International Journal of Interdisciplinary Research Methods
Vol.7, No.3, pp. 31-47, December 2020
Published by ECRTD-UK
Print ISSN: ISSN 2398-712X, Online ISSN: ISSN 2398-7138

CONNECTING DIFFERENT BRANCHES DOMAINS THROUGH MATHEMATICAL MODELLING: AN INTERDISCIPLINARY APPROACH

Dr. Sellamuthu Prabakaran
Pontificia Universidad Javeriana Cali
Department of Accounting & Finance
School of Economics and Business Administration
Calle 18 No. 118 - 250 Cali, Colombia
prabakaran@javerianacali.edu.co

Dr. Animesh Acharjee, PhD
University of Birmingham,Institute of Cancer and Genomic Sciences,
Centre for Computational Biology, UK,
Institute of Translational Medicine,
University Hospitals Birmingham NHS Foundation Trust,
animesh.acharjee@gmail.com

ABSTRACT*:* *In twenty first century, science and technology has get in to an enormous change in terms of different disciplines and inter relationships. Today the boundaries between all disciplines overlap and converge at an accelerating pace. Progress in one area seeds advances in another. Such types of disciplines are called as interdisciplinary science and have a huge potential to solve problems in many disciplines. Although such disciplines came in to literature and practice few decades ago, but still the broad aspect of such fields need to be studied. In this study we tried to show interconnectivity and dependencies among different science like natural science, social science, engineering science etc. and their application simultaneously in finance and plant science through some case studies. The main goal of this study is four fold: 1) First we begin our approach to brief and how make a distinction between disciplinary sciences and interdisciplinary science. 2) Next we extend this approaches and discuss about relation among natural science, social science and engineering science. 3) We introduce the concept through the analogy relation between statistical thermodynamics and economics 4) finally, application of the mathematical description in plant growth and to construct the dynamic models of plant growth.*

KEYWORDS*:* Interdisciplinary sciences, mathematical modelling, drug discovery, statistical mechanics and finance.

INTRODUCTION

In last few decades, new emerging disciplines are trying to make their place to solve real world problems. One observation we can make out easily that the breaking of barriers in traditional sciences and combined approaches to solve a common problem in different disciplines. Such kinds of combinations of branches are called interdisciplinary sciences [1-2]. Although interdisciplinary sciences are more pronounced in the social science context but there are substantial connection among different other domains like biological science, computational science, social science and engineering sciences. In this short communication, we attempt to show connections among different branches of sciences through interdisciplinary approaches.

INTERDISCIPLINARY VS. DISCIPLINARY SCIENCES

It is not the aim of every scientific community to understand each and every factor participating in the interrelated domains. The disciplinary studies are those that take place within the bounds of a single currently recognized academic discipline. The research activity in the disciplinary sciences is orientated towards one specific goal and looking for an answer related to specific problem in a specific research area [3]. Individual sciences select a part of the interrelated matrix of natural happenings as their subject matter.

An interdisciplinary or multidisciplinary science is a ground of study that crosses traditional boundaries with academic disciplines of thought and creates connectivity. This approaches focus on problems thought, too complex or vast for adequate understanding of a single disciple. Logically, interdisciplinary involves aggressively involves a subject from various angles and methods, eventually cutting across disciplines and forming a new method for understanding the subject. The main goal to understand this unites with the various methods and acknowledges a common, shared subject or problem, even if it spreads to other disciplines. In the fig. 1 the relation or association between other branches of sciences and interconnection between them are shown. So, combination of all these ideas gives a platform to solve the intensity of the problem.

Interdisciplinary sciences are distinguished from disciplinary sciences by the source of the derivation of their subject matters. As we have mentioned, the subject matters of disciplinary sciences are derived from the multi-factored field comprising the natural world of nature. However, this is not the case for interdisciplinary sciences. Rather, the subject matters of interdisciplinary sciences are derived from the subject matters of already existing disciplinary sciences.

Similar to disciplinary sciences, interdisciplinary sciences achieve progress by their continued focus on the same subject matter over a period of time. It is only through this prolonged focus that a coherent body of knowledge may develop and evolve. The sustained focus on the same objects of study facilitates the development of a larger set of theories, laws, and more. Interdisciplinary sciences face additional concerns involving the systemization and general strength of the participating disciplinary sciences, as well as the manner in which the participating disciplines are positioned with respect to one another within the interdisciplinary system. While subject matter issues may become extremely complex within disciplinary systems, they are even more so within the context of interdisciplinary systems. The uniqueness of the interdisciplinary subject matter can be compromised in many ways the consideration of both disciplinary and interdisciplinary subject matters. Given this, it is not surprising that a number of missteps might occur along the way toward developing valuable interdisciplinary enterprises.
The concept of interdisciplinary sciences are not only combine or contrast the concepts and methods drawn from the expertise of social, natural and engineering sciences, but also integrate their divergent prospective, even while remaining anchored in their own relevant fields. The scientific and engineering

discipline represents external intimidation to traditional social sciences, like cybernetics applied for the analysis of social phenomenon such as conceptualizing, organization of information processing system or modeling, social interactions by artificial intelligence [4]. Collection and organization of data about the information society may come to be dominated by information scientist and computer scientist rather than sociologist and anthropologist [5].

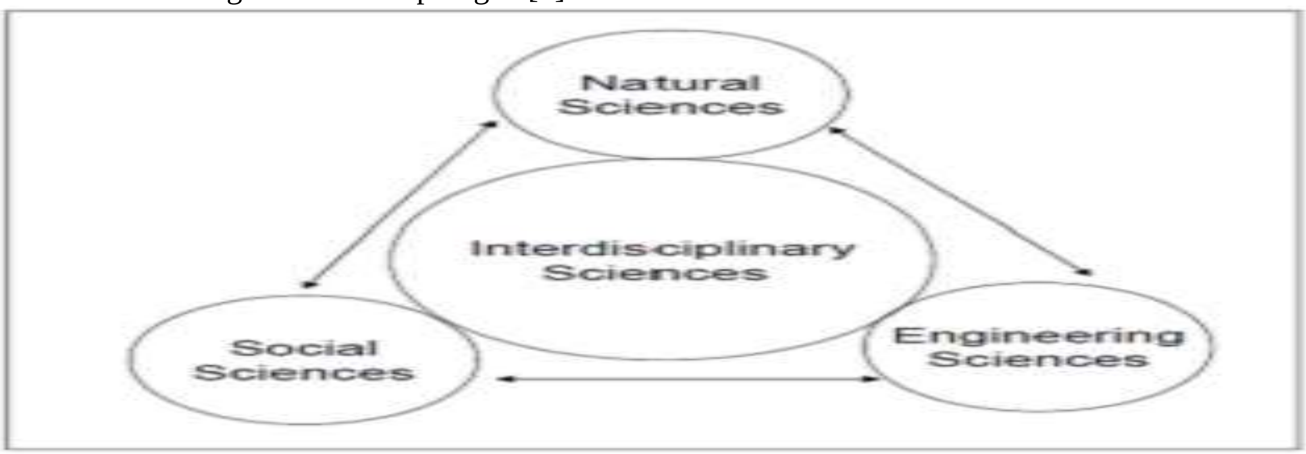

Fig.1 Connectivity between natural sciences, engineering sciences and social sciences. Combination of all makes interdisciplinary science.

The aim of social science information system is to access the data, ethics of social scientist, privacy, freedom of information, legislation and encourage a dialogue between social science and engineering, to provide a background where information, ideas and theory can be shared [6]. The progress in sequencing of human genetics and their comparison methodology develops physical anthropology. Progress in biology, transforms its idea to social sciences and other interdisciplinary sciences, like magnetic resonance energy (MRI) and positron emission tomography (PET) have replaced the traditional methods of research in social sciences, for collecting the information from human behavior to specific brain structure and functions.

The progress in behavioral and social sciences, leads to the progress in the biological sciences by effective treatment of not only mental illness and disorders but also treatment of AIDS and HIV that is spread by drug abuse, decision making and sexual behaviors [7]. The new research tools not only facilitate the scientific progress but also confers the prestige upon biologically oriented studies on social sciences.

CASE STUDY-1: BIOLOGY AND LINKING OTHER DOMAINS

Over the past few years there were tremendous progress in the field of high throughput experimental biology due to technological advancement and decreasing cost of the experimental setups for example the cost of the sequencing technology reduced significantly last five years which provided lots of opportunities for collecting multidimensional data. Such massive data generation shifted the perception of looking at the biologically Other high throughput omics analysis like large scale studies of transcript called as transcriptomic [8-9], all possible metabolites called as metabolomics [10-11] all possible proteins label denoted as proteomics [12], microbiome [13] , methylation data [14] etc. Those data sets provided an opportunity to translate knowledge from basic science into translational aspects using better identification of molecular signature or biomarker [15]

Due to massive amount of data sets computational, statistical and mathematical methods become important part of the high throughput modern biology. More specifically, different approaches like mathematical modeling, bioinformatics, sensor science, data analytics, medical imaging, nanotechnology, spatial coding, cyber infrastructure and communication tool [16] are applied to solve the data related aspects. The predictability of risk of disease in patients and identification of the signatures and then stratify patients group becomes more and more important task. These new technologies and tools provided a view on bottom-up approach on the systems biology from cells, organs and organism physiology. These will improve understanding the etiology of disease and provide clues to cure it. These trends not only facilitate the cause of prevention of chronic diseases but also prevention from AIDS and HIV [17] . The mechanism of actions of disease identification and cure has been explained by Fig 2.

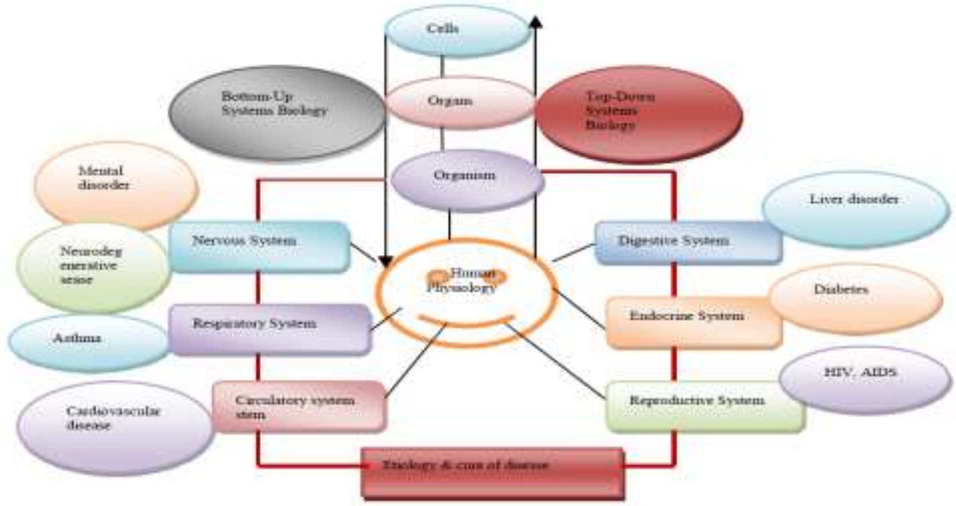

Fig 2: Schematic diagram represent the two approaches of system biology (top-up and bottom-up), starting from cells to human physiology for understanding the etiology and cure of disease.

Although human genome sequencing has been done still diseases like cancer, AIDS, diabetes are pronouncing in every country. The main issues of such types of diseases are less understanding about the complex bimolecular processes. To solve such type of problems one of the approach may be combined knowledge of all the sciences. This leads to the birth of different new disciplines in the biology like systems biology, synthetic biology, data science and engineering biology which are highly interdisciplinary in nature. Some of the aspects of the molecular biology is listed here to justify the importance of the interdisciplinary sciences. For example, protein folding [18] is still an unsolved problem in biology, it deals with the relationship between gene and structure of protein it encodes, how can a linearly arrange sequence of amino acid encode the information for three dimensional structure of protein? It does so in a medium that decodes the information on which amino acid residues can interact closely together or not. Although the concept is very being simple and referred to as a hydrophobic consequence, but the actual dynamics of the system's behavior is spectacularly complex, for eg: Protein, DNA, polysaccharides and membrane are the major biological macromolecules that is composed of monomers and covalently linked together; these molecular entities exhibit larger complexes and linked together by non-covalent interactions. These supra-molecular structures are self-assembly systems which represent the fundamental unit of living system. Biosynthesis of these polymers is the central importance of the life and is responsible for the transformation and accuracy of molecular structures and function that are stored in the genes and transfer from one generation to next generation, based upon the Darwin's theory of evolution. As a result, it creates the complexity in biology to understand how macromolecular structure related to their functions and how the function of individual macromolecules related to working of an organism through the dynamic interactions of genes. The dynamics of gene interaction network biology is much more complex, because biological functions do not exist by the activation of one gene, but they arise by the multiple interactions of the genes, proteins and metabolites. From the available data shows the actively participating genes in biological function and it helped behavior of each gene in the dynamics [19] from robustness [20] to perturbation (information of underlying network topology) with external stimuli. Control of such a network is crucial, and it gives the information, how group of genes control cellular response to external stimuli. Dysfunction of one gene in gene network creates new asymptotic behavior of the system, which leads to the cause of epigenetic diseases, which is caused by environmental influence on the genes, that generate gene alterations in other words it affects the life of healthy individuals, these genes can then be transfer into the future generations as a hereditary disease [21]. This problem is not only limited to biology, to solve these problem it requires a complex system scientist (system biologist).

CASE STUDY -2: ANALOGY BETWEEN STATISTICAL THERMODYNAMICS AND ECONOMICS

The specialty of "physics" is the study of interactions between the various manifestations of matter and its constituents. The development of this subject over the last several centuries has led to a gradual refining of our understanding of natural phenomena. Accompanying this has been a spectacular evolution of sophisticated mathematical tools for the modeling of complex systems. These analytical tools are versatile enough to find application not only in point processes involving particles but also aggregates thereof leading to field theoretic generalizations and condensed matter physics.

The origin of the association between physics and finance, though, can be traced way back to the seminal works [22 & 23], the former being instrumental in establishing empirically that the distribution of wealth in several nations follows a power law with an exponent of, while the latter pioneered the modeling of speculative prices by the random walk and Brownian motion. The cardinal contribution of physicists to the world of finance came from Fischer Black & Myron Scholes through the option pricing formula which bears their epitaph and which won them the Nobel Prize for economics in 1997 together with Robert Merton. They obtained closed form expressions for the pricing of financial derivatives by converting the problem to a heat equation and then solving it for specific boundary conditions.

The description of statistical physics approaches to economics possibilities new insights into problems traditionally not associated with physics [21]. Both statistical mechanics and economics study big groups: collections of atoms or economic agents, respectively.
The fundamental law of equilibrium statistical mechanics is the Boltzmann-Gibbs law, which states that the probability distribution of energy ε is

International Journal of Interdisciplinary Research Methods
Vol.7, No.3, pp. 31-47, December 2020
Published by ECRTD-UK
Print ISSN: ISSN 2398-712X, Online ISSN: ISSN 2398-7138

$$P(\varepsilon) = \frac{C}{e^{\varepsilon/T}} \tag{1}$$

Where T is the temperature and C is a normalizing constant [24].
The main ingredient that is essential for the textbook derivation of the Boltzmann- Gibbs law [24] is the conservation of energy [25]. Thus, one may generalize that any conserved quantity in a big statistical system should have an exponential probability distribution in equilibrium.
We claim that, in a closed economic system, the total amount of money is conserved. Thus the equilibrium probability distribution of money $P(m)$ should follow the Boltzmann-Gibbs law $P(m) = \frac{C}{e^{m/T}}$. Here m is money, and T is an effective temperature equal to the average amount of money per economic agent. The conservation law of money [26] reflects their fundamental property that, unlike material wealth, money (more precisely the fiat, "paper" money) is not allowed to be manufactured by regular economic agents, but can only be transferred between agents. Our approach here is very similar to that of Ispolatov *et al*. [27]. However, they considered only models with broken time-reversal symmetry, for which the Boltzmann-Gibbs law typically does not hold. It is tempting to identify the money distribution $P(m)$ with the distribution of wealth [27]. However, money is only one part of wealth, the other part being material wealth. Material products have no conservation law: They can be manufactured, destroyed, consumed, etc. Moreover, the monetary value of a material product (the price) is not constant. The same applies to stocks, which economics textbooks explicitly exclude from the dentition of money [28]. So, in general, we do not expect the Boltzmann- Gibbs law for the distribution of wealth. Some authors believe that wealth is distributed according to a power law (Pareto-Zipf), which originates from a multiplicative random process [29]. Such a process may reflect, among other things, the fluctuations of prices needed to evaluate the monetary value of material wealth.

Examples of economic systems of interest are an individual consumer or a small country, each of which is embedded within a larger economic system. Consider an individual consumer. A fundamental assumption in economics is that the consumer employs a *utility function* U to choose to purchase one good over another. For many purposes, it is sufficient for the utility to be an ordinal quantity (that is, it specifies only relative ordering). However, to make the full analogy to thermodynamics, we must take the utility U to be a real number. We assume U to be given in a convenient set of units, such as dollars, and we also assume that U is measurable [30]. The formalism we develop is falsifiable, and can be over determined by a proper set of measurements, thus providing constraints on its consistency.
To explain the analogy, we begin by discussing certain fundamental relations in economics.
First consider the Measurable Economic Quantity Known as Wealth,

$$W = \lambda M + pN \text{ (Economics)} \tag{2}$$

Where λ and M represents the value and amount of money, and p and N represents vectors of prices and numbers of goods.
Economics assumes that the value of an individual consumer's money and goods is summarized by the value of U, which typically exceeds W. The excess is known as the surplus for which we introduce the notation Ψ (p surplus) [31 & 32]. Thus

$$\Psi = U - W \text{ (Economics)} \tag{3}$$

International Journal of Interdisciplinary Research Methods
Vol.7, No.3, pp. 31-47, December 2020
Published by ECRTD-UK
Print ISSN: ISSN 2398-712X, Online ISSN: ISSN 2398-7138

In a primitive or very poor economy, there is no surplus, so $\Psi = 0$. In this case, every individual performs the same economic function at the same efficiency, and there is no benefit from specialization and trade. The surplus Ψ cannot be negative; for typical economic systems $\Psi > 0$. Although Equation (3) appears only to define another unknown quantity, Ψ, in terms of U, this economic relationship is useful because it has a thermodynamic analogue.

The Helmholtz free energy of a system with N identical particles is defined as

$$F = -PV + \mu N \text{ (Thermodynamics)} \quad (4)$$

Where P is the pressure, V is the volume, and μ is the chemical potential of the particles. We may think of $-PV$ as analogous to λM. The quantity in thermodynamics analogous to the price p is the chemical potential μ. The energy E is related to F in terms of the temperature T and entropy S via

$$TS = E - F \text{ (Thermodynamics)} \quad (5)$$

Note that, according to the third law of thermodynamics, $S = 0$ for a system at $T = 0$

A comparison of Equation (3) and (5) suggests another analogy, that of Ψ and TS. By taking a system with zero surpluses (and thus zero economic temperature) to have zero economic entropy, we assume the economic analogue of the third law of thermodynamics. Here we summarize the thermodynamic and economic analogies in Table 1.

THERMODYNAMICS	$-F$	$-E$	TS	μ	N
ECONOMICS	$W_{(Wealth)}$	$U_{(Utility)}$	$\Psi_{(Surplus)}$	$P_{(\text{Price})}$	$N_{(\#of Goods)}$

Table 1: Summary Of The Suggested Analogies Between Thermodynamic And Economic Systems

Our goal in making an analogy between economics and thermodynamics is to provide a theoretical framework so that economics measurements can determine the functional dependence of the utility U on the economic parameters that specify the state of an economic system. A knowledge of the state function as a function of the appropriate economic parameters completely characterizes the economic system.

From the economic relations we introduce

$$\Psi = TS \quad \text{and} \quad (6)$$

$$U = TS + W = TS + \lambda M + pN \quad (7)$$

For E suggests that, from the point of view of its natural set of variables, we have

$$U = U(S, M, N) \quad (8)$$

Relation (8) is our fundamental assumption.

The economic equivalent is

$$dU + TdS + \lambda dM + pdN \quad (9)$$

Where $T = \left(\frac{\partial U}{\partial S}\right)_{M,N}, \quad \lambda = \left(\frac{\partial U}{\partial M}\right)_{S,N}, \quad p = \left(\frac{\partial U}{\partial N}\right)_{S,M}$ (10)

Let us now apply this theoretical structure

In the Wealth of Nations, Adam Smith distinguishes between two measures of utility [33]. One measure is the ‘‘value in exchange.’’ In economics it is conventional to identify the value in exchange with the price p. From (9), we take this measure to be the marginal utility per good dU/dN at fixed S and M

International Journal of Interdisciplinary Research Methods
Vol.7, No.3, pp. 31-47, December 2020
Published by ECRTD-UK
Print ISSN: ISSN 2398-712X, Online ISSN: ISSN 2398-7138

. Another measure is the ‘‘value in use,’’ which is less readily identified. We will identify ‘‘value in use’’ with the marginal utility per good dU/dN for another set of fixed variables. For simplicity, we will take M to be fixed, but we cannot be explicit about the second variable that is to be held fixed, and will simply denote it as x.
From Eq. (9) we then have

$$\left(\frac{\partial U}{\partial N}\right)_{x,M} = T\left(\frac{\partial S}{\partial N}\right)_{x,M} + p \qquad (11)$$

For fixed market values of goods, U be maximized for each good. Fixed market value means that the goods 1 and 2 are exchanged in the marketplace subject to the condition (9)

$$0 = p_1 dN_1 + p_2 dN_2 \qquad (12)$$

The maximization of U require that

$$0 = \left(\frac{\partial U}{\partial N_1}\right) dN_1 + \left(\frac{\partial U}{\partial N_2}\right) dN_2 \qquad (13)$$

Combining equation (12) and (13) then yields

$$\frac{1}{p}\left(\frac{\partial U}{\partial N}\right) = Cons\tan t \qquad (14)$$

for each good. Thus, as desired, the ratio of value in use to price is a constant.
Using equation (11) and (14) can be expressed as

$$\frac{1}{p}\left(\frac{\partial U}{\partial N}\right)_{x,M} = \frac{T}{p}\left(\frac{\partial S}{\partial N}\right)_{x,M} + 1 = Cons\tan t \qquad (15)$$

From Equation (15) the constancy of this ratio for all goods does not depend on whether the price is included in the computation of utility. The present formalism helps us focus on the issue of ‘‘what is held constant.’’ Let m represent the value in use (marginal utility per good, at fixed x and M):

$$m = \left(\frac{\partial U}{\partial N}\right)_{x,M} \qquad (16)$$

Note that m is specified in monetary units, from equation (14) and (15), the ratio m/p has the same dimensionless value for all goods.
The ratio m/p can be generalized to include the value of currency, thus permitting the study of saving. Specifically, define

$$m_\lambda = \left(\frac{\partial U}{\partial M}\right)_{x,N} \qquad (17)$$

Then the ratio of value in use to value in exchange for money, m_λ/λ, takes on the same value as m/p for goods.
If we use equation (7) to relate W and U, the analogy to the development associated with F leads to

$$dW = -SdT + \lambda dM + pdN \qquad (18)$$

Where $S = -\frac{\partial W}{\partial T}, \quad \lambda = \frac{\partial W}{\partial M}, \quad p = \frac{\partial W}{\partial N}$ (19)

It is implicit that two of the three quantities (T, M, N) are held constant in the partial derivatives. From Equation (18) we may write the functional dependence

$$W = W(T, M, N) \tag{20}$$

CASE STUDY-3: DYNAMICAL MODELS FOR PLANTS

Plants are very complex systems. If agronomic plants (rice, maize or corn) are essential to provide food or other kind of goods, trees are also essential to preserve the carbon balance, or even to absorb carbon surplus. Despite the great importance of plants, only a small number of modelers, and applied mathematicians are involved in the modelling, the development of mathematical tools, the simulation of plant growth, and, in general, in problems related to Agronomy or Forestry. In fact, the amount of knowledge necessary to understand how a plant is growing is huge and only a multidisciplinary approach can be used to overcome the encountered difficulties.

Growth is regarded as one of the most fundamental and conspicuous characteristics of a living being. What is growth? Growth can be defined as an irreversible permanent increase in size of an organ or its parts or even of an individual cell. Generally, growth is accompanied by metabolic processes (both anabolic and catabolic), that occur at the expense of energy.
Look at Figure 3. On plotting the length of the organ against time, a linear curve is obtained.

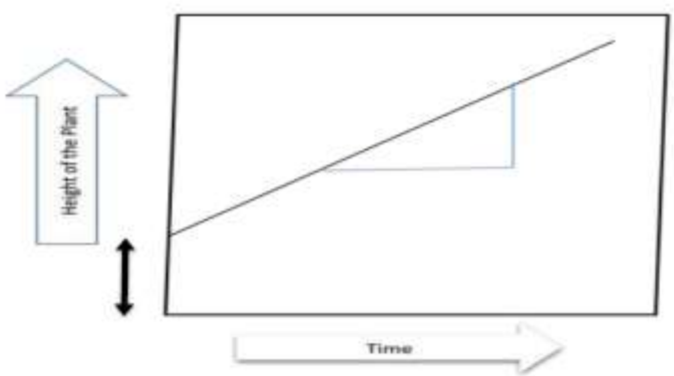

Fig.3 - Constant linear growth, a plot of length L against time t

Mathematically, it is expressed as

$$L_t = L_0 + rt \tag{21}$$

Where

$L_t =$ Length at time t

$L_t =$ Length at time Zero

$r =$ Growth/Elongation per unit time

Let us now see what happens in geometrical growth. In most systems, the initial growth is slow (lag phase), and it increases rapidly thereafter – at an exponential rate (log or exponential phase). Here, both the progeny cells following mitotic cell division retain the ability to divide and continue to do so. However, with limited nutrient supply, the growth slows down leading to a stationary phase. If we plot the parameter of growth against time, we get a typical sigmoid or S-curve (Figure 4).

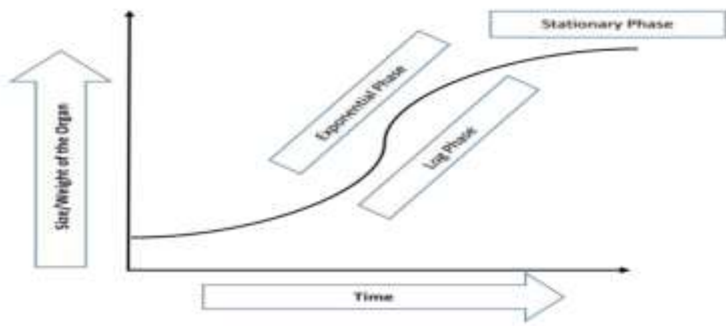

Fig. 4 - Graphical representation of ´Grand period of growth´ (sigmoid curve) A sigmoid curve is a characteristic of living organism growing in a natural environment. It is typical for all cells, tissues and organs of a plant.

The exponential growth can be expressed as $W_1 = W_0 e^{rt}$ (22)

Where

$W_1 =$ Final Size (Weight, Height, Number and etc.)

$W_0 =$ Initial Size at the beginning of the period

$r =$ Growth Rate

$t =$ Time of Growth

$e =$ Base of Natural Logarithms

Here, r is the relative growth rate and is also the measure of the ability of the plant to produce new plant material, referred to as efficiency index. Hence, the final size of W_1 depends on the initial size, W_0.

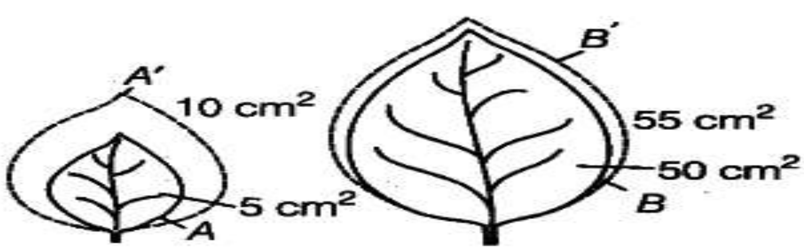

Fig. 5 - Diagrammatic comparison of absolute and relative growth rates. Both leaves A and B have increased their area by 5 cm2 in a given time to produce A1, B1 leaves.
Source:http://www.yourarticlelibrary.com/biology/plants/plants-growth-and-development-explained-with-diagram/29741

Quantitative comparisons between the growth of living system can also be made in two ways: (i) measurement and the comparison of total growth per unit time is called the absolute growth rate. (ii) The growth of the given system per unit time expressed on a common basis, e.g., per unit initial parameter is called the relative growth rate. In Figure 5 two leaves, A and B, are drawn that are of different sizes but shows absolute increase in area in the given time to give leaves, A1 and B1. However, one of them shows much higher relative growth rate.

Studies of plant growth probably begin from ancient times. In the middle ages, Leonardo da Vinci observed the seasonal periodicity of growth and some features of plant forms [34]. Theories of phyllotaxis, which can be defined as a construction determined by organs, parts of organs, or primordia of plants" [35] appear already in the 18th century. D'Arcy Thompson reviewed early theories [34] and contemporary theories of phyllotaxis [35] . Phyllotaxis often considers a plant as a given geometrical object without specifying the underlying biological mechanisms that result in the appearance of the observed patterns. There are several approaches in which plants are considered as dynamic objects, which change their size and form over time based on some growth mechanism. The best-known mechanism of pattern formation in mathematical biology is related to reaction - diffusion systems and Turing structures [36 & 37]. Another approach to plant modelling is based on attempt to describe the kinetic of plant growth.

Let $L(t)$ is the plant size that depends on time t, and then we can consider the empirical equation

$$\frac{dL}{dt} = F(L) \qquad (23)$$

where F can be proportional to L (autocatalytic growth), or be some constant (linear growth), or

$$F(L) = aL(L_0 - L) \qquad (24)$$

Where a and L_0 are parameters [38 & 39]. Such kinetic equations have been proposed since the early 20th century [34] no significant progress since then. It is interesting to note that D'Arcy Thompson discusses autocatalytic growth in relation to chemical kinetics and plant hormones.

In this part, we model the dynamics of growing plants, i.e., the evolution of their size in time and the emergence of their forms. To suggest a mathematical model of plant growth, we need to identify the most essential features of the growth mechanism. The simplest and schematic description takes into account nutrients coming from the root along the xylem, metabolites produced by the plant and distributed throughout the plant through the phloem, and plant cells that proliferate and consume nutrients and metabolite.

The growing plant is represented as an interval with its left end and point fixed at $x = 0$ and its right end point at $x = L(t)$. The length $L(t)$ is a function of time. Nutrients enter through $x = 0$ and are transported through the interval by convective and diffusive fluxes. The speed of growth $V(t) = L'(t)$ depends on the concentration C of nutrients and on the concentration $R(t)$ of the Granulocyte-macrophage (GM) factor at $x = L(t)$. The production of the GM - factor is described by the equation

$$\frac{dR}{dt} = Cg(R) - \sigma R \qquad (25)$$

International Journal of Interdisciplinary Research Methods
Vol.7, No.3, pp. 31-47, December 2020
Published by ECRTD-UK
Print ISSN: ISSN 2398-712X, Online ISSN: ISSN 2398-7138

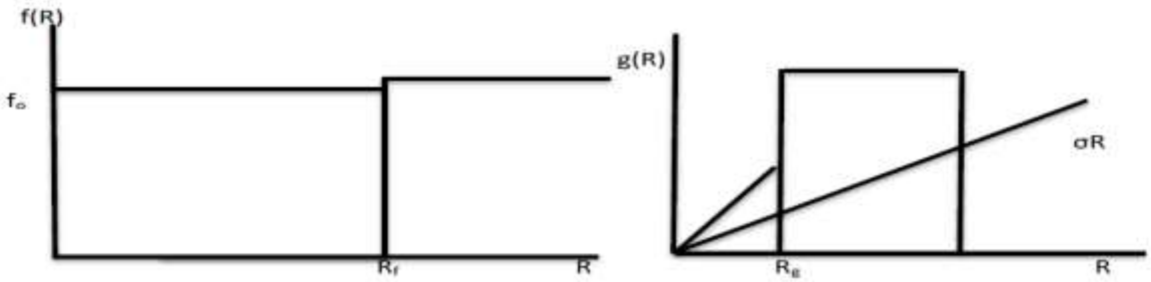

Fig. 6 - Functions f and g.

The typical form of the function $g(R)$ is shown in Figure 6 though we often use a smooth function, σ is a parameter. Its first derivative increases at some interval of R. This allows us to describe an auto - catalytic production of the GM-factor. The second term in the right-hand side of this equation describes consumption or destruction of the factor. $g'(0) > \sigma$

Another essential property of the function $g(R)$ is related to the value of its derivative at $R = 0$ Assuming that the dimensionless concentration C changes between 0 and 1 with $C = 1$ at $x = 1$, we choose $g'(0)$ slightly greater than σ. Therefore, if the concentration C of nutrients at the growing end is small, then the GM-factor will not be produced. Moreover, its concentration will be decreasing. If C is close to its maximal value, then the right-hand side in (5) becomes positive, and the concentration of the GM-factor will grow.

The growth rate V is considered as a given function of the GM-factor, $V = f(R)$. For simplicity, we suppose that it is zero for $R \le R_0$ and equals some positive constant for $R \ge R_1 \ge R_0$. Thus, the rate of plant growth equals zero for small concentrations of the GM-factor, and some positive constant for large concentrations.

We consider in this section the one-dimensional case justified if the length (or height) L of the plant is essentially greater than the diameter of its trunk. Hence, we consider the interval $0 \le x \le L(t)$ with the length depending on time. The left endpoint $x = 0$ corresponds to the root. Its role is to provide the flux of nutrients taken into account through the boundary condition. We do not model the root growth here. Therefore, the left boundary is fixed. The right end point, $x = L(t)$ corresponds to the apex. Its width is much less than that of the plant. We suppose in the model that it is a mathematical point. The value $L(t)$ increases over time. According to the assumption above, the growth rate is determined by the concentration of metabolites at $x = L(t)$, which we denote by R. Thus

$$\frac{dL}{dt} = f(R) \qquad (26)$$

The function $f(R)$ will be specified below.

We recall that the interval $0 < x < L(t)$ corresponds to differentiated cells that conduct nutrients from the root to the apex. We suppose that they are in a liquid solution. Denote by C their concentration, which is a function of x and t. Its evolution is described by the diffusion - advection equation

$$\frac{\partial C}{\partial t} = u\frac{\partial C}{\partial x} = d\frac{\partial^2 C}{\partial x^2} \quad (27)$$

Here u is the velocity of the fluid, and d is the diffusion coefficient. Assuming that the fluid is incompressible and fills the xylem uniformly (the part of the plant tissue conducting nutrients from below to above and located inside the cambium layer), we obtain

$$u = \frac{dL}{dt} \quad (28)$$

We complete equation (7) by setting the boundary conditions

$$x = 0: \quad C = 1, \quad x = L(t): \quad d\frac{\partial C}{\partial x} = -g(R)C \quad (29)$$

The second boundary condition shows that the flux of nutrients from the main body of the plant to the meristem is proportional to the concentration $C(L,t)$. This is a conventional relation for mass exchange at the boundary, Robin boundary conditions. The factor $g(R)$ shows that this flux can be regulated by proliferating cells. We discuss this assumption as well as the form of the function $g(R)$ below.

We now derive the equation describing the evolution of R. At this point, we need to return to the model in which the width of the meristem is finite. We denote it by h. Then we have

$$h\frac{dR}{dt} = g(R)C = \sigma R \quad (30)$$

The first term in the right-hand side of this equation describes production of the GM-factor R in the meristem. The second term corresponds to its consumption. System of equations (26 – 29) is a generic one-dimensional model of plant growth based on:

a. Continuous medium" assumptions of mass conservation (for $C + R$) and of the proportionality of the flux $\partial C / \partial x$ at the boundary to the value of C; and

b. A "biological" assumption that there is a chemical species R, the plant growth and mitosis factor, which is produced in the meristem and which determines the plant growth.

We note that the conservation of mass in the case $\sigma = 0$ implies that the term $g(R)C$ enters both the boundary condition and equation (30). Therefore, the assumption that the rate of the plant growth factor production depends on its concentration R makes the boundary condition depend on it also. We will see below that properties of the function g can be crucial for plant growth. In particular, if it is constant (the production rate is not auto - catalytic), we will not be able to describe the essential difference in plant sizes.

We now specify the form of the functions f and g. We will consider f as a piecewise constant function equal to 0 if R is less than a critical value R_f and equal to some positive constant f_0 if R is greater than R_f (Figure 1a). This means that the growth begins if the concentration of the plant growth factor exceeds some critical value. The production of the growth factor R is assumed to be auto - catalytic. To simplify the modelling, we consider a piecewise linear function $g(R)$ (Figure 1b). In some cases, we also consider smooth functions f and g.

Since $f(R)=0$ in this case, we obtained from equation (36) $C(x)=1-\frac{1-C(L)}{L}$. Then from equation (38 & 39) $C(L)=1-\frac{1-\sigma L}{d}R$. Finally, from equation (39)

$$\frac{\sigma R}{1-\frac{\sigma L}{d}R}=g(R) \tag{31}$$

This equation should be completed by the condition

$$R<R_f \tag{32}$$

Such that $L'(t)=0$.

We assume in what follows that $\sigma<g'(0)$. Then for all L sufficiently large, there exists a solution R of equation (31) with condition (32). Depending on the function $g(R)$, there can exist more than one solution with the same value of L. Denote by $F(R)$ the left-hand side in (31). The standard linear stability analysis shows that the stationary solution is stable if $F'(R)>g'(R)$ for a solution R.

DISCUSSION

In this piece of work, we tried to show how different disciplines are connected or can be connected through different branches of science and technology. We showed modern biology that is high through put in nature is heavily depends upon computation and mathematical modelling and add value to the knowledge and complement different domain. Also from other case studies like thermodynamics [40] that represents an abstract and generalized approach that enables one to analyze the basic regularities of various energy processes, even under conditions where the details of their intrinsic mechanisms are unknown. The methods of thermodynamics are applicable to systems that belong to very diverse classes of objects from starts to living cells with several concepts of fundamental physics like quantum mechanics, field theory and related tools of non-commutative probability , gauge theory , path integral etc. being applied for pricing of contemporary financial products and for explaining various phenomena of financial markets like stock price patterns, critical crashes etc. [41]. Phynance [42] has often been apparent as a field which integrates both finance and physics aspects for the resolve of simplifying proper understanding promotion concerning various economic facets.

Another example in physics, finance and economics where interconnection or linking is driven by mathematical modelling of the systems and hence knowledge transfer among disciplines are feasible. In addition to that, from all of the above it can be seen that there exists a significant body of opinion that acknowledges that analogies or isomorphic links between the disciplines of thermodynamics and economics can be observed.

From the other case study on plant modelling again Mathematics is acting like a communication tool and linking two diverse domains through modelling and allows us to pronounce the performance of a phenomenon or system in the real world, particular in Plant Science.

CONCLUSIONS

In conclusions, we can say that a mathematical model can be used to solve real world situations is usually called the mathematical modeling process. In addition, we set out to demonstrate and discuss how the mathematical modelling could be applying to aid our understanding of plant growth. The process by which the application of mathematical models to plant phenotyping and how the equations are derived and assembled in plant science.

Such interdisciplinary approaches also made an impact on research funding bodies. A recent article by [43] shows the values of the interdisciplinary article.

However, such approaches are not free from issues for example one potential problem is to break the barrier among different sciences. People working in the same domain for decades tend to believe in their domain more compared to the complementary knowledge coming from other domains. Such barriers are observed in general in the interdisciplinary science. One way to break such barriers would be to involve customized training and education and regular communication among them. Communication on knowledge transfer and common language using mathematical science would be very much important and will smooth the process. Given the technological growth in the multiple domains like telecommunication, social networks, digital health care etc. there will be great demand for interdisciplinary science and hence to meet this demand there will be need for appropriate skill sets in future.

REFERENCES

1. Pedersen, D. B. (2016). Integrating social sciences and humanities in interdisciplinary research. Palgrave Communications, 2(16036). https://doi.org/10.1057/palcomms.2016.36
2. Sandro Serpa, Carlos Miguel Ferreira, Ana Isabel Santos, Fostering Interdisciplinary: Implications for Social Sciences, International Journal of Social Science Studies Vol. 5, No. 12; December 2017, ISSN 2324-8033
3. Tress B, Tress G, Fry G (2006) Defining concepts and process of knowledge production in integrative research. In: Tress B. Tress G. Fry G, and Opdam P (eds.), From landscape research to landscape planning: aspects of integration, education and application. Springer; The Netherlands, PP 16-296.
4. Carley K (1998) A theory of group stability; American Sociological Review; 56:331–54.
5. Bainbridge W S (2003) The future in the social sciences. Futures 35: 633–650.
6. Wilson T D (1980) On information science and social science. Social Science information Studies, 1: 5- 12.
7. Patricia J. Rogers (2008) Using Programme Theory to Evaluate Complicated and Complex Aspects of Interventions, SAGE Publications (Los Angeles, London, New Delhi and Singapore) DOI: 10.1177/1356389007084674, Vol 14(1): 29 – 48

8. Brazma, A., Vilo, J., 2000. Gene expression data analysis. FEBS Lett. 480, 17–24.
9. Gaasterland, T., Bekiranov, S., 2000. Making the most of microarray data. Nat. Genet. 24, 204–206. https://doi.org/10.1038/73392
10. Acharjee, A., Prentice, P., Acerini, C., Smith, J., Hughes, I.A., Ong, K., Griffin, J.L., Dunger, D., Koulman, A., 2017b. The translation of lipid profiles to nutritional biomarkers in the study of infant metabolism. Metabolomics Off. J. Metabolomic Soc. 13, 25.
11. Clish, C.B., 2015. Metabolomics: an emerging but powerful tool for precision medicine. Cold Spring Harb. Mol. Case Stud. 1, a000588. https://doi.org/10.1101/mcs.a00058
12. Meng, C., Bai, C., Brown, T.D., Hood, L., Tian, Q., 2018. Human Gut Microbiota and Gastrointestinal Cancer. Genomics Proteomics Bioinformatics. https://doi.org/10.1016/j.gpb.2017.06.002
13. Zhang, W., Feng, H., Wu, H., Zheng, X., 2017. Accounting for tumor purity improves cancer subtype classification from DNA methylation data. Bioinforma. Oxf. Engl. 33, 2651–2657. Strimbu, K., Tavel, J.A., 2010. What are biomarkers? Curr. Opin. HIV AIDS 5, 463–466.
14. Glass TA, McAtee MJ (2006) Behavioral science at the crossroads in public health: extending horizons, envisioning the future. Soc Sci Med; 62: 1650–71.
15. Butler A C, Chapman J E, Forman E M, Beck A T (2006) The empirical status of cognitive-behavioral therapy: a review of meta-analyses. Clin Psychol Rev 26:17–31.
16. Dill KA, Ozkan SB, Weikl TR, Chodera JD, Voelz VA (2007) The protein folding problem: when will it be solved? Current Opinion in Structural Biology, 17:1–5.
17. Chaos A, Aldana M, Espinosa-Soto C, Leon B G P D, Arroyo A G, Alvarez- Buylla E R (2006). From Genes to Flower Patterns and Evolution: Dynamic Models of Gene Regulatory Networks, J Plant Growth Regul 25:278–289.
18. Kitano H (2002) Systems biology: a brief overview. Science 295, 5560, 1662-1664.
19. Hood L(2009) Systems biology and the “new biology”. FALL volume III.
20. J.D. Farmer (1999) Computing in Science and Engineering 1, 26; R.N. Mantegna, H.E. Stanley, An Introduction to Econophysics (Cambridge University Press, Cambridge, (2000).
21. V. Pareto, Cours d’Economie Politique (Lausannes and Paris),(1897);
22. L. Batchlier, Annelas Scientifiques de l’Normal Superieure III-17 21-86,(1900); P. Cootner, The Random Character of Stock Market Prices (Cambridge, MA: MIT Press) Reprint ,(1964);
23. G.H. Wannier (1987) Statistical Physics Dover, New York.
24. C. Tsallis (1999) J. Stat. Phys. 52, 479 (1988); Braz. J. Phys. 29, 1.
25. M. Shubik (1997) in The Economy as an Evolving Complex System II, edited by W.B. Arthur, S.N. Durlauf, D.A. Lane (Addison-Wesley, Reading,, p. 263.
26. S. Ispolatov, P. L. Krapivsky, and S. Redner (1998) Wealth distribution in asset exchange models, Eur. Phys. J. B 2, 267.
27. C.R. McConnell, S.L. Brue (1996) Economics: Principles, Problems, and Policies (McGraw-Hill, New York,.
28. E.W. Montroll, M.F. Shlesinger (1982) Proc. Natl. Acad. Sci. USA 79, 3380; O. Malcai, O. Biham, S. Solomon (1999) Phys. Rev. E 60, 1299; D. Sornette, R. Cont (1997) J. Phys. I France 7, 431; J.-P. Bouchaud, M. Mezard, cond-mat/0002374.
29. P. Samuelson, Economics (McGraw–Hill, New York, 1970), 8th ed., p. 410.
30. E. Silberberg, The Structure of Economics: A Mathematical Analysis (McGraw–Hill, New York, 1990), p. 396.

31. E. K. Browning and J. M. Browning, Microeconomic Theory and Applications (HarperCollins, New York, 1992), 4th ed., p. 87.
32. Adam Smith (1776) An Inquiry into the Nature and Causes of the Wealth of Nations, published in Dublin, printed for Messrs. Whitestone and others.
33. D'Arcy Thompson (1992) On growth and forms. The complete revised edition. Dover, New York.
34. R. V. Jean. Phyllotaxis (1994) A systematic study in plant morphogenesis. Cambridge University Press, New York.
35. H. Meinhardt (2003) The algorithmic beauty of sea shells. Springer.
36. J. Murray. Mathematical biology. Introduction 3rd ed, Springer 2001, 575p.
37. R. Heller, R. Esnault, C. Lance (1995) Physologie vegetale. 2. Developpement. 6 - eme edition, Dunod, Paris.
38. P. Mazliak (1998) Physiologie vegetale. II. Croissance et developement. Hermann, Paris.
39. ÇENGEL, YUNUS A., and Michael A. Boles. "6-7." Thermodynamics: An Engineering Approach. 7th ed. New York: McGraw-Hill, 2011. 299. Print.
40. B. Baaquie et al, Quantum Mechanics, Path Integration and Option Pricing: Reducing the Complexity of Finance, arXiv:cond-mat/0208191v2, 11 Aug 2002;
41. Roehner, B. (2002). Patterns of Speculation - A Study in Observational Econophysics, Cambridge: Cambridge University Press.
42. Alberico L Catapano et al, (2016) ESC/EAS Guidelines for the Management of Dyslipidaemias, European Heart Journal, Volume 37, Issue 39, 14, Pages 2999–3058, https://doi.org/10.1093/eurheartj/ehw272.