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Comparing Methods of Estimation for Two-Parameter Gamma Distribution Using Rainfall Patterns in Nigeria

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ABSTRACT: *In other to improve the ability of decision-makers to prepare for and deal with the unforeseen circumstances resulting from climate change as consequences of precipitation fluctuations, extreme and torrential rainfall. It is important to provide a more complete understanding of the range and likelihood of rainfall patterns a location could receive using a probabilistic model whose parameters might complement or even replace such common measures as the mean, median, variance, minimum, maximum and quartile values as major descriptors of rainfall at such location. Daily precipitation totals can be approximated by the gamma distribution as it is bounded on the left at zero and positively skewed indicating an extended tail to the right which suit the distribution of daily rainfall and accommodate the lower limit of zero which constrains rainfall values. This paper presents the comparison between Maximum Likelihood Estimation (MLE) of closed & open form solutions and Method of Moment Estimation (MME) of location and scaling parameters of the two-parameter gamma distribution, the parameters were estimated using MME and MLE with their performance adjudged and the result obtained showed that the closed-form solution of the MLE outperformed the open form solution and MME by comparing their estimates for the scaling parameter.*

KEY WORDS. Generalized Gamma Distribution, Maximum Likelihood, Closed-Form Solution Open Form Solution, Method of Moments, Positively Skewed, Rainfall Patterns

INTRODUCTION

Climate change seems to be the foremost global challenge facing the human race at this present moment as extreme and torrential rainfall is a climate parameter that affected the way and manner man lives [3]. Rainfall is an important element of the climate that is beneficial to the earth and the amount of rainfall received over an area is also a factor for accessing availability of water to meet various demands and needs of agriculture, industry and other human activities in such an environment. Therefore, the need to study the variability of rainfall pattern cannot be overemphasized due to its effect on the ecological system or ecosystem [18] which necessitated many researchers to carry out studies on the said subject which revealed that the changing pattern of the annual rainy season and monthly rainfalls indicate a long run of dry years for Saharan West Africa [11]; declining rainfall pattern [1, 2]; fluctuations in the most month within the decades as rainfall variability continues to be on the increase.

Modelling rainfall patterns is directly proportional to the suitability of rainfall data on a particular distribution suggested through Exploratory Data Analysis (EDA) and estimating parameters of such corresponding expected distribution is based on probability models as it could be positively or negatively skewed. A good model can be used to forecast rainfall density for several periods ahead as the case of the gamma distribution is often used to model the distribution of wet-day rainfall amount because it is distinctly skewed to the right which suits the distribution of daily rainfall and accommodates the lower limit of zero that constrains rainfall values [14].

Establishing a probability distribution that provides a good fit for rainfall has long been a topic of interest, the stochastic analysis of rainfall reveals that log Pearson III distribution suit the maximum daily rainfall data [19] whereas gamma and log normal distribution fit well with the probability distribution of rainfall data [6]. A simple regression showed that the beta parameter of the gamma distribution for a given month can be predicted reasonably well by the average rainfall per wet day [10] with the Weibull and exponential distribution also suitable for modelling daily rainfall amounts [9]. Other distributions such as mixed geometric with truncated Poisson [8] and with Pareto distribution [16] using Mann Kendall test [18] to affirm its suitability. Whereas, Reference [13 & 20] affirmed among other distributions that have been employed to fit rainfall data with log Pearson III distribution opined as best in modelling daily rainfall data or patterns.

Reference [14, 15 & 23] suggestion of the gamma distribution function for wet day amount seems to carry considerable weight because it is bounded on the left at zero and positively skewed. Several researchers working on modelling rainfall agreed with the suitability of the distribution to its frequency. Reference [11] established that gamma distribution can represent a variety of distribution shapes and suits rainfall data by 98% with the ratio of the coefficient of variation to the coefficient of skewness being closed to the value of 2 for a gamma distribution [5]. The distribution of daily precipitation totals can also be approximated by the gamma distribution [12] with gamma distribution being the best model or fit for analyzing daily, quarterly, annual and seasonal rainfall data [21], [26], [24], [27], [22],[4], [6], [25].

2 METHODOLOGY

In the classical system of densities introduced by Pearson in 1894, the gamma density is characterized as type III indicating a random variable X to have a gamma distribution with two parameters α, β and denoted by $X \sim \Gamma(\alpha,\beta)\ iff\ X$ has the probability density function

$$f(x) = \frac{\beta^{\alpha}\, x^{\alpha-1} e^{-\beta x}}{\Gamma\alpha} \ \ or \ \ \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^{\alpha}\Gamma\alpha}$$

where $\alpha > 0, \beta > 0$ are the location & scaling parameters respectively and Γ is the gamma function defined by $\Gamma(t) = \int_0^{\infty} X^{t-1} e^{-x} dx \ , t > 0$

2.1 METHOD OF MAXIMUM LIKELIHOOD IN ESTIMATING PARAMETERS OF GAMMA DISTRIBUTION

The maximum likelihood method (ML) is based upon maximizing what is known as the likelihood function. The joint density function of a set of random variables $X_1, X_2, X_3, \dots, X_n$ evaluated at $x_1, x_2, x_3, \dots, x_n$ which may call $f(x_1, x_2, x_3, \dots, x_n; \theta)$ is referred to as a likelihood function $L(\theta)$. The value of $\hat{\theta}$ at which $L(\theta)$ is maximized is called maximum likelihood estimate (MLE) of θ. That is $f\left(x_1, x_2, x_3, \dots, x_n; \hat{\theta}\right) = \max_{\theta \epsilon \Omega} f(X_1, X_2, X_3, \dots, X_n; \theta)$ where Ω is the parameter space.

OPEN FORM SOLUTION

Finding the maximum likelihood estimator for the gamma distribution, we consider;

$$f(x;\alpha,\beta) = \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^{\alpha}\Gamma\alpha} \tag{1}$$

where $\alpha > 0, \beta > 0$ and defined for $0 < x < \infty$.

Considering that we have a random sample $X = (x_1, x_2, x_3, \ldots, x_n)$ from a gamma probability density, then our likelihood function for this data is given by the product of the density function which is:

$$L(\alpha,\beta;X) = \prod_{i=1}^{n} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^{\alpha}\Gamma\alpha} \tag{2}$$

$$= \prod_{i=1}^{n} (\beta^{\alpha}\Gamma\alpha)^{-1} x^{\alpha-1} e^{-x/\beta}$$

$$= (\beta^{\alpha}\Gamma\alpha)^{-n} \sum x^{\alpha-1}\, e^{-\sum x/\beta} \tag{3}$$

$$Log[L(\alpha,\beta;X)] = \log[(\beta^{\alpha}\Gamma\alpha)^{-n} \sum x^{\alpha-1}\ e^{-\sum x/\beta}]$$

$$= log(\beta^{\alpha}\Gamma\alpha)^{-n} + log \sum x^{\alpha-1} + loge^{-\sum x/\beta}$$

$$= -nlog(\beta^{\alpha}\Gamma\alpha) + (\alpha-1)log \sum x - \sum x/\beta$$

$$= -n[log\beta^{\alpha} + log\Gamma\alpha] + (\alpha-1)log \sum x - \sum x/\beta$$

$$= -nlog\beta^{\alpha} - nlog\Gamma\alpha + (\alpha-1)log \sum x - \sum x/\beta$$

$$= -n\alpha log\beta - nlog\Gamma\alpha + (\alpha-1)log \sum x - \sum x/\beta \tag{4}$$

$$\frac{\partial(4)}{\partial\alpha} = -nlog\beta - n\Psi(\alpha) + \sum logx$$

$$0 = -nlog\beta - n\Psi(\alpha) + \sum logx \tag{5}$$

also,

$$\frac{\partial(4)}{\partial\beta} = \frac{-n\alpha}{\beta} + \frac{\sum x}{\beta^2}$$

$$0 = \frac{-n\alpha\beta + \sum x}{\beta^2}$$

$$0 = -n\alpha\beta + \sum x$$

$$n\alpha\beta = \sum x$$

$$\alpha\beta = \frac{\sum x}{n}$$

$$\alpha\beta = \bar{x}$$

$$\alpha = \frac{\bar{x}}{\beta}$$

$$\frac{\bar{x}}{\hat{\beta}} - \hat{\alpha} = 0 \tag{6}$$

Employing Thom's approximation [23], we consider taking logarithm of equation (6) and substituting for $log\hat{\beta}$ in equation (5),

$$\log[{}^{\bar{x}}/_{\hat{\beta}} - \hat{\alpha}] = 0$$

$$log\bar{x} - log\hat{\beta} - log\hat{\alpha} = 0$$

$$log\hat{\beta} = log\bar{x} - log\hat{\alpha}$$

substituting $log\hat{\beta}$ into equation (5)

$$0 = -nlog\beta - n\Psi(\alpha) + \sum logx \tag{7}$$

$$log\beta + \Psi(\alpha) - {}^{\sum logx}/_{n} = 0$$

$$log\bar{x} - log\hat{\alpha} + \Psi(\alpha) - {}^{\sum logx}/_{n} = 0$$

$$log\bar{x} - {}^{\sum logx}/_{n} = log\hat{\alpha} - \Psi(\alpha) \tag{8}$$

Reference [17] showed that

$$\Psi(\alpha) = log\hat{\alpha} - {}^{1}/_{2}\,\hat{\alpha} - \textstyle\sum_{k=1}^{m}(-1)^{k-1}B_k/\left(2k\hat{\alpha}^{2k}\right) + R_m \tag{9}$$

is an asymptotic expansion in which B_k are the Bernoulli numbers, $B_1 = {}^{1}/_{6}, B_2 = {}^{1}/_{30}$, ... and R_m is the remainder after m terms.

from equation (9) for $m = 1$, we have

$$\Psi(\hat{\alpha}) = log\hat{\alpha} - {}^{1}/_{(2\hat{\alpha})} - {}^{1}/_{12\hat{\alpha}^2} \tag{10}$$

substituting equation (10) into equation (8), we have

$$log\bar{x} - {}^{1}/_{n}\sum logx = log\hat{\alpha} - [log\hat{\alpha} - {}^{1}/_{(2\hat{\alpha})} - {}^{1}/_{12\hat{\alpha}^2}]$$

$$log\bar{x} - {}^{1}/_{n}\sum logx = log\hat{\alpha} - log\hat{\alpha} + {}^{1}/_{(2\hat{\alpha})} + {}^{1}/_{12\hat{\alpha}^2}]$$

$$log\bar{x} - {}^1/_n \sum logx = \frac{6\hat{\alpha} + 1}{12\hat{\alpha}^2}$$

$$12[log\bar{x} - {}^1/_n \sum logx]\hat{\alpha}^2 = 6\hat{\alpha} + 1$$

$$12\left[log\bar{x} - {}^1/_n \sum logx\right]\hat{\alpha}^2 - 6\hat{\alpha} - 1 = 0 \tag{11}$$

let $A = \ log\bar{x} - {}^1/_n \sum logx$

then, equation (11) becomes

$$12A\hat{\alpha}^2 - 6\hat{\alpha} - 1 = 0 \tag{12}$$

$$\hat{\alpha} = \frac{1+\sqrt{1+4A/3}}{4A} \tag{13}$$

where $A = \ log\bar{x} - {}^1/_n \sum logx \ \ or \ log({}^{\bar{x}}/_{\tilde{x}})$

$$\bar{x} = mean$$

$$\tilde{x} = \left(\prod_{i=1}^{n} x_i\right)^{1/n} = geometric\ mean$$

Recall from equation (6) that

$$\frac{\bar{x}}{\hat{\beta}} - \hat{\alpha} = 0$$

$$\frac{\bar{x}}{\hat{\beta}} = \hat{\alpha}$$

$$\hat{\beta} = {}^{\bar{x}}/_{\hat{\alpha}} \tag{14}$$

indicating equation (11) and equation (12) are the estimators of $\hat{\alpha}$ and $\hat{\beta}$ respectively.

CLOSED-FORM SOLUTION

It is well known that maximum likelihood (ML) estimator of the two-parameter gamma distribution do not have a closed-form solution. Surprisingly, two out of the three likelihood equations of the generalized gamma distribution can be used to estimate a closed-form solution for the gamma distribution [7]. In other to obtain simple, unbiased and efficient estimators of parameters for the gamma distribution, we use the generalized gamma distribution denoted as $gg(\alpha, \beta, \gamma)$ where $\gamma > 0$ is a power parameter. It is a useful extension of the gamma distribution with (p.d.f)

$$f_{gg}(x) = \frac{\gamma x^{\alpha\gamma-1}}{\beta^{\alpha\gamma}\Gamma\alpha} e^{-(x/\beta)^{\gamma}} \tag{15}$$

$$x > 0, \gamma = 1$$

taking the likelihood of equation (15),

$$L[f_{gg}(x)] = \prod_{i=1}^{n} \frac{\gamma x^{\alpha\gamma-1}}{\beta^{\alpha\gamma}\Gamma\alpha} e^{-(x/\beta)^\gamma}$$

$$= \prod_{i=1}^{n} (\beta^{\alpha\gamma}\Gamma\alpha)^{-1}\gamma x^{\alpha\gamma-1} e^{(-x/\beta)^\gamma}$$

$$= (\beta^{\alpha\gamma}\Gamma\alpha)^{-n}\gamma^n(\sum x)^{\alpha\gamma-1} e^{(-\sum x/\beta)^\gamma} \quad (16)$$

$$log[L\{f_{gg}(x)\}] = \log[(\beta^{\alpha\gamma}\Gamma\alpha)^{-n}\gamma^n\left(\sum x\right)^{\alpha\gamma-1} e^{(-\sum x/\beta)^\gamma}]$$

$$= log(\beta^{\alpha\gamma}\Gamma\alpha)^{-n} + log\gamma^n\left(\sum x\right)^{\alpha\gamma-1} + loge^{(-\sum x/\beta)^\gamma}$$

$$= -nlog(\beta^{\alpha\gamma}\Gamma\alpha) + log\gamma^n + log\left(\sum x\right)^{\alpha\gamma-1} - \left({\sum x}/{\beta}\right)^\gamma$$

$$= -n[log\beta^{\alpha\gamma} + log\Gamma\alpha] + log\gamma^n + log\left(\sum x\right)^{\alpha\gamma-1} - \left({\sum x}/{\beta}\right)^\gamma$$

$$= -n\alpha\gamma log\beta - nlog\Gamma\alpha + log\gamma^n + (\alpha\gamma-1)log\sum x - \left({\sum x}/{\beta}\right)^\gamma$$

$$= nlog\gamma - n\alpha\gamma log\beta - nlog\Gamma\alpha + \sum(\alpha\gamma-1)logx - \sum\left({x}/{\beta}\right)^\gamma$$

$$= log\gamma - \alpha\gamma log\beta - log\Gamma\alpha + {1}/{n}\sum_{i=1}^{n}\left[(\alpha\gamma-1)logx - \left({x}/{\beta}\right)^\gamma\right] \quad (17)$$

differentiating equation (17) with respect to α and β

$$\frac{\partial(17)}{\partial\alpha} = -\gamma log\beta - \Psi(\alpha) + {\gamma}/{n}\sum_{i=1}^{n} logx \quad (18)$$

$$\frac{\partial(17)}{\partial\beta} = -\frac{\alpha\gamma}{\beta} + \frac{\gamma}{n\beta}\sum\left({x}/{\beta}\right)^\gamma \quad (19)$$

$$= \frac{-n\alpha\gamma + \gamma\sum\left({x}/{\beta}\right)^\gamma}{n\beta}$$

$$= -n\alpha\gamma + \gamma\sum\left({x}/{\beta}\right)^\gamma$$

$$= -n\alpha + \sum\left({x}/{\beta}\right)^\gamma$$

$$= -\alpha + {1}/{n}\sum\left({x}/{\beta}\right)^\gamma \quad (20)$$

also,

$$\frac{\partial(17)}{\partial\gamma} = \frac{\partial}{\partial\gamma}\left\{log\gamma - \alpha\gamma log\beta - log\Gamma\alpha + {}^1/_n\sum(\alpha\gamma - 1)\, logx - {}^1/_n\sum\left({}^x/_\beta\right)^\gamma\right\}$$

$$= \frac{\partial}{\partial\gamma}\left\{log\gamma - \alpha\gamma log\beta - log\Gamma\alpha + {}^1/_n\sum\alpha\gamma\, logx - {}^1/_n\sum logx - {}^1/_n\sum\left({}^x/_\beta\right)^\gamma\right\}$$

$$= \frac{1}{\gamma} - \alpha log\beta + \frac{1}{n}\sum\alpha\, logx - \frac{1}{n}\sum\left({}^x/_\beta\right)^\gamma log\left({}^x/_\beta\right)$$

$$= \frac{1}{\gamma} + \frac{1}{n}\sum\alpha\, logx - \alpha log\beta - \frac{1}{n}\sum\left({}^x/_\beta\right)^\gamma log\left({}^x/_\beta\right)$$

$$= \frac{1}{\gamma} + \frac{\sum\alpha logx - \sum\alpha\, log\beta}{n} - \frac{1}{n}\sum\left({}^x/_\beta\right)^\gamma log\left({}^x/_\beta\right)$$

$$= \frac{1}{\gamma} + \frac{1}{n}\alpha\sum log\left({}^x/_\beta\right) - \frac{1}{n}\sum\left({}^x/_\beta\right)^\gamma log\left({}^x/_\beta\right)$$

$$\frac{\partial(17)}{\partial\gamma} = \frac{1}{\gamma} + \frac{\alpha}{n}\sum log\left({}^x/_\beta\right) - \frac{1}{n}\sum\left({}^x/_\beta\right)^\gamma log\left({}^x/_\beta\right) \quad (21)$$

setting equation (20) to zero and expressing β as a function of $\alpha\ and\ \gamma$:

$$0 = -\alpha + {}^1/_n\sum\left({}^x/_\beta\right)^\gamma$$

$$\alpha = {}^1/_n\sum\left({}^x/_\beta\right)^\gamma$$

$$n\alpha = \frac{\sum x^\gamma}{\beta^\gamma}$$

$$\beta^\gamma n\alpha = \sum x^\gamma$$

$$\beta^\gamma = \frac{\sum(x)^\gamma}{n\alpha}$$

$$\hat{\beta} = \left[\frac{\sum x^\gamma}{n\alpha}\right]^{1/\gamma} \quad (22)$$

substituting equation (22) into equation (21), we have

$$\alpha = \frac{n\sum x^\gamma}{n\gamma\sum x^\gamma logx_i - \gamma\sum logx_i\sum x^\gamma} \quad (23)$$

as we already know that $\gamma = 1$, we then use this fact to obtain the new estimators for $\alpha\ and\ \beta$

$$\hat{\alpha} = \frac{n\sum x_i}{n\sum x_i log x_i - \sum log x_i \sum x_i} \quad (24)$$

$$\hat{\beta} = \frac{1}{n^2}(n\sum x_i \, log x_i - \sum log x_i \sum x_i) \quad (25)$$

equations (24) and (25) are the estimators of the closed form solution for gamma distribution.

METHOD OF MOMENTS IN ESTIMATING PARAMETERS OF GAMMA DISTRIBUTION

Method of Moments estimation provides us with a method that is easy to apply and widely applicable. Method of Moments Estimation is the solution of a series of equations where we equate the theoretical moment with the corresponding sample moments.

We define moment such that the j^{th} moment is given by

$$U_j(\theta_1, \theta_2, \ldots, \theta_k) = E[X^j]$$

Our sample moments are such that the J^{th} sample moment is given by $M_j = \frac{\sum x_i^j}{n}$ which is based upon a random sample $x_1, x_2, \ldots, x_n$ from a distribution $f(X; \theta_1, \theta_2, \ldots, \theta_n)$. If we have k parameters, say$\theta_1, \theta_2, \ldots, \theta_k$, then we require k equations to solve. Thus, our estimates $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k$ for the parameters are the solutions of the equations. To find the method of moments estimators, we must equate the moments for the population distribution with the sample moments. Firstly, let us consider the moment generating function for the gamma distribution

$$M_x(t) = E[e^{tx}]$$

$$= \int_0^\infty \frac{e^{tx} x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma\alpha} dx$$

$$= \frac{1}{\beta^\alpha \Gamma\alpha} \int_0^\infty e^{tx} x^{\alpha-1} e^{-x/\beta} \, dx$$

$$let\ U = -\left(1/_\beta - t\right)x, then\ du = -\left(1/_\beta - t\right)dx$$

$$M_x(t) = \int_0^\infty e^{-u} \frac{u^{\alpha-1}}{\left(1/_\beta - t\right)^\alpha} du$$

where $x = -\frac{u}{\left(1/_\beta - t\right)}$

$$M_x(t) = \frac{1}{\beta^\alpha \Gamma\alpha} \int_0^\infty \frac{u^{\alpha-1}}{-\left(1/_\beta - t\right)^{\alpha-1}} e^{-u} - \left(1/_\beta - t\right)^{-1} du$$

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$$= \frac{1}{\beta^{\alpha}\Gamma\alpha}\int_{0}^{\infty}\frac{u^{\alpha-1}}{\left(1/_{\beta}-t\right)^{\alpha-1}}e^{-u}\frac{1}{\left(1/_{\beta}-t\right)}du$$

$$= \frac{1}{\beta^{\alpha}\Gamma\alpha}\int_{0}^{\infty}\frac{u^{\alpha-1}}{\left(1/_{\beta}-t\right)^{\alpha}}e^{-u}\,du$$

$$= \frac{\left(1/_{\beta}-t\right)^{-\alpha}}{\beta^{\alpha}\Gamma\alpha}\int_{0}^{\infty}e^{-u}u^{\alpha-1}du$$

$$= \frac{\left(1/_{\beta}-t\right)^{-\alpha}}{\beta^{\alpha}\Gamma\alpha}\Gamma\alpha$$

where $\Gamma\alpha = \int_{0}^{\infty} e^{-u}u^{\alpha-1}\,du$

$$M_x(t) = \frac{\left(1/_{\beta}-t\right)^{-\alpha}}{\beta^{\alpha}}$$

$$= \beta^{-\alpha}\left(1/_{\beta}-t\right)^{-\alpha}$$

$$= \left[\beta\left(1/_{\beta}-t\right)\right]^{-\alpha}$$

$$M_x(t) = (1-t\beta)^{-\alpha} \tag{26}$$

differentiating equation (26) with respect to t

$$M_x'(t) = \alpha\beta(1-t\beta)^{-\alpha-1} \tag{27}$$

setting $t = 0$, then

$$M_x'(t=0) = \alpha\beta(1-(0)\beta)^{-\alpha-1}$$

$$= \alpha\beta(1)^{-\alpha-1}$$

$$= \alpha\beta \tag{28}$$

differentiating equation (27) with respect to t

$$M_x''(t) = (\alpha\beta)(-\alpha-1)(1-t\beta)^{-\alpha-2}(-\beta) \tag{29}$$

$$= (\alpha\beta^2)(\alpha+1)(1-t\beta)^{-\alpha-2}$$

setting $t = 0$, then

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$$M_x''(t) = (\alpha\beta^2)(\alpha+1)(1-(0)\beta)^{-\alpha-2}$$
$$= (\alpha\beta^2)(\alpha+1)(1)^{-\alpha-2}$$
$$= (\alpha\beta^2)(\alpha+1) \tag{30}$$

finding the moment estimators, we equate (28) and (30) to the first and second moment respectively,

$$\frac{\sum x_i}{n} = \alpha\beta$$
$$\bar{x} = \alpha\beta$$
$$\hat{\alpha} = \frac{\bar{x}}{\hat{\beta}} \tag{31}$$

also,

$$\frac{\sum x^2}{n} = \alpha\beta^2(\alpha+1) \tag{32}$$

substituting equation (31) into equation (32), we have

$$\frac{\sum x^2}{n} = \frac{\bar{x}}{\beta}\beta^2(\hat{\alpha}+1)$$
$$\frac{\sum x^2}{n} = \frac{\bar{x}}{\beta}\beta^2\left({}^{\hat{\alpha}}/_{\beta}+1\right)$$
$$= \bar{x}\beta\left({}^{\bar{x}}/_{\beta}+1\right)$$
$$E(x^2) = (\bar{x})^2 + \bar{x}\beta$$
$$\bar{x}\beta = E(x^2) - (\bar{x})^2$$
$$\bar{x}\beta = E(x^2) - (E[x])^2$$
$$\hat{\beta} = \frac{E(x^2) - (E[x])^2}{\bar{x}}$$
$$\hat{\beta} = \frac{\sum[x_i-\bar{x}]^2}{n\bar{x}} \tag{33}$$

The moment estimators of $\hat{\alpha}$ and $\hat{\beta}$ are given in equations (31 and 33) respectively.

3 DISCUSSION OF RESULTS

Table 1: Estimates of Location and Scaling Parameters of Two-Parameter Gamma Distribution for Six Geo-Political Zones including Abuja (FCT) in Nigeria

		Maximum Likelihood Estimate		Method Of Moment Estimate
		Open form	Closed form	
$\hat{\alpha}$	Kaduna State	1.1	1.3	0.0079
	Taraba State	1.3	1.6	0.0080
	Edo State	0.9	1.0	0.0080
	Enugu State	4.9	5.4	0.0080
	Plateau State	2.3	2.7	0.0080
	FCT, Abuja	3.8	3.9	0.0080
	Lagos State	3.2	3.6	0.0080
$\hat{\beta}$	Kaduna State	284.7	234.9	38713.8
	Taraba State	209.1	170.8	33415.8
	Edo State	294.0	247.0	31504.1
	Enugu State	148.4	135.4	91179.3
	Plateau State	190.4	159.4	54514
	FCT, Abuja	95.8	93.62	46000.5
	Lagos State	729.8	659.6	295038.2

Six geo-political zones of the country were sampled including Abuja (FCT) and for each zone, a state was selected using simple random sampling with consideration given to Abuja to be part of samples to be studied. States considered in the country were Kaduna, Taraba, Edo, Enugu, Plateau and Lagos with FCT and their rainfall data of 44 years on monthly basis from 1976 to 2020 modelled after two-parameter gamma distribution of location and scaling parameters.

Results showed closed-form solution of maximum likelihood method[7] of the two-parameter gamma distribution to be most efficient for the six states considered including Abuja (FCT) compared to the open form solution of maximum likelihood method that is more efficient when evaluated with the method of moments as the estimates of the scaling parameter for closed-form solution has the least values for the states under consideration but it is worthy of noting that values were extremely on the high side which is an indication for the possibilities of torrential rainfall leading to an emergency of extreme variability in the rainfall patterns among the six states considered from each geo-political zone including Abuja (FCT) in the country thereby making flooding inevitable in all parts of Nigeria.

In classical inference, estimators of maximum likelihood (ML) perform better than that of the Method of Moments (MM), same was observed for the estimation of two-parameter gamma distribution as seen in table 1 which is in agreement with the findings of reference [28], however, the closed-form solution of the

MLE outperformed the open form solution[7] as attested to by the smaller scaling parameter for each state of geo-political zones in Nigeria including Abuja (FCT). In addition, it is observed that each of the methods considered yielded a larger scaling parameter and lower location parameter which is an indication that the considered areas are receiving varying degree amounts of rainfall patterns.

CONCLUSION

This research work has demonstrated a greater alignment for the classical inference of statistics as a course of study hereby reflecting the practical demonstration of theories for better understanding and expanding of knowledge in all spheres of livelihood as reflected in the assessment of climate change for rainfall variability through estimation of two-parameter gamma distribution via maximum likelihood (closed and open form solutions) and methods of moments to explain extreme and torrential rainfall likelihood in all parts of the country as encountered across the six geo-political zones of Nigeria thereby giving a red alert for flooding.

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