s) \text{ to } (X_s, Y_d)\} + OPL\{(X_s, Y_d) \text{ to } (X_d, Y_d)\} \\
2. & \quad OPL\{(X_s, Y_s) \text{ to } (X_s^*, Y_d)\} + OPL\{(X_s^*, Y_d) \text{ to } (X_d, Y_d)\} \\
& \quad \text{[If HOZL move exists along the optimal path]} \\
3. & \quad OPL\{(X_s, Y_s) \text{ to } (X_d, Y_s^*)\} + OPL\{(X_d, Y_s^*) \text{ to } (X_d, Y_d)\} \\
& \quad \text{[If VERT move exists along the optimal path]} \\
4. & \quad \text{Direct Move from } (X_s, Y_s) \text{ to } (X_d, Y_d) \\
& \quad \text{[If intermediate node doesn’t match with either } X_d \text{ or } Y_d]\end{align*}
\]

The \(OPL \{ (X_s, Y_s) \text{ to } (X_s^*, Y_d) \}\) or \(\{ (X_s, Y_s) \text{ to } (X_d, Y_s^*) \}\) or \(\{ (X_s, Y_s) \text{ to } (X_d, Y_d) \}\) =

\[
\begin{cases} 
(n - 1) + (p \times 2) & \text{if } (Y_s \text{ is odd and } (Y_s^* \text{ or } Y_d \text{ is even})) \\
(n + 1) + (p \times 2) & \text{if } [(Y_s \text{ is odd and } (Y_s^* \text{ or } Y_d \text{ is odd}) \text{ or } (Y_s \text{ and } (Y_s^* \text{ or } Y_d \text{ are even})]}
\end{cases}
\]

Where \(n\) is the number of nodes with line number belongs to \(N\) \((N \mod 3 \not= 1)\) and \(n = |X_d - X_s| - p\).
P is the number of horizontal lines belong to M (M mod 3 = 1, excluding the line of source node) and  
\[ p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor \]

**Test Case** \((X_s, Y_s) = (1, 9), (X_d, Y_d) = (7, 3)\)

\(Y_s - 1\) doesn’t match with \(Y_d\).

Now \(\left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1 = 3\) gives the number of lines belongs to M. They are 1, 4 and 7.

Here the signal always moves to left. Some required nodes in the path are (1, 9), (4, 7), (7, 6). Now \(X_d\) matches. So a horizontal move exists in optimal path.

Therefore  \(OPL = OPL\{(1, 9)\ to\ (7, 6)\} + OPL\{(7, 6)\ to\ (7, 3)\}\)

Here \(n = 4, p = 2\). So \(OPL = 11\)

For the case \((X_s > X_d)\), the test cases are shown in table 1.

**Theorem 12**

If \((X_s \ mod\ 3 = 1 \ \&\& \ X_d \ mod\ 3 \neq 1 \ \&\& \ (X_s < X_d \ || \ X_s > X_d) \ \&\& \ Y_s > Y_d)\), then the optimal path length is given by

\[ OPL = \left\{ \begin{array}{l}
1. OPL\{(X_s, Y_s)\ to\ (X_s^*, Y_d)\} + OPL\{(X_s^*, Y_d)\ to\ (X_d, Y_d)\} \\
\hspace{1cm} [\text{If HOZL move exist on the optimal path}]
2. OPL\{(X_s, Y_s)\ to\ (X_d, Y_s^*)\} + OPL\{(X_d, Y_s^*)\ to\ (X_d, Y_d)\} \\
\hspace{1cm} [\text{If VERT move exist on the optimal path}]
3. \text{Direct Move from} \ (X_s, Y_s) \ \text{to} \ (X_d, Y_d) \\
\hspace{1cm} [\text{If intermediate node doesn’t match with either} \ X_d \ \text{or} \ Y_d]\end{array} \right. \]

The \(OPL \{(X_s, Y_s)\ to\ (X_s^*, Y_d)\}\) or \(\{(X_s, Y_s)\ to\ (X_d, Y_s^*)\}\) or \(\{(X_s, Y_s)\ to\ (X_d, Y_d)\}\) =

\[ \left\{ \begin{array}{l}
1. (n + 1) + (p \times 2) \\
2. n + (p \times 2) \\
\end{array} \right. \]

\[ [\text{If} \ (2Y_d - 1 \leq Y_s \ \text{and} \ Y_s \ \text{is even}) \ \text{or} \ (2Y_d - 1 > Y_s \ \text{and} \ Y_s \ \text{is odd})] \]

Where \(n\) is the total number of nodes with line numbers belong to \(N\) (\(N \ mod\ 3 \neq 1\)) and  
\[ n = |X_d - X_s| - p \]

\(P\) is the total number of lines belong to \(M\) (\(M \ mod\ 3 = 1\), excluding line of source node) and  
\[ p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor \]

**Test Case** \((X_s, Y_s) = (4, 11), (X_d, Y_d) = (9, 4)\)

Now \(\left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1 = 2\) gives the number of lines belongs to \(M\) including source node. They are 4, and 7.
2Y_d - 1 < Y_s. So the signal moves left. As Y_s is odd, Y_s/2 + 1 = 6 is the column number of first node, whose line number belongs to N in optimal path. So some of the required nodes along the optimal path are (4, 11), (6, 6), (9, 5). So X_d matches and horizontal move exists.

Therefore \( OPL = OPL \{(4, 11) \to (9, 5)\} + OPL \{(9, 5) \to (9, 4)\} \)

Here \( n = 4, p = 1 \). So \( OPL = 9 \)

For the case \( (X_s > X_d) \), the test cases are shown in table 1.

**Theorem 13**

If \( (X_s \, \text{mod} \, 3 \neq 1 \& \& X_d \, \text{mod} \, 3 = 1 \& \& (X_s < X_d \| X_s > X_d) \& \& Y_s > Y_d) \), then the optimal path length is given by

\[
OPL = \begin{cases} 
1. OPL\{(X_s, Y_s) \to (X_s^*, Y_s^*)\} + OPL\{(X_s^*, Y_d) \to (X_d, Y_d)\} & \text{[If HOZL move exist on the optimal path]} \\
2. OPL\{(X_s, Y_s) \to (X_d, Y_s^*)\} + OPL\{(X_d, Y_s^*) \to (X_d, Y_d)\} & \text{[If VERT move exist on the optimal path]} \\
3. Direct Move from (X_s, Y_s) to (X_d, Y_d) & \text{[If intermediate node doesn’t match with either X_d or Y_d]} 
\end{cases}
\]

The \( OPL \{(X_s, Y_s) \to (X_s^*, Y_d)\} \) or \( \{(X_s, Y_s) \to (X_d, Y_s^*)\} \) or \( \{(X_s, Y_s) \to (X_d, Y_d)\} \)

\[
= \begin{cases} 
n + (p \times 2) & \text{If} \ (Y_s^* \, \text{or} \ Y_d \, \text{is even}) \\
(n + 1) + (p \times 2) & \text{If} \ (Y_s^* \, \text{or} \ Y_d \, \text{is odd}) 
\end{cases}
\]

Where \( n \) is the number of node with line number belongs to \( N \) \( (N \, \text{mod} \, 3 \neq 1) \) and \( n = |X_d - X_s| - p \)

\( P \) is the number of horizontal lines belong to \( M \) along \( OP \) (Excluding the line of destination node and \( M \, \text{mod} \, 3 = 1 \)) and \( p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor \)

**Test Case** \( (X_s, Y_s) = (3, 4), (X_d, Y_d) = (10, 3) \)

Now \( \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor + 1 = 3 \) gives the number of lines belongs to \( M \). They are 4, 7 and 10. The signal always moves left. So \( 2Y_s - 2 = 6 \) is the column number of the 1st node, with line number belongs to \( M \). Some of the required nodes along the optimal path are (4, 6), (7, 4), (7, 3). So \( Y_d \) matches and vertical move exists.

Therefore \( OPL = OPL \{(3, 4) \to (7, 3)\} + OPL \{(7, 3) \to (10, 3)\} \)

Here \( n = 3, p = 1 \), \( OPL = 9 \)

For the case \( (X_s > X_d) \), the test cases are shown in table 1.

**Test Case1:** \( (X_s, Y_s) = (12, 7), (X_d, Y_d) = (7, 5) \)

**Theorem 14**
If \( (X_s \mod 3 \neq 1 \&\& X_d \mod 3 \neq 1 \&\& X_s > X_d \&\& (Y_s < Y_d \| Y_s > Y_d) \), then the optimal path length is given by

\[
OPL = \begin{cases} 
1. & \text{OPL} \{ (X_s, Y_s) \to (X_s', Y_d) \} + \text{OPL}\{ (X_s', Y_d) \to (X_d, Y_d) \} \\
& \text{[If HOZL move exists along OP]} \\
2. & \text{OPL}\{ (X_s, Y_s) \to (X_d, Y_s') \} + \text{OPL}\{ (X_d, Y_s') \to (X_d, Y_d) \} \\
& \text{[If VERT move exists along OP]} \\
3. & \text{Direct Move from} \ (X_s, Y_s) \ \text{to} \ (X_d, Y_d) \\
& \text{[If intermediate node doesn’t match with either} \ X_d \ \text{or} \ Y_d) \\
\end{cases}
\]

The \( \text{OPL} \ \{ \ (X_s, Y_s) \to (X_s', Y_d) \} \) or \( \{ (X_s, Y_s) \to (X_d, Y_s') \} \) or \( \{ (X_s, Y_s) \to (X_d, Y_d) \} \)

\[
= n + (2 \times p)
\]

Here \( n \) is the number of nodes decreased by one with line number belongs to \( N \) (\( N \mod 3 \neq 1 \))

and \( n = |X_d - X_s| - p \). \( p \) is the total number of horizontal lines with line number belongs to \( M \) (\( M \mod 3 = 1 \)).

\[
p = \left\lfloor \frac{|X_d - X_s|}{3} \right\rfloor \text{ if } X_d \text{ or } X_s^* \neq 3t \text{ (t is any natural number)}
\]

\[
p = \left\lfloor \frac{|(X_d - 1) - X_s|}{3} \right\rfloor \text{ if } X_d \text{ or } X_s^* = 3t \text{ (t is any natural number)}
\]

Here \( X_s^* \) corresponds to \( X_d \) and \( Y_s^* \) corresponds to \( Y_d \).

**Test Case** \( (X_s, Y_s) = (8, 5), (X_d, Y_d) = (3, 10) \)

Here \( X_d \) matches. So a horizontal move exists along optimal path.

Therefore we have \( \text{OPL} = \text{OPL}\{ (8, 5) \to (3, 7) \} + \text{OPL}\{ (3, 7) \to (3, 10) \} \)

Here \( n = 3, p = 2 \)

So \( \text{OPL} = 16 \)

For the case \( (Y_s > Y_d) \), the tase cases are shown in table 1.

**RESULTS/FINDINGS**

The optimal routing algorithm for OCN has been presented in [1]. In this paper very interesting theorems have been proved and explained, which are the correctness of optimal routing algorithm that have been presented in [1]. The first four basic theorems “Theorem-1, 2, 3 and 4” have been proved in this paper to find optimal path in horizontal direction \( (X_s=X_d) \) and in vertical direction \( (Y_s=Y_d) \). These four theorems are used in other theorems if there exists a horizontal move or vertical move. Theorems 5 and 6 are the cases of vertical moves but the
line number $X_s$ of source and the line number $X_d$ of destination are of different categories. In our theorems, it is found that a horizontal move or a vertical move may exist in between the source and destination. If the line number of any intermediate node matches with $X_d$, then this is considered as a horizontal move. But in case of existence vertical move, it is very important that to check the line number of the node. If vertical move exist, then to check whether the line number of the intermediate node and $X_d$ are of same category. Because direct formulae have been presented for the case of vertical move where the line numbers are of same category. So no more calculations are required. The basic idea of our theorems is that whether a node exists in optimal path such that its line number or column number matches with $X_d$ or $Y_d$ respectively. Some of the test cases are presented in figures.

<table>
<thead>
<tr>
<th>Test Cases for $(X_s \neq X_d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cases</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>01</td>
</tr>
<tr>
<td>02</td>
</tr>
</tbody>
</table>
## Test Cases for \( X_s = X_d \)

<table>
<thead>
<tr>
<th>Cases</th>
<th>Source Node</th>
<th>Destination Node</th>
<th>OPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>(10,4)</td>
<td>(10,1)</td>
<td>4</td>
</tr>
<tr>
<td>32</td>
<td>(7,6)</td>
<td>(7,2)</td>
<td>6</td>
</tr>
<tr>
<td>33</td>
<td>(4,1)</td>
<td>(4,8)</td>
<td>10</td>
</tr>
<tr>
<td>34</td>
<td>(4,2)</td>
<td>(4,8)</td>
<td>9</td>
</tr>
<tr>
<td>35</td>
<td>(13,1)</td>
<td>(13,9)</td>
<td>12</td>
</tr>
<tr>
<td>36</td>
<td>(13,5)</td>
<td>(13,1)</td>
<td>6</td>
</tr>
<tr>
<td>37</td>
<td>(13,7)</td>
<td>(13,2)</td>
<td>8</td>
</tr>
<tr>
<td>38</td>
<td>(13,4)</td>
<td>(13,9)</td>
<td>8</td>
</tr>
<tr>
<td>39</td>
<td>(1,12)</td>
<td>(1,7)</td>
<td>7</td>
</tr>
<tr>
<td>40</td>
<td>(1,3)</td>
<td>(1,10)</td>
<td>10</td>
</tr>
<tr>
<td>41</td>
<td>(1,2)</td>
<td>(1,5)</td>
<td>5</td>
</tr>
<tr>
<td>42</td>
<td>(1,9)</td>
<td>(1,4)</td>
<td>8</td>
</tr>
<tr>
<td>43</td>
<td>(2,2)</td>
<td>(2,5)</td>
<td>9</td>
</tr>
<tr>
<td>44</td>
<td>(5,9)</td>
<td>(5,3)</td>
<td>18</td>
</tr>
<tr>
<td>45</td>
<td>(3,5)</td>
<td>(3,7)</td>
<td>6</td>
</tr>
<tr>
<td>46</td>
<td>(9,10)</td>
<td>(9,2)</td>
<td>24</td>
</tr>
</tbody>
</table>
Test Cases for ($Y_s = Y_d$)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>(7.5)</td>
<td>(13.5)</td>
<td>6</td>
</tr>
<tr>
<td>48</td>
<td>(16.9)</td>
<td>(4.9)</td>
<td>12</td>
</tr>
<tr>
<td>49</td>
<td>(3.9)</td>
<td>(8.9)</td>
<td>5</td>
</tr>
<tr>
<td>50</td>
<td>(11.7)</td>
<td>(2.7)</td>
<td>9</td>
</tr>
</tbody>
</table>

*Table.1 Fifty Test Cases with OPL*

**IMPLICATION TO RESEARCH AND PRACTICE**

The architecture of OCN contains some of the important characters of other interconnected network topologies such as hex-cell, binary tree, star, linear array etc. This structure is proposed as a good candidate for massively parallel systems, as OCN is assumed a type of integrated circuit with an array of hundreds or thousands of central processing units (CPUs) and random-access memory banks. Many researchers have proposed different architectures to be useful in wide range of network applications but OCN has efficient routing scheme, recursive nature and can connect hundreds of thousands nodes with only 3 links per node.

**DISCUSSION AND CONCLUSION**

The fundamental theorems discussed in this paper can find the optimum path length between the source and destination in case of routing of information. From the optimal routing algorithm and the addressing scheme which are the key points of these theorems, the optimal path length can easily be found. The idea behind the theorem is that it always finds the address of the intermediate node of which either the line number matches with $X_d$ or the column number matches with $Y_d$. If this match does not occur then there exists a direct move in between the source and destination. In few cases the line number of intermediate node matches with $X_s$. These types of cases are clearly presented above. The theorems found in OCN are very interesting, as all these maintain a special property and applicable for the OCN of any depth.

**FUTURE RESEARCH**

OCN has comparable properties with different topological structures. It can be used in mobile networks, massively parallel systems etc. The future research includes the fault tolerant scheme in routing of information in OCN in all directions of network and its usefulness in wide range of networks.

**REFERENCES**


