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EVALUATION OF SOME ESTIMATORS PERFORMANCE ON LINEAR MODELS WITH HETEROSCEDASTICITY AND SERIAL AUTOCORRELATION

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ABSTRACT: *In many, if not most, econometric applications, economic data arises from time-series or cross-sectional studies which typically exhibit some form of autocorrelation and/or heteroskedasticity. If the covariance structure were known, it could be taken into account in a (parametric) model, but more often than not the form of autocorrelation and heteroskedasticity is unknown. In such cases, model parameters can typically still be estimated consistently using the usual estimating functions, but for valid inference in such models a consistent covariance matrix estimate is essential. In this study, the strength of some methods of estimating classical linear regression model with both negative and positive autocorrelation in the presence of heteroscedasticity were investigated. The Ordinary Least Square (OLS) estimator, Heteroskedasticity and Autocorrelation (HAC) estimators which includes Cluster-Robust Standard Errors estimators, Newey-West standard errors and Feasible Generalized Least Squares Estimator (FGLS) were considered in this study. Monte-Carlo experiments were conducted and the study further identifies the best estimator that can be used for prediction purpose by adopting the goodness of fit statistics of the estimators. The result revealed the superiority of the Newey-West standard errors over others using root mean squared error (RMSE) of the parameter estimates and relative efficiency (RR) as assessment criteria among others over various considerations for the distribution of the serial correlation and heteroskedasticity.*

KEYWORDS: ordinary least squares estimation, heteroscedasticity, autocorrelation, Panel data, Robust Regression.

INTRODUCTION

Linear regression model is probably the most widely used statistical technique for solving functional relationship problems among variables. It helps to explain observations of a dependent variable, y, with observed values of one or more independent variables, X_1, X_2,…., X_p. In an attempt to explain the dependent variable, prediction of its values often becomes very essential and necessary. Moreover, the linear regression model is formulated under some basic assumptions. Among these assumptions are regressors being assumed to be non-stochastic (fixed in repeated sampling) and independent. The error terms also assumed to be independent, have constant variance and are also independent of the regressors. When all these assumptions of the classical linear regression model are satisfied, the

Ordinary Least Square (OLS) estimator given as: $$\hat{\beta} = (X^T X)^{-1} X^T Y \quad (1)$$

is known to possess some ideal or optimum properties of an estimator which include linearity, unbiasedness and efficiency [1]. When the regression model does not meet the fundamental assumptions, the prediction and estimation of the model may become biased. Disturbances are heteroscedastic when they have different variances. Heteroscedasticity arises in volatile high-frequency time-series data such as daily observations in financial markets and in cross-section data where the scale of the dependent variable and the explanatory power of the model tend to vary across observations. Microeconomic data such as expenditure surveys are typical. Economic time series also often display a "memory" in that variation around the regression function is not independent from one period to the next (autocorrelation). The seasonally adjusted price and quantity series published by government agencies are examples. A single atypical observation can in fact cause this estimator to break down. Moreover, the consistency of this estimator requires a moment condition on the error distribution. Data described by econometric models typically contains autocorrelation and/or heteroscedasticity of unknown form and for inference in such models it is essential to use covariance matrix estimators that can consistently estimate the covariance of the model parameters. In the econometric literature less attention is given to robust estimators of regression, but the concept of robust standard errors is well established and can be found even in introductory textbooks [2, 3]. Here the estimator being used is often the ordinary least squares (OLS) estimator, but its standard errors are estimated without relying on assumption of OLS. Failures of these assumptions can predispose output toward false statistical significance. Residuals, differences between the values predicted by the model and the real data, that are very large can seriously distort the prediction. Among typical challenges in numerous multiple regression models are those of heteroscedasticity and autocorrelation which have created undesirable consequences for ordinary least squares (OLS) estimator**.** Accurate inference about the estimated coefficients from an ordinary least squares (OLS) regression model relies crucially on a consistent estimator of the coefficient covariance matrix. It is widely understood that if the OLS residuals have heteroskedasticity and/or serial correlation, then the usual coefficient covariance matrix will not be consistent and if it is used to perform inference, then it will lead to erroneous conclusions. To overcome this, alternative covariance matrix estimators have been proposed in the literature which are consistent to unknown forms of heteroskedasticity (HC), [see 4,5] and/or autocorrelation (HAC), [6]. However, the performance of these so-called robust estimators can differ significantly in finite samples encountered in empirical applications. Derivation of appropriate corrections to standard errors when conducting inference with autocorrelated data is a standard problem in time series econometrics. Classical references include [6 – 8] among many others. It is well known that ordinary least squares estimation in the linear regression model is not robust to autocorrelation and heteroscedasticity. Robust regression analysis provides an alternative to a least squares regression model when these fundamental assumptions are unfulfilled by the nature of the data. Considerable attention however, has been paid in recent years to the estimation of covariance matrices in the presence of heteroscedasticity and autocorrelation of unknown form, [6, 9-15]. The standard approach to statistical inference based on robust regression methods is to derive the limiting distribution of the robust estimator from assumption OLS, and to compute the standard errors of the estimated regression coefficients from the formula for

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the asymptotic variance matrix. To satisfy the regression assumptions and be able to trust the results, the residuals should have a constant variance. This work examines the effect of heteroscedasticity and autocorrelation on the efficiency of the estimates of the regression coefficients in multiple regression modelling. This paper is about computing estimators for the covariance matrix of parameters in a linear panel model, of the kind commonly used in applied practice to produce 'robust' root mean square errors since standard errors for heteroskedasticity/autocorrelation can take a number of different forms and result from a variety of different processes. Literature has it that the variability in the errors may increase or decrease linearly as a function of one or more of the predictors, or variability might be larger for moderate values of one or more of the predictors. Given that heteroskedasticity can affect the validity or power of statistical tests when using OLS regression, it behooves researchers to test the tenability of this assumptions. The remainder of the paper is organized as follows: In the next section, we introduced the general linear model based on Heteroscedastic-Autocorrelation consistent (HAC) covariance matrix estimator. In Section 3, we present brief overview of some estimators of heteroscedastic-autocorrelation consistent. Section 4 deals with the material and methods with Monte Carlo simulation design and how to offset it while in section 5, the results of the simulation comparing the OLS residuals based on HAC estimator to the other more commonly employed covariance matrix estimation techniques when the cross-sectional units are spatially dependent. Section 6 concludes.

The Linear Model

Consider a linear model

$$y = X\beta + e \qquad (2)$$

and the OLS estimator $\hat{\beta}_{OLS} = (X^TX)^{-1}X^TY$

If one is interested in making inference on β, then an estimate of $Var(\hat{\beta})$ is required. If the error terms ε are independent and identically distributed, then the covariance matrix takes the general form of $ar(\hat{\beta}) = \sigma^2(X^TX)^{-1}$, where $\hat{\sigma}^2$ is an estimate of the error variance. This case is synthetically dubbed spherical errors, and the relative formulation of $Var(\hat{\beta}_{OLS})$ is often referred to, somewhat inappropriately, as "OLS covariance" [16]. In the general linear model, it is typically assumed that the errors have zero mean and variance $ar(\mu) = \Omega$. Under suitable regularity conditions [3, 17], the coefficients β can be consistently estimated by OLS giving the well-known OLS estimator $\hat{\beta}$ with corresponding OLS residuals $\hat{\mu}$.

$$\hat{\beta}_{OLS} = (X^TX)^{-1}X^TY \qquad (3)$$

$$\mu = (I_n = H)Y = (I_n - X(X^TX)^{-1}X^T)Y \qquad (4)$$

where I_n is the n-dimensional identity matrix and H is usually called hat matrix. The estimates $\hat{\beta}$ are unbiased and asymptotically normal [17]. Their covariance matrix Ψ is usually denoted in one of the two following ways:

$$\psi = Var(\hat{\beta}) = (X^T X)^{-1} X^T \Omega X (X^T X)^{-1} \qquad (5)$$

$$= \left(\frac{1}{n} X^T X\right)^{-1} \frac{1}{n} \phi \left(\frac{1}{n} X^T X\right)^{-1} \qquad (6)$$

where $\phi = n^{-1} X^T \Omega X$ is essentially the covariance matrix of the scores or estimating functions $V_i(\beta) = x_i(y_i - x_i^T \beta)$. The estimating functions evaluated at the parameter estimates $V_i(\beta) V_i(\beta)$ have then sum zero. For inference in the linear regression model, it is essential to have a consistent estimator for Ψ. What kind of estimator should be used for Ψ depends on the assumptions about Ω : In the classical linear model independent and homoscedastic errors with variance σ^2 are assumed yielding $\Omega = \sigma^2 I_n$ and $\psi = \sigma^2 (X^T X)^{-1}$ which can be consistently estimated by plugging in the usual OLS estimator $\hat{\sigma}^2 = (n-k)^{-1} \sum_i^n \mu_i^2$. But if the independence and/or homoscedasticity assumption is violated, inference based on this estimator $\psi_{const} = \sigma^2 (X^T X)^{-1}$ will be biased. HC and HAC estimators tackle this problem by plugging an estimate $\widehat{\Omega}\ or\ \hat{\phi}$ into (5) or (6) respectively which are consistent in the presence of heteroscedasticity and autocorrelation respectively. As a means of achieving a more efficient estimator the HAC literature has devoted much attention to the problem of selecting the optimal kernel [18], the right bandwidth (Andrews 1991 and Newey and West 1994) or whether or not to pre-whiten the OLS residuals beforehand [19-20]. However, only minor attention has been given to potential problems caused from using the OLS residuals as an estimate for the unobserved stochastic error term in the regression model. The problem at hand is to estimate the covariance matrix of the OLS estimator relaxing the assumptions of serial correlation and/or homoskedasticity without imposing any particular structure to the errors' variance or interdependence.

Brief Overview of Some Estimators of Models Considered

In this section, we provide brief theoretical formulations of the four estimators of panel data models as considered in this study.

Newey-West Errors

Newey and West [6] propose an alternative kernel-based estimation technique to obtain heteroskedasticity and autocorrelation robust standard errors, known as HAC standard errors in cross-section parlance. They extended this heteroscedastically consistent variance estimator to handle residual autocorrelation as well. Newey-West standard errors are calculated using the VCV matrix (5) with

$$X'\hat{\Omega}X = \left(\sum_{i=1}^{N} \sum_{i=1}^{T} \varepsilon_{it}^2 x'_{it} x_{it} + \sum_{i=1}^{N} \sum_{m=1}^{M} \sum_{t=m+1}^{T} \left(1 - \frac{m}{M+1}\right) \hat{\varepsilon}_{it} \hat{\varepsilon}_{it-m} \left(x'_{it} x_{it-m} + x'_{it-m} x_{it} \right) \right) \qquad (7)$$

where m = 1,...,M denotes the mth lag. Newey-West standard errors are regularly used in the finance literature [21] but since they were originally designed to address heteroskedasticity and autocorrelation in a single time series, they play a minor role in other panel data applications. The most likely reason is that within-firm autocorrelated errors in panel data are usually assumed to have a time persistent component. By construction, Newey West standard error estimation assumes autocorrelation up to some lag M, where weights are assigned to each lag through a decreasing function in M. Hence, standard errors using (11) are biased if autocorrelation is induced by a constant firm-specific component. Therefore, it is generally preferable to use cluster-robust standard errors instead, since in this case standard error consistency does not hinge on the assumed functional form of autocorrelation.

Cluster-Robust Standard Errors

One-way cluster-robust VCV matrix estimation is designed to correct for both heteroskedasticity and within-cluster autocorrelation of any form, i.e. violation in assumption of $Var\left(\varepsilon_{it} / X_i\right) = \sigma^2$ and $Cov\left(\varepsilon_{it}, \varepsilon_{is} / x_{it}, x_{is}\right) = 0$ for $t \neq s$. According to White [5] a consistent estimate for the VCV matrix in presence of heteroskedasticity and within-group autocorrelation is

$$\psi = Var\big(\hat{\beta}\big) = (X^T X)^{-1} \widehat{H} (X^T X)^{-1}$$

$$= (X^T X)^{-1} \sum_{g=1}^{G} X_g^T u_g \, u_g^T X_g (X^T X)^{-1} \qquad (8)$$

Where $\widehat{H} = \sum_{g=1}^{G} X_g^T u_g \, u_g^T X_g$ and G is the number of clusters.

In panel data applications we usually set G = N such that the total number of clusters G equals the number of firms in our dataset. These standard errors are also known as Rogers standard errors or Huber-White standard errors, even though the latter term is often used for White standard errors as well. To avoid confusion, we only use the term cluster-robust to refer to standard errors calculated in the above fashion. The cluster-robust approach is a generalization of the Eicker-Huber-White-“robust” to the case of observations that are correlated within but not across groups. Instead of just summing across observations, we take the cross products of x and ˆ ν for each group m to get what looks like (but isn’t) a within-group correlation matrix, and sum these across all groups M.

Feasible Generalized Least Squares Estimator (FGLS):

An alternative to HAC estimators is FGLS estimators (also known as Estimated GLS, or EGLS, estimators), for both regression coefficients and their standard errors. These estimators make use of revised formulas which explicitly incorporate the innovations covariance matrix. The generalized least squares estimator for the model parameters is obtained from the OLS estimation of the transformed model as shown below:

$$y_{it}^* = \alpha_i^* + X_{it}^{*T} \beta + w_{it}^* \qquad (9)$$

for i = 1, 2, …., n and t = 1, 2, …, T

where $y_{it}^{*} = y_{it} - \lambda_i; X_{it}^{*T} = X_{it}^{T} - \lambda_i;\ w_{it}^{*} = \lambda_i and\ \alpha_i^{*} = 1 - \lambda$

The term λ gives a measure of the relative sizes of the within and between unit variances. Which is compactly written as $GLS = (X^{*T}\Omega^{-1}X^{*})^{-1}X^{*T}\Omega^{-1}y^{*}$ (10)

Since Ω is often unknown, FGLS is more frequently used rather than GLS. The difficulty of using FGLS estimators, in practice, is providing an accurate estimate of the covariance. Again, various models are employed, and estimated from the residual series, but numerical sensitivities often provide challenges.

MATERIAL AND METHODS

This work considers performance of some estimators on linear model in the presence of heteroscedasticity and serial autocorrelation. Heteroscedasticity was implanted into the model via the individual-specific error component. This is in line with the works of [22- 25]. We considered second-order serial correlation as done by [22, 26, 27], The autocorrelated disturbances and heteroscedasticity focused on single exogenous variable with respect to stability while efficiency of the estimation methods for panel data models were examined.

Simulation Scheme

The datasets used for this work were simulated using Monte Carlo experiments in the environment of SAS version 9.0 statistical package. The series Y is a time trend plus a second-order autoregressive error. The model simulated is

$$y_{it} = \beta_o + \beta_{it} + e_t$$

$$v_t = \rho_1 v_{t-1} - \rho_2 v_{t-2} + e_t$$

$$e_t \sim IN(0,4)$$

For each replication, the value of the parameters $\beta = (\beta_0, \beta_1,)$ were fixed at 10 and 0.5 while five levels of autocorrelation (ρ = ±1.9, ±1.6 and 0.9), were estimated for the following estimators: I. Ordinary Least Squares (OLS) II. Newey West Estimator (NEW) III. Feasible Generalized Least Squares (FGLS) and IV. Cluster-Robust Standard Errors (CRSE). Three different scenarios are considered from the simulation above with different level of autocorrelation structures and degree of heteroscedasticity. The performances of the estimators at different heteroscedastic structures, positive and negative autocorrelation and sample sizes were evaluated using RMSE and their relative efficiency to OLS. Durbin-Watson test statistic was used for level of autocorrelation with Pr<DW from SAS 9.0 for testing positive autocorrelation, and Pr>DW for testing negative autocorrelation while the Q-statistic was used

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for degree of heteroscedasticity. The relative efficiency of an estimator is the measure of the degree to which the estimator performs similar to common method (OLS). Following Afolayan, and Adeleke, [28], the relative efficiency of two unbiased estimators, θ_1 and θ_2, of the parameter θ, is defined as:

$$RE = \frac{RMSE(\theta_1)}{RMSE(\theta_2)}$$

where, RMSE is the Root mean square error, θ_1 is OLS estimator and θ_2 is any other estimator. If the relative efficiency is 1, then it means that the estimator is as efficient as the OLS if the error distribution of the data is normal. A relative efficiency above 1, implies that the estimator is more efficient than OLS estimator. Starting with the ideal case which assumes that there is strong and high level of autocorrelation and heteroscedasticity among the series. In this scenario, we investigate the effect of bias in estimates of the panel data in the presence of strong and severe autocorrelation and heteroscedasticity. For the next group, we relax the first scenario by allowing minor heteroscedasticity with severe autocorrelation. And in the last group, interchange the degree of severity to minimal autocorrelation and high heteroscedasticity. This to enable us capture which of the estimators perform best in each case. Each of the combinations was iterated 1000 times and the assessments of the various estimators considered in this work were based on the absolute bias, variance and RMSE of parameter estimates.

RESULTS AND DISCUSSION

The results of the performances of the estimators considered at various levels of autocorrelation and heteroscedasticity considered in this work are presented and discussed here. To compare the efficiency of each estimator the OLS, NEW, FGLS and CRSE models we comparatively studied their error mean squares as shown in tables 1,2 and 3 based on each of the criteria stated in section 4 above. Those parameter estimates that have smaller root mean squares are deemed to be more efficient. This is due to their small deviance from the true mean.

Case 1: HAC estimator under strong (positive) autocorrelation and high degree of Heteroscedasticity

This scenario assumes strong and high positive autocorrelation and high heteroscedasticity among the panel data. As shown in table 1, the coefficients and standard errors of the performance estimators at different samples with Q-stat to capture the degree of heteroscedasticity and Durbin Watson statistic for autocorrelation where Pr < DW is the p-value for testing positive autocorrelation, and Pr > DW is the p-value for testing negative autocorrelation. A close look at the table reveals that CRSE appears to show better estimator in terms of standard error since it relaxes the assumption that the error terms are independent of each other. However, it is important to note that using standard error to determine the robust estimator might lead to wrong estimation since the estimates of the standard errors of the coefficients in any econometric model are biased downward if the residuals are positively autocorrelated. Based on this, we shall consider RMSE and RR criteria for robust estimator. Table 2 reveals that NEW are fairly consistent in terms of having smallest RMSE on the original data and is relatively more efficient than any other estimator all through the samples. This is followed by FGLS. Meanwhile the performances of CRSE estimator are equivalent with that of OLS. All through the samples using RMSE criterion, NEW is consistently performed better than FGLS and CRSE estimators.

Table 1: Summary Estimate under strong (positive) Autocorrelation and High Heteroscedasticity

		OLS					**NEW**		**FGLS**		**CRSE**	
N	**P> Q_{sata}**	**DW**	**P < DW**	**True value**	**Coef.**	**SE**	**Coef.**	**SE**	**Coef.**	**SE**	**Coef.**	**SE**
20	0.0003	0.18	< 0.001	β_0 = 10	-22.36	0.99	-22.36	1.70	-22.31	1.09	-22.36	1.04
				β_1 =0.5	-1.64	0.08	-1.64	0.12	-1.64	0.08	-1.64	0.07
50	< 0.0001	0.05	<0.001	β_0 = 10	-28.66	1.97	-28.66	3.28	-29.43	1.98	-28.66	1.82
				β_1 =0.5	-1.23	0.07	-1.23	0.16	-1.19	0.10	-1.23	0.09
100	< 0.0001	0.03	<0.001	β_0 = 10	-30.65	2.80	-30.65	3.61	-30.64	2.17	-30.65	1.72
				β_1 =0.5	-0.97	0.05	-0.97	0.08	-0.97	0.05	-0.97	0.04
200	< 0.0001	0.004	<0.001	β_0 = 10	-82.70	6.69	-82.70	12.97	-82.69	8.49	-82.70	6.04
				β_1 =0.5	-0.05	0.06	-0.05	0.159	-0.055	0.10	-0.05	0.07
500	< 0.0001	0.001	<0.001	β_0 = 10	-114.96	6.16	-114.96	12.56	-114.96	7.47	-114.96	5.68
				β_1 =0.5	0.61	0.02	0.61	0.04	0.61	0.02	0.61	0.02
1000	< 0.0001	0.001	<0.001	β_0 = 10	-49.88	4.26	-49.88	11.23	-50.03	8.49	-49.88	5.03
				β_1 =0.5	0.29	0.01	0.29	0.02	0.29	0.01	0.29	0.01

Table 2 also showed that NEW estimator under strong positive autocorrelation accompanied by high degree of heteroscedasticity, an efficiency gains of 5% (i.e. 1.0541) over OLS followed by FGLS which has a gain of over 2% (RE = 1.0268) when the sample size is small (n = 20).

Table 2: RMSE and RR of the selected estimators for high degree of Autocorrelation and Heteroscedasticity

Sample size	**OLS**	**NEW**		**FGLS**		**CRSE**	
	RMSE	**RMSE**	**RE**	**RMSE**	**RE**	**RMSE**	**RE**
20	2.1469	**2.0367**	1.0541	2.0908	1.0268	2.1469	1.0000
50	6.8736	**6.7348**	1.0206	6.8239	1.0073	6.8736	1.0000
100	13.8988	**13.7570**	1.0103	13.8263	1.0052	13.8966	1.0002
200	47.1289	**46.8927**	1.0050	47.0103	1.0025	47.1289	1.0000
500	68.7183	**68.5807**	1.0020	68.6494	1.0010	68.7183	1.0000
1000	67.3815	**67.3141**	1.0010	67.3478	1.0005	67.3815	1.0000

Case 2: Estimate under strong (negative) Autocorrelation with High degree of Heteroscedasticity

In this case, we considered strong negative autocorrelation high degree of heteroscedasticity among the panel data. Table 3 shows similar picture as Table 1 since here the standard errors are biased upward if the residuals are negatively autocorrelated. Table 4 results further showed the superiority of NEW estimator over the other estimators. Interestingly, NEW estimator consistently maintained minimum RMSE even as the sample increased with efficiency gain of 5% (RE=1.05) over OLS estimator when the sample size is small (n = 20) and gain of 2% (RE = 1.02) when the sample size is large (n =50). A close examination of the table showed that FGLS closely followed NEW and even outperformed NEW as the sample increased to 500 with efficiency gain above 2% (RE = 1.028) over OLS. Meanwhile the performances of CRSE estimator are equivalent with that of OLS.

Table 3: Summary Estimate under strong (negative) Autocorrelation and High Heteroscedasticity

		OLS					**NEW**		**FGLS**		**CRSE**	
n	**P > Q_{stat}**	**DW**	**P > DW**	**True value**	**Coef.**	**SE**	**Coef.**	**SE**	**Coef.**	**SE**	**Coef.**	**SE**
20	0.0004	3.72	0.0004	$\beta_0 = 10$	8.89	5.24	8.89	1.40	9.12	1.61	8.89	3.44
				$\beta_1 = 0.5$	0.65	0.44	0.65	0.19	0.61	0.18	0.65	0.46
50	< 0.0001	3.84	<0.001	$\beta_0 = 10$	8.87	7.29	8.87	2.22	9.18	3.11	8.87	5.12
				$\beta_1 = 0.5$	0.56	0.25	0.56	0.12	0.54	0.17	0.56	0.29
100	< 0.0001	3.94	<0.001	$\beta_0 = 10$	10.67	8.80	10.67	2.27	10.69	2.37	10.67	5.19
				$\beta_1 = 0.5$	0.48	0.15	0.48	0.06	0.48	0.06	0.48	0.13
200	< 0.0001	3.966	<0.001	$\beta_0 = 10$	10.78	13.27	10.78	3.93	10.64	2.37	10.78	8.86
				$\beta_1 = 0.5$	0.49	0.11	0.49	0.06	0.49	0.03	0.49	0.13
500	< 0.0001	3.994	<0.001	$\beta_0 = 10$	10.37	13.41	10.37	3.88	10.37	0.70	10.37	8.71
				$\beta_1 = 0.5$	0.49	0.05	0.49	0.02	0.49	0.003	0.50	0.04
1000	< 0.0001	3.999	<0.001	$\beta_0 = 10$	10.08	10.70	10.08	3.58	10.08	0.10	10.08	8.01
				$\beta_1 = 0.5$	0.49	0.02	0.49	0.006	0.49	0.0003	0.49	0.01

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Table 4: RMSE and RR of the selected estimators for high degree of (negative) Autocorrelation and Heteroscedasticity

Sample size	OLS	NEW		FGLS		CRSE	
	RMSE	**RMSE**	**RE**	**RMSE**	**RE**	**RMSE**	**RE**
20	11.2889	**10.7096**	1.0541	10.9906	1.0271	11.2889	1.0000
50	25.3839	**24.8710**	1.0206	25.1259	1.0103	25.3839	1.0000
100	43.6524	**43.2137**	1.0101	43.4314	1.0051	43.6524	1.0000
200	93.5002	**93.0316**	1.0050	93.2651	1.0025	93.5002	1.0000
500	149.7272	**149.400**	1.0022	145.600	1.0283	149.730	0.9999
1000	169.1199	**169.000**	1.0007	169.000	1.0007	169.120	1.0000

Case 3: Estimate under high Heteroscedasticity but minimal Autocorrelation

This is a scenario where there is high Heteroscedasticity among the series accompanied by minimal autocorrelation. When the model has high level of non- constant variance accompanied by minimum autocorrelation, New estimator provided good estimates of the regression parameters than any other estimators considered as shown in Table 5. The result in table 5 reveals that the standard error of NEW estimator was lower than all the other estimators and converge to their true value as the sample sizes become large (i.e. 20 to 1000).

Table 5: Summary Estimate under high Heteroscedasticity but minimal Autocorrelation

n	**OLS**					**NEW**		**FGLS**		**CRSE**	
	DW	**P >Q**	**True value**	**Coef.**	**SE**	**Coef.**	**SE**	**Coef.**	**SE**	**Coef.**	**SE**
20	1.58	0.008	$\beta_0 = 10$	10.26	1.37	10.26	**0.57**	10.26	1.27	10.25	1.39
			$\beta_1 = 0.5$	0.46	0.11	0.46	**0.05**	0.46	0.09	0.46	0.10
50	1.49	0.000	$\beta_0 = 10$	9.82	1.51	9.82	**0.33**	9.88	0.99	9.82	1.34
			$\beta_1 = 0.5$	0.50	0.05	0.50	**0.01**	0.50	0.03	0.50	0.04
100	1.77	0.01	$\beta_0 = 10$	9.85	0.31	9.85	**0.16**	9.85	0.19	9.86	0.19
			$\beta_1 = 0.5$	0.50	0.005	0.50	**0.003**	0.50	0.004	0.50	0.004
200	1.03	0.01	$\beta_0 = 10$	9.69	0.35	9.69	**0.23**	9.69	0.34	9.69	0.29
			$\beta_1 = 0.5$	0.50	0.003	0.50	**0.002**	0.50	0.002	0.50	0.02
500	1.47	0.004	$\beta_0 = 10$	10.03	0.1057	10.00	**0.1112**	10.00	0.1177	10.00	0.1151
			$\beta_1 = 0.5$	0.49	0.0003	0.49	**0.0003**	0.49	0.0003	0.49	0.0003
1000	1.69	0.00	$\beta_0 = 10$	9.98	0.0675	9.98	**0.0745**	9.98	0.0758	9.98	0.0746
			$\beta_1 = 0.5$	0.50	0.0001	0.49	**0.0001**	0.49	0.0001	0.49	0.0001

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Table 6 further confirmed the superiority of NEW estimator with least RMSE with high RR over other selected estimators. This indicates the relative consistency of NEW estimator compared to other estimators considered in the study. This is followed by FGLS estimator which also showed a lower RMSE than CRSE and OLS estimators

Table 6: RMSE and RR of the selected estimators for high Heteroscedasticity but minimal Autocorrelation

Sample size	**OLS**	**NEW**		**FGLS**		**CRSE**	
	RMSE	**RMSE**	**RE**	**RMSE**	**RE**	**RMSE**	**RE**
20	2.95526	**2.8036**	1.0541	2.8764	1.0274	2.9553	1.0000
50	5.25136	**5.1453**	1.0206	5.1976	1.0103	2.2514	2.3325
100	1.53379	**1.5184**	1.0101	1.5261	1.0050	1.5338	1.0000
200	2.45000	**2.4341**	1.0065	2.4402	1.0040	2.4464	1.0015
500	1.17972	**1.1774**	1.0020	1.1785	1.0010	1.1797	1.0000
1000	1.06574	**1.0647**	1.0010	1.0652	1.0005	1.0657	1.0000

CONCLUSION

Various results obtained in this work generally showed that the behaviors of the four estimators investigated for modeling various panel data vary as the violations are varied. Failure of the orthogonality assumption makes the OLS estimators to be biased and imprecise. For OLS to be accurately used in estimating the parameters of panel data models, errors have to be independent and homoscedastic. These conditions are so atypical and mostly unrealistic in many real-life situations that would have warranted the use of OLS for modeling panel data efficiently. The efficiency of the three methods of estimating panel data models with violations of homoscedasticity and no autocorrelation (both positive and negative) were considered. Our findings from Monte Carlo experiments for several combinations of violations such as heteroscedasticity and (negative and positive) autocorrelation, revealed the superiority of the Newey-West standard errors (NEW) over FGLS and CRSE estimators using root mean squared error (RMSE) of the parameter estimates and relative efficiency (RR) as assessment criteria for the distribution of the serial correlation and heteroskedasticity. The performances of CRSE estimator are equivalent with that of OLS. For small samples, NEW estimator accounted for an average efficiency gain of 5% over OLS in all the three cases considered.

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