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EFFECT OF MATHEMATICS METHOD COURSE ON PRE-SERVICE TEACHERS' KNOWLEDGE OF CONTENT AND TEACHING FRACTIONS

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ABSTRACT*: The study sought to examine the effects of mathematics teaching methods course on pre-service mathematics teachers' knowledge of content of teaching mathematics. The data were collected from 120 level 300 pre-service mathematics teachers. Participants were given Fraction Knowledge Questionnaire to test their knowledge of content of teaching mathematics at the beginning and after method course. The purpose of the pre-test and post-test assessment was to measure the amount of change in the participants' knowledge for mathematics teaching. The result of the study revealed pre-service teacher recorded gains in their knowledge of content and teaching fraction operations from content level to epistemic level. An eta squared statistic of 0.63 revealed a large effect size of mathematics teaching methods course on pre-service teachers' knowledge of content and teaching fraction operations. Consequently, it was recommended that other studies be done in the other content domains of the basic mathematics in order to find the effect Mathematics Method course might have on those contents.*

KEYWORDS*:* Teacher knowledge, pre-service elementary mathematics teachers, mathematics teaching methods course, content knowledge for teaching mathematics.

INTRODUCTION

With the emergence of mathematical knowledge for teaching teachers (Zopf, 2010; Kim, 2013), it was indicated that the fact that teachers have adequate content knowledge (CK) on a subject does not mean they can teach this subject efficiently (Kahan, Cooper, & Bethea, 2003). Thus, adequate content knowledge is not sufficient on its own for effective teaching (Tamisli & Kose, 2013). For this reason, many researchers have focused on how teachers teach a field and replicate their content knowledge during the teaching process in addition to having acceptable content knowledge (Gökkurt & Soylu, 2016a).

Emphasizing the content knowledge for teaching mathematics, Shulman (1986) divided teacher knowledge into three main categories; namely, subject matter content knowledge (SMCK), pedagogical content knowledge (PCK), and curricular knowledge (CK). SMCK refers to the knowledge about the mathematical content that will be taught, whereas PCK means the knowledge about the ways of teaching the mathematics concerning multiple representations, materials, appropriate and alternative solutions, and students' preconceptions and misconceptions.

The type of mathematical knowledge teachers should have to be able to teach the subject proficiently and efficiently is a fundamental question that is currently being explored by

researchers and mathematics educators worldwide. Recent developments in the field of teaching and learning mathematics saw the emergence and conceptualization of a framework for teaching mathematics called mathematical knowledge for teaching (MKT) (Pelingon, 2019).

The MKT framework is categorized into two major categories, namely; subject matter content knowledge and pedagogical content knowledge. Under the subject matter knowledge domain are common content knowledge (CCK) - the mathematical knowledge and skill used in settings other than teaching and specialized content knowledge (SCK) - the knowledge and expertise unique to teaching. Pedagogical content knowledge (PCK) includes knowledge of content and students (KCS) which is the knowledge that combines knowing about students and knowing about mathematics and knowledge of content and teaching (KCT) which combines knowing about teaching and knowing about mathematics (Ball et al., 2008).

As part of ensuring quality education, Ghana implemented teacher education reform in 2004 which saw the introduction of content and methodology courses which are taught in teacher training colleges and examined separately by Institute of Education of the University of Cape Coast to equip pre-service teachers with the pre-requisite content knowledge and pedagogical content knowledge (CK) and (PCK). Meanwhile, researchers (Cardetti & Truxaw, 2014; Goodson-Epsy, Lynch-Davis, Cifarelli, Morge, Pugalee, & Salinas, 2014; Tyminski, Zambak, Drake, & Land, 2014) observed that mathematics methods coursework has shown to improve the CK and PCK of teacher candidates and the ability to lead discussions.

Hill (2007) investigated middle school teachers' mathematical knowledge for teaching, and relationships between teachers' mathematical knowledge for teaching and their subject matter preparation with more than 80% of teachers studying three or more mathematics courses and most of the teachers studying mathematics methods courses in their education program. When Hill tried to answer the question of whether teachers' training influences their content knowledge for teaching, the results showed that teachers who took mathematics courses and mathematics methods courses performed better on the test. Based on these results, Hill argued that the number of methods and mathematics course work taken can be used to predict levels of mathematical content knowledge for teaching

However, Moss (2006) investigated pre-service teachers' mathematical content knowledge for teaching, revealed that pre-service teachers have limited understanding about content knowledge for teaching mathematics. Especially, pre-service teachers and in-service teachers do not have sufficient specialized knowledge of content for teaching mathematics. Researchers have suggested to teacher educators should take the mathematical content knowledge for teaching into consideration seriously while designing their courses. In addition, the studies highlighted the role of methods courses in improving pre-service teachers' content knowledge for teaching mathematics.

Further, studies showed that the mathematics teaching methods course plays an important role in improving pre-service teachers' mathematical content knowledge for teaching (Kinach,2002;

Moss, 2006; Quinn, 1997). However, with the recent transformation of teacher education and learning in Ghana, some mathematics educators have deemphasized the teaching of mathematics method course separately. Hence, this study aims to investigate the effect of mathematics teaching methods course on pre-service mathematics teachers' content knowledge for teaching mathematics.

Statement of the Problem

To researchers, content knowledge alone is insufficient to support the teaching of mathematics and a lack of pedagogical content negatively affects a teacher's instructional practice (Baki & Arslan, 2016). In addition, teachers who lack high MKT, specifically in the KCT domain, may struggle to plan effective lessons (Linder & Simpson, 2017) and may not have the ability to adjust curriculum or instruction to best meet the needs of their students (Lui & Bonner, 2016).Asiedu–Addo and Yidana (2000) investigated low performance in mathematics at the pre-tertiary level of the Education system. In the study however, the problem of poor performance in mathematics was traced to pre-service teachers' lack of "Mathematical Knowledge for Teaching" (MKT). In addition, Mereku (2004) also debunked the educational administrators' claims that the poor performance of pupils in the subject was due mainly to lack of confidence and competence on the part of teachers to teach the content of the materials and to use the prescribed methods to facilitate the learning of mathematics in primary schools.

To establish whether a relationship exists between the mathematics content knowledge for teaching and method of teaching mathematics courses, Asante and Mereku (2012) analyzed the performance of 100 randomly selected pre-service teachers enrolled in the Colleges of Education in Ghana using two data sets, mathematical knowledge for teaching (MKT) scores and DBE examination results. The MKT scores were measured through an adapted instrument from the MKT instrument developed by Ball, et al (2005) while DBE examination results were obtained from the previous records of respondents' first year mathematics content examination. The overall performance of the pre-service teachers in the MKT test was low with only 8% of them obtained marks from 60% - 73% of the items while 75% of them got marks from 32% - 51%. Meanwhile, the mathematics courses offered in Colleges of Education in Ghana comprises Content and Methodology to equip pre-service teachers with mathematical knowledge for teach in order to teach mathematics in the basic school curriculum. Therefore, this present study sought to examine the effects of mathematics teaching methods course on pre-service mathematics teachers' knowledge of content of teaching mathematics.

Research Questions

1. What extent do pre-service teachers' make gain in their knowledge of content and teaching following their participation in method of teaching mathematics course?
2. What extent do Mathematics Method courses have effect on pre-service teachers' knowledge of content teaching operation of fractions?

Mathematics Teaching Methods Course

The Methods of Mathematics Teaching course is a last year course in the mathematics teacher education program. The general goal of the course was to make pre-service teachers understand mathematics concepts and the ideas behind the standard algorithms, rules and formulas and get insight about how to teach a particular mathematical topic.

LITERATURE

Role of Methods Courses on Teachers' Content Knowledge for Teaching

It is an undeniable fact that mathematics teaching methods course plays an important role in teacher preparations since it can influence preservice teachers' content knowledge (Ball, 1989). Meanwhile, Seviş (2008) examined the effects of a Mathematics Teaching Methods course on pre-service elementary mathematics teachers' content knowledge for teaching mathematics (CKTM). The data were collected from 43 senior pre-service mathematics teachers from a teacher education program at a large public university. The participants were given an 83-item test to measure their content knowledge for mathematics teaching at the beginning and after mathematics methods course was taught. The purpose of the pre- and post-test assessment was to measure the amount of change in the participants' knowledge for mathematics teaching. The results indicated that there was a significant effect of the mathematics teaching methods course on pre-service teachers' content knowledge for teaching mathematics Number concepts and Operations, Geometry and Algebra.

Quinn (1997) investigated the effects of methods courses on mathematical content knowledge for teaching of elementary and secondary mathematics teachers. A sample 28 pre-service elementary mathematics teachers took part in the study. The results of the study showed that at the end of the methods course which promoted students' questions, hands-on activities and cooperative group activities, pre-service elementary teachers' knowledge of content increased significantly. Quinn suggested that during method courses, pre-service teachers should experience using multiple representations in teaching, evaluating students' representations and answers, and evaluating textbooks, activities and instructional tools.

In another study, Ball (1989) explained the function of methods course on pre-service teachers' learning to teach. More specifically, Ball mentioned about the role of methods courses on reflecting fundamental nature of teaching to foster students' learning mathematics. Ball put forward that the methods courses give pre-service teachers opportunities to revisit and reconstruct their own past learning and experiences and to learn about appropriate ways of using manipulatives to teach mathematics effectively.

Moss (2006) used Ball and her colleagues' framework to investigate strengths and weaknesses of 244 pre-service teachers' specialized understanding of mathematics (SUM) and the contribution of methods course on their improvement of SUM. Paired sample t-test results showed that pre-service teachers exhibited statistically significant improvement in their content knowledge to teach mathematics effectively even if their improvement was varied in item based analysis.

Nolan, Dempsey, Lovatt, and O'Shea, (2015) studied the development of mathematical knowledge for teaching of two groups of pre-service teachers with pre-and post-test. For MKT awareness, pre-service teachers were asked to list down different teaching situations using their knowledge of mathematics. The researchers' intervention strategy was the delivery of a specially designed mathematics pedagogy course intended to improve pre-service MKT. After the intervention strategy, MKT level of the pre-service teachers significantly increased.

The results of the studies mentioned above revealed the role of methods courses in developing pre-service teachers' mathematical content knowledge. These studies also implied that teacher educators should not overlook the value of subject matter understanding of pre-service teachers in preparing them to teach for understanding (McDiarmid, Ball, & Anderson, 1989)

METHODOLOGY

Research Design

The design of this quantitative study was a single- group pre-test-post-test pre-experimental design. In this type of design, one group of subject was given a pre-test, then the treatment and then the post. Pre-test and posttest are the same just given at different times (McMillan & Schumacher, 2010). In most of the Colleges of Education in Ghana, a packet programme is implemented where method of teaching mathematics course is compulsory to attend. For this reason, there was no opportunity to assign participants into control or experimental groups by random selection, and single group pre-test-post-test pre-design was carried out by researchers.

Population and Sample

The accessible population is all level 300 pre-service teachers specializing in mathematics as their major course for junior high school level in Akatsi College of Education. Purposive sampling method was used to obtain the representative sample of this study. Purposive sampling is appropriate in occasions where investigators need to select participants who have particular characteristics needed for the study. Regarding purposive sampling method, investigators select people who are believed to provide the data needed (Fraenkel & Wallen, 2006). Therefore, one hundred and twenty (120) level 300 pre-service teachers who took part method of teaching Mathematics course and microteaching activities in their first semester were selected to constitute the sample for the study.

Instrument

The instrument for the study was Fraction Knowledge Questionnaire (FKQ). The items in the questionnaire concerning misconceptions/ mistakes related to fractions comprised comparison of unlike fractions, addition of unlike fractions and multiplication of fractions was adapted from a similar study conducted by Eroglu in 2012. The completed questionnaire was given to supervisors in the field of mathematics education in the Mathematics Department of University of Education, Winneba for expert judgment in order to ensure content validity and item relevance. The questionnaire administered consisted of three open ended-in class problems that contained

students' misconceptions/errors. Pre-service teachers were asked to use their knowledge of content and teaching and show step by step how they would assist students overcome their misconceptions/errors. The first item in the FKQ evaluated pre-service teachers' reasoning on students' formal knowledge about comparison of fractions. Students usually thought $\frac{1}{3} > \frac{1}{2}$ since 3 is greater than 2. The second item in the FKQ evaluated pre-service teachers' knowledge on students' misconception/mistakes based on whole number bias (WNB). Whole number bias involves performing the arithmetic operation on the numerators and denominators separately as if they were independent whole numbers (Braithwaite et al., 2017); Van Steenbrugge, Lesage, Valcke, & Desoete, 2014). For example, $\frac{1}{3} + \frac{1}{4} = \frac{1+1}{3+4} = \frac{2}{7}$ leads to an incorrect answer. In this case, students added denominators alongside the numerators.

The third item in the FKQ evaluated incorrect fraction operation strategies (IFO). This item explored the prospective teachers' knowledge about students' erroneous use of numerator or denominator. Wrong fractions operations strategies are concerned with how students treat the numerators or denominators of fractions incorrectly due to their misconceptions, but in a way that would be correct for a different fraction arithmetic operation (Braithwaite, Pyke, & Soldier, 2017; Van Steenbrugge et al., 2014). For example, students would generalize the ideas of common denominators in multiplication contexts such as $\frac{3}{5} \times \frac{4}{5} = \frac{12}{5}$.
Pre-service teachers' knowledge of content and teaching was scored and classified into five levels (1= Content level, 2 = Concept level, 3= Problem-solving, 4 = Epistemic level, 5 = Inquiry level). The scores were interpreted as follows: one is the least possible score and five is the highest. On the scale of 1 to 5, 3 being the median, when the mean is above 3 it implies the pre-service displayed relational understanding, but when the mean is below 3, it implies the pre-service teachers exhibited instrumental understanding.

Data Collection Procedure
The researcher administered the Fraction Knowledge Questionnaire to the pre-service having explained the purpose of the study and the sampling techniques. The Fraction Knowledge Questionnaire was administered two times as pretest and post-test to compare pre-service mathematics teachers' knowledge of content of teaching fractions, and identify the effect of mathematics teaching methods course. The pretest was administered at the beginning of the second semester of the second year and the posttest at the first semester of the third year which was the beginning of teaching the mathematics teaching method course. Both pretest and posttest were collected by the researcher. All retrieved questionnaires were adequately completed and were found usable for the study.

The study employed Kinach (2002a, 2002b) framework which focused on Perkins and Simmons's (1988) levels of mathematical knowledge for scoring of pre-service teachers' knowledge of content and teaching fractions. Kinach (2002a, 2002b) focuses on Perkins and Simmons's (1988) levels of mathematical understanding and groups those under two headings: instrumental understanding and relational understanding. Instrumental understanding contains what and how knowledge, while relational understanding shows the reasons underlying what and how. Within

this perspective, instrumental understanding is handled at the context level, which (aims to explain rules and practices superficially). Relational understanding is formed of four levels of understanding: concept level, problem-solving level, the epistemic level.
After examining the answers given by prospective pre-service teachers, the researcher sub-coded the levels of understanding as follows:

a) Relational Understanding
Inquiry level: Advanced problem-solving level in which new knowledge or theorems are suggested
Epistemic level: Correct use of visual elements and showing their grounds.
Problem-solving: Use of concept-specific strategies for fraction problems.
Concept level: Correct use of visual elements and creating patterns.
b) Instrumental Understanding
Content level: Superficial explanation of rules or given expressions, misuse of visual elements and solving equations by giving direct answers.

Data Analysis
The data collected were organized and analyzed using statistical tools such as mean, standard deviation and paired samples t-test.

Descriptive Statistics The mean, standard deviation, minimum and maximum scores and skewness of the pretest, posttest, and gain scores, and the pretest, posttest, and gain scores in operation of fractions. These descriptive statistics indicated a general picture about the pre-service teachers' knowledge of content of teaching fraction improvement.

Inferential Statistics The main goal of this study is to explore the effect of mathematics teaching methods course on pre-service teachers' content knowledge for teaching mathematics. To investigate this, paired-samples t-test was conducted for pretest and posttest scores. Also, paired samples t-tests were employed to examine the mean difference between pretest and posttest of pre-service teachers knowledge of content of teaching operation fractions

RESULTS AND FINDINGS

Reseaer1: What extent do pre-service teachers' make gain in their knowledge of content and teaching following their participation in method of teaching mathematics course?

Table 4.1 Pretest, Posttest, and Gain Scores for the Whole Group

	Pre-test	Post-test	Gain (Posttest- Pretest)
N	120	120	120
Mean	4.56	6.96	2.40
Standard Deviation	1.94	1.14	-0.80
Minimum	2	3	1
Maximum	8	8	0
Skewness	0.36	-1.47	-1.84

As shown in Table 4.1, whilst pre-service mathematics teachers have obtained a mean score of 4.56 (SD = 1.94) for their pretest, their mean score in the posttest is 2.398 (SD = 1.14) out of 120. Thus, the average score of pre-service mathematics teachers is found to be 2.40 (SD = -0.80. The gain score constitutes 2% of 120, the highest possible score in the test. Moreover, the minimum score in the pretest (2) went up to 3 in the posttest. However, the maximum score recorded no gain

Table 4.1 showed a negative coefficient of skewness (Sk = -1.837) which revealed that the distribution is skewed to the left with the mean< median < mode indicating a very high level of pre-service teachers' knowledge of content of teaching fraction operations. The pre-service teacher also made gains in their knowledge of content of teaching fraction operations from content level (Superficial explanation of rules or given expressions, misuse of visual elements and solving equations by giving direct answers) to epistemic level (Correct use of visual elements and showing their grounds). Table 4.1 showed that the distributions for the pretest and posttest scores, and gain scores for the whole group are approximately normally distributed.

Research Question 2: What extent do Mathematics Method course have effect on pre-service teachers' knowledge of content teaching operation of fractions?

Table 4.2: Paired Samples Results for Pre-test and Post test

	M	SD	SEM	Lower	Upper	t	df	p(.05)
Pre-test Post-test	-2.54	1.99	0.19	-2.91	-2.17	-13.48	110	0.000

Table 4.2 shows a paired samples t-test conducted to evaluate the impact of the mathematics teaching methods course on pre-service teachers' content knowledge for teaching mathematics. Before conducting paired samples t-test, the normality assumption was verified.

The findings showed that there was a statistically significant increase in total scores from pretest (M = 4.56, SD = 1.94) to posttest (M = 6.96, SD = 1.14), t (110) = -13.48, p < .05. The average

gain score was found as 2.398. The effect size interpreted by eta squared statistic which was computed by Formula 4.1 (Pallant, 2001).

$$\text{Eta squared} = \frac{t^2}{t^2 + N - t} \qquad (4.1)$$

The eta squared statistic was found as 0.63 which indicates a large effect size according to Cohen's (1977) guidelines for paired samples t-test.

DISCUSSION

The results of the study indicated that the mathematics teaching methods course has potential to influence pre-service teachers' content knowledge for teaching mathematics positively. More specifically, pre-service teachers' pretest scores on content knowledge for teaching fraction operation significantly increased at the end of the method course as seen in the posttest scores and gain scores.

It can be inferred from the results of the study conducted that pre-service teachers' knowledge about solving a mathematical problem in different ways, carrying out the algorithms correctly, evaluating students' solutions appropriately, using physical materials in mathematical expressions or operations appropriately and accurately, and explaining the meaning of the mathematical concepts, procedures and algorithms, showed significant improvement at the end of the methods course (Hill, Schilling, & Ball, 2004).

More briefly, the mathematics teaching methods course influenced pre-service mathematics teachers' knowledge related to the fractions they will teach at basic schools. It might be because the methods course dealt with most of the mathematical topics in fractions pre-service teachers will teach in elementary schools, like Numbers and Operations.

Moreover, pre-service teachers' have experienced discussing the mathematical tasks conceptually, representing mathematical operations or expressions in multiple ways, evaluating students' solution strategies, and handling students' misconceptions in each mathematical topic in the methods course. Therefore, pre-service teachers could have gained both common knowledge of content and specialized knowledge of content for teaching mathematics in this course.
These studies emphasized the role of methods courses play in developing pre-service teachers' mathematical content knowledge. To be more specific, Seviş (2008) reported that pre-service teachers' content knowledge for teaching mathematics related to Number and Operation showed significant improvement at the end of the methods course. The findings of the present study are consistent with Savis's study since it showed significant difference between total pretest and posttest scores, as compared with the present study.

Other studies which showed significant improvement in teachers' knowledge of content and teaching as result of mathematics method course and are consistent with the findings of this study are Seviş (2008), Quinn (1997), Moss (2006) and Nolan, Dempsey, Lovatt, and O'Shea (2015).

In another study, Hill (2007) put forward that the middle school teachers who took mathematics courses and mathematics methods courses performed better on the test, one of the CKTM measures. By considering this argument as an indicator of role of methods course on content knowledge development, this result can be viewed as another agreement with the results of the present study.

CONCLUSION

The evidence available from the findings of the study provided the basis of a number of evidence for a number of conclusion to be made. It was found that pre-service teachers made sufficient gains from content level to epistemic level after they were taught method of teaching fraction since the scores are approximately normally distributed. The mathematics teaching methods course had a potential influence on pre-service teachers' content knowledge for teaching mathematics from statistical perspective.

Recommendations and Implications

Even though the study underlines that the mathematics teaching methods courses have potential to influence 120 pre-service teachers' content knowledge for teaching mathematics from a statistical perspective, pre-service teachers need more opportunities to understand mathematics conceptually. According to Moss (2006) the improvement of mathematical content knowledge is an important issue for pre-service teachers since they will teach mathematics more meaningfully. As part of transforming teacher education and learning, Teacher Education Programme has recently changed. The programme has placed more emphasis on teaching mathematics in a blended format which researchers (Auslander, Smith, Smith, Hart, and Carothers, 2016; Hoover, Mosvold, Ball, Lai, 2016; Son and Lee, 2016) observed affect the development of mathematical knowledge of teaching more than addressing the concepts individually.

Therefore, teacher education institution in collaboration affiliated colleges of education should fashion out mathematics content and methodology courses which should be taught and examined separately to equip pre-service teachers with the knowledge about explaining meanings of the concepts and procedures, using multiple representations, using materials, and evaluating students' solutions.

Similarly, Mapolelo and Akinsola (2015) found that the inadequacy of teachers' knowledge of mathematics and how they teach it, is one of the major reasons why students are not learning the mathematics they are supposed to learn in school. To improve pre-service teachers' knowledge of content of teaching mathematics, more emphasis should be placed on quality content courses and mathematics teaching courses to equip them with the pre-requisite conceptual and procedural knowledge.

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