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DEVELOPMENT OF 3D DATUM TRANSFORMATION MODEL BETWEEN WGS 84 AND CLARKE 1880 FOR CROSS RIVERS STATE, NIGERIA

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ABSTRACT: *The need to have unified 3D datum transformation parameters for Nigeria for converting coordinates from Minna to WGS84 datum and vice-versa in order to overcome the ambiguity, inconsistency and non-conformity of existing traditional reference frames within national and international mapping system is long overdue. This study therefore develops the optimal transformation parameters between Clarke 1880 and WGS84 datums and vice-versa for Cross River State in Nigeria using the Molodensky-Badekas model. One hundred (100) first order 3D geodetic controls common in the Clarke 1880 and WGS84 datums were used for the study. Least squares solutions of the model was solved using MATLAB programming software. The datum shift parameters derived in the study were ΔX=99.388653795075243m ± 2.453509278, ΔY = 15.027733957346365m ± 2.450564809, ΔZ = -60.390012806020579m ± 2.450556881,α=-0.000000601338389±0.000004394,β=0.000021566705811 ± 0.00004133728, γ = 0.000034795781381 ± 0.00007348844, S(ppm) = 0.9999325233 ± 0.00003047930445. The results of the computation showed roughly good estimates of the datum shift parameters (dX, dY, dZ, K, R_X , R_Y , R_Z, K) and standard deviation of the parameters. The computed residuals of the XYZ parameters were relatively good. The result of the test computation of the shift parameters using the entire 107 points were however not significantly different from those obtained with the 100 points, as the results showed good agreement between them. Seven reserved points (xsw148, xsw117, xsw126, xsw97, xsw82, xsw64, xsw155) were used to validate the model.*

KEYWORDS: Datum; transformation; MATLAB; Clark 1880; WGS84.

INTRODUCTION

Background of study

The science of geodesy has provided us with two different types of coordinate systems. These are geocentric and regional (local) coordinate systems (Sella *et al*., 2002). The origins and axes of these coordinate systems are different. While the geocentric coordinate system has its origin at the centre of the mass of the earth and the regional coordinate system has its centre different from the geocentre. These coordinate systems are associated with the term 'datum', which uses coordinates referred to the surface of defined ellipsoid of revolution. Historically, different ellipsoids have been chosen by different countries of the world in order to simplify surveying and mapping in their region and as such these ellipsoids are not necessarily geocentric (Rollins & Avouac, 2019). In Cross Rivers, Nigeria, the regional (local) coordinate system is the Minna Datum based on CLARKE 1880 ellipsoid. The geocentric system of Cross Rivers, Nigeria is the WGS84 ellipsoid. These datums are defined using two parameters i.e. Semi-major axis (a) and flattening (f). Several assumptions were made in the definition.

Geospatial Cartesian coordinate is a geocentric coordinate system having the earth centre of mass at its origin. This makes it a valid and unified reference system for the World Geodetic Systems

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1984, which is an earth-centered, earth fixed coordinate system. Datum transformation using Molodensky equations are becoming increasingly important, because of the growing use of GNSS data (Trojanowiczl *et al*., 2018). Very often the spatial data is captured using GNSS whose reference datum is the earth-centered WGS84 ellipsoid, and have to be transformed to a local projection with its own ellipsoid and datum (e.g. Clarke 1880 Ellipsoid and Minna datum). Heiskanen and Moritz (1967) gave the forward transformation from geodetic coordinates (φ,λ,h) to cartesian coordinates (X,Y,Z) as,

$$X = (v + h) \cos\varphi \cos\lambda$$
$$Y = (v + h) \cos\varphi \sin\lambda \qquad (1)$$
$$Z = [v(1 - e^2) + h] \sin\varphi$$

Where the prime vertical radius of curvature (v) is given by,

$$v = a(1 - e^2\sin^2\varphi)^{-1/2} \qquad (2)$$

Where a and e represent the semi-major axis and the first eccentricity of the reference ellipsoid respectively. The parameters in Table 1 were used in equations 1 and 2 to compute forward transformation from geodetic coordinates (φ,λ,h) to Cartesian coordinates (X,Y,Z). For Minna datum, it is assumed that ellipsoidal height (h) is equal to orthometric height (h). This implies that geoidal height (H) is zero; the normal and vertical coincided. To fully describe positions in relation to the earth, the geodetic coordinate system and Cartesian coordinate system is employed. The geodetic coordinate system comprises a right–handed orthogonal three-dimensional coordinates made up of geodetic latitude (φ), geodetic longitude (λ) and ellipsoidal height (h). They refer to the surface of specific ellipsoid of revolution about its minor axis. The Cartesian coordinate system is the three-dimensional orthogonal axes in the x, y, and z directions. Thus, corresponding triplets of Cartesian coordinates refer to these axes. The x-axis is directed towards the intersection of the Greenwich meridian and equatorial plane. The z-axis is aligned towards the north pole of the Earth’s rotation. The y-axis is orthogonal to x and z axes and completes the right–handed coordinate system (Figure 1).

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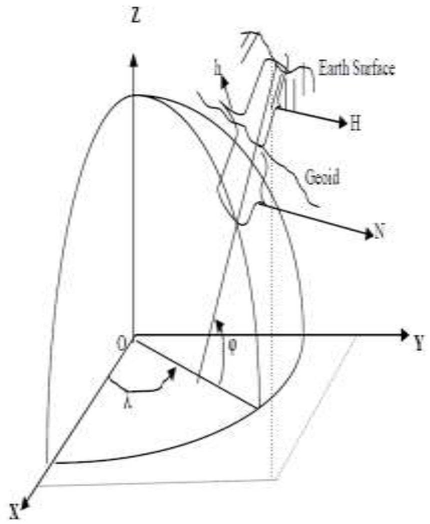

Figure 1: A corresponding triplet of Cartesian coordinates

Usually, the more easily potable and understandable coordinates are the Eastings (E) and Northings (N), which leads us again to another system, called plane (rectangular) coordinate system. The geodetic coordinate system or Cartesian coordinate system can be projected to plane coordinate system through appropriate projection models. The need to integrate data between the WGS84 (UTM) ellipsoid and the Clarke 1880 (NTM) ellipsoid has been an issue of major concern in the country over the years (Uzodinma, 2005). Unfortunately, there is no generally acceptable transformation parameter to perform this transformation yet as there has been no consensus on the particular set of parameters to use (Uzodinma, 2013). Therefore this study aims to concentrate on a smaller division of the country in order to have a more concentrated area and increase accuracy.

Due to the advent of modern space based method, data in Nigeria is now being captured in the WGS84 reference system as opposed to the local datum capture which has been the norm in data capture in Nigeria. This therefore calls for a major need to integrate the data in both systems in order to create homogeneity in our reference systems as well as ensuring the integrity of geospatial information while promoting the sharing and exchange of data across ministries, agencies, and between the public and private sector and most importantly ensure that end users achieve the transformation of geospatial information using only one methodology (or tool set) which will result in transformations of known accuracy, with repeatable and consistent results that are compatible across state boundaries. (Okeke, 2013).

A coordinate system forms a common frame of reference for the description of positions and on the other hand, coordinates are simply an ordered set of numbers that are used to describe the positions or features in a coordinate system (Moritz, 1980, Laundal & Richmond, 2017). Transformation parameters are required to move from one system to another. In Nigeria, we have different coordinate systems based on different origins which are used for various mapping purposes. Also, new technologies like global positioning system have provided new methods of coordinates' determination. The map production, update and revision are based on geographical

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coordinates; map-grid coordinates or coordinates in an arbitrary system. Some others are based on old (local) system. There are no truly accepted transformation parameters the consequences are obvious confusion and misrepresentation of features. In fact surveyors and survey practitioners are already using the new technology based on geocentric system while most available maps and map coordinates are in local system. The effect or implication is multiple data sets on different systems (Ojigi & Dodo, 2013).

Thus, this project has emphasized on a clear choice of coordinate systems and coordinates especially as new methods of spatial information capture emerge. It described the methodology of making different coordinates compatible to be employed in spatial referencing by determination of transformation parameters. This therefore will help in generalizing features for representation in two dimensions on flat piece of paper. Hence, a recommendation has been made to unify all the different coordinates or made to be compatible and flexible by employing least squares adjustment principles to determine the transformation parameters. Seven parameters of Helmert transformation are estimated using three-dimensional Cartesian coordinates in Nigeria. Here, two cases are studied. Cartesian coordinates of WGS 84 and Clarke 1880 in mm level accuracy, where the information from both cases was generated from secondary source. It provides the coordinates in millimeter level accuracy. Helmert transformation parameters are estimated by applying MATLAB code. Seven parameters of Helmert transformation between WGS 84 and Clarke 1880 datums, and Clarke 1880 and WGS84 datums, and vice-versa are estimated. Due to lack of large data, the estimation for this project might not be exactly accurate as estimations of Helmert transformation parameters requires sizable numbers of Cartesian coordinates based on the project area with high accuracy.

Statement of the problem

The existence of multiple coordinate systems has proved to be a major setback in our map production, map updates and map revision in our country as well as states. Due to lack of a perfectly universally adopted set of transformation parameters, it has been problematic transferring coordinates from one system to another. This has in fact led to a situation where known points in a certain coordinate system have to be re-observed just to determine their coordinates in a different coordinates system.

It is therefore necessary to create a good relationship between systems so that we can move from one system to another easily and maintain a certain level of accuracy. The development of the 7 parameters which would most likely coincide between two systems have been a major project for Geodesists over the years and therefore this project bases at the realization of an acceptable connection between the Minna Datum based on the Clarke 1880 Ellipsoid and the Global Datum based on the WGS84 ellipsoid in Cross Rivers State so that coordinate conversion between the two systems using certain parameters coincides exactly or very closely to each other.

Aim

This work was aimed at developing a simple, efficient, unique and accurate transformation model for coordinates between WGS 84 and Clarke 1880 in Cross Rivers State.

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Objectives

The following were the main objectives of this work:

i. The organization of data containing 100 common points in both Clarke 1880 (φ, λ,H) and WGS 84 (φ, λ,h) geodetic systems.
ii. Compute and transform from geodetic coordinates (φ, λ,h) to Cartesian coordinates (X,Y,Z)
iii. Develop adequate algorithms and simulations for the determination of the seven Parameter datum transformation in Cross Rivers State, Nigeria.
iv. Determine the seven optimal transformation parameters between Clarke 1880 and WGS84 Reference Ellipsoids for Cross Rivers State using the Molodensky Badekas model.
v. Carry out the validation of the developed transformation parameters with some reserved control points (testing points) in the network.
vi. Submission of findings and recommendations

METHODOLOGY

Study area

Cross River is a coastal state in southern part of Nigeria, bordering Cameroon to the east. It has Its capital city at Calabar, and it derived its named from the Cross River, which passes through the state. Its coordinates are 5°45'0" N and 8°30'0" E in DMS (Degrees Minutes Seconds) or 5.75 and 8.5 (in decimal degrees). The state was created in 1967 from part of the former Eastern Region, and was known as the 'South-Eastern State until 1976 when it adopted its present name. The state originally included what is now Akwa Ibom State. Cross River State is located in Nigeria's Delta region. It is bounded on the North by Benue State, on the South by Akwa Ibom State, on the East by Cameroun Repulic and on the West by Anambra and Imo States.

Selection of points

A set of 107 points involving coordinates in both the Nigerian Geodetic Network/System established on the Clarke 1880 spheroid and the world/Global System located on the WGS 84 spheroid were used for this project where seven points was reserved to be used as validation points (testing samples) therefore, only one hundred (100) points were used in the estimation (Figure 2). The data was in the geographic units and the ellipsoidal heights were in meters. However some assumptions were made during the process of this computation as stated in Table1.

Table 1: Ellipsoid parameters of clarke 1880 and WGS84

Ellipsoid	Semi-major axis (a) (m)	1/f	f	e^2
Clarke 1880	6378249.145	293.465	0.003407561	0.006803511
WGS 84	6378137.000	298.257223563	0.003352811	0.006694380

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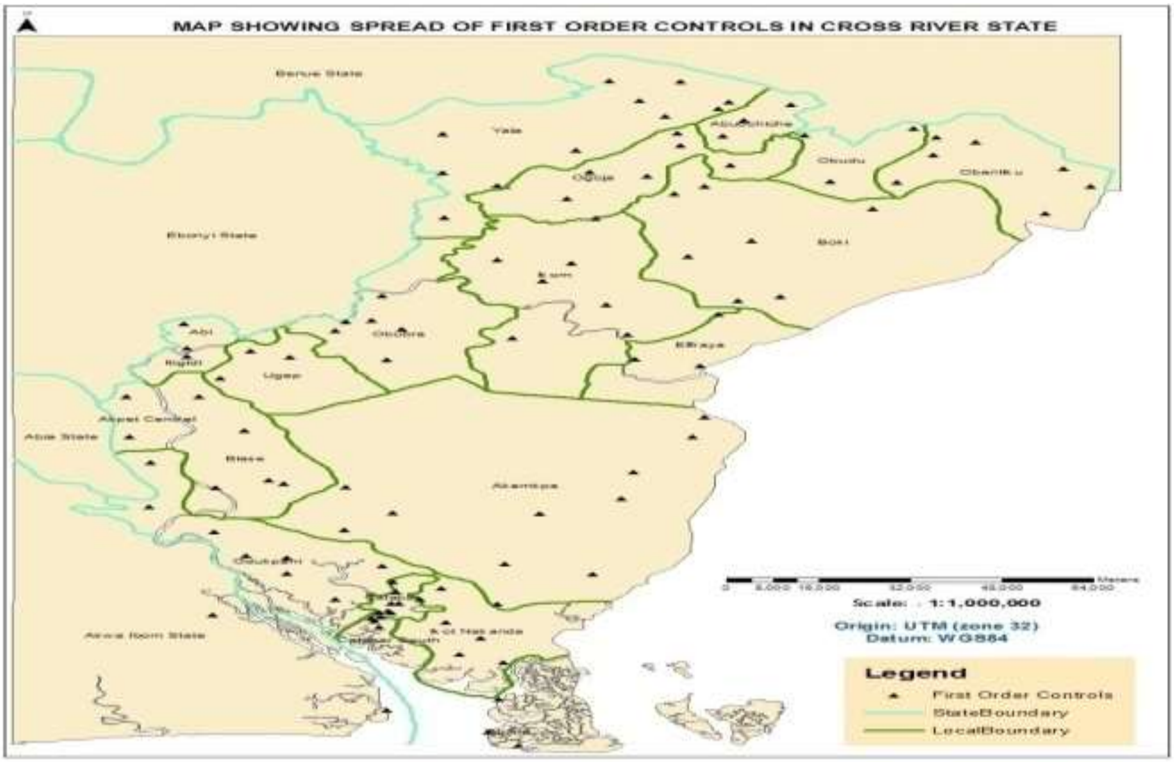

Figure 2: Map showing spread of first order controls in Cross River State

Data Acquisition and Evaluation

The data are first order controls across Cross Rivers State acquired in 2000. The data was carefully evaluated before using it for the purpose of this exercise.

Transformation of 3D Cartesian coordinates between two Datums

One of the many ways to mathematically transform positions from one datum to another is the requirement of 'common points'. The common points are surveyed points that have coordinates in both local and global datums. The achievable accuracy of the datum transformation will be determined by the number, distribution and transformation technique adopted. Hence, the greater accuracy required, the more common points are needed.
Further, the choice also depends on the following:

i. The extents of the area for which it is to be applied
ii. The presence of distortion in either of the reference systems
iii. The dimensions of the reference systems i.e. whether it is two or three dimensional and again
iv. The accuracy requirements.

The 3D similarity transformation was chosen for this study for the obvious reasons that include the following: it preserves shape and angles while lengths of lines and the positions of points may be changed. Also, it assumes that there are systematic distortions within the two networks. The general similarity transformation is the one defined as:

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$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = S_F R \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} \tag{3}$$

Where,

X_B, Y_B, Z_B = Coordinates in coordinate system B
X_A, Y_A, Z_A = Coordinates in coordinate system A
T_X, T_Y, T_Z = Translations terms in the three respective axes
R = 3×3 orthogonal matrix
S_F = Scale factor = 1+Δs where Δs is the differential scale

There are seven parameters which are usually associated with a similarity transformation; three rotation angles, three translational components and one scale factor (Shen *et al*., 2006). For small rotation angles, the rotation matrix R is approximated by,

$$R = \begin{bmatrix} 1 & \alpha_z & -\alpha_y \\ -\alpha_z & 1 & \alpha_x \\ \alpha_y & -\alpha_x & 1 \end{bmatrix} \tag{4}$$

Where α_x , α_y , α_z are the rotation angles in the three axes.

The least squares estimation of the transformation parameters

The least squares solution of the unknown parameters or the estimate of the correction to approximate parameter vector (x) is given by (Mahboub, 2012).

$$\dot{X} = -(A^T PA)^{-1} A^T PL \tag{5}$$

Where, A^TPA is a non-singular matrix called Normal equation coefficient matrix, and A^TPL is the normal equations constant (or absolute) term vector. Equation 5 was given by Ghilani (2000) as equation 6 in which the weight (P) and the column matrix (L) were presented as W and b respectively,

$$x = -(A^T WA)^{-1} A^T Wb \tag{6}$$

Also, the least squares observation equation for a linear mathematical model is given as,

$$Ax - b = V \tag{7}$$

$$V = Ax - b \tag{8}$$

Where x = column matrix of the unknown parameters [in this case, dX, dY, dZ; Rx, Ry, Rz, K]
b = column matrix of absolute or differences in Cartesian coordinates local Clarke 1880 system
v = column matrix of the residuals

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A = Design coefficient matrix of the unknown parameters in the observation equation. The Molodensky-Badekas model as,

$$\begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & D_X & 0 & -D_Z & D_Y \\ 0 & 1 & 0 & D_Y & D_Z & 0 & -D_X \\ 0 & 0 & 1 & D_Z & -D_Y & D_X & 0 \end{bmatrix} \begin{bmatrix} dX \\ dY \\ dZ \\ K \\ R_X \\ R_Y \\ R_Z \end{bmatrix} - \begin{bmatrix} X_{WGS84} & - & X_{CLK} \\ Y_{WGS84} & - & Y_{CLK} \\ Z_{WGS84} & - & Z_{CLK} \end{bmatrix} \quad (9)$$

Where,

$$\begin{bmatrix} D_X \\ D_Y \\ D_Z \end{bmatrix} = \begin{bmatrix} X_{CLK} & - & X_M \\ Y_{CLK} & - & Y_M \\ Z_{CLK} & - & Z_M \end{bmatrix}$$

Computing the variables and coefficients of the design matrix

Putting the dimension of each matrix in equation 9 yields,

$$X_1 = ({}_mA_n^T\ {}_nW_n\ {}_nA_m)^{-1}\ {}_mA_n^T\ {}_nW_n\ {}_nb_1 \quad (10)$$

In order to extend the matrix to accommodate the number of common points, the number of observation equations has to be determined. Each point provides 3 observation equations; hence the number of observation equations (n) were 300, with seven (7) unknown parameters (dX, dY, dZ; Rx, Ry, Rz and K). From equation 10 the least squares solution for the unknown parameters (x) together with their statistics can be computed. However, the weight (W) in this study is assumed same (unit); hence equation 10 becomes,

$$x = -(A^TA)^{-1}A^Tb \quad (11)$$

Therefore equation 11 becomes the solution vector to the normal equation for the determination of the estimates of the approximate or probable parameters (x). The coordinates of 107 common points in Clarke 1880 and WGS 84 were fully used for an initial quick computation of the mean shift parameters (dX, dY, dZ) in order to ascertain if the exclusion of 7 points from the network would make significant difference in the values of the transformation parameters in comparison with those derived from 100 control points. The main round of computation used 100 common points, while seven (xsw148, xsw117, xsw126, xsw97, xsw82, xsw64, xsw155) were reserved for validation or check computations. All computations were carried out using the MATLAB programming software.

Variance factor and standard deviation

A basic procedure in error analysis is finding the variance factor (σ_0^2) as derived from the adjustment that allows calculation of standard deviation of an observation. The variance of unit weight for weighted observation (A-posteriori) is given as,

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$$\sigma_0^2 = \frac{V^T W V}{n-m} \qquad (12)$$

Where σ_0^2 is the estimated variance factor, n is the number of observation and m is the number of unknown transformation parameters; hence n-m is the degree of freedom, V^TWV is the weighted sum of the residuals. The observations were assumed to carry equal weights (unit weight); hence the sum of the residual shall be without weight matrix. Therefore, the standard deviation of unit weight for the observation is the square root of equation 12, but without the weight element,

$$\sqrt{\sigma_0^2} = \sqrt{\frac{V^T V}{n-m}} \qquad (13)$$

Determination of accuracy by variance-covariance estimation

Variances and covariance of transformation parameters in the adjustment is the basis for calculation of the absolute and relative error and accuracy. Apart from the solution vector x, the matrix $(A^TA)^{-1}$ is of great importance. Now to compute the estimated variances and covariance for the determination of standard deviations of estimated parameters and their residual, the equations 12 and 13 were used respectively. The variance-covariance matrix is the product of the unit variance and the inverse of the normal matrix, given by equation 14,

$$C_X = \sigma_0^2 (A^T A)^{-1} = \sigma_0^2 Q \qquad (14)$$

Therefore, equation 14 is the variance-covariance matrix of the least squares solution in the adjustment computation.

Validation of the transformation parameters (back-substitution)

In this work, the test of the accuracy of the Molodensky Badekas computed transformation parameters for Cross River State was achieved by comparing the observed coordinates of the reserved seven (7) points (xsw148, xsw117, xsw126, xsw97, xsw82, xsw64, xsw155) (Figure 3) with the back computation coordinates using MATLAB programming software.

RESULTS AND DISCUSSION

The geodetic coordinates were converted to Cartesian coordinates as they have the same ellipsoid and the axes are aligned with minor and major axis of the ellipsoid. Heiskanen and Moritz (1967) gave the conversion models as,

$$X = (N + h) \cos\varphi \cos\lambda$$
$$Y = (N + h) \cos\varphi \sin\lambda \qquad (15)$$
$$Z = [N(1 - (2f - f^2)) + h] \sin\varphi$$

Where N is the radius of curvature in the prime vertical given as,

$$N = a(1 - e^2 \sin^2\varphi)^{-1/2} \qquad (16)$$

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The constants a and f are the dimensional parameters of either the regional or geocentric ellipsoids. In local ellipsoids, the parameter h is not known but if Geoid-ellipsoid separation is known along with orthometric height (H), then we can use the relation below to find h, as given by Heiskanen and Moritz (1967).

$$h = H + N \tag{17}$$

Where H = Orthometric height, and N = Geoid – ellipsoid separation. This formula was applied to the data in Microsoft Excel to derive the following set of data where the necessary conversions were made before computation.

Computation of ellipsoidal heights for Minna Datum

In order to derive the ellipsoidal height for the sets of Minna Datum coordinate provided by "source", the undulations of all 107 points were computed using the 5-parameter molodensky standard formula (Agajelu & Moka, 1989),

$$\Delta h = T_x cos\varphi cos\lambda + T_y cos\varphi sin\lambda + T_z sin\varphi - \Delta a\left(\frac{a}{R_N}\right) + \Delta f\left(\frac{b}{a}\right) R_N sin^2\varphi \,. \tag{18}$$

Where,

Φ,λ = WGS84 coordinates of the station
T_x , T_y , T_z = Datum shifts to transform WGS84 datum to Minna datum
Δa, Δf = (Minna minus WGS84) semi-major and flattening respectively
a_{WGS84} = semi-major axis radius of WGS84 ellipsoid = 6378137m
f_{WGS84} = flattening of WGS84 ellipsoid = 1/298.257223563
a_{MINNA} = 6378249.145m
f_{MINNA} = 1/293.465
Δa = 112.145m
$\Delta f*10^4$ = 0.54750714 (Δf = 0.000054750714)
R_N = radius of curvature of the prime vertical
b/a = 1 – f
$e^2 = 2f - f^2$

Since there were no official transformation parameters as at the time, the values adopted for T_x , T_y and T_z were applied at the origin point of Minna datum,

$$T_x = a\delta\varphi_0 sin\varphi_0 cos\lambda_0 + a\delta\lambda_0 cos\lambda_0 sin\varphi_0 - cos\varphi_0 cos\lambda_0(\delta h_0 + \delta a + a\delta f sin^2\varphi_0) \quad (4.3b)$$
$$T_y = a\delta\varphi_0 sin\varphi_0 sin\lambda_0 + a\delta\lambda_0 cos\varphi_0 cos\lambda_0 - cos\varphi_0 sin\lambda_0(\delta h_0 + \delta a + a\delta f sin^2\varphi_0) \quad (4.3c)$$
$$T_z = -a\delta\varphi_0 cos\varphi_0 - sin\varphi_0(\delta h_0 + \delta a + a\delta f sin^2\varphi_0) + 2a\delta f sin\varphi_0 \tag{19}$$

Where,
$\varphi_0\ \lambda_0$ = latitude and longitude respectively of the origin point.
$\delta\varphi_0\ \delta\lambda_0\ \delta h_0$ = differences between Minna datum and WGS84 datum latitudes, longitudes and ellipsoidal heights respectively of the origin point.
a = semi-major radius of the reference ellipsoid used.

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δa, δf = difference between the semi-major radius and flattening respectively of the Clarke 1880 ellipsoid (Minna datum) and the WGS84 ellipsoid. From the computations carried out, the following values were derived for Tx, Ty and Tz: Tx = 93.708m; Ty = 92.626m; Tz = -121.330m. Minna Datum ellipsoidal heights were gotten direct from the WGS84 ellipsoidal heights.

Applying least squares adjustment

The least squares solution of the unknown parameters (seven parameters) including three translational, three rotational and one scale factor or the estimate of the correction to approximate parameter vector (x) is given by $\dot{X} = -(A^TPA)^{-1}A^TPL$.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -(Z-Zb) & (Y-Yb) & (X-Xb) \\ 0 & 1 & 0 & (Z-Zb) & 0 & -(X-Xb) & (Y-Yb) \\ 0 & 0 & 1 & -(Y-Yb) & (X-Xb) & 0 & (Z-Zb) \end{bmatrix}$$

(20)

Where the X,Y,Z values are the Cartesian coordinates ranging from point 1 to 100. The value for Xb, Yb and Zb is derived from the mean of the X,Y, and Z parameters in the local ellipsoid (CLARKE 1880). This was computed using Microsoft Excel and the following values were derived, Xb = 6274890.52, Yb = 942160.5278, Zb = 640909.2331. In this project, the parameters were assumed to be weighted equally. Therefore, the value of P = I; L was developed using the formula below,

$$\begin{bmatrix} X_G - Xb & - & X_L - Xb \\ Y_G - Yb & - & Y_G - Yb \\ Z_G - Zb & - & Z_G - Zb \end{bmatrix} \quad (21)$$

Where,
X_G , Y_G , Z_G = The X, Y and Z data from the global system
X_L , Y_L , Z_L = The X, Y and Z data from the local system
Xb, Yb , Zb = Mean of the X,Y,Z parameters from the local system.
This was computed using MATLAB to derive the 7 datum transformation parameters.

RESULTS

The computed values of datum transformation parameters are presented in Table 2.

Table 2: Datum Transformation Parameters/Std. Deviations for Cross River derived using Molodensky-Badekas model (100 stations)

Parameters	Estimated Values	Approx.	Std Dev.
ΔX(m)	99.344640001658689	99.3446	± 2.433105012
ΔY(m)	14.883071267146580	14.8831	± 2.445608309
ΔZ(m)	-60.279356996844790	-60.2794	± 2.433084462
R_x (°)	-0.000000651559345	-6.51×10^{-7}	±0.000004385
R_y (°)	0.000021346404411	2.13×10^{-5}	± 0.00003947894
R_z (°)	0.000035924462804	3.59×10^{-5}	± 0.000074430417
K (ppm)	-0.000069847626647	6.98×10^{-5}	± 0.000029443607

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The variance covariance matrix (Table 3) was computed using the formula given in equations 12, 13 and 14. The degree of freedom is given by n – m.
Where,
n = number of observations and m = no of parameters
Since n = 100, m = 7
Therefore, df = 93
After computations, it was observed that;
A-posteriori variance scale = 5.9199720579536 53e+004
Using the "*format short*" command, the covariance matrix is given as;

Table 3: Covariance matrix

-5.9200	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.0000
0.0038	-5.9810	0.0000	-0.0000	-0.0000	-0.0000	-0.0000
0.0000	0.0000	-5.9199	0.0000	0.0000	0.0000	0.0000
-0.0000	-0.0000	0.0000	-1.9×10^{-11}	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	0.0000	-0.0000	-1.6×10^{-9}	-0.0000	-0.0000
-0.0000	-0.0001	0.0000	-0.0000	-0.0000	-5.54×10^{-9}	-0.0000
-0.0000	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	-8.67×10^{-10}

The above set of seven parameter optimal transformation parameters for Cross River State were computed using Molodensky-Badekas Model. All computations were carried out using MATLAB. 100 common points in the Clarke 1880 and WGS84 were used for the initial determination of the 3D Datum Transformation parameters for Nigeria. The results of the computation shown above showed an acceptable estimate of the datum shift parameters using the available data (dX, dY, dZ, K, R_X, R_Y, R_Z) and standard deviation of the parameters. The computed residuals were fairly good. The result of the test computation of the shift parameters using the entire 107 points were however not significantly different from those obtained with the 100 points.

Validation of points using the training samples

There was need to validate the parameters obtained from this project by testing them against the seven coordinates which were specifically set aside for confirmation in the Clarke 1880 Ellipsoid. These points were carefully selected in order to spread across the entire state (North, South, West, East and Centre) and they include; xsw148, xsw117, xsw126, xsw97, xsw82, xsw64, xsw155 (Figure 3).

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Figure 3: Validation points

The formula given below was used for this exercise,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} + \begin{bmatrix} 1+\Delta L & R_Z & -R_Y \\ -R_Z & 1+\Delta L & R_X \\ R_Y & -R_X & 1+\Delta L \end{bmatrix} \begin{bmatrix} X' & - & X_M \\ Y' & - & Y_M \\ Z' & - & Z_M \end{bmatrix} \quad (22)$$

Where,
$X_M = {}^{1}/_{n} \sum_{i=1}^{n} X_i,$
$Y_M = {}^{1}/_{n} \sum_{i=1}^{n} Y_i$
$Z_M = {}^{1}/_{n} \sum_{i=1}^{n} Z_i$

Where,
n = number of common points
X_M ,Y_M , Z_M = the mean of the cartesian coordinates of common points in the local datum (Minna)
X,Y,Z = Cartesian coordinates in the global datum
ΔX, ΔY, ΔZ = the translational parameters

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R_X , R_Y , R_Z = the rotation parameter
ΔL = the scale factor
X',Y',Z' = Cartesian coordinates in the local datum
Table 4 is the expected coordinates in WGS 84 while Table 5 is the computed coordinates in WGS 84.

Table 4: Expected coordinates in WGS84

Stn ID	X	Y	Z
xsw148	6258493.911	969338.772	754436.0547
xsw117	6254271.032	1033136.284	703269.6455
xsw126	6273891.225	941839.4031	655283.4031
xsw99	6285872.13	875352.4007	632557.009
xsw82	6272979.892	973430.1241	616972.012
xsw64	6286219.834	923121.6107	556858.7991
xsw155	6288045.065	944929.2052	497179.467

Table 5: Computed coordinates in WGS84

Stn ID	X	Y	Z
xsw148	6258274.547	969076.065	754386.435
xsw117	6254055.828	1032870.413	703228.420
xsw126	6273671.465	941577.586	655250.262
xsw99	6285648.958	875093.640	632527.719
xsw82	6272762.468	973166.709	616945.185
xsw64	6286000.561	922860.385	556842.017
xsw155	6287828.141	944666.833	497172.553

Comparison of the observed and computed WGS84 coordinates for the seven validation points

Tables 6 to 12 expressed the difference between observed and computed WGS84 coordinates.

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Table 6: Difference between observed and computed WGS84 coordinates of Point xsw148 for Point xsw148

Latitude (Degrees)	Longitude (Degrees)	Results
6.8387293	8.80422641	Expected
6.839305608	8.802181377	Computed
-0.000576308	*+0.002045033*	*Difference*

Table 7: Difference between observed and computed WGS84 coordinates of Point xsw117

Latitude (Degrees)	Longitude (Degrees)	Results
6.372982095	9.379922272	Expected
6.373563455	9.377868247	Computed
-0.00058136	*+0.002054025*	*Difference*

Table 8: Difference between observed and computed WGS84 coordinates of Point xsw126

Latitude (Degrees)	Longitude (Degrees)	Results
5.936555594	8.537515166	Expected
5.937145463	8.535471400	Computed
-0.000589869	*+0.002043766*	*Difference*

Table 9: Difference between observed and computed WGS84 coordinates of Point xsw99

Latitude (Degrees)	Longitude (Degrees)	Results
5.729963949	7.927860033	Expected
5.730558281	7.925824104	Computed
-0.000594332	*+0.002035929*	*Difference*

Table 10: Difference between observed and computed WGS84 coordinates of Point xsw82

Latitude (Degrees)	Longitude (Degrees)	Results
5.588355912	8.820706812	Expected
5.588950061	8.818658260	Computed
-0.000594149	*+0.002048552*	*Difference*

Table 11: Difference between observed and computed WGS84 coordinates of Point xsw64

Latitude (Degrees)	Longitude (Degrees)	Results
5.042420957	8.354087155	Expected
5.043024196	8.352043680	Computed
-0.000603239	*+0.002043475*	*Difference*

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Table 12: Difference between observed and computed WGS84 coordinates of Point xsw155

Latitude (Degrees)	Longitude (Degrees)	Results
4.500839505	8.546114313	Expected
4.501450148	8.544066797	Computed
-0.000610643	*+0.002047516*	*Difference*

Judging from the results of the validation which was done by transferring coordinates in the global system (WGS84) to the local system (CLARKE 1880), the transformation model gives a rough estimate based on the available data of the validation points xsw148, xsw117, xsw126, xsw97, xsw82, xsw64, xsw155 in both horizontal and vertical plane and can be accepted for the purpose of this project.

CONCLUSION

This study has estimated the optimal transformation parameters for Cross River State between Minna and WGS84 Datums using Molodensky-Badekas Models. One hundred (100) of the One hundred and seven (107) common point coordinates in Minna and WGS84 Datums provided through secondary means were used for the computations of the parameters. From the results of the validation, the transformation parameter determined in this study defines roughly the spatial locations of the validation points (xsw148, xsw117, xsw126, xsw97, xsw82, xsw64, xsw155) in horizontal plane even though the ellipsoidal heights are in variation to some meters. However, these height disparities are likely to reduce when a local and more precise geoid model is in place for Cross River State; from which subsequent undulations would be derived as inputs for future versions of geodetic datum Transformations. Also, considering the size of the State, more common point's data would perhaps produce better results in the future. Achieving better results in subsequent determination would require many more common points than the present. Also, continuously quality assurances of all geodetic coordinates before and after computations are imperative in order to ensure internal accuracy and homogeneity. Noting that, the control points used for the 7-paramter transformation in this work are acceptable as it did aid the accuracy achieved.

This research makes the flowing recommendations:

i. Efforts should be made by the Office of the Surveyor General of the Federation to ensure that other states in the country develops a suitable model for transformation as this will have increased accuracy than relying on the parameters developed for the entire country
ii. The determination of precise geoid model for Nigeria is imperative.
iii. It is pertinent for other researchers to validate the findings of this study by using other models such as the Bursa Wolf or the Veiss model.

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