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EFFECTS OF MULTICOLLINEARITY AND AUTOCORRELATION ON SOME ESTIMATORS IN A SYSTEM OF REGRESSION EQUATION

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ABSTRACT: *When dealing with time series data, some of these assumptions especially that of independence of regressors and error terms leading to multicolinearity and autocorrelation respectively, are not often satisfied in Economics, Social Sciences, Agricultural Economics and some other fields. This study therefore examined the effect of correlation between the error terms, multicollinearity and autocorrelation on some methods of parameter estimation in SUR model using Monte Carlo approach. A two equation model in which the first equation was having multicollinearity and autocorrelation problems while the second has no correlational problem was considered. The error terms of the two equations were also correlated. The levels of correlation between the error terms, multicolinearity and autocorrelation were specified between* ± 1 *at interval of 0.2 except when the correlation tends to unity. A Monte Carlo experiment of 1000 trials was carried out at five levels of sample sizes 20, 30, 50, 100 and 250 at two runs. The performances of seven estimation methods; Ordinary Least Squares (OLS), Cochran – Orcut (COCR), Maximum Likelihood Estimator (MLE), Multivariate Regression, Full Information Maximum Likelihood (FIML), Seemingly Unrelated Regression (SUR) Model and Three Stage Least Squares (3SLS) were examined by subjecting the results obtained from each finite properties of the estimators into a multi factor analysis of variance model. The significant factors were further examined using their estimated marginal means and the Least Significant Difference (LSD) methodology to determine the best estimator. The results generally show that the estimators' performances are equivalent asymptotically but at low sample sizes, the performances differ. Moreover, when there is presence of multicollinearity and autocorrelation in the seemingly unrelated regression model, the estimators of MLE, SUR, FIML and 3SLS are preferred but the most preferred among them is MLE.*

KEYWORDS: Multicollinearity, Autocorrelation Estimators, Regression Equation

INTRODUCTION

The SUR estimation procedures which enable an efficient joint estimation of all the regression parameters was first reported by Zellner (1962) which involves the application of Aitken's Generalised Least squares(AGLS), (Powell 1965) to the whole system of equations. Zellner (1962 & 1963), Zellner&Theil (1962) submitted that the joint estimation procedure of SUR is more efficient than the equation-by-equation estimation procedure of the Ordinary Least Square (OLS) and the gain in efficiency would be magnified if the contemporaneous correlation between each pair of the disturbances in the SUR system of equations is very high and explanatory variables (covariates) in different equations are uncorrelated. In other words, the efficiency in the SUR formulation increases the more the correlation between error vector differs from zero and the closer the explanatory variables for each response are to being uncorrelated.

After the much celebrated Zellner's joint generalized least squares estimator, several other estimators for different SUR systems were developed by many scholars to address different situations being investigated. For instance, Jackson (2002) developed an estimator for SUR system that could be used to model election returns in a multiparty election. Sparks (2004) developed a SUR procedure that is applicable to environmental situations especially when missing and censored data are inevitable. In share equation systems with random coefficients, Mandy & Martins-Filho (1993) proposed a consistent and asymptotically efficient estimator for SUR systems that have additive heteroscedastic contemporaneous correlation. They followed Amemiya (1977) by using Generalized Least Squares (GLS) to estimate the parameters of thecovariance matrix. Furthermore, Lang, Adebayo &Fahrmeir (2002), Adebayo (2003), and Lang *et al* (2003) in their works also extended the usual parametric SUR model to Semiparametric SUR (SSUR) and Geoadditive SUR models withina Bayesian context. Also O'Donnell *et al* (1999) and Wilde *et al* (1999) developed SUR estimators that are applicable in Agricultural Economics. More recently, Foschi (2004) provided some new numerical procedures that could successively and efficiently solve a large scale of SUR model.. In all the estimation procedures developed for different SUR situations as reported above, Zellner's basic recommendation for high contemporaneous correlation between the error vectors with uncorrelated explanatory variables within each response equations was also maintained. However, in most practical situations, the explanatory variables across the different equations in SUR systems are often correlated. Also, it may be necessary to jointly regress the demand for two or more complementary products like automobiles and gasoline on peoples' income and expenditures on other products within the SUR framework. While the two demands (responses) would obviously correlate through their error, satisfying the first basic requirement of SUR estimation, people's income and their expenditure on other products should not be expected to be uncorrelated thereby, violating the second important condition. Therefore, the existence of this kind of relationship needed to be recognized and accorded proper management within the SUR context such that the efficiency of SUR estimator would not be compromised. It is now obvious, due to several instances of SUR highlighted above, that the independent variables are often correlated (collinear).

SINGLE REGRESSION EQUATION MODEL AND ITS ASSUMPTION

Classical linear regression Equation

The Classical Linear Regression Model (CLRM) is specified as

$$Y_t = ß_0 + ß_1 X_{1t} + ß_2 X_{2t} + \cdots + ß_{(k-1)} X_{(k-1)t} + U_t \quad (1.1)$$

Where Y_t is the t^{th} response variable, $X_1, X_2, \cdots, X_{(k-1)}$ are the (k – 1) explanatory variables, U_t is the t^{th} disturbance (error) term and $ß_0, ß_1, ß_3, \cdots, ß_{(k-1)}$ are the unknown parameters to be estimated, for $t = 1, 2, \cdots, n$.

In matrix form, the model can be written as

$$Y = X ß + U \quad (1.2)$$

Where $Y' = [y_1, y_2, \cdots, y_n]$, $U' = [u_1, u_2, \cdots, u_n]$, $ß' = [ß, ß_2, \cdots, ß_{(k-1)}]$ and

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$$X = \begin{bmatrix} 1 & x_{11} & x_{21} & \cdots & x_{(k-1)1} \\ 1 & x_{12} & x_{22} & \cdots & x_{(k-1)2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{1n} & x_{2n} & \cdots & x_{(k-1)n} \end{bmatrix}$$

That is X matrix is an n x k matrix of observable and fixed values.

Here are the assumptions made about the variables and the error terms in the model

Assumptions about Error Terms

i. U is a random vector

ii. E(U) = 0

iii. $E(U^iU) = \sigma^2 In$ i.e $E(U_iU_j) = \begin{cases} \sigma^2 & i = j \\ 0 & i \neq j \end{cases}$

iv. $U \neg NID(0, \sigma^2 I_n)$

Assumptions about the regressors

1. X is non – stochastic matrix i.e. the values of X variables are fixed or selected in advance.
2. There is no correlation between the non- stochastic X (regressors) and the stochastic U (error terms) i.e. $E(X'U) = 0$
3. The X variables are linearly independent, so $|X'X| \neq 0$

Thus, X matrix has rank r = (k – 1)< n.

Generalized Regression Equation

The generalized linear regression model

$$Y = X\beta + \varepsilon \qquad (1.3)$$

differs from the standard one considered before in three of the five underlying assumptions:

1. $E(\varepsilon) = 0$
2. X is non – stochastic and is of full column rank i.e. the rank of X is (k – 1)< n
3. $Var(\varepsilon) = \sigma^2\Sigma$, where Σ is p.d. matrix
4. $\varepsilon \sim N(0, \sigma^2\Sigma)$
5. $\lim_{n\to\infty} \frac{1}{n} X'\Sigma^{-1}X = Q$, where Q is a finite and p.d. matrix

Assumptions 1 and 2 are the same as those of the standard linear regression model. Assumption 3 generalizes the variance - covariance matrix of the disturbance terms $Var(\varepsilon)$ from the

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spherical form σ^2 into the non – spherical form $\sigma^2\Sigma$ and is the key feature of the generalized linear regression model. By assuming a more general form of Var(ε), we allow the variances of individual disturbance terms, i.e., the diagonal terms in Var(ε), to differ (which results in a model that is referred to as heteroscedasticity) and covariance between any two disturbance terms, i.e., the off – diagonal terms in Var(ε), to be non – zero (which gives a model that is called autocorrelation). Besides these two possible specifications of Σ, there are other econometric models where Var(ε) is more complicated than the simple spherical form so that the results from the standard linear regression cannot apply. Assumption 5 is also new; it imposes certain restrictions on how the relationship between the data matrix X and the variance – covariance matrix$\sigma^2\Sigma$ should evolve as the sample size increases.

Estimation methods under multicollinearity in singleequation

One of the major assumptions of the explanatory variables in the classical linear regression model is that they are independent (orthogonal). Orthogonal variables may be set up in experimental designs, but such variables are not common in business and economic data. Thus when the explanatory variables are strongly interrelated we have the problem of multicollinearity. When multicollinearity is not exact (i.e. the linear relationship between two between two explanatory variables is not perfect) but strong, the regression analysis is not affected; however, its results become ambiguous. Consequently, interpreting a regression coefficient as measuring the change in the response variable when the corresponding independent variable is increased by one unit while other predictors are held constant is incorrect. This is because the OLS estimator of β

$$\hat{\beta}_{(OLS)} = (X'X)^{-1}X'Y \tag{2.1}$$

and

$$V(\hat{\beta}_{(OLS)}) = \sigma^2(X'X)^{-1} \tag{2.2}$$

are affected by the sample value of the explanatory variables. Precisely, in this case

$$|X'X| \to 0$$

When multicollinearity is exact (perfect), the assumption that X has a full column rank break down and therefore$|X'X| = 0$. Consequently, the OLS estimate of (2.1) and (2.2) cannot be obtained. The concept of estimable function in which (2.1) and (2.2) now have an infinite solution of vectors is used.

Estimation methods under autocorrelation in single equation

If the error terms are correlated in a sequential order then we have autocorrelation. Autocorrelation of the error terms may occur for several reasons. Successive residual in economic time series tend to be positively correlated (Chattterjee et al 2000). In experiments, correlated observations may be due to the nature of the plots, the layout of plots, some cumulative effects through time, pest infections from the neighboring plots, or some local factors which blocking cannot remove (Berenlut and Web, 1974, Williams, 1952; Papadakis, 1937).

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Autocorrelation can arise as a result of:

- Omitted explanatory variables
- Misspecification of the mathematical form of the model
- Interpolation in the statistical observations
- Misspecification of the true random error (Johnson, 1984)

The simplest form of the classical linear regression model with autocorrelation error terms assumed to follow the first order autoregressive (AR(1)) process is given as

$$y_t = B_0 + B_1 x_{1t} + u_t \qquad (2.3)$$

Where

$u_t = \rho u_{t-1} + \varepsilon_t |\rho| < 1$ t = 1,2,⋯, n $\qquad \varepsilon_t = N(0, \sigma^2 I_n)$

It can be shown that $u_t = \left(0, \frac{\sigma_\varepsilon^2}{1-\rho^2}\right)$ and that $E(u_t u_{t-s}) = \rho^s \sigma_u^2$

The consequence of applying OLS estimator to model (1.1) according to Johnson (1984), Fomby et al (1984) and many others include

1. The ordinary least square estimator $\hat{\beta}_{(OLS)} = (X'X)^{-1}X'Y$ remains unbiased and consistent.
2. The variance covariance of $\hat{\beta}$ is biased. The true variances and standard errors are being underestimated and the t and F tests are no more reliable.
3. The variances of the error term may also be seriously underestimated (biased). Thus, R^2 also becomes unreliable.

The monte - carlo approach

Monte-Carlos is a mathematical technique based on experiment for evaluation and estimation of problems which are intractable by probabilistic or deterministic approach. By probabilistic Monte-Carlo experiment, random numbers are observed and chosen in such a way that they directly simulate the physical random process of the original problem. The desired solutions from the behavior of these random numbers are then inferred. The idea of Monte-Carlo approach to deterministic problems is to exploit the strength of theoretical Mathematics which cannot be solved by theoretical means but now being solved by a numerical approach.

The Monte-Carlo approach has been found useful to investigate the small (finite) sample properties of these estimators. The use of this approach is due to the fact that real life observation on economic variables are in most cases plagued by one or all of the problem of nonspherical disturbances and measurement and misspecification errors. By this approach, data sets and stochastic terms are generated which are free from all the problems listed above and therefore can be regarded as data obtained from controlled laboratory experiment.

In a Monte-Carlo experiment, the experimenter artificially sets up a system (model) and specifies the distribution of the independent variables alongside with the values of the model parameters. Values are then generated for the error term and the independent variables as specified for a specified sample size. Using the generated values and the parameter values, the value of the dependent variable is thus determined. Next is to treat the generated data as if they are real life data by estimating the parameters of the model via the estimation methods (estimators). This process of generating values for the disturbance term, independent variables and estimating the parameters of the model is then replicated a large number of times. The experimenter then builds up empirical distributions of the parameter estimates which are then used to evaluate the performance of the estimators in estimating the parameter values.

The Monte – Carlo studies can be designed generally by using the following summarized five steps as given below:

(a) The researcher specifies a model and assigns specific numeric values as in parameters. The assigned values are assumed to be the true values of the parameter

(b) The distribution of error terms is also specified by the researcher

(c) He uses the distribution of U's with the random drawings from the distribution to obtain different values for the error terms.

(d) The experimenter now selects or generates values for the regressors (X's) depending on the specifications of the model.

(e) The researcher obtains or generates values for the dependent variable using the true values of the regressors and the error terms.

Steps (a) to (e) are repeated several times, say R, to have R replications.

Thus, the experimenter obtains estimate of the model parameters for each replication treating the generated data as real life data.

The model formulation

The system of regression equation used in this research work is given as

$$y_{1t} = \beta_{01} + \beta_{11}x_{1t} + \beta_{12}x_{2t} + u_{1t} \tag{3.1}$$

where $u_{1t} = \rho u_{1(t-1)} + e_{1t}$, $e_{1t} \approx (0, \sigma^2)$.

$$y_{2t} = \beta_{02} + \beta_{21}x_{1t} + \beta_{22}x_{3t} + u_{2t} \;,\quad u_{2t} \approx N(0, \sigma^2) \tag{3.2}$$

NOTE: (1) Multicollinearity exists between X_1 and X_2 in equation (3.1)

(2) Autocorrelation exists in equations (3.1)

(3) There is correlation between U_1 and U_2 of the two equations

(4) There is no correlation between X_1 and X_3 in equation (3.2), thus, equation (3.2)

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appears as control equation.

Specifications and choice of parameters for simulationstudy

For the simulation study, the parameters of the model in equations 3.1 and 3.2 are fixed as $\beta_{01} = 0.4$; $\beta_{11} = 1.8$; $\beta_{21} = 2.5$; $\beta_{02} = 2.0$; $\beta_{12} = 4.5$; $\beta_{22} = -1.2$. The Multicollinearity(δ) levels are -0.99, -0.9,-0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99. The Autocorrelation(ρ) levels are -0.99, -0.9,-0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8,0.9, 0.99 and that of Correlation between error terms (ƛ) levels are -0.99, -0.9,-0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99. The sample sizes (n) are 20, 30, 50, 100 and 250 were used in the simulation. At a particular choice of sample size, multicollinearity level, autocorrelation level and correlation between the error terms, a Monte-Carlo experiment is performed 1000 times at two runs which were averaged at analysis stage.

The data generation for the simulation study

The generation of the data used in this simulation study is in three stages which are:

(i) Generation of the independent variables

(ii) Generation of the error terms

(iii) Generation of dependent variables

Estimation methods used for the simulation study

The following estimation methods were considered for the simulation study in this research

1. Ordinary Least Squares (OLS)
2. Cochran – Orcut (CORC)
3. Maximum Likelihood Estimator (MLE)
4. Multivariate Regression Estimator (MRE)
5. Full Information Maximum Likelihood (FIML)
6. Seemingly Unrelated Regression Estimator (SUR)
7. Three Stage Least Squares (3SLS)

Evaluation, comparism and preference of estimators

Evaluation and comparison of the seven (7) estimators listed in section 3.5 were examined using the finite sampling properties of estimators which include Bias (BB), Absolute Bias (AB), Variance (VAR) and the Mean Square Error (MS) criteria.

Mathematically, for any estimator $\hat{\beta}_{ij}$ of Model (3.1) & (3.2)

(i) $$\bar{\hat{\beta}}_{ij} = \frac{1}{R}\sum_{l=1}^{R}\hat{\beta}_{ijl}$$

$$(ii) \qquad Bias\left(\hat{\beta}_{ij}\right) = \frac{1}{R}\sum_{l=1}^{R}\left(\hat{\beta}_{ijl} - \beta_{ij}\right) = \bar{\hat{\beta}}_{ij} - \beta_{ij}$$

$$(iii) \qquad AB\left(\hat{\beta}_{ij}\right) = \frac{1}{R}\sum_{l=1}^{R}\left|\hat{\beta}_{ijl} - \beta_{ij}\right|$$

$$(iv) \qquad VAR\left(\hat{\beta}_{ij}\right) = \frac{1}{R}\sum_{l=1}^{R}\left(\hat{\beta}_{ijl} - \bar{\hat{\beta}}_{ij}\right)^2$$

$$(v) \qquad MSE\left(\hat{\beta}_{ij}\right) = \frac{1}{R}\sum_{l=1}^{R}\left(\hat{\beta}_{ijl} - \beta_{ij}\right)^2$$, for i = 0, 1, 2 ; j = 1,2 and l = 1,2,…,R.

Using a computer program which was written with TSP software package to estimate all the model parameters and the criteria, the performances of seven estimation methods; Ordinary Least Squares (OLS), Cochran – Orcut (CORC), Maximum Likelihood Estimator (MLE), Multivariate Regression Estimator (MRE), Full Information Maximum Likelihood (FIML), Seemingly Unrelated Regression (SUR) and Three Stage Least Squares (3SLS) were examined by subjecting the results obtained from each finite properties of the estimators into a multi factor analysis of variance model. Consequently, the highest order significant interaction effect which has "method" as a factor is further examined using Duncan Multiple Range Test and the Least Significance Difference (LSD) test. The estimated marginal mean of the factor was investigated out at a particular combination of levels of the correlations in which estimators were preferred. An estimator is most preferred at a particular combination of levels of the correlation if the marginal means is the smallest. All estimators whose estimated marginal means are not significantly different from the most preferred are also preferred.

RESULTS WHEN THERE IS MULTICOLLINEARITY & AUTOCORRELATION IN THE MODEL

The performances of the estimators under the influence of multicollinearity and autocorrelation at various sample sizes on the basis of finite sampling properties of estimators using the Analysis of Variance technique are presented and discussed.

Effect on β_0:

The effect of estimators, multicollinearity and autocorrelation on estimating β_0 based on the sampling properties of the estimators as revealed by Analysis of Variance technique are shown in Table 4.1.1

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TABLE 4.1.1: ANOVA Table showing the effect of estimators, multicollinearity and autocorrelation on β_0 in the model

n	Factor	df	Value of F – Statistic							
			Equation 1				Equation 2			
			BB	A B	VAR	MS	BB	A B	VAR	MS
20	E	6,1183	17.325***	1.882E3***	197.227***	197.625***	.125	.366	442.022***	.692
	δ	12, 1183	2.3087E-6	5.665E-7	20.824***	20.813***	0.000001	0.00001	0.00001	.000001
	ρ	12, 1183	24.536***	1.306E3***	102.197***	102.417***	.001	4.0383E-4	31.674***	.001
	E* δ	72.1183	2.2728E-6	5.8262E-7	20.824***	20.813***	0.000001	0.00001	0.00001	.000001
	E* ρ	72,1183	25.387***	852.788***	101.927***	102.147***	1.28E-4	5.5773E-5	5.473***	1.4451E-4
	$\delta * \rho$	144,1183	1.7357E-6	8.8777E-7	10.840***	10.835***	0.000001	0.00001	0.00001	.000001
	E* $\delta * \rho$	864,1183	1.7087E-6	8.8471E-7	10.840***	10.835***	0.000001	0.00001	0.00001	.00001
30	E	6,1183	11.491***	3.467E3***	2.62E3***	2.586E3***	.850	284.664***	.017	236.136***
	δ	12, 1183	3.3567E-6	9.1226E-6	.002	.002	.048	.507	17.814***	1.593
	ρ	12, 1183	43.851***	3.413E3***	2.01E3***	1.985E3***	.018	.779	8.394***	1.862**
	E* δ	72.1183	3.2939E-6	1.6097E-6	.002	.002	3.6663E-4	.035	.012	.033
	E* ρ	72,1183	39.539***	2.397E3***	2.01E3***	1.983E3***	2.6479E-4	.012	.013	.011
	$\delta * \rho$	144,1183	3.1313E-6	8.3802E-6	.002	.002	.020	.423	5.708***	.874
	E* $\delta * \rho$	864,1183	3.1339E-6	1.7533E-6	.002	.002	2.572E-4	.015	.013	.014
50	E	6,1183	4.010***	434.752***	55.698***	55.696***	2.160**	.011	.010	1.473
	δ	12, 1183	.074	.693	.938	.935	1.181	5.242***	6.265***	4.513***
	ρ	12, 1183	1.927**	511.34***	54.193***	54.180***	.025	.869	6.545***	4.181***
	E* δ	72.1183	.048	.472	.938	.935	.087	.505	.012	.061
	E* ρ	72,1183	2.273***	344.062***	54.079***	54.066***	.001	.003	.012	.006
	$\delta * \rho$	144,1183	.031	.887	.918	.915	.040	.648	6.951***	4.037***
	E* $\delta * \rho$	864,1183	.027	.831	.918	.915	.001	.004	.013	.006
100	E	6,1183	1.122	165.362***	10.127***	10.220***	.054	.001	1.596	.049
	δ	12, 1183	.010	.635	.085	.085	.001	.004	2.761***	.004
	ρ	12, 1183	1.501	344.256***	12.442***	12.550***	3.2298E-5	3.3587E-4	.902	.001
	E* δ	72.1183	.043	1.127	.265	.266	1.5070E-4	.001	1.375**	.002
	E* ρ	72,1183	1.124	165.237***	10.127***	10.220***	9.3208E-6	7.8318E-5	.896	.001
	$\delta * \rho$	144,1183	.010	.638	.085	.085	3.0467E-5	2.3933E-4	1.099	.001
	E* $\delta * \rho$	864,1183	.043	1.126**	.265	.266	1.0753E-5	7.9543E-5	.927	.001
250	E	6,1183	.678	107.848***	129.134***	129.131***	.013	.385	.988	.990
	δ	12, 1183	.189	1.930**	1.031	1.032	.111	2.210**	1.071	1.070
	ρ	12, 1183	17.167***	271.972***	133.043***	133.051***	.108	1.975**	1.071	1.070
	E* δ	72.1183	.234	.769	1.002	1.002	.065	.286	.988	.988
	E* ρ	72,1183	.562	108.06***	129.137***	129.134***	.065	.285	.988	.988
	$\delta * \rho$	144,1183	.150	1.762***	1.031	1.031	.116	2.071**	1.072	1.070
	E* $\delta * \rho$	864,1183	.232	.769	1.002	1.002	.065	.286	.988	.988

**** Result is significant at** $\alpha = 0.05$ **and** ***** Result is significant at** $\alpha = 0.01$

From Table 4.1.1, the following are observed:

- The effect of multicollinearity is occasionally significant under all criteria except bias criterion in both equation.
- The effect of autocorrelation is generally significant under all criteria in equations 1 and occasionally significant under some criteria in equation 2.

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- The effect of estimators is generally significant under all the criteria in equation 1 but occasionally significant in equation 2.
- The interaction effect of estimators and multicollinearity are significant under variance and mean square criteria when the sample size is 20(low) in equation 1.
- The interaction effect of estimators and autocorrelation are generally significant under all criteria in equation 1 alone.
- The interaction effect of estimators, autocorrelation and multicollinearity are occasionally significant under absolute bias, variance and mean square error criteria when the sample size is low and when it is high in equation 1.

Consequently, it can be inferred that the performances of the estimators are affected by autocorrelation and multicollinearity under all criteria. The results of the LSD further test visa-vice their estimated marginal means revealed that MLE, MR, FIML, SUR and 3SLS estimators are preferred to estimate β_0.

Effect on β_1:

The effect of estimators, multicollinearity and autocorrelation on estimating β_1 based on the sampling properties of the estimators as revealed by Analysis of Variance technique are shown in Table 4.2.1

TABLE 4.2.1: ANOVA Table showing the effect of estimators,multicollinearity and autocorrelation on β_1 in the model

n	Factor	df	Value of F – Statistic							
			Equation 1				Equation 2			
			BB	A B	VAR	MS	BB	A B	VAR	MS
20	E	6,1183	.340	70.297***	11.817***	11.833***	11.629***	55.951***	5.004E3***	53.75***
	δ	12, 1183	188.21***	740.896***	110.836***	111.005***	0.00001	.0001	.0001	.0001
	ρ	12, 1183	6.912***	101.610***	31.361***	31.395***	.006	.012	316.268***	.044
	E* δ	72.1183	4.709***	6.235***	3.407***	3.411***	.00001	.0001	.0001	.0001
	E* ρ	72,1183	.478	9.752***	2.819***	2.821***	.001	.002	53.121***	.006
	$\delta * \rho$	144,1183	3.453***	6.967***	8.207***	8.215***	0.00001	.0001	.0001	.0001
	E* $\delta * \rho$	864,1183	.481	.745	.802	.803	0.00001	.0001	.0001	.0001
30	E	6,1183	2.243**	44.674***	5.846***	5.835***	1.313	1.317	.041	.568
	δ	12, 1183	18.109***	388.248***	44.467***	44.365***	.005	.074	15.381***	.085
	ρ	12, 1183	4.145***	124.878***	38.553***	38.504***	.002	.027	5.432***	.034
	E* δ	72.1183	1.858***	3.871***	1.742***	1.738***	1.2487E-4	4.7427E-4	.228	.001
	E* ρ	72,1183	.313	9.639***	2.641***	2.637***	7.7578E-5	2.978E-4	.067	.001
	$\delta * \rho$	144,1183	17.963***	10.648***	11.399***	11.386***	.002	.030	5.801***	.034
	E* $\delta * \rho$	864,1183	.953	.813	.781	.780	7.03E-5	3.0312E-4	.066	4.1938E-4
50	E	6,1183	.634	62.115***	5.408***	5.049***	2.330**	23.824***	1.255	33.945***
	δ	12, 1183	1.167	447.428***	32.68***	38.950***	28.794***	43.797***	6.908***	43.239***
	ρ	12, 1183	1.351	89.877***	14.979***	18.231***	.100	.527	6.085***	.463
	E* δ	72.1183	.949	4.738***	1.286	1.458***	.018	.165	.205	.078
	E* ρ	72,1183	.868	7.835***	1.276	1.305**	.001	.004	.162	.004
	$\delta * \rho$	144,1183	1.167	7.402***	5.572***	6.606***	.142	.428	5.553***	.330
	E* $\delta * \rho$	864,1183	.901	.606	.557	.461	.001	.007	.190	.007
	E	6,1183	2.269**	126.713***	10.625***	10.652***	.684	6.361***	3.643***	2.946***

100	δ	12, 1183	15.419***	615.873***	61.012***	61.112***	1.4238E-4	.042	4.020***	.870
	ρ	12, 1183	5.881***	285.204***	56.819***	56.959***	.001	.007	.402	.123
	E* δ	72.1183	2.704***	9.074***	2.841***	2.845***	3.2275E-4	.007	1.257	.276
	E* ρ	72,1183	.358	22.024***	3.892***	3.902***	9.5039E-5	.001	.726	.167
	$\delta * \rho$	144,1183	6.975***	21.579***	15.607***	15.626***	4.9797E-5	.004	1.096	.250
	E* $\delta * \rho$	864,1183	.578	1.680***	1.071	1.072	3.8279E-5	3.0964E-4	.912	.209
250	E	6,1183	.019	61.037***	7.365***	7.565***	1.119	.225	1.000	.016
	δ	12, 1183	6.159***	268.229***	40.675***	41.853***	.621	.454	1.000	.381
	ρ	12, 1183	7.259***	123.750***	29.579***	30.214***	.590	.043	1.000	.040
	E* δ	72.1183	1.317**	3.941***	1.916***	1.975***	.652	.004	1.000	.004
	E* ρ	72,1183	.721	9.624***	2.046***	2.088***	.652	.002	1.000	.002
	$\delta * \rho$	144,1183	4.917***	7.625***	7.698***	7.863***	.610	.052	.999	.039
	E* $\delta * \rho$	864,1183	.536	.618	.540	.550	.650	.001	1.000	.001

**** Result is significant at $\alpha = 0.05$ and** ***** Result is significant at $\alpha = 0.01$**

From Table 4.2.1, the following are observed:

- The effect of multicollinearity is generally significant under all criteria in equation1and occasionally significant under all criteria equation 2.
- The effect of autocorrelation is generally significant under all criteria in equations 1 and occasionally significant under variance criterion in equation 2.
- The effect of estimators is generally significant under all the criteria in equation 1 butoccasionally significant in equation 2. The results of the further test as shown in Table 4.2.2a revealed that CORC and MLE are the most preferred estimators
- The interaction effect of estimators and multicollinearity are significant under all criteria in equation 1 only
- The interaction effect of estimators and autocorrelation are generally significant under allcriteria except under bias in equation 1.
- The interaction effect of multicollinearity and autocorrelation are generally significant under all criteria in equation 1.
- The interaction effect of estimators, multicollinearity and autocorrelation is only significant under absolute bias criterion when the sample size is 100 in equation 1.

Consequently, it can be inferred that the performances of the estimators are affected by autocorrelation and multicollinearity under all criteria. The results of the LSD further test visa-vice their estimated marginal means revealed that CORC and MLE estimators are preferred to estimate β_1.

TABLE4.2.1a: Results of further test on β_1 to identify Means that are not significantly different

n	Criterion	Equation	Means of the Estimators						
			OLS	CORC	MLE	MR	FIML	SUR	3SLS
20	BB	2	-6.7484E-2[a]	-1.510E-3[b]	-2.640E-3[b]	-6.7503E-2[a]	-6.7497E-2[a]	-6.7058E-2[a]	-6.7058E-2[a]

	VAR	2	4.1333E-7^{a}	-7.7E-8^{a}	1.4404E-6^{a}	3.4522E-4^{c}	3.4565E-4^{c}	1.8372E-4^{b}	1.8372E-4^{b}
	MS	2	.0144^{a}	.0274^{b}	.0273^{b}	.0145^{a}	.0145^{a}	.0144^{a}	.0144^{a}
50	AB	1	.4454^{b}	.2376^{a}	.2372^{a}	.45297^{b}	.45297^{b}	.4515^{b}	.4515^{b}
	BB	2	.0591^{a}	.0842^{ab}	.0862^{b}	.0596^{a}	.0596^{a}	.05956^{a}	.05956^{a}
	AB	2	.1258^{a}	.1506^{b}	.1529^{b}	.1256^{a}	.1256^{a}	.1256^{a}	.1256^{a}
	MS	2	.0176^{a}	.0244^{b}	.0255^{b}	.0176^{a}	.0176^{a}	.0176^{a}	.0176^{a}
100	AB	2	.0173^{a}	.0213^{b}	.0184^{a}	.01728^{a}	.0172^{a}	.0173^{a}	.0173^{a}
	VAR	2	.00000969^{a}	.00000951^{a}	.00000142^{a}	.00001863^{a}	.00001862^{a}	.0000186^{a}	.0000724^{b}
	MS	2	.0003762^{a}	.00049315^{b}	.00038186^{a}	.0003832^{a}	.00038115^{a}	.00038188^{a}	.0004354^{ab}

NOTE: Means with the same alphabets (superscript) are not significantly different.

Effect on β2:

The effect of estimators, multicollinearity and autocorrelation on estimating β_2 based on the sampling properties of the estimators as revealed by Analysis of Variance technique are shown in Table 4.3.1

TABLE 4.3.1: ANOVA Table showing the effect of estimators, multicollinearity and autocorrelation on β_2 in the model

n	Factor	df	Value of F – Statistic							
			Equation 1				Equation 2			
			BB	A B	VAR	MS	BB	A B	VAR	MS
20	E	6,1183	72.772***	88.409***	12.952***	12.967***	83.118***	63.049***	920.7***	36.029***
	δ	12, 1183	247.67***	772.511***	117.691***	117.868***	0.0001	0.0001	.001	0.0001
	ρ	12, 1183	59.587***	96.987***	29.220***	29.248***	3.103***	3.613***	53.525***	6.685***
	E* δ	72.1183	6.117***	6.114***	3.517***	3.521***	0.0001	.0001	.0001	0.0001
	E* ρ	72,1183	8.163***	10.297***	2.858***	2.860***	.403	.491	8.827	.959
	$\delta * \rho$	144,1183	4.169***	6.705***	7.933***	7.941***	.0001	.001	.0001	.0001
	E* $\delta * \rho$	864,1183	.590	.712	.776	.776	.00001	.00001	.00001	.0001
30	E	6,1183	13.535***	45.648***	5.720***	5.711***	.012	3.67***	8.642***	4.480***
	δ	12, 1183	5.417***	322.711***	39.995***	39.906***	.057	1.156	8.196***	.636
	ρ	12, 1183	108.38***	122.761***	36.800***	36.762***	.019	.628	.933	.364
	E* δ	72.1183	.916	3.205***	1.555***	1.552***	2.7395E-4	.016	6.574***	.045
	E* ρ	72,1183	6.416***	9.502***	2.523***	2.520***	4.5278E-4	.034	.520	.024
	$\delta * \rho$	144,1183	9.435***	8.455***	9.990***	9.980***	.024	.471	.834	.260
	E* $\delta * \rho$	864,1183	.441	.654	.685	.684	2.3022E-4	.008	.455	.006
50	E	6,1183	.505	85.349***	7.334***	7.375***	1.818	2.947***	39.293***	4.398***
	δ	12, 1183	2.268***	584.717***	60.141***	60.075**	53.409***	74.808***	27.962***	81.457***
	ρ	12, 1183	5.263***	92.694***	19.946***	19.988***	.413	2.625***	2.160**	2.832***
	E* δ	72.1183	.399	5.964***	2.101***	2.103***	.047	.440	3.755***	.763
	E* ρ	72,1183	.420	8.439***	1.461***	1.464***	.003	.146	.380	.222
	$\delta * \rho$	144,1183	2.620***	7.650***	7.049***	7.064***	.276	.527	.397	.436
	E* $\delta * \rho$	864,1183	.290	.668	.504	.505	.002	.012	.105	.023
100	E	6,1183	7.569***	157.158***	11.818***	11.850***	.007	4.750***	781.69***	4.462***
	δ	12, 1183	17.264***	753.051***	70.295***	70.384***	6.3886E-5	.003	.0001	.002
	ρ	12, 1183	32.122***	337.035***	58.960***	59.122***	.003	.059	24.422***	.050
	E* δ	72.1183	2.365***	11.086***	3.239***	3.242***	1.2608E-4	.006	.001	.004

Print ISSN: ISSN 2055-0154(Print), Online ISSN: ISSN 2055-0162(Online)

	E* ρ	72,1183	1.769***	26.144***	4.049***	4.060***	4.5586E-4	.009	.00001	.008
	$\delta * \rho$	144,1183	3.694***	23.274***	15.999***	16.015***	7.9134E-6	3.7903E-4	.0001	2.2266E-4
	E* $\delta * \rho$	864,1183	.405	1.849***	1.102	1.103	1.237E-5	.001	.0001	4.0052E-4
250	E	6,1183	3.923***	62.212***	7.350***	7.569***	.375	1.676	3.518***	3.735***
	δ	12, 1183	3.83***	270.233***	40.624***	41.918***	.048	.185	11.717***	.120
	ρ	12, 1183	7.088***	125.657***	29.458***	30.172***	.042	.191	1.398	.169
	E* δ	72.1183	.969	3.964***	1.915***	1.979***	.001	.039	2.778***	.042
	E* ρ	72,1183	.718	9.781***	2.040***	2.088***	.001	.004	.316	.003
	$\delta * \rho$	144,1183	4.748***	7.548***	7.587***	7.784***	.029	.211	1.997***	.121
	E* $\delta * \rho$	864,1183	.543	.616	.534	.546	3.1346E-4	.007	.528	.006

**** Result is significant at $\alpha = 0.05$ and** ***** Result is significant at $\alpha = 0.01$**

From Table 4.3.1, the following are observed:

- The effect of multicollinearity is generally significant under all criteria in equation1and occasionally significant under all criteria in equation 2.
- The effect of autocorrelation is generally significant under all criteria in equations 1 and occasionally significant under variance criterion in equation 2.
- The effect of estimators is generally significant under all the criteria in equations 1and 2. The results of the further test as shown in Table 4.3.1a revealed that CORC and MLE are the most preferred estimators
- The interaction effect of estimators and multicollinearity are significant under all criteriain equation 1 and occasionally significant under variance criterion in equation2.
- The interaction effect of estimators and autocorrelation are generally significant under allcriteria except under bias in equation 1 only.
- The interaction effect of multicollinearity and autocorrelation are generally significantunder all criteria in equation 1.
- The interaction effect of estimators, multicollinearity and autocorrelation is only significant under absolute bias criterion when the sample size is 100 in equation 1.

Consequently, it can be inferred that the performances of the estimators are affected by autocorrelation under all criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that CORC and MLE estimators are preferred to estimate β_2.

TABLE4.3.1a: Results of further test on β_2 to identify Means that are not significantly different

n	Criterion	Equation	Means of the Estimators						
			OLS	CORC	MLE	MR	FIML	SUR	3SLS
20	BB	2	.1183^{a}	.1603^{c}	.1461^{b}	.1176^{a}	.1176^{a}	.1159^{a}	.1159^{a}
	AB	2	.1183^{a}	.1603^{d}	.1461^{c}	.1266^{b}	.1266^{b}	.11967^{a}	.11967^{a}
	VAR	2	8.86205 E-7^{a}	1.15E-7^{a}	- 5.226E-7^{a}	7.3863E -3^{b}	7.3918E -3^{b}	4.1621E -3^{b}	4.1621E -3^{b}
	MS	2	.01512^{a}	.02579^{d}	.02149^{c}	.02207^{c}	.02207^{c}	.0185^{b}	.0185^{b}
	AB	2	.3095^{b}	.2876^{a}	.2956^{ab}	.3096^{b}	.3096^{b}	.3094^{b}	.3086^{b}

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30	MS	2	.1002[b]	.0862[a]	.0908[a]	.1013[b]	.1013[b]	.1009[b]	.0991[b]
50	AB	2	.0996[a]	.0982[a]	.1085[b]	.1002[a]	.1002[a]	.0993[a]	.0993[a]
	MS	2	.0115[ab]	.01051[a]	.01258[b]	.01315[c]	.01315[c]	.01249[bc]	.01249[bc]
100	AB	2	.09997[a]	.1060[b]	.1023[a]	.09957[a]	.0996[a]	.09947[a]	.09945[a]
	VAR	2	.00000314[a]	.00000338[b]	.00000142[a]	.00008072[b]	.00008054[b]	.00007707[b]	.00007651[b]
	MS	2	.010198[a]	.01146[b]	.010732[a]	.01021[a]	.01019[a]	.0102[a]	.0102[a]
250	MS	2	.0038[b]	.0036[b]	.00314[a]	.00314[a]	.00314[a]	.00313[a]	.00313[a]

NOTE: Means with the same alphabets (superscript) are not significantly different.

However, observing the two equations together, we can conclude that MLE is the most preferred in estimating all the parameters of the two equations among all the estimation methods used.

RECOMMENDATION

The research work has revealed that MLE method of estimation is the most preferred estimator in estimating all the parameters of the model based on the four criteria used namely; Bias, Absolute Bias, Variance and Mean Square Error under the five level of sample sizes considered. It can therefore be recommended that when the validity of other correlation assumptions cannot be authenticated in seemingly unrelated regression model, the most preferred estimator to use is MLE. Meanwhile, for any SUR model without any form of correlation, SUR estimation method is most preferred.

SUGGESTION FOR FURTHER STUDY

This study considered two- equation model with two depended variables in each equation, a future research may consider situation in which more than two equations and as many depended variables as possible.One may still consider a Bayesian estimation approach as one of the estimation methods in order to test its own potential.

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