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COMPARATIVE ANALYSIS OF SOME SELECTED CLASSES OF RATIO ESTIMATORS

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ABSTRACT: *Many ratio type estimators for population mean have come into play in the past. Researchers over the years have been making efforts to improve the efficiency of thee estimators. There has been a lot of modification of some of these estimators. Some forms of comparison have been done in the literature. There is need to further compare these estimators with other existing estimators at varying sample sizes and also considering discrete and continuous distribution. Thirty-eight estimators, five different sample sizes and seven distributions were considered. The population mean estimates and their Bias were computed for the thirty- eight estimators at varying sample sizes under various distribution. The efficiency of the estimator was computed using Mean Square Error (MSE). Using simulation study, it was observed that the efficiency of the estimators increase as sample sizes increases and the estimator performed alike in most distributions.*

KEYWORDS: Mean, Sampling, Auxiliary variable, Estimators, Constants, Bias, MSE.

INTRODUCTION

The primary goal of a survey sampling is to generally collect data that could be used to describe features of the population of interest. To achieve this goal, one must be able to use units observed in the sample to make inferences about units in the entire population. Estimation of population mean is the persistent issue in sampling practices and many efforts have been made by various statisticians to improve the precision of the estimates by using the auxiliary information. Auxiliary information is any information closely related to the study variate, used to improve the precision of its estimates. Auxiliary information may be used at the planning or designing stage, selection stage and estimation stage.

At the estimation stage, auxiliary information is used as ratio estimators, regression estimators and product estimators. Proper use of auxiliary information may result to appreciable gain in precision while indiscriminate use may lead to a loss in precision. Ratio estimation is a very important method in sample surveys which utilizes the information on auxiliary variable which is positively and highly correlated with the study variable. Hence, a ratio estimator is an estimator which utilizes the information on an auxiliary variable, X to estimate the population parameter of the variable of interest Y. For X to serve as an auxiliary variable, its value must be known for the entire population. In sampling it is a well-known fact that the proper use of auxiliary information supplied by auxiliary variable improves the efficiency of the estimators of population parameters of the study variable. Ratio estimation is one such example where this criterion is fulfilled. It is used for estimation of population mean when study variable and auxiliary variable are highly correlated with each other. An important objective in any statistical estimation procedure is to obtain the estimators of parameters of interest with more precision. It is also well understood that incorporation of more information in the estimation procedure yield better estimators, provided the information is valid and proper.

Use of such auxiliary information is made through the ratio method of estimation to obtain an improved estimator of population mean. In ratio method of estimation, auxiliary information on a variable is available which is linearly related to the variable under study and is utilized to estimate the population mean.

In survey sampling, bias refers to the tendency of a sample statistic to systematically over-or under-estimate a population parameter. It is the difference between the expected value of the estimator and the true value of the parameter being estimated. Bias can occur due to unrepresentative of the sample (selection bias), poor measurement process (response bias) and differences in how the treatment or intervention was carried out or how subjects were exposed to the factor of interest.

Increasing the sample size tends to reduce sampling error which is the variability among statistics from different samples drawn from the same population. However, increasing the sample size does not affect survey bias, a large sample cannot correct the methodological problems that produce survey bias.

In statistics, the Mean Squared Error (MSE) is another concept used in assessing the performance of an estimator. The Mean Squared Error, is quite important for relaying the concepts of precision, bias and accuracy during the statistical estimation. The measure of Mean Squared Error requires a target of prediction or estimation along with a predictor or estimator which is said to be the function of the given data. Mean Squared Error is defined as the average of squares of the "errors". The error is said to be the difference between the attribute which is to the estimated and the estimator. The (MSE) Mean Squared Error may be called a risk function which corresponds to the expected value of the loss of squared error. This difference could be developed due to the randomness or due to the estimator does not depict the information which could provide a more accurate estimate. It can be referred to the second moment of the error measured about the origin. It incorporates both the variance and bias of the estimator. If an estimator is unbiased estimator, then its Mean Squared Error is same as the variance of the estimator. The unit of MSE is the same as the unit of measurement for the quantity which is being estimated.

To reduce the Bias, MSE and hence improve the precision of estimates gotten by ratio method, researchers began to modify the original ratio estimator using median, quartiles, deciles, coefficient of correlation, coefficient of variation, coefficient of Kurtosis, quartile deviation, ranked set sampling and so on. Among such researchers are Abid et al (2016), Cochran (1940), Singh and Tailor (2003), Etaga et al (2015), Amogu (2012 and 2013), Bhat et al (2018), Jeelani et al (2016), Subramani & Ajith (2017), Okafor (2002). Robson (1957), Agrawal and Roy (1999) Singh (2003) Singh et al (2004), ,Subramani and Kumarapandiyan (2012), Subzar et at (2016) ,Rao (1991) ,Upadhyaya and Singh (1999) , Singh et al (1973), Rajesh (2008), Rajesh et al (2011), Subzar et al (2017a,b) Searls and Intarapanich (1990), Sarbjit and Jasleen (2016), Mclntyre (1952), and many other researchers have suggested some new modified ratio estimators in survey sampling.

MATERIAL AND METHOD

The population considered in this study is of size 260 drawn from the following distributions:
Poison, Hypergeometric, Binomial, Normal, Exponential, Continuous Uniform, Chi-square population.
Sample sizes of 17, 25, 39, 50 and 100 were drawn from each of the distribution.
Simple Random Sampling without replacement (SRSWOR) technique was used to select sample from the populations. Data were simulated from Minitab software.

The data used in work were simulated using the following values of the needed parameters.

Poisson

F(x) = $\frac{\lambda^x e^{-\lambda}}{x!}$; $x = 0,1,2, \ldots$

The mean, $\lambda = 12$

Hypergeometric distribution

$$F(x) = \frac{\binom{m}{x}\left[N-M\right]}{\left[N\right]} \quad ; \quad x = 0, 1, 2, \ldots, \min(m, n)$$

Population size (N) = 16, Event count in population (M) = 37, Sample size (n) = 12

Normal distribution

$$F(x) = \frac{1}{\sigma\sqrt{2x}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty$$

Mean, N = 15, Standard deviation, $\sigma = 2.4$

Exponential distribution

$$F(x) = \frac{1}{\lambda} e^{-x/\lambda} \; ; \quad \lambda > 0, x > 0$$

The scale parameter, λ = 3.0, Threshold = 0.0

Continuous Uniform Distribution

$$F(x) = \frac{1}{b-9} \; ; \; a \leq x \leq b, -\infty < a < b < \infty$$

Lower endpoint, a = 1, Upper endpoint, b = 17

Binomial Distribution

$$f(x) = \begin{cases} \binom{n}{x} p^x q^{n-x} & x = 1,2,3, \ldots, n; \; 0 \leq p \leq 1; \; p + q = 1 \\ 0 \; elsewhere \end{cases}$$

Number of trial, n = 25, Event probability, p = 0.6

Chi-square Distribution

$$f(x) = \begin{cases} \frac{1}{2^{\frac{k}{2}} \Gamma\frac{k}{2}} x^{\frac{k}{2}} e^{\frac{k}{2}-1} ; & x > 0 \\ 0 \; elsewhere \end{cases}$$

With k degrees of freedom

ESTIMATORS

$\hat{\bar{Y}}_{P^1} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{X}QD+D_1)}(\bar{X}QD+D_1)$

$R_1 = \frac{\bar{X}QD}{\bar{X}QD+D_1}$

$B\left(\hat{\bar{Y}}_{P^1}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_1^2$

$MSE\left(\hat{\bar{Y}}_{P^1}\right) = \frac{(1-f)}{n}\left[R_1^2 S_x^2 + S_y^2(1-\rho^2)\right]$

$\hat{\bar{Y}}_{P^2} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_2)}(\bar{X}QD+D_2)$

$R_2 = \frac{\bar{X}QD}{\bar{X}QD+D_2}$

$B\left(\hat{\bar{Y}}_{P^2}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_2^2$

$MSE\left(\hat{\bar{Y}}_{P^2}\right) = \frac{(1-f)}{n}\left[R_2^2 S_x^2 + S_y^2(1-\rho^2)\right]$

$\hat{\bar{Y}}_{P^3} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_3)}(\bar{X}QD+D_3)$

$R_3 = \frac{\bar{X}QD}{\bar{X}QD+D_3}$

$B\left(\hat{\bar{Y}}_{P^3}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_3^2$

$MSE\left(\hat{\bar{Y}}_{P^3}\right) = \frac{(1-f)}{n}\left[R_3^2 S_x^2 + S_y^2(1-\rho^2)\right]$

$\hat{\bar{Y}}_{P^4} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_4)}(\bar{X}QD+D_4)$

$R_4 = \frac{\bar{X}QD}{\bar{X}QD+D_4}$

$B\left(\hat{\bar{Y}}_{P^4}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_4^2$

$MSE\left(\hat{\bar{Y}}_{P^4}\right) = \frac{(1-f)}{n}\left[R_4^2 S_x^2 + S_y^2(1-\rho^2)\right]$

$\hat{\bar{Y}}_{P^5} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_5)}(\bar{X}QD+D_5)$

$R_5 = \frac{\bar{X}QD}{\bar{X}QD+D_5}$

$$B\left(\hat{\bar{Y}}_{P^5}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_5^2$$
$$MSE\left(\hat{\bar{Y}}_{P^5}\right) = \frac{(1-f)}{n}\left[R_5^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^6} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_6)}(\bar{X}QD+D_6)$$
$$R_6 = \frac{\bar{X}QD}{\bar{X}QD+D_6}$$
$$B\left(\hat{\bar{Y}}_{P^6}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_6^2$$
$$MSE\left(\hat{\bar{Y}}_{P^6}\right) = \frac{(1-f)}{n}\left[R_6^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^7} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_7)}(\bar{X}QD+D_7)$$
$$R_7 = \frac{XQD}{\bar{X}QD+D_7}$$
$$B\left(\hat{\bar{Y}}_{P^7}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_7^2$$
$$MSE\left(\hat{\bar{Y}}_{P^7}\right) = \frac{(1-f)}{n}\left[R_7^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^8} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_8)}(\bar{X}QD+D_8)$$
$$R_8 = \frac{\bar{X}QD}{\bar{X}QD+D_8}$$
$$B\left(\hat{\bar{Y}}_{P^8}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_8^2$$
$$MSE\left(\hat{\bar{Y}}_{P^8}\right) = \frac{(1-f)}{n}\left[R_8^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^9} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_9)}(\bar{X}QD+D_9)$$
$$R_9 = \frac{\bar{X}QD}{\bar{X}QD+D_9}$$
$$B\left(\hat{\bar{Y}}_{P^9}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_9^2$$
$$MSE\left(\hat{\bar{Y}}_{P^9}\right) = \frac{(1-f)}{n}\left[R_9^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{10}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}QD + D_{10})} (\bar{X}QD + D_{10})$$

$$R_{10} = \frac{\bar{X}QD}{\bar{X}QD + D_{10}}$$

$$B\left(\hat{\bar{Y}}_{P^{10}}\right) = \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{10}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{10}}\right) = \frac{(1-f)}{n} \left[R_{10}^2 S_x^2 + S_y^2 (1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{11}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_1 + QD)} (\bar{X}D_1 + QD)$$

$$R_{11} = \frac{XD_1}{\bar{X}D_1 + QD}$$

$$B\left(\hat{\bar{Y}}_{P^{11}}\right) = \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{11}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{11}}\right) = \frac{(1-f)}{n} \left[R_{11}^2 S_x^2 + S_y^2 (1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{12}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_2 + QD)} (\bar{X}D_2 + QD)$$

$$R_{12} = \frac{\bar{x}X}{\bar{X}D_2 + QD}$$

$$B\left(\hat{\bar{Y}}_{P^{12}}\right) = \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{12}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{12}}\right) = \frac{(1-f)}{n} \left[R_{12}^2 S_x^2 + S_y^2 (1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{13}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_3 + QD)} (\bar{X}D_3 + QD)$$

$$R_{13} = \frac{\bar{X}D_3}{\bar{X}D_3 + QD}$$

$$B\left(\hat{\bar{Y}}_{P^{13}}\right) = \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{13}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{13}}\right) = \frac{(1-f)}{n} \left[R_{13}^2 S_x^2 + S_y^2 (1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{14}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_4 + QD)} (\bar{X}D_4 + QD)$$

$$R_{14} = \frac{\bar{X}D_4}{\bar{X}D_4 + QD}$$

$$B\left(\hat{\bar{Y}}_{P14}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{14}^2$$
$$MSE\left(\hat{\bar{Y}}_{P14}\right) = \frac{(1-f)}{n}\left[R_{14}^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P15} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_5+QD)}(\bar{X}D_5+QD)$$
$$R_{15} = \frac{\bar{X}D_5}{\bar{X}D_5+QD}$$
$$B\left(\hat{\bar{Y}}_{P15}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{15}^2$$
$$MSE\left(\hat{\bar{Y}}_{P15}\right) = \frac{(1-f)}{n}\left[R_{15}^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P16} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_6+QD)}(\bar{X}D_6+QD)$$
$$R_{16} = \frac{\bar{X}D_6}{\bar{X}D_6+QD}$$
$$B\left(\hat{\bar{Y}}_{P16}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{16}^2$$
$$MSE\left(\hat{\bar{Y}}_{P16}\right) = \frac{(1-f)}{n}\left[R_{16}^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P17} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_7+QD)}(\bar{X}D_7+QD)$$
$$R_{13} = \frac{\bar{X}D_7}{\bar{X}D_7+QD}$$
$$B\left(\hat{\bar{Y}}_{P17}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{17}^2$$
$$MSE\left(\hat{\bar{Y}}_{P17}\right) = \frac{(1-f)}{n}\left[R_{17}^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P18} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_8+QD)}(\bar{X}D_8+QD)$$
$$R_{18} = \frac{\bar{X}D_8}{\bar{X}D_8+QD}$$
$$B\left(\hat{\bar{Y}}_{P18}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{18}^2$$
$$MSE\left(\hat{\bar{Y}}_{P18}\right) = \frac{(1-f)}{n}\left[R_{18}^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{19}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_9 + QD)} (\bar{X}D_9 + QD)$$

$$R_{19} = \frac{\bar{X}D_9}{\bar{X}D_9 + QD}$$

$$B\left(\hat{\bar{Y}}_{P^{19}}\right) = \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{19}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{19}}\right) = \frac{(1-f)}{n} \left[R_{19}^2 S_x^2 + S_y^2 (1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{20}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_{10} + QD)} (\bar{X}D_{10} + QD)$$

$$R_{20} = \frac{\bar{X}D_{10}}{\bar{X}D_{10} + QD}$$

$$B\left(\hat{\bar{Y}}_{P^{20}}\right) = \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{20}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{20}}\right) = \frac{(1-f)}{n} \left[R_{20}^2 S_x^2 + S_y^2 (1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{21}} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n + \phi_1)} (\bar{X}_N + \phi_1)$$

$$R_{21} = \frac{\bar{Y}_N}{\bar{X}_{N+}\phi_1}$$

$$B\left(\hat{\bar{Y}}_{P^{21}}\right) = \frac{(1-f_n)}{n_n} \frac{S_{x_n}^2}{\bar{Y}_N} R_{21}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{21}}\right) = \frac{(1-f_n)}{n_n} \left[R_{21}^2 S_{x_n}^2 + S_{y_n}^2 (1-\rho_{xy}^2)\right]$$

$$\hat{\bar{Y}}_{P^{22}} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n + \phi_2)} (\bar{X}_N + \phi_2)$$

$$R_{22} = \frac{\bar{Y}_N}{\bar{X}_N + \phi_2}$$

$$B\left(\hat{\bar{Y}}_{P^{22}}\right) = \frac{(1-f_n)}{n_n} \frac{S_{x_n}^2}{\bar{Y}_N} R_{22}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{22}}\right) = \frac{(1-f_n)}{n_n} \left[R_{31}^2 S_{x_n}^2 + S_{y_n}^2 (1-\rho_{xy}^2)\right]$$

$$\hat{\bar{Y}}_{P^23} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n\rho_{xy} + \phi_1)} \left(\bar{X}_N\rho_{xy} + \phi_1\right)$$

$$R_{23} = \frac{\bar{Y}_N\rho_{xy}}{\bar{X}_N\rho_{xy} + \phi_1}$$

$$B\left(\hat{\bar{Y}}_{P^{23}}\right) = \frac{(1-f_n)}{n_n}\frac{S_{x_n}^2}{\bar{Y}_N}R_{23}^2$$

$$MSE\left(\hat{\bar{Y}}_{23}\right) = \frac{(1-f_n)}{n_n}\left[R_{23}^2 S_{x_n}^2 + S_{y_n}^2(1-\rho_{xy}^2)\right]$$

$$\hat{\bar{Y}}_{P^{24}} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n\rho_{xy} + \phi_2)}\left(\bar{X}_N\rho_{xy} + \phi_2\right)$$

$$R_{24} = \frac{\bar{Y}_N\rho_{xy}}{\bar{X}_N\rho_{xy} + \phi_2}$$

$$B\left(\hat{\bar{Y}}_{P^{33}}\right) = \frac{(1-f_n)}{n_n}\frac{S_{x_n}^2}{\bar{Y}_N}R_{24}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{24}}\right) = \frac{(1-f_n)}{n_n}\left[R_{24}^2 S_{x_n}^2 + S_{y_n}^2(1-\rho_{xy})\right]$$

$$\hat{\bar{Y}}_{P^{25}} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n C_{x_n} + \phi_1)}\left(\bar{X}_N C_{x_n} + \phi_1\right)$$

$$R_{25} = \frac{\bar{Y}_N C_{x_n}}{\bar{X}_N C_{x_n} + \phi_1}$$

$$B\left(\hat{\bar{Y}}_{P^{25}}\right) = \frac{(1-f_n)}{n_n}\frac{S_{x_n}^2}{\bar{Y}_N}R_{25}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{25}}\right) = \frac{(1-f_n)}{n_n}\left[R_{25} S_{x_n}^2 + S_{y_n}^2(1-\rho_{xy})\right]$$

$$\hat{\bar{Y}}_{P^{26}} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n C_{x_n} + \phi_2)}\left(\bar{X}_N C_{x_n} + \phi_2\right)$$

$$R_{26} = \frac{\bar{Y}_N C_{x_n}}{\bar{X}_N C_{x_n} + \phi_2}$$

$$B\left(\hat{\bar{Y}}_{P^{26}}\right) = \frac{(1-f_n)}{n_n}\frac{S_{x_n}^2}{\bar{Y}_N}R_{26}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{26}}\right) = \frac{(1-f_n)}{n_n}\left[R_{26}^2 S_{x_n}^2 + S_{y_n}^2(1-\rho_{xy}^2)\right]$$

$$\hat{\bar{Y}}_{P^{27}} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n\beta_1 + \phi_1)}\left(\bar{X}_N\beta_1 + \phi_1\right)$$

$$R_{27} = \frac{\bar{Y}_N\beta_1}{\bar{X}_N\beta_1 + \phi_2}$$

$$B\left(\hat{\bar{Y}}_{P^{27}}\right) = \frac{(1-f_n)}{n_n}\frac{S_{x_n}^2}{\bar{Y}_N}R_{27}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{27}}\right) = \frac{(1-f_n)}{n_n}\left[R_{27}^2\, S_{x_n}^2 + S_{y_n}^2\,(1-\rho_{xy}^2)\right]$$

$$\hat{\bar{Y}}_{P^{28}} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n\beta_1 + \phi_2)}\,(\bar{X}_N\beta_1 + \phi_2)$$

$$R_{28} = \frac{\bar{Y}_N\beta_1}{\bar{X}_N\beta_1 + \phi_2}$$

$$B\left(\hat{\bar{Y}}_{P^{28}}\right) = \frac{(1-f_n)}{n_n}\frac{S_{x_n}^2}{\bar{Y}_N}\,R_{28}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{28}}\right) = \frac{(1-f_n)}{n_n}\left[R_{28}^2\, S_{x_n}^2 + S_{y_n}^2\,(1-\rho_{xy}^2)\right]$$

$$\hat{\bar{Y}}_{P^{29}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_1)}\,(\bar{X}C_x + D_1)$$

$$R_{29} = \frac{\bar{X}C_x}{\bar{X}C_x + D_1}$$

$$B\left(\hat{\bar{Y}}_{P^{29}}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}\,R_{29}$$

$$MSE\left(\hat{\bar{Y}}_{P^{29}}\right) = \frac{(1-f)}{n}\left[R_{29}^2\, S_x^2 + S_y^2\,(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{30}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_2)}\,(\bar{X}C_x + D_2)$$

$$R_{30} = \frac{\bar{X}C_x}{\bar{X}C_x + D_2}$$

$$B\left(\hat{\bar{Y}}_{P^{30}}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}\,R_{30}^2$$

$$MSE\left(\hat{\bar{Y}}_{30}\right) = \frac{(1-f)}{n}\left[R_{30}^2\, S_x^2 + S_y^2\,(1-\rho_{xy}^2)\right]$$

$$\hat{\bar{Y}}_{P^{31}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_3)}\,(\bar{X}C_x + D_3)$$

$$R_{31} = \frac{\bar{X}C_x}{\bar{X}C_x + D_3}$$

$$B\left(\hat{\bar{Y}}_{P^{31}}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}\,R_{31}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{31}}\right) = \frac{(1-f)}{n}\left[R_{31}^2\, S_x^2 + S_y^2\,(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{32}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_4)}\,(\bar{X}C_x + D_4)$$

$$R_{32} = \frac{\bar{X}C_x}{\bar{X}C_x + D_4}$$
$$B\left(\hat{\bar{Y}}_{P32}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{32}^2$$
$$MSE\left(\hat{\bar{Y}}_{P32}\right) = \frac{(1-f)}{n}\left[R_{32}^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P33} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_5)}(\bar{X}C_x + D_5)$$
$$R_{33} = \frac{\bar{X}C_x}{\bar{X}C_x + D_5}$$
$$B\left(\hat{\bar{Y}}_{P33}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{33}$$
$$MSE\left(\hat{\bar{Y}}_{P33}\right) = \frac{(1-f)}{n}\left[R_{33}^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P34} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_6)}(\bar{X}C_x + D_6)$$
$$R_{34} = \frac{\bar{X}C_x}{\bar{X}C_x + D_6}$$
$$B\left(\hat{\bar{Y}}_{P34}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{34}^2$$
$$MSE\left(\hat{\bar{Y}}_{P34}\right) = \frac{(1-f)}{n}\left[R_{34}^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P35} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_7)}(\bar{X}C_x + D_7)$$
$$R_{35} = \frac{\bar{X}C_x}{\bar{X}C_x + D_7}$$
$$B\left(\hat{\bar{Y}}_{P35}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{35}^2$$
$$MSE\left(\hat{\bar{Y}}_{P35}\right) = \frac{(1-f)}{n}\left[R_{35}^2 S_x^2 + S_y^2(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P36} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_8)}(\bar{X}C_x + D_8)$$
$$R_{36} = \frac{\bar{X}C_x}{\bar{X}C_x + D_8}$$
$$B\left(\hat{\bar{Y}}_{P36}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{36}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{36}}\right) = \frac{(1-f)}{n}\left[R_{36}^2\, S_x^2 + S_y^2\,(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{37}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_9)}\,(\bar{X}C_x + D_9)$$

$$R_{37} = \frac{\bar{X}C_x}{\bar{X}C_x + D_9}$$

$$B\left(\hat{\bar{Y}}_{P^{37}}\right) = \frac{(1-f)}{n}\,\frac{S_x^2}{\bar{Y}}\,R_{37}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{37}}\right) = \frac{(1-f)}{n}\left[R_{37}^2\, S_x^2 + S_y^2\,(1-\rho^2)\right]$$

$$\hat{\bar{Y}}_{P^{38}} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_{10})}\,(\bar{X}C_x + D_{10})$$

$$R_{38} = \frac{\bar{X}C_x}{\bar{X}C_x + D_{10}}$$

$$B\left(\hat{\bar{Y}}_{P^{38}}\right) = \frac{(1-f)}{n}\,\frac{S_x^2}{\bar{Y}}\,R_{38}^2$$

$$MSE\left(\hat{\bar{Y}}_{P^{38}}\right) = \frac{(1-f)}{n}\left[R_{38}^2\, S_x^2 + S_y^2\,(1-\rho^2)\right]$$

Where ϕ_1 = $(DM_N\, x\, Q_{2N})$ and
ϕ_2 = $(DM_N\, x\, QD_N)$
ρ_{xy} - Population correlation coefficient
C_{x_n}, C_{y_n} - Population coefficient of variation of the response and the auxiliary variable.
$s_{y_n x_n}$ - Population covariance between response and the auxiliary variables
S_{x_n}, S_{y_n} - Population standard deviations of the response and the auxiliary variable
$y,\ x$ - Sample totals of the response and the auxiliary variable.
$\bar{y}_n,\ \bar{x}_n$ - Sample means of the response and the auxiliary Variable.
$\bar{Y}_N,\ \bar{X}_N$ - Population mean of the response and the auxiliary variable.
X_N - auxiliary variable
Y_N - Response variable
$f_n = \frac{n_n}{N_N}$ - - Fraction of sampling
n_n - Size of the sample
N_N - Size of the population
M_d - Population median of X
β_2 - Population Kurtosis
β_1 - Population Skewness

D_q - Population Deciles q = 1, 2, … 10.

Q_D = $\frac{Q_3 - Q_1}{2}$ - Population quartile deviation

M_R = $\frac{X_{(1)} - X_{(N)}}{2}$ Population mid-range

RESULTS

The Results are presented in Table 1 – Table 22 below shows these computed values of the population parameters:

Table 1: Computed Values of Parameters

N/S		Values of the parameters						
		Poisson	**Hyper Geometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi – Square**
1	$\bar{Y}$	12.277	7.192	14.908	3.035	8.967	15.165	22.247
2	$\bar{X}$	12.073	7.3154	15.031	3.187	9.238	15.0188	21.697
3	S_y	3.451	1.623	2.456	2.662	4.504	2.366	6.774
4	S_x	3.345	1.5018	2.459	3.233	4.629	2.549	7.094
5	S_y^2	11.908	2.635	6.030	7.089	20.284	5.598	45.882
6	S_x^2	11.188	2.2554	6.045	10.450	21.424	6.496	50.330
7	S_{xy}	11.1877	2.35224	5.94108	8.52315	20.7438	5.95049	47.8037
8	QD	2	1	2	1.5785	3.9925	1.865	4.2645
9	ρ	0.983	0.965	0.984	0.990	0.995	0.987	0.995
10	b	1.0145	1.043	0.9827	0.81563	0.96827	0.91602	0.9498
11	D_1	8	6	12	0.34337186	2.431807199	11.98655216	13.2660441
12	D_2	9	6	13	0.705561473	4.283708332	12.77677047	15.83079446
13	D_3	10	7	14	1.130192142	6.295871773	13.51511462	17.62813422
14	D_4	11	7	14	1.71839885	8.36835318	14.06408899	19.34967226
15	D_5	12	7	15	2.274413113	9.574666074	14.83508165	20.53629098
16	D_6	13	8	16	2.702891015	11.33341484	15.50309732	22.26868765
17	D_7	14	8	16	3.474499939	12.51046248	16.25050319	24.13409374
18	D_8	15	9	17	4.993709298	13.77257233	17.25010088	27.41673898
19	D_9	17	9	18	7.782496416	15.2731954	18.53429153	31.68868868
20	D_{10}	24	11	23	15.9650015	16.98740078	25.31483454	49.30336421
21	C_x	27.70	20.53	16.36	101.44	50.11	16.97	32.70
22	C_y	28.11	22.57	16.47	87.72	50.22	15.60	30.45
23	$\emptyset_1$	145.333	52.1111	225	6.348838495	89.20075491	222.0562153	438.3731959
24	$\emptyset_2$	24.222	7.4444	30	4.406739509	37.19415289	27.91606617	91.03245491
25	B_1	0.34	0.13	0.20	1.72	-0.19	0.43	0.74
26	B_2	0.20	0.08	0.28	2.83	-1.18	0.45	0.70
27	$\bar{y}_{17}$	13.06	6.824	14.882	3.324	9.91	15.306	24.25
28	$\bar{x}_{17}$	12.824	6.941	15.176	3.612	10.20	15.224	23.63
29	$\bar{y}_{25}$	12.880	7.640	16.080	4.341	10.099	15.180	22.77
30	$\bar{x}_{25}$	12.720	7.640	16.240	4.745	10.331	15.030	22.07
31	$\bar{y}_{39}$	11.923	7.513	16.641	2.703	8.816	15.626	21.317

32	$\bar{x}_{39}$	11.769	7.641	14.821	2.773	9.063	15.459	20.666
33	$\bar{y}_{50}$	12.500	7.360	15.100	2.992	8.838	15.280	21.328
34	$\bar{x}_{50}$	12.180	7.440	15.180	3.178	9.147	15.163	20.65
35	$\bar{y}_{100}$	12.180	7.230	14.910	2.655	8.551	15.093	23.105
36	$\bar{x}_{100}$	11.940	7.350	15.030	2.707	8.819	14.948	22.669

The ratio estimators were used to estimate the population mean of the study variable ($\bar{Y}$) on the basis of the information provided by the various samples and the auxiliary information. This was done for each of the seven distributions and five samples. Their related constants, Biases and Mean Squared Errors (MSEs) were calculated.
The results obtained are as shown the Tables 2 – Table 20:

Table 2: Estimates of the population mean for sample size n=17

	Estimates						
	Poisson	**Hyper Geometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi - Square**
$\hat{\bar{Y}}_{P1}$	11.74914	7.4232241	14.638582	2.6469311	8.1794434	14.973466	20.793804
$\hat{\bar{Y}}_{P2}$	11.764984	7.4232241	14.64091	2.6656099	8.212323	14.976227	20.829442
$\hat{\bar{Y}}_{P3}$	11.779939	7.4082521	14.643133	2.6849868	8.2451121	14.978712	20.853497
$\hat{\bar{Y}}_{P4}$	11.794079	7.4082521	14.643133	2.7081641	8.2760741	14.980504	20.875863
$\hat{\bar{Y}}_{P5}$	11.807467	7.4082521	14.645258	2.7269299	8.2929211	14.982944	20.890909
$\hat{\bar{Y}}_{P6}$	11.820162	7.3952842	14.647291	2.7396973	8.3160843	14.984989	20.912355
$\hat{\bar{Y}}_{P7}$	11.832217	7.3952842	14.647291	2.759682	8.3307314	14.987204	20.934782
$\hat{\bar{Y}}_{P8}$	11.843678	7.3839433	14.649239	2.7906017	8.345734	14.990053	20.972663
$\hat{\bar{Y}}_{P9}$	11.864989	7.3839433	14.651106	2.8292268	8.3626918	14.993535	21.019149
$\hat{\bar{Y}}_{P10}$	11.926056	7.3650543	14.65939	2.8851692	8.3809841	15.00916	21.182872
$\hat{\bar{Y}}_{P11}$	11.591679	7.5858196	14.600209	2.8432646	8.1949096	14.914961	20.59213
$\hat{\bar{Y}}_{P12}$	11.590175	7.5858196	14.600092	2.781072	8.1687902	14.914826	20.590257
$\hat{\bar{Y}}_{P13}$	11.588967	7.5882664	14.599992	2.7422275	8.1573023	14.914713	20.589267
$\hat{\bar{Y}}_{P14}$	11.587976	7.5882664	14.599992	2.7124054	8.1511132	14.914637	20.588491
$\hat{\bar{Y}}_{P15}$	11.587148	7.5882664	14.599906	2.6956207	8.1487197	14.914539	20.588031
$\hat{\bar{Y}}_{P16}$	11.586445	7.5901227	14.599829	2.6865963	8.1461274	14.914462	20.587448
$\hat{\bar{Y}}_{P17}$	11.585842	7.5901227	14.599829	2.6751865	8.1447934	14.914384	20.586914
$\hat{\bar{Y}}_{P18}$	11.585319	7.5915793	14.599762	2.6619988	8.1436127	14.91429	20.586149
$\hat{\bar{Y}}_{P19}$	11.584455	7.5915793	14.599703	2.6503009	8.1424596	14.914183	20.58539
$\hat{\bar{Y}}_{P20}$	11.582558	7.5937183	14.599481	2.6387744	8.1413889	14.9138	20.583648
$\hat{\bar{Y}}_{P21}$	12.239714	7.2602404	14.73061	2.8503163	8.8916301	15.104175	22.320257
$\hat{\bar{Y}}_{P22}$	12.048802	7.4022718	14.6922	2.8195548	8.7962794	15.045113	22.036177
$\hat{\bar{Y}}_{P23}$	12.240627	7.2588218	14.730743	2.851129	8.8920202	15.104335	22.320702
$\hat{\bar{Y}}_{P24}$	12.05159	7.3988124	14.692705	2.8204259	8.7969953	15.045723	22.037678
$\hat{\bar{Y}}_{P25}$	11.787013	7.4994477	14.66563	2.6329979	8.2575502	15.007295	21.24419

$\hat{\bar{Y}}_{P26}$	11.623882	7.5843321	14.613861	2.6311948	8.1891674	14.932691	20.773749
$\hat{\bar{Y}}_{P27}$	12.277133	7.2211229	14.737634	2.8040873	8.9973306	15.111442	22.343705
$\hat{\bar{Y}}_{P28}$	12.188245	7.2565709	14.726569	2.7724067	9.025072	15.078443	22.11859
$\hat{\bar{Y}}_{P29}$	11.59377	7.5879284	14.605172	2.6273594	8.135736	14.921814	20.611453
$\hat{\bar{Y}}_{P30}$	11.595704	7.5879284	14.605686	2.6277047	8.1387642	14.922383	20.617316
$\hat{\bar{Y}}_{P31}$	11.597627	7.5854305	14.606196	2.6281087	8.1420299	14.922913	20.621402
$\hat{\bar{Y}}_{P32}$	11.59954	7.5854305	14.606196	2.6286668	8.1453671	14.923304	20.625299
$\hat{\bar{Y}}_{P33}$	11.601442	7.5854305	14.606703	2.6291926	8.1472973	14.923852	20.627974
$\hat{\bar{Y}}_{P34}$	11.603334	7.5829658	14.607205	2.6295968	8.1500954	14.924324	20.631867
$\hat{\bar{Y}}_{P35}$	11.605216	7.5829658	14.607205	2.6303223	8.1519576	14.924849	20.636039
$\hat{\bar{Y}}_{P36}$	11.607087	7.5805337	14.607704	2.6317419	8.1539451	14.925547	20.643333
$\hat{\bar{Y}}_{P3s7}$	11.6108	7.5805337	14.608199	2.6343177	8.1562958	14.926436	20.652737
$\hat{\bar{Y}}_{P38}$	11.623487	7.5757645	14.610619	2.6416586	8.1589647	14.930999	20.69048

Table 3: Estimates of the population mean for sample size n=25

	Estimates						
	Poisson	**Hyper Geometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi – Square**
$\hat{\bar{Y}}_{P1}$	11.750611	7.127685	14.082369	2.1063356	8.1374469	15.160524	22.083683
$\hat{\bar{Y}}_{P2}$	11.764345	7.127685	14.100169	2.1489342	8.1741852	15.160689	22.091429
$\hat{\bar{Y}}_{P3}$	11.777305	7.1395537	14.117203	2.1943183	8.2108584	15.160837	22.096645
$\hat{\bar{Y}}_{P4}$	11.789553	7.1395537	14.117203	2.2502703	8.2455212	15.160943	22.101486
$\hat{\bar{Y}}_{P5}$	11.801146	7.1395537	14.133519	2.2969624	8.2643954	15.161088	22.104738
$\hat{\bar{Y}}_{P6}$	11.812137	7.1499046	14.149163	2.329469	8.2903614	15.16121	22.109367
$\hat{\bar{Y}}_{P7}$	11.82257	7.1499046	14.149163	2.3816002	8.3067902	15.161341	22.1142
$\hat{\bar{Y}}_{P8}$	11.832487	7.1590114	14.164174	2.4654056	8.3236253	15.161511	22.122344
$\hat{\bar{Y}}_{P9}$	11.850919	7.1590114	14.178591	2.5758512	8.3426634	15.161717	22.132306
$\hat{\bar{Y}}_{P10}$	11.903688	7.1742937	14.242877	2.7483266	8.3632107	15.162644	22.167115
$\hat{\bar{Y}}_{P11}$	11.613851	7.0041954	13.794537	2.6176815	8.1547237	15.15703	22.039452
$\hat{\bar{Y}}_{P12}$	11.612543	7.0041954	13.793679	2.4391554	8.1255513	15.157022	22.039038
$\hat{\bar{Y}}_{P13}$	11.611491	7.0024095	13.792942	2.335984	8.1127279	15.157015	22.038819
$\hat{\bar{Y}}_{P14}$	11.610629	7.0024095	13.792942	2.2607123	8.1058211	15.15701	22.038648
$\hat{\bar{Y}}_{P15}$	11.609908	7.0024095	13.792303	2.2197592	8.1031503	15.157004	22.038546
$\hat{\bar{Y}}_{P16}$	11.609297	7.0010559	13.791743	2.1981442	8.100258	15.157	22.038417
$\hat{\bar{Y}}_{P17}$	11.608772	7.0010559	13.791743	2.1712094	8.0987697	15.156995	22.038299
$\hat{\bar{Y}}_{P18}$	11.608317	6.9999946	13.791248	2.1406122	8.0974525	15.156989	22.03813
$\hat{\bar{Y}}_{P19}$	11.607565	6.9999946	13.790809	2.1139391	8.0961662	15.156983	22.037962
$\hat{\bar{Y}}_{P20}$	11.605914	6.9984374	13.789179	2.0880764	8.0949717	15.15696	22.037577
$\hat{\bar{Y}}_{P21}$	12.17358	7.2617768	14.817283	2.6390503	8.9414014	15.16824	22.397566
$\hat{\bar{Y}}_{P22}$	12.009535	7.1443189	14.502549	2.5475667	8.8327602	15.164769	22.3418
$\hat{\bar{Y}}_{P23}$	12.174364	7.262993	14.818398	2.6415285	8.9418465	15.168249	22.397652

$\hat{\bar{Y}}_{P24}$	12.011935	7.1470818	14.506613	2.5500967	8.8335747	15.164805	22.342098
$\hat{\bar{Y}}_{P25}$	11.783432	7.0685888	14.291633	2.0752688	8.2247794	15.162533	22.180042
$\hat{\bar{Y}}_{P26}$	11.64186	7.0052821	13.895756	2.0712915	8.1483084	15.158092	22.079316
$\hat{\bar{Y}}_{P27}$	12.205651	7.2956403	14.87614	2.5032153	9.0622023	15.168665	22.402118
$\hat{\bar{Y}}_{P28}$	12.129424	7.2649246	14.783612	2.4156147	9.093971	15.166731	22.358097
$\hat{\bar{Y}}_{P29}$	11.61567	7.002656	13.831184	2.0628644	8.0886662	15.15744	22.04372
$\hat{\bar{Y}}_{P30}$	11.617353	7.002656	13.83499	2.0636214	8.0920439	15.157475	22.045013
$\hat{\bar{Y}}_{P31}$	11.619026	7.0044796	13.838769	2.0645073	8.0956868	15.157506	22.045915
$\hat{\bar{Y}}_{P32}$	11.62069	7.0044796	13.838769	2.065732	8.0994097	15.15753	22.046774
$\hat{\bar{Y}}_{P33}$	11.622345	7.0044796	13.842521	2.0668869	8.1015633	15.157562	22.047364
$\hat{\bar{Y}}_{P34}$	11.623991	7.006281	13.846247	2.0677751	8.1046854	15.157591	22.048221
$\hat{\bar{Y}}_{P35}$	11.625627	7.006281	13.846247	2.0693706	8.1067633	15.157622	22.049141
$\hat{\bar{Y}}_{P36}$	11.627255	7.0080607	13.849946	2.0724972	8.1089812	15.157664	22.050747
$\hat{\bar{Y}}_{P3s7}$	11.630485	7.0080607	13.853619	2.0781861	8.1116045	15.157717	22.052817
$\hat{\bar{Y}}_{P38}$	11.641517	7.0115565	13.871602	2.0945094	8.1145832	15.15799	22.061108

Table 4: Estimates of the population mean for sample size n=39

Estimators	Estimates						
	Poisson	**Hyper Geometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi-Square**
$\hat{\bar{Y}}_{P1}$	12.467209	7.0021758	14.997117	3.4616121	9.1480236	14.915315	23.263044
$\hat{\bar{Y}}_{P2}$	12.459962	7.0021758	14.993605	3.4316164	9.1405837	14.92114	23.239193
$\hat{\bar{Y}}_{P3}$	12.453147	7.0138706	14.990254	3.4014736	9.1332353	14.926386	23.223168
$\hat{\bar{Y}}_{P4}$	12.446727	7.0138706	14.990254	3.3666571	9.1263605	14.93017	23.208321
$\hat{\bar{Y}}_{P5}$	12.440668	7.0138706	14.987054	3.3394076	9.1226457	14.935323	23.198361
$\hat{\bar{Y}}_{P6}$	12.434941	7.02407	14.983993	3.3213283	9.1175676	14.939643	23.184205
$\hat{\bar{Y}}_{P7}$	12.429519	7.02407	14.983993	3.2937473	9.1143739	14.944325	23.16945
$\hat{\bar{Y}}_{P8}$	12.424378	7.0330435	14.981064	3.2527185	9.1111167	14.950349	23.144643
$\hat{\bar{Y}}_{P9}$	12.414858	7.0330435	14.978258	3.204086	9.1074519	14.957715	23.114395
$\hat{\bar{Y}}_{P10}$	12.387845	7.0481023	14.965826	3.1383533	9.1035186	14.990815	23.009533
$\hat{\bar{Y}}_{P11}$	12.54078	6.8805021	15.055401	3.1870883	9.144515	14.792432	23.400519
$\hat{\bar{Y}}_{P12}$	12.541497	6.8805021	15.055579	3.2651577	9.1504496	14.792148	23.401816
$\hat{\bar{Y}}_{P13}$	12.542073	6.8787426	15.055732	3.3177885	9.1530742	14.791912	23.402501
$\hat{\bar{Y}}_{P14}$	12.542546	6.8787426	15.055732	3.3604265	9.154492	14.791753	23.403039
$\hat{\bar{Y}}_{P15}$	12.54294	6.8787426	15.055865	3.3853362	9.155041	14.791549	23.403357
$\hat{\bar{Y}}_{P16}$	12.543275	6.877409	15.055981	3.399013	9.1556359	14.791389	23.403761
$\hat{\bar{Y}}_{P17}$	12.543563	6.877409	15.055981	3.4165979	9.1559423	14.791225	23.404132
$\hat{\bar{Y}}_{P18}$	12.543813	6.8763634	15.056084	3.4373421	9.1562136	14.791028	23.404662
$\hat{\bar{Y}}_{P19}$	12.544225	6.8763634	15.056175	3.4561306	9.1564786	14.790805	23.405188

$\hat{\bar{Y}}_{P20}$	12.545131	6.8748292	15.056514	3.4750113	9.1567248	14.790003	23.406395
$\hat{\bar{Y}}_{P21}$	12.255076	7.13431	14.860368	3.1786803	9.0014496	15.193772	22.346321
$\hat{\bar{Y}}_{P22}$	12.334721	7.018566	14.916931	3.2160028	9.019441	15.067271	22.502043
$\hat{\bar{Y}}_{P23}$	12.254704	7.1355085	14.860173	3.1777169	9.001377	15.194116	22.346082
$\hat{\bar{Y}}_{P24}$	12.333533	7.0212885	14.916182	3.2149225	9.0193042	15.068572	22.501203
$\hat{\bar{Y}}_{P25}$	12.449933	6.9439471	14.956485	3.484614	9.1304661	14.986859	22.970918
$\hat{\bar{Y}}_{P26}$	12.525501	6.8815729	15.034578	3.4876311	9.1458158	14.829559	23.276523
$\hat{\bar{Y}}_{P27}$	12.239874	7.1676815	14.850102	3.2354191	8.982032	15.209414	22.33374
$\hat{\bar{Y}}_{P28}$	12.276202	7.137412	14.866284	3.2766266	8.9770247	15.13852	22.456231
$\hat{\bar{Y}}_{P29}$	12.539785	6.8789855	15.04782	3.4940797	9.1580257	14.806769	23.387159
$\hat{\bar{Y}}_{P30}$	12.538864	6.8789855	15.047035	3.4934974	9.1573285	14.807962	23.383114
$\hat{\bar{Y}}_{P31}$	12.537949	6.8807822	15.046257	3.4928166	9.1565774	14.80907	23.380296
$\hat{\bar{Y}}_{P32}$	12.53704	6.8807822	15.046257	3.4918768	9.1558106	14.80989	23.377612
$\hat{\bar{Y}}_{P33}$	12.536135	6.8807822	15.045484	3.4909921	9.1553674	14.811037	23.375769
$\hat{\bar{Y}}_{P34}$	12.535236	6.882557	15.044718	3.4903127	9.1547254	14.812025	23.37309
$\hat{\bar{Y}}_{P35}$	12.534343	6.882557	15.044718	3.4890943	9.1542984	14.813125	23.37022
$\hat{\bar{Y}}_{P36}$	12.533454	6.8843104	15.043957	3.4867147	9.153843	14.814587	23.365206
$\hat{\bar{Y}}_{P3s7}$	12.531693	6.8843104	15.043203	3.4824116	9.1533046	14.81645	23.358751
$\hat{\bar{Y}}_{P38}$	12.525688	6.8877546	15.039514	3.470252	9.1526939	14.82601	23.332939

Table 5: Estimates of the population mean for sample size n=50

	Estimates						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi-square**
$\hat{\bar{Y}}_{P1}$	12.309503	7.1630137	14.84838	3.0072906	9.0093709	15.045448	23.30606
$\hat{\bar{Y}}_{P2}$	12.311959	7.1630137	14.850806	3.0067873	9.0055921	15.047406	23.281778
$\hat{\bar{Y}}_{P3}$	12.314272	7.1676555	14.853123	3.0062729	9.0018569	15.049169	23.265464
$\hat{\bar{Y}}_{P4}$	12.316455	7.1676555	14.853123	3.0056675	8.9983597	15.050439	23.250349
$\hat{\bar{Y}}_{P5}$	12.318518	7.1676555	14.855338	3.005185	8.996469	15.052169	23.24021
$\hat{\bar{Y}}_{P6}$	12.32047	7.1716961	14.857457	3.0048606	8.9938832	15.053619	23.2258
$\hat{\bar{Y}}_{P7}$	12.32232	7.1716961	14.857457	3.004359	8.9922562	15.055189	23.21078
$\hat{\bar{Y}}_{P8}$	12.324076	7.1752452	14.859486	3.0035973	8.9905963	15.057209	23.185528
$\hat{\bar{Y}}_{P9}$	12.327334	7.1752452	14.861432	3.0026699	8.988728	15.059677	23.154738
$\hat{\bar{Y}}_{P10}$	12.336615	7.1811884	14.870066	3.0013715	8.9867219	15.070747	23.048011
$\hat{\bar{Y}}_{P11}$	12.28478	7.1141507	14.808394	3.0023391	9.0075892	15.003905	23.446034
$\hat{\bar{Y}}_{P12}$	12.284541	7.1141507	14.808273	3.0038302	9.0106025	15.003808	23.447355
$\hat{\bar{Y}}_{P13}$	12.28435	7.1134364	14.808169	3.0047967	9.0119345	15.003728	23.448053
$\hat{\bar{Y}}_{P14}$	12.284192	7.1134364	14.808169	3.0055578	9.0126539	15.003674	23.4486

$\hat{\bar{Y}}_{P15}$	12.28406	7.1134364	14.808078	3.0059938	9.0129324	15.003605	23.448924
$\hat{\bar{Y}}_{P16}$	12.283949	7.1128948	14.807999	3.0062305	9.0132342	15.00355	23.449336
$\hat{\bar{Y}}_{P17}$	12.283853	7.1128948	14.807999	3.0065321	9.0133897	15.003495	23.449713
$\hat{\bar{Y}}_{P18}$	12.28377	7.1124701	14.807929	3.0068841	9.0135273	15.003428	23.450253
$\hat{\bar{Y}}_{P19}$	12.283632	7.1124701	14.807867	3.0071992	9.0136617	15.003352	23.450788
$\hat{\bar{Y}}_{P20}$	12.283331	7.1118469	14.807636	3.0075126	9.0137866	15.003079	23.452018
$\hat{\bar{Y}}_{P21}$	12.383031	7.2149146	14.944301	3.0021743	8.9343718	15.137918	22.373357
$\hat{\bar{Y}}_{P22}$	12.355025	7.1695166	14.904262	3.0028997	8.9436408	15.096193	22.531709
$\hat{\bar{Y}}_{P23}$	12.383163	7.21538	14.94444	3.0021553	8.9343343	15.138031	22.373113
$\hat{\bar{Y}}_{P24}$	12.355439	7.1705949	14.904789	3.0028789	8.9435703	15.096625	22.530855
$\hat{\bar{Y}}_{P25}$	12.315365	7.1397598	14.876569	3.0076707	9.0004485	15.069426	23.008712
$\hat{\bar{Y}}_{P26}$	12.289883	7.1145853	14.82262	3.0077202	9.0082499	15.016505	23.319783
$\hat{\bar{Y}}_{P27}$	12.388433	7.2278358	14.951624	3.0032706	8.924348	15.143045	22.360564
$\hat{\bar{Y}}_{P28}$	12.375555	7.2161188	14.940089	3.0040434	8.9217598	15.119752	22.48512
$\hat{\bar{Y}}_{P29}$	12.285112	7.113535	14.813566	3.0078257	9.0144464	15.008776	23.43243
$\hat{\bar{Y}}_{P30}$	12.285419	7.113535	14.814101	3.0078162	9.0140928	15.00918	23.428311
$\hat{\bar{Y}}_{P31}$	12.285725	7.1142644	14.814633	3.007805	9.0137118	15.009557	23.425442
$\hat{\bar{Y}}_{P32}$	12.286028	7.1142644	14.814633	3.0077897	9.0133228	15.009835	23.422708
$\hat{\bar{Y}}_{P33}$	12.28633	7.1142644	14.815161	3.0077752	9.013098	15.010224	23.420832
$\hat{\bar{Y}}_{P34}$	12.28663	7.1149846	14.815684	3.0077641	9.0127723	15.01056	23.418104
$\hat{\bar{Y}}_{P35}$	12.286928	7.1149846	14.815684	3.0077441	9.0125557	15.010933	23.415182
$\hat{\bar{Y}}_{P36}$	12.287225	7.115696	14.816204	3.0077051	9.0123246	15.011429	23.410077
$\hat{\bar{Y}}_{P3s7}$	12.287813	7.115696	14.81672	3.0076345	9.0120514	15.012061	23.403504
$\hat{\bar{Y}}_{P38}$	12.28982	7.1170926	14.819241	3.0074339	9.0117415	15.015302	23.377223

Table 6: Estimates of the population mean for sample size n=100

	Estimates						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi - Square**
$\hat{\bar{Y}}_{P1}$	12.417682	7.1752673	14.911692	3.5465215	9.3547558	15.206759	21.345454
$\hat{\bar{Y}}_{P2}$	12.414557	7.1752673	14.911675	3.5101452	9.3360908	15.205794	21.36452
$\hat{\bar{Y}}_{P3}$	12.411616	7.1765666	14.91166	3.473708	9.3176985	15.204926	21.377371
$\hat{\bar{Y}}_{P4}$	12.408844	7.1765666	14.91166	3.431767	9.3005304	15.2043	21.389307
$\hat{\bar{Y}}_{P5}$	12.406226	7.1765666	14.911645	3.3990499	9.2912692	15.203448	21.397331
$\hat{\bar{Y}}_{P6}$	12.403751	7.1776966	14.91163	3.3773955	9.2786269	15.202735	21.408756
$\hat{\bar{Y}}_{P7}$	12.401406	7.1776966	14.91163	3.3444403	9.2706865	15.201962	21.420693

$\hat{\bar{Y}}_{P8}$	12.399182	7.1786884	14.911616	3.2955956	9.2625963	15.200969	21.440827
$\hat{\bar{Y}}_{P9}$	12.39506	7.1786884	14.911603	3.2379731	9.2535036	15.199756	21.465487
$\hat{\bar{Y}}_{P10}$	12.383345	7.1803477	14.911545	3.1605588	9.2437564	15.194321	21.551923
$\hat{\bar{Y}}_{P11}$	12.449292	7.1615162	14.911964	3.2179032	9.345948	15.227317	21.236968
$\hat{\bar{Y}}_{P12}$	12.449598	7.1615162	14.911965	3.310382	9.3608518	15.227365	21.235956
$\hat{\bar{Y}}_{P13}$	12.449845	7.1613142	14.911965	3.3731605	9.3674522	15.227405	21.235421
$\hat{\bar{Y}}_{P14}$	12.450047	7.1613142	14.911965	3.4242778	9.3710197	15.227432	21.235002
$\hat{\bar{Y}}_{P15}$	12.450216	7.1613142	14.911966	3.454249	9.3724016	15.227466	21.234753
$\hat{\bar{Y}}_{P16}$	12.45036	7.161161	14.911967	3.4707388	9.3738995	15.227493	21.234438
$\hat{\bar{Y}}_{P17}$	12.450483	7.161161	14.911967	3.4919759	9.374671	15.227521	21.234149
$\hat{\bar{Y}}_{P18}$	12.45059	7.1610408	14.911967	3.5170798	9.3753541	15.227554	21.233736
$\hat{\bar{Y}}_{P19}$	12.450766	7.1610408	14.911968	3.5398653	9.3760215	15.227592	21.233325
$\hat{\bar{Y}}_{P20}$	12.451154	7.1608645	14.911969	3.5628088	9.3766415	15.227728	21.232384
$\hat{\bar{Y}}_{P21}$	12.325343	7.1897261	14.911045	3.2079888	8.9949919	15.161598	22.135029
$\hat{\bar{Y}}_{P22}$	12.360222	7.1770872	14.911314	3.2520653	9.0382657	15.181874	21.992169
$\hat{\bar{Y}}_{P23}$	12.325179	7.1898551	14.911044	3.2068532	8.9948176	15.161544	22.135252
$\hat{\bar{Y}}_{P24}$	12.359703	7.1773887	14.91131	3.2507871	9.0379358	15.181664	21.992929
$\hat{\bar{Y}}_{P25}$	12.410228	7.1687401	14.911501	3.5744957	9.3107786	15.194969	21.58413
$\hat{\bar{Y}}_{P26}$	12.442744	7.1616391	14.911867	3.5781701	9.3492123	15.221064	21.334711
$\hat{\bar{Y}}_{P27}$	12.318656	7.1933024	14.910996	3.2750643	8.9485584	15.159118	22.14674
$\hat{\bar{Y}}_{P28}$	12.334619	7.1900598	14.911073	3.3240323	8.9366299	15.170406	22.0338
$\hat{\bar{Y}}_{P29}$	12.448865	7.1613421	14.911929	3.5860277	9.3799188	15.224898	21.247407
$\hat{\bar{Y}}_{P30}$	12.448471	7.1613421	14.911925	3.5853179	9.3781623	15.224697	21.250572
$\hat{\bar{Y}}_{P31}$	12.448079	7.1615483	14.911921	3.5844881	9.3762703	15.22451	21.252778
$\hat{\bar{Y}}_{P32}$	12.447689	7.1615483	14.911921	3.5833428	9.3743392	15.224372	21.25488
$\hat{\bar{Y}}_{P33}$	12.447302	7.1615483	14.911918	3.5822648	9.3732233	15.224179	21.256324
$\hat{\bar{Y}}_{P34}$	12.446917	7.161752	14.911914	3.5814369	9.3716071	15.224013	21.258424
$\hat{\bar{Y}}_{P35}$	12.446534	7.161752	14.911914	3.5799525	9.3705325	15.223827	21.260674
$\hat{\bar{Y}}_{P36}$	12.446153	7.1619531	14.911911	3.577054	9.3693863	15.223581	21.264608
$\hat{\bar{Y}}_{P3s7}$	12.445399	7.1619531	14.911907	3.5718142	9.3680318	15.223268	21.269677
$\hat{\bar{Y}}_{P38}$	12.442824	7.162348	14.91189	3.557021	9.3664953	15.22166	21.289999

The related constants of each of the estimators where computed for the different samples and distribution. It was discovered that the relative constant of an estimator in a particular distribution is not affected by the sample size, it is independent of the sample size. They are shown in the table below:

Table 7: Related constants of the estimators for the different distributions

	Related Constants						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi - Square**
$\hat{\bar{Y}}_{P1}$	0.751135	0.5493939	0.7147069	0.9361056	0.9381448	0.7002996	0.8746036
$\hat{\bar{Y}}_{P2}$	0.728474	0.5493939	0.6981097	0.8769993	0.8959417	0.6867313	0.8539024
$\hat{\bar{Y}}_{P3}$	0.70714	0.5110161	0.6822659	0.8165532	0.85419	0.6745203	0.8399697
$\hat{\bar{Y}}_{P4}$	0.68702	0.5110161	0.6822659	0.7453876	0.8150684	0.665719	0.8270444
$\hat{\bar{Y}}_{P5}$	0.668013	0.5110161	0.6671253	0.6886538	0.7939043	0.6537391	0.8183644
$\hat{\bar{Y}}_{P6}$	0.65003	0.4776499	0.6526421	0.6504989	0.7649456	0.6437025	0.8060144
$\hat{\bar{Y}}_{P7}$	0.632989	0.4776499	0.6526421	0.5914842	0.7467168	0.6328323	0.7931262
$\hat{\bar{Y}}_{P8}$	0.616819	0.4483739	0.6387744	0.501844	0.7281119	0.6188553	0.7714197
$\hat{\bar{Y}}_{P9}$	0.586837	0.4483739	0.6254838	0.3926177	0.7071627	0.6017801	0.7448895
$\hat{\bar{Y}}_{P10}$	0.501516	0.3994125	0.5665448	0.2396054	0.68466	0.5252585	0.6523776
$\hat{\bar{Y}}_{P11}$	0.979713	0.9564198	0.9890334	0.3536557	0.9182507	0.9890118	0.9930995
$\hat{\bar{Y}}_{P12}$	0.981926	0.9564198	0.9898684	0.5292594	0.9518917	0.9896844	0.994211
$\hat{\bar{Y}}_{P13}$	0.983704	0.9624115	0.9905853	0.6429797	0.966756	0.9902425	0.9947981
$\hat{\bar{Y}}_{P14}$	0.985164	0.9624115	0.9905853	0.7324965	0.9747815	0.9906198	0.9952588
$\hat{\bar{Y}}_{P15}$	0.986383	0.9624115	0.9912074	0.78375	0.9778885	0.9911029	0.9955315
$\hat{\bar{Y}}_{P16}$	0.987417	0.9669548	0.9917524	0.8115716	0.9812555	0.991483	0.9958777
$\hat{\bar{Y}}_{P17}$	0.988306	0.9669548	0.9917524	0.8470156	0.9829891	0.9918716	0.9961951
$\hat{\bar{Y}}_{P18}$	0.989077	0.9705183	0.9922338	0.8883614	0.9845238	0.992339	0.9966491
$\hat{\bar{Y}}_{P19}$	0.990349	0.9705183	0.9926621	0.925381	0.9860232	0.992866	0.9970996
$\hat{\bar{Y}}_{P20}$	0.993145	0.9757486	0.9942481	0.9621788	0.9874158	0.9947668	0.9981339
$\hat{\bar{Y}}_{P21}$	0.077996	0.1210237	0.0621086	0.3182881	0.0910922	0.0639673	0.0483557
$\hat{\bar{Y}}_{P22}$	0.338256	0.4872827	0.3310608	0.3996713	0.1931205	0.353216	0.1973486
$\hat{\bar{Y}}_{P23}$	0.07677	0.1172932	0.0611762	0.316162	0.0906793	0.0631878	0.0481252
$\hat{\bar{Y}}_{P24}$	0.334397	0.4785291	0.327513	0.3973422	0.1923462	0.3502168	0.196551
$\hat{\bar{Y}}_{P25}$	0.708847	0.7298792	0.5179256	0.9339661	0.8138428	0.5396178	0.6337652
$\hat{\bar{Y}}_{P26}$	0.948218	0.9367033	0.8839744	0.9395	0.8984745	0.9100989	0.9087505
$\hat{\bar{Y}}_{P27}$	0.027933	0.0176201	0.0130768	0.441267	0.0194833	0.0285363	0.0362274
$\hat{\bar{Y}}_{P28}$	0.147358	0.111371	0.0903345	0.5279126	0.0480751	0.189707	0.153731
$\hat{\bar{Y}}_{P29}$	0.976637	0.9615841	0.9534716	0.998939	0.9947742	0.95508	0.9816452

$\hat{\bar{Y}}_{P30}$	0.973793	0.9615841	0.949789	0.9978223	0.9908311	0.95226	0.9781741
$\hat{\bar{Y}}_{P31}$	0.970966	0.9554665	0.9461346	0.9965163	0.986582	0.9496401	0.9757562
$\hat{\bar{Y}}_{P32}$	0.968155	0.9554665	0.9461346	0.9947127	0.9822435	0.9477015	0.9734515
$\hat{\bar{Y}}_{P33}$	0.96536	0.9554665	0.9425083	0.9930139	0.9797358	0.9449922	0.9718692
$\hat{\bar{Y}}_{P34}$	0.962582	0.9494264	0.9389097	0.9917087	0.9761024	0.9426573	0.9695683
$\hat{\bar{Y}}_{P35}$	0.959819	0.9494264	0.9389097	0.9893669	0.9736858	0.9400585	0.967103
$\hat{\bar{Y}}_{P36}$	0.957072	0.9434621	0.9353384	0.9847884	0.9711078	0.9366051	0.9627949
$\hat{\bar{Y}}_{P3s7}$	0.951625	0.9434621	0.9317942	0.976493	0.9680604	0.9322056	0.9572457
$\hat{\bar{Y}}_{P38}$	0.93304	0.9317555	0.9144686	0.9529409	0.9646025	0.9096448	0.9350242

The biases of the estimators were equally computed for each of the seven distributions and five samples, the results are presented in the following tables:

Table 8: Biases of the different estimators for each distribution at n = 17

	Biases						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi -square**
$\hat{\bar{Y}}_{P1}$	0.0282671	0.005204	0.0113872	0.1658789	0.1156051	0.0115493	0.0951398
$\hat{\bar{Y}}_{P2}$	0.0265872	0.005204	0.0108645	0.1455929	0.1054379	0.0111061	0.0906893
$\hat{\bar{Y}}_{P3}$	0.0250527	0.004502	0.0103769	0.1262149	0.0958399	0.0107146	0.087754
$\hat{\bar{Y}}_{P4}$	0.0236474	0.004502	0.0103769	0.1051734	0.0872621	0.0104369	0.0850741
$\hat{\bar{Y}}_{P5}$	0.022357	0.004502	0.0099215	0.0897725	0.0827892	0.0100646	0.0832977
$\hat{\bar{Y}}_{P6}$	0.0211695	0.003934	0.0094954	0.0801004	0.0768597	0.0097579	0.0808025
$\hat{\bar{Y}}_{P7}$	0.0200741	0.003934	0.0094954	0.0662259	0.0732402	0.0094312	0.0782391
$\hat{\bar{Y}}_{P8}$	0.0190616	0.003466	0.0090961	0.0476737	0.069636	0.0090192	0.0740152
$\hat{\bar{Y}}_{P9}$	0.0172536	0.003466	0.0087215	0.0291797	0.0656865	0.0085283	0.0690118
$\hat{\bar{Y}}_{P10}$	0.0126013	0.00275	0.0071553	**0.0108676**	0.0615726	0.0064973	0.0529343
$\hat{\bar{Y}}_{P11}$	0.0480886	0.015771	0.0218063	0.0236757	0.1107541	0.0230351	0.1226662
$\hat{\bar{Y}}_{P12}$	0.0483061	0.015771	0.0218432	0.0530248	0.119018	0.0230665	0.122941
$\hat{\bar{Y}}_{P13}$	0.0484812	0.015969	0.0218748	0.0782593	0.122764	0.0230925	0.1230862
$\hat{\bar{Y}}_{P14}$	0.0486251	0.015969	0.0218748	0.101567	0.1248107	0.0231101	0.1232002
$\hat{\bar{Y}}_{P15}$	0.0487456	0.015969	0.0219023	0.1162778	0.1256077	0.0231326	0.1232678
$\hat{\bar{Y}}_{P16}$	0.0488479	0.01612	0.0219264	0.1246796	0.1264741	0.0231504	0.1233535
$\hat{\bar{Y}}_{P17}$	0.0489358	0.01612	0.0219264	0.1358077	0.1269214	0.0231685	0.1234321
$\hat{\bar{Y}}_{P18}$	0.0490122	0.016239	0.0219477	0.1493898	0.127318	0.0231904	0.1235447
$\hat{\bar{Y}}_{P19}$	0.0491384	0.016239	0.0219666	0.1620999	0.1277061	0.023215	0.1236564
$\hat{\bar{Y}}_{P20}$	0.0494162	0.016415	0.0220369	0.1752481	0.1280671	0.023304	0.1239131

$\hat{\hat{Y}}_{P21}$	0.0003048	0.000253	8.599E-05	0.0191771	0.0010899	9.636E-05	0.0002908
$\hat{\hat{Y}}_{P22}$	0.0057324	0.004094	0.0024433	0.0302376	0.0048988	0.0029381	0.004844
$\hat{\hat{Y}}_{P23}$	0.0002953	0.000237	8.343E-05	0.0189217	0.0010801	9.403E-05	0.0002881
$\hat{\hat{Y}}_{P24}$	0.0056023	0.003948	0.0023912	0.0298862	0.0048596	0.0028884	0.004805
$\hat{\hat{Y}}_{P25}$	0.0251738	0.009185	0.0059799	0.1651216	0.0869998	0.0068574	0.049957
$\hat{\hat{Y}}_{P26}$	0.0450465	0.015127	0.0174197	0.1670841	0.1060349	0.0195058	0.1027138
$\hat{\hat{Y}}_{P27}$	**3.91E-05**	**5.35E-06**	**3.812E-06**	0.0368591	**4.986E-05**	**1.918E-05**	**0.0001632**
$\hat{\hat{Y}}_{P28}$	0.0010879	0.000214	0.0001819	0.0527552	0.0003036	0.0008475	0.0029394
$\hat{\hat{Y}}_{P29}$	0.0477871	0.015942	0.0202664	0.1888946	0.129983	0.0214816	0.1198529
$\hat{\hat{Y}}_{P30}$	0.0475092	0.015942	0.0201101	0.1884725	0.1289545	0.021355	0.1190068
$\hat{\hat{Y}}_{P31}$	0.0472337	0.015739	0.0199557	0.1879795	0.1278509	0.0212376	0.1184192
$\hat{\hat{Y}}_{P32}$	0.0469606	0.015739	0.0199557	0.1872996	0.1267289	0.021151	0.1178604
$\hat{\hat{Y}}_{P33}$	0.0466899	0.015739	0.019803	0.1866604	0.1260826	0.0210302	0.1174776
$\hat{\hat{Y}}_{P34}$	0.0464215	0.015541	0.0196521	0.1861701	0.1251492	0.0209264	0.116922
$\hat{\hat{Y}}_{P35}$	0.0461554	0.015541	0.0196521	0.1852919	0.1245303	0.0208112	0.1163282
$\hat{\hat{Y}}_{P36}$	0.0458916	0.015346	0.0195029	0.1835809	0.1238718	0.0206586	0.1152941
$\hat{\hat{Y}}_{P3s7}$	0.0453708	0.015346	0.0193553	0.1805011	0.1230955	0.020465	0.1139689
$\hat{\hat{Y}}_{P38}$	0.0436159	0.014968	0.0186423	0.1718991	0.1222177	0.0194864	0.108739

It was discovered that the twenty-seventh estimator has the least bias in each of the seven distributions considered. These values are shown in bold print.

Table 9: Biases of the different estimators for each distribution at n = 25

	Biases						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi – Square**
$\hat{\hat{Y}}_{P1}$	0.0185888	0.0034221	0.0074884	0.1090842	0.0760235	0.007595	0.0625652
$\hat{\hat{Y}}_{P2}$	0.0174841	0.0034221	0.0071446	0.0957438	0.0693374	0.0073035	0.0596385
$\hat{\hat{Y}}_{P3}$	0.016475	0.0029607	0.006824	0.0830006	0.0630256	0.0070461	0.0577082
$\hat{\hat{Y}}_{P4}$	0.0155508	0.0029607	0.006824	0.0691634	0.0573847	0.0068634	0.0559458
$\hat{\hat{Y}}_{P5}$	0.0147023	0.0029607	0.0065245	0.0590356	0.0544433	0.0066186	0.0547777
$\hat{\hat{Y}}_{P6}$	0.0147023	0.0025867	0.0062443	0.0526751	0.0505439	0.0064169	0.0531368
$\hat{\hat{Y}}_{P7}$	0.0139213	0.0025867	0.0062443	0.043551	0.0481637	0.0062021	0.0514511
$\hat{\hat{Y}}_{P8}$	0.0125352	0.0022793	0.0059817	0.0313509	0.0457935	0.0059311	0.0486734
$\hat{\hat{Y}}_{P9}$	0.0113462	0.0022793	0.0057354	0.019189	0.0431963	0.0056083	0.0453831
$\hat{\hat{Y}}_{P10}$	0.0082868	0.0018087	0.0047054	**0.0071467**	0.0404909	0.0042727	0.0348103
$\hat{\hat{Y}}_{P11}$	0.0316237	0.0103711	0.0143401	0.0155695	0.0728334	0.0151482	0.0806669
$\hat{\hat{Y}}_{P12}$	0.0317667	0.0103711	0.0143644	0.0348698	0.0782678	0.0151688	0.0808476

$\hat{\hat{Y}}_{P13}$	0.0318819	0.0105015	0.0143852	0.0514644	0.0807312	0.0151859	0.0809431
$\hat{\hat{Y}}_{P14}$	0.0319765	0.0105015	0.0143852	0.0667918	0.0820772	0.0151975	0.0810181
$\hat{\hat{Y}}_{P15}$	0.0320557	0.0105015	0.0144033	0.0764658	0.0826012	0.0152123	0.0810625
$\hat{\hat{Y}}_{P16}$	0.0320557	0.0106009	0.0144191	0.0819909	0.083171	0.015224	0.0811189
$\hat{\hat{Y}}_{P17}$	0.032123	0.0106009	0.0144191	0.0893089	0.0834652	0.0152359	0.0811706
$\hat{\hat{Y}}_{P18}$	0.0322311	0.0106791	0.0144331	0.0982407	0.083726	0.0152503	0.0812446
$\hat{\hat{Y}}_{P19}$	0.0323141	0.0106791	0.0144456	0.106599	0.0839812	0.0152665	0.0813181
$\hat{\hat{Y}}_{P20}$	0.0324967	0.0107945	0.0144918	0.1152454	0.0842186	0.015325	0.0814869
$\hat{\hat{Y}}_{P21}$	0.0002004	0.0001661	5.655E-05	0.0126111	0.0007168	6.337E-05	0.0001913
$\hat{\hat{Y}}_{P22}$	0.0037697	0.0026921	0.0016067	0.0198847	0.0032215	0.0019321	0.0031855
$\hat{\hat{Y}}_{P23}$	0.0001942	0.000156	5.487E-05	0.0124432	0.0007103	6.183E-05	0.0001894
$\hat{\hat{Y}}_{P24}$	0.0036842	0.0025962	0.0015725	0.0196536	0.0031958	0.0018995	0.0031598
$\hat{\hat{Y}}_{P25}$	0.0165546	0.0060399	0.0039325	0.1085861	0.0572122	0.0045095	0.0328524
$\hat{\hat{Y}}_{P26}$	0.0296232	0.0099479	0.0114554	0.1098767	0.06973	0.0128273	0.0675459
$\hat{\hat{Y}}_{P27}$	**2.571E-05**	**3.52E-06**	**2.507E-06**	0.024239	**3.279E-05**	**1.261E-05**	**0.0001073**
$\hat{\hat{Y}}_{P28}$	0.0007154	0.0001406	0.0001196	0.0346925	0.0001996	0.0005573	0.001933
$\hat{\hat{Y}}_{P29}$	0.0314254	0.0104834	0.0133274	0.1242196	0.0854785	0.0141266	0.0788169
$\hat{\hat{Y}}_{P30}$	0.0312427	0.0104834	0.0132247	0.123942	0.0848022	0.0140433	0.0782604
$\hat{\hat{Y}}_{P31}$	0.0310615	0.0103505	0.0131231	0.1236178	0.0840764	0.0139661	0.077874
$\hat{\hat{Y}}_{P32}$	0.0308819	0.0103505	0.0131231	0.1231707	0.0833386	0.0139092	0.0775066
$\hat{\hat{Y}}_{P33}$	0.0307039	0.0103505	0.0130227	0.1227504	0.0829136	0.0138298	0.0772548
$\hat{\hat{Y}}_{P34}$	0.0305274	0.01022	0.0129235	0.1224279	0.0822998	0.0137615	0.0768895
$\hat{\hat{Y}}_{P35}$	0.0303524	0.01022	0.0129235	0.1218504	0.0818928	0.0136857	0.0764989
$\hat{\hat{Y}}_{P36}$	0.0301789	0.010092	0.0128253	0.1207252	0.0814597	0.0135854	0.0758189
$\hat{\hat{Y}}_{P3s7}$	0.0298364	0.010092	0.0127283	0.1186999	0.0809492	0.013458	0.0749474
$\hat{\hat{Y}}_{P38}$	0.0286824	0.0098431	0.0122594	0.1130431	0.080372	0.0128145	0.0715082

Table 10: Biases of the different estimators for each distribution at n = 39

	Biases						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi – Square**
$\hat{\hat{Y}}_{P1}$	0.011206	0.002063	0.0045143	0.06576	0.0458297	0.0045785	0.0377166
$\hat{\hat{Y}}_{P2}$	0.01054	0.002063	0.004307	0.0577179	0.0417991	0.0044028	0.0359523
$\hat{\hat{Y}}_{P3}$	0.0099317	0.0017848	0.0041138	0.0500358	0.0379941	0.0042476	0.0347886
$\hat{\hat{Y}}_{P4}$	0.0093746	0.0017848	0.0041138	0.0416943	0.0345936	0.0041375	0.0337262
$\hat{\hat{Y}}_{P5}$	0.0088631	0.0017848	0.0039332	0.0355888	0.0328204	0.0039899	0.033022

$\hat{\bar{Y}}_{P6}$	0.0083923	0.0015594	0.0037643	0.0317545	0.0304697	0.0038684	0.0320328
$\hat{\bar{Y}}_{P7}$	0.0079581	0.0015594	0.0037643	0.0262542	0.0290348	0.0037388	0.0310166
$\hat{\bar{Y}}_{P8}$	0.0075567	0.0013741	0.003606	0.0188995	0.027606	0.0035755	0.0293421
$\hat{\bar{Y}}_{P9}$	0.0068399	0.0013741	0.0034575	0.0115678	0.0260403	0.0033809	0.0273586
$\hat{\bar{Y}}_{P10}$	0.0049956	0.0010904	0.0028366	**0.0043083**	0.0244094	0.0025758	0.0209849
$\hat{\bar{Y}}_{P11}$	0.0190639	0.0062521	0.0086448	0.0093858	0.0439066	0.0091319	0.048629
$\hat{\bar{Y}}_{P12}$	0.0191501	0.0062521	0.0086594	0.0210208	0.0471827	0.0091443	0.0487379
$\hat{\bar{Y}}_{P13}$	0.0192196	0.0063307	0.0086719	0.0310246	0.0486678	0.0091546	0.0487955
$\hat{\bar{Y}}_{P14}$	0.0192766	0.0063307	0.0086719	0.0402646	0.0494792	0.0091616	0.0488407
$\hat{\bar{Y}}_{P15}$	0.0193244	0.0063307	0.0086828	0.0460964	0.0497951	0.0091706	0.0488675
$\hat{\bar{Y}}_{P16}$	0.0193649	0.0063906	0.0086924	0.0494272	0.0501386	0.0091776	0.0489015
$\hat{\bar{Y}}_{P17}$	0.0193998	0.0063906	0.0086924	0.0538387	0.0503159	0.0091848	0.0489326
$\hat{\bar{Y}}_{P18}$	0.0194301	0.0064378	0.0087008	0.0592231	0.0504731	0.0091934	0.0489772
$\hat{\bar{Y}}_{P19}$	0.0194801	0.0064378	0.0087083	0.0642618	0.050627	0.0092032	0.0490215
$\hat{\bar{Y}}_{P20}$	0.0195902	0.0065073	0.0087362	0.0694742	0.0507701	0.0092385	0.0491233
$\hat{\bar{Y}}_{P21}$	0.0001208	0.0001001	3.409E-05	0.0076024	0.0004321	3.82E-05	0.0001153
$\hat{\bar{Y}}_{P22}$	0.0022725	0.0016229	0.0009686	0.0119872	0.0019421	0.0011648	0.0019203
$\hat{\bar{Y}}_{P23}$	0.0001171	9.403E-05	3.307E-05	0.0075012	0.0004282	3.728E-05	0.0001142
$\hat{\bar{Y}}_{P24}$	0.0022209	0.0015651	0.000948	0.0118479	0.0019265	0.0011451	0.0019048
$\hat{\bar{Y}}_{P25}$	0.0099797	0.0036411	0.0023706	0.0654597	0.0344896	0.0027185	0.0198046
$\hat{\bar{Y}}_{P26}$	0.0178579	0.005997	0.0069057	0.0662377	0.0420358	0.0077328	0.0407192
$\hat{\bar{Y}}_{P27}$	**1.55E-05**	**2.122E-06**	**1.511E-06**	0.0146122	**1.977E-05**	**7.602E-06**	**6.471E-05**
$\hat{\bar{Y}}_{P28}$	0.0004313	8.478E-05	7.212E-05	0.0209139	0.0001204	0.000336	0.0011653
$\hat{\bar{Y}}_{P29}$	0.0189444	0.0063198	0.0080343	0.0748841	0.0515296	0.008516	0.0475137
$\hat{\bar{Y}}_{P30}$	0.0188342	0.0063198	0.0079723	0.0747168	0.0511219	0.0084658	0.0471783
$\hat{\bar{Y}}_{P31}$	0.018725	0.0062396	0.0079111	0.0745213	0.0506844	0.0084193	0.0469453
$\hat{\bar{Y}}_{P32}$	0.0186168	0.0062396	0.0079111	0.0742518	0.0502396	0.008385	0.0467238
$\hat{\bar{Y}}_{P33}$	0.0185094	0.0062396	0.0078506	0.0739984	0.0499834	0.0083371	0.0465721
$\hat{\bar{Y}}_{P34}$	0.018403	0.006161	0.0077907	0.073804	0.0496133	0.0082959	0.0463518
$\hat{\bar{Y}}_{P35}$	0.0182976	0.006161	0.0077907	0.0734559	0.049368	0.0082503	0.0461164
$\hat{\bar{Y}}_{P36}$	0.018193	0.0060838	0.0077316	0.0727776	0.0491069	0.0081898	0.0457064
$\hat{\bar{Y}}_{P3s7}$	0.0179865	0.0060838	0.0076731	0.0715567	0.0487992	0.008113	0.0451811
$\hat{\bar{Y}}_{P38}$	0.0172908	0.0059338	0.0073904	0.0681465	0.0484512	0.0077251	0.0431078

Table 11: Biases of the different estimators for each distribution at n = 50

	Biases						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi - Square**
$\hat{\bar{Y}}_{P1}$	0.0083056	0.001529	0.0033459	0.0487397	0.0339679	0.0033935	0.0279546
$\hat{\bar{Y}}_{P2}$	0.007812	0.001529	0.0031923	0.0427791	0.0309805	0.0032633	0.026647
$\hat{\bar{Y}}_{P3}$	0.0073612	0.0013229	0.003049	0.0370854	0.0281604	0.0031483	0.0257845
$\hat{\bar{Y}}_{P4}$	0.0069482	0.0013229	0.003049	0.0309028	0.02564	0.0030666	0.0249971
$\hat{\bar{Y}}_{P5}$	0.0065691	0.0013229	0.0029152	0.0263776	0.0243257	0.0029573	0.0244751
$\hat{\bar{Y}}_{P6}$	0.0062202	0.0011558	0.00279	0.0235357	0.0225835	0.0028671	0.023742
$\hat{\bar{Y}}_{P7}$	0.0058983	0.0011558	0.00279	0.019459	0.0215199	0.0027711	0.0229888
$\hat{\bar{Y}}_{P8}$	0.0056008	0.0010184	0.0026727	0.0140078	0.0204609	0.0026501	0.0217477
$\hat{\bar{Y}}_{P9}$	0.0050696	0.0010184	0.0025626	0.0085738	0.0193005	0.0025059	0.0202775
$\hat{\bar{Y}}_{P10}$	0.0037026	0.0008082	0.0021024	**0.0031932**	0.0180917	0.0019091	0.0155535
$\hat{\bar{Y}}_{P11}$	0.0141297	0.0046339	0.0064073	0.0069566	0.0325426	0.0067683	0.0360427
$\hat{\bar{Y}}_{P12}$	0.0141936	0.0046339	0.0064181	0.0155801	0.0349707	0.0067776	0.0361234
$\hat{\bar{Y}}_{P13}$	0.0142451	0.0046921	0.0064274	0.0229947	0.0360714	0.0067852	0.0361661
$\hat{\bar{Y}}_{P14}$	0.0142874	0.0046921	0.0064274	0.0298431	0.0366728	0.0067904	0.0361996
$\hat{\bar{Y}}_{P15}$	0.0143228	0.0046921	0.0064355	0.0341656	0.0369069	0.006797	0.0362194
$\hat{\bar{Y}}_{P16}$	0.0143528	0.0047366	0.0064426	0.0366342	0.0371615	0.0068022	0.0362446
$\hat{\bar{Y}}_{P17}$	0.0143787	0.0047366	0.0064426	0.039904	0.0372929	0.0068075	0.0362677
$\hat{\bar{Y}}_{P18}$	0.0144011	0.0047715	0.0064488	0.0438948	0.0374095	0.006814	0.0363008
$\hat{\bar{Y}}_{P19}$	0.0144382	0.0047715	0.0064544	0.0476294	0.0375235	0.0068212	0.0363336
$\hat{\bar{Y}}_{P20}$	0.0145198	0.0048231	0.006475	0.0514926	0.0376296	0.0068473	0.036409
$\hat{\bar{Y}}_{P21}$	8.955E-05	7.42E-05	2.527E-05	0.0056347	0.0003203	2.831E-05	8.545E-05
$\hat{\bar{Y}}_{P22}$	0.0016843	0.0012029	0.0007179	0.0088846	0.0014394	0.0008633	0.0014233
$\hat{\bar{Y}}_{P23}$	8.676E-05	6.969E-05	2.451E-05	0.0055597	0.0003174	2.763E-05	8.464E-05
$\hat{\bar{Y}}_{P24}$	0.0016461	0.00116	0.0007026	0.0087814	0.0014279	0.0008487	0.0014118
$\hat{\bar{Y}}_{P25}$	0.0073968	0.0026987	0.0017571	0.0485172	0.0255629	0.0020149	0.0146787
$\hat{\bar{Y}}_{P26}$	0.0132359	0.0044448	0.0051184	0.0490938	0.0311559	0.0057313	0.0301801
$\hat{\bar{Y}}_{P27}$	**1.149E-05**	**1.573E-06**	**1.12E-06**	0.0108302	**1.465E-05**	**5.635E-06**	**4.796E-05**
$\hat{\bar{Y}}_{P28}$	0.0003197	6.283E-05	5.345E-05	0.0155009	8.92E-05	0.000249	0.0008637
$\hat{\bar{Y}}_{P29}$	0.0140411	0.0046841	0.0059548	0.0555024	0.0381925	0.0063119	0.035216
$\hat{\bar{Y}}_{P30}$	0.0139595	0.0046841	0.0059089	0.0553783	0.0378903	0.0062747	0.0349674
$\hat{\bar{Y}}_{P31}$	0.0138785	0.0046247	0.0058635	0.0552335	0.0375661	0.0062402	0.0347948
$\hat{\bar{Y}}_{P32}$	0.0137983	0.0046247	0.0058635	0.0550337	0.0372364	0.0062147	0.0346306

$\hat{\bar{Y}}_{P33}$	0.0137188	0.0046247	0.0058187	0.0548459	0.0370465	0.0061793	0.0345181
$\hat{\bar{Y}}_{P34}$	0.0136399	0.0045664	0.0057743	0.0547018	0.0367722	0.0061488	0.0343549
$\hat{\bar{Y}}_{P35}$	0.0135617	0.0045664	0.0057743	0.0544438	0.0365904	0.0061149	0.0341804
$\hat{\bar{Y}}_{P36}$	0.0134842	0.0045092	0.0057305	0.053941	0.0363969	0.0060701	0.0338765
$\hat{\bar{Y}}_{P3s7}$	0.0133312	0.0045092	0.0056871	0.0530361	0.0361688	0.0060132	0.0334872
$\hat{\bar{Y}}_{P38}$	0.0128155	0.004398	0.0054776	0.0505086	0.0359109	0.0057256	0.0319505

Table 12: Biases of the different estimators for each distribution at n = 100

	Biases						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi - Square**
$\hat{\bar{Y}}_{P1}$	0.0031641	0.0005825	0.0012746	0.0185675	0.0129402	0.0012928	0.0106494
$\hat{\bar{Y}}_{P2}$	0.002976	0.0005825	0.0012161	0.0162968	0.0118021	0.0012432	0.0101512
$\hat{\bar{Y}}_{P3}$	0.0028043	0.000504	0.0011615	0.0141278	0.0107278	0.0011993	0.0098227
$\hat{\bar{Y}}_{P4}$	0.0026469	0.000504	0.0011615	0.0117725	0.0097676	0.0011682	0.0095227
$\hat{\bar{Y}}_{P5}$	0.0025025	0.000504	0.0011106	0.0100486	0.0092669	0.0011266	0.0093239
$\hat{\bar{Y}}_{P6}$	0.0023696	0.0004403	0.0010629	0.008966	0.0086032	0.0010922	0.0090446
$\hat{\bar{Y}}_{P7}$	0.002247	0.0004403	0.0010629	0.0074129	0.0081981	0.0010557	0.0087576
$\hat{\bar{Y}}_{P8}$	0.0021336	0.000388	0.0010182	0.0053363	0.0077946	0.0010096	0.0082848
$\hat{\bar{Y}}_{P9}$	0.0019313	0.000388	0.0009762	0.0032662	0.0073526	0.0009546	0.0077248
$\hat{\bar{Y}}_{P10}$	0.0014105	0.0003079	0.0008009	**0.0012165**	0.0068921	0.0007273	0.0059252
$\hat{\bar{Y}}_{P11}$	0.0053828	0.0017653	0.0024409	0.0026501	0.0123972	0.0025784	0.0137305
$\hat{\bar{Y}}_{P12}$	0.0054071	0.0017653	0.002445	0.0059353	0.0133222	0.0025819	0.0137613
$\hat{\bar{Y}}_{P13}$	0.0054267	0.0017875	0.0024485	0.0087599	0.0137415	0.0025848	0.0137776
$\hat{\bar{Y}}_{P14}$	0.0054428	0.0017875	0.0024485	0.0113688	0.0139706	0.0025868	0.0137903
$\hat{\bar{Y}}_{P15}$	0.0054563	0.0017875	0.0024516	0.0130155	0.0140598	0.0025893	0.0137979
$\hat{\bar{Y}}_{P16}$	0.0054677	0.0018044	0.0024543	0.0139559	0.0141568	0.0025913	0.0138075
$\hat{\bar{Y}}_{P17}$	0.0054776	0.0018044	0.0024543	0.0152015	0.0142068	0.0025933	0.0138163
$\hat{\bar{Y}}_{P18}$	0.0054861	0.0018177	0.0024567	0.0167218	0.0142512	0.0025958	0.0138289
$\hat{\bar{Y}}_{P19}$	0.0055003	0.0018177	0.0024588	0.0181445	0.0142947	0.0025986	0.0138414
$\hat{\bar{Y}}_{P20}$	0.0055314	0.0018374	0.0024667	0.0196162	0.0143351	0.0026085	0.0138701
$\hat{\bar{Y}}_{P21}$	3.412E-05	2.827E-05	9.626E-06	0.0021466	0.000122	1.079E-05	3.255E-05
$\hat{\bar{Y}}_{P22}$	0.0006416	0.0004582	0.0002735	0.0033846	0.0005483	0.0003289	0.0005422
$\hat{\bar{Y}}_{P23}$	3.305E-05	2.655E-05	9.339E-06	0.002118	0.0001209	1.052E-05	3.224E-05
$\hat{\bar{Y}}_{P24}$	0.0006271	0.0004419	0.0002677	0.0033453	0.000544	0.0003233	0.0005378
$\hat{\bar{Y}}_{P25}$	0.0028178	0.0010281	0.0006694	0.0184827	0.0097383	0.0007676	0.0055919

$\hat{\bar{Y}}_{P26}$	0.0050422	0.0016933	0.0019499	0.0187024	0.0118689	0.0021834	0.0114972
$\hat{\bar{Y}}_{P27}$	**4.376E-06**	**5.992E-07**	**4.267E-07**	0.0041258	**5.581E-06**	**2.147E-06**	**1.827E-05**
$\hat{\bar{Y}}_{P28}$	0.0001218	2.394E-05	2.036E-05	0.0059051	3.398E-05	9.487E-05	0.000329
$\hat{\bar{Y}}_{P29}$	0.005349	0.0017844	0.0022685	0.0211438	0.0145495	0.0024045	0.0134156
$\hat{\bar{Y}}_{P30}$	0.0053179	0.0017844	0.002251	0.0210965	0.0144344	0.0023903	0.0133209
$\hat{\bar{Y}}_{P31}$	0.0052871	0.0017618	0.0022337	0.0210413	0.0143109	0.0023772	0.0132552
$\hat{\bar{Y}}_{P32}$	0.0052565	0.0017618	0.0022337	0.0209652	0.0141853	0.0023675	0.0131926
$\hat{\bar{Y}}_{P33}$	0.0052262	0.0017618	0.0022166	0.0208937	0.014113	0.002354	0.0131498
$\hat{\bar{Y}}_{P34}$	0.0051962	0.0017396	0.0021997	0.0208388	0.0140085	0.0023424	0.0130876
$\hat{\bar{Y}}_{P35}$	0.0051664	0.0017396	0.0021997	0.0207405	0.0139392	0.0023295	0.0130211
$\hat{\bar{Y}}_{P36}$	0.0051368	0.0017178	0.002183	0.020549	0.0138655	0.0023124	0.0129053
$\hat{\bar{Y}}_{P3s7}$	0.0050785	0.0017178	0.0021665	0.0202042	0.0137786	0.0022907	0.012757
$\hat{\bar{Y}}_{P38}$	0.0048821	0.0016754	0.0020867	0.0192414	0.0136803	0.0021812	0.0121716

Table 13: MSE of the different estimators for each distribution at n = 17

	MSE						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi - Square**
$\hat{\bar{Y}}_{P1}$	0.3691044	0.0473893	0.1802839	0.5111983	1.0477551	0.1830949	2.1417359
$\hat{\bar{Y}}_{P2}$	0.3484805	0.0473893	0.172491	0.4496301	0.9565857	0.1763737	2.042726
$\hat{\bar{Y}}_{P3}$	0.3296419	0.0423431	0.1652226	0.3908179	0.87052	0.1704374	1.9774241
$\hat{\bar{Y}}_{P4}$	0.3123883	0.0423431	0.1652226	0.326957	0.7936027	0.1662247	1.9178041
$\hat{\bar{Y}}_{P5}$	0.2965469	0.0423431	0.1584327	0.2802153	0.7534945	0.1605795	1.8782853
$\hat{\bar{Y}}_{P6}$	0.2819675	0.0382527	0.1520803	0.2508605	0.7003243	0.155929	1.8227758
$\hat{\bar{Y}}_{P7}$	0.2685196	0.0382527	0.1520803	0.2087514	0.6678682	0.1509733	1.7657478
$\hat{\bar{Y}}_{P8}$	0.2560891	0.0348912	0.1461284	0.1524455	0.6355494	0.1447253	1.671778
$\hat{\bar{Y}}_{P9}$	0.2338919	0.0348912	0.1405442	0.0963162	0.6001345	0.1372818	1.5604666
$\hat{\bar{Y}}_{P10}$	0.1767753	0.0297442	0.1171952	**0.040739**	0.563245	0.1064816	1.202792
$\hat{\bar{Y}}_{P11}$	0.6124528	0.123387	0.3356125	0.0796115	1.004256	0.3572775	2.7541171
$\hat{\bar{Y}}_{P12}$	0.6151234	0.123387	0.3361617	0.1686859	1.0783578	0.3577528	2.7602291
$\hat{\bar{Y}}_{P13}$	0.617273	0.1248126	0.3366335	0.2452728	1.1119489	0.3581474	2.7634606
$\hat{\bar{Y}}_{P14}$	0.6190404	0.1248126	0.3366335	0.3160116	1.1303016	0.3584143	2.765997
$\hat{\bar{Y}}_{P15}$	0.6205193	0.1248126	0.3370433	0.3606588	1.1374475	0.3587563	2.7674994
$\hat{\bar{Y}}_{P16}$	0.621775	0.1258996	0.3374025	0.3861582	1.145217	0.3590254	2.769407
$\hat{\bar{Y}}_{P17}$	0.6228545	0.1258996	0.3374025	0.4199321	1.1492278	0.3593006	2.7711566
$\hat{\bar{Y}}_{P18}$	0.6237923	0.1267556	0.3377199	0.4611538	1.1527844	0.3596318	2.7736603

$\hat{\hat{Y}}_{P19}$	0.6253418	0.1267556	0.3380024	0.4997289	1.1562643	0.3600055	2.7761451
$\hat{\hat{Y}}_{P20}$	0.6287524	0.1280179	0.3390497	0.5396336	1.1595013	0.3613548	2.7818554
$\hat{\hat{Y}}_{P21}$	0.0258114	0.0117793	0.0118056	0.0659582	0.0208972	0.0094112	0.0316317
$\hat{\hat{Y}}_{P22}$	0.0924461	0.0394053	0.0469483	0.0995269	0.0550517	0.0525063	0.132927
$\hat{\hat{Y}}_{P23}$	0.0256947	0.011669	0.0117674	0.0651832	0.0208088	0.0093758	0.0315702
$\hat{\hat{Y}}_{P24}$	0.0908493	0.038357	0.0461717	0.0984604	0.0547002	0.0517528	0.1320577
$\hat{\hat{Y}}_{P25}$	0.3311285	0.0760187	0.0996723	0.5088997	0.7912512	0.1119425	1.1365546
$\hat{\hat{Y}}_{P26}$	0.5751053	0.1187588	0.2702163	0.514856	0.9619388	0.303756	2.3102356
$\hat{\hat{Y}}_{P27}$	**0.0225495**	**0.0100016**	**0.0105804**	0.119623	**0.0115708**	**0.0082407**	**0.0287932**
$\hat{\hat{Y}}_{P28}$	0.0354258	0.0115011	0.0132356	0.1678679	0.013846	0.0208026	0.090555
$\hat{\hat{Y}}_{P29}$	0.6087517	0.1246152	0.312655	0.5810508	1.176681	0.3337186	2.6915295
$\hat{\hat{Y}}_{P30}$	0.60534	0.1246152	0.3103256	0.5797698	1.1674591	0.3317977	2.6727063
$\hat{\hat{Y}}_{P31}$	0.6019579	0.1231611	0.308023	0.5782734	1.1575628	0.3300182	2.6596337
$\hat{\hat{Y}}_{P32}$	0.5986052	0.1231611	0.308023	0.5762102	1.147502	0.3287046	2.6472031
$\hat{\hat{Y}}_{P33}$	0.5952815	0.1231611	0.3057469	0.5742701	1.1417068	0.3268733	2.638686
$\hat{\hat{Y}}_{P34}$	0.5919865	0.1217344	0.3034968	0.5727819	1.1333369	0.3252992	2.626326
$\hat{\hat{Y}}_{P35}$	0.5887198	0.1217344	0.3034968	0.5701166	1.1277871	0.3235518	2.6131147
$\hat{\hat{Y}}_{P36}$	0.585481	0.1203345	0.3012723	0.5649237	1.1218818	0.3212373	2.5901094
$\hat{\hat{Y}}_{P3s7}$	0.5790864	0.1203345	0.2990731	0.5555766	1.1149213	0.318301	2.5606274
$\hat{\hat{Y}}_{P38}$	0.5575417	0.1176125	0.2884424	0.5294694	1.1070499	0.3034609	2.4442771

Table 14: MSE of the different estimators for each distribution at n = 25

	MSE						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi - Square**
$\hat{\hat{Y}}_{P1}$	0.2427279	0.0311638	0.1185571	0.3361708	0.6890175	0.1204056	1.4084337
$\hat{\hat{Y}}_{P2}$	0.2291653	0.0311638	0.1134323	0.2956827	0.6290633	0.1159857	1.3433235
$\hat{\hat{Y}}_{P3}$	0.2167768	0.0278454	0.1086526	0.257007	0.5724654	0.1120819	1.3003801
$\hat{\hat{Y}}_{P4}$	0.2054307	0.0278454	0.1086526	0.2150112	0.5218836	0.1093116	1.2611733
$\hat{\hat{Y}}_{P5}$	0.1950131	0.0278454	0.1041875	0.1842733	0.4955079	0.1055992	1.2351852
$\hat{\hat{Y}}_{P6}$	0.1854255	0.0251555	0.10001	0.1649692	0.4605425	0.1025409	1.1986814
$\hat{\hat{Y}}_{P7}$	0.176582	0.0251555	0.10001	0.1372776	0.4391989	0.099282	1.161179
$\hat{\hat{Y}}_{P8}$	0.1684076	0.0229449	0.096096	0.1002502	0.4179457	0.0951733	1.0993832
$\hat{\hat{Y}}_{P9}$	0.1538104	0.0229449	0.0924237	0.0633388	0.3946564	0.0902783	1.0261834
$\hat{\hat{Y}}_{P10}$	0.1162498	0.0195602	0.0770691	**0.0267905**	0.3703974	0.0700237	0.7909719
$\hat{\hat{Y}}_{P11}$	0.402757	0.0811409	0.2207032	0.0523536	0.660412	0.2349504	1.8111437

$\hat{\bar{Y}}_{P12}$	0.4045133	0.0811409	0.2210644	0.1109301	0.7091423	0.235263	1.815163
$\hat{\bar{Y}}_{P13}$	0.4059268	0.0820784	0.2213746	0.1612946	0.7312322	0.2355224	1.8172881
$\hat{\bar{Y}}_{P14}$	0.4070891	0.0820784	0.2213746	0.2078134	0.7433012	0.235698	1.818956
$\hat{\bar{Y}}_{P15}$	0.4080617	0.0820784	0.2216441	0.2371739	0.7480005	0.2359228	1.819944
$\hat{\bar{Y}}_{P16}$	0.4088874	0.0827932	0.2218803	0.2539427	0.7531098	0.2360998	1.8211985
$\hat{\bar{Y}}_{P17}$	0.4095973	0.0827932	0.2218803	0.2761529	0.7557473	0.2362808	1.8223491
$\hat{\bar{Y}}_{P18}$	0.410214	0.0833562	0.222089	0.3032608	0.7580862	0.2364986	1.8239955
$\hat{\bar{Y}}_{P19}$	0.411233	0.0833562	0.2222748	0.3286283	0.7603746	0.2367443	1.8256296
$\hat{\bar{Y}}_{P20}$	0.4134758	0.0841862	0.2229635	0.3548701	0.7625034	0.2376317	1.8293847
$\hat{\bar{Y}}_{P21}$	0.0169739	0.0077462	0.0077635	0.043375	0.0137422	0.0061889	0.0208014
$\hat{\bar{Y}}_{P22}$	0.0607937	0.0259134	0.0308738	0.0654502	0.0362027	0.0345288	0.0874145
$\hat{\bar{Y}}_{P23}$	0.0168972	0.0076737	0.0077384	0.0428653	0.0136841	0.0061656	0.020761
$\hat{\bar{Y}}_{P24}$	0.0597437	0.025224	0.0303631	0.0647489	0.0359716	0.0340333	0.0868429
$\hat{\bar{Y}}_{P25}$	0.2177545	0.0499909	0.0655458	0.3346591	0.5203372	0.0736148	0.7474133
$\hat{\bar{Y}}_{P26}$	0.3781968	0.0780974	0.1776978	0.3385761	0.6325836	0.1997539	1.5192413
$\hat{\bar{Y}}_{P27}$	**0.0148289**	**0.0065772**	**0.0069578**	0.0786657	**0.0076091**	**0.0054192**	**0.0189348**
$\hat{\bar{Y}}_{P28}$	0.0232965	0.0075633	0.0087039	0.1103921	0.0091053	0.0136801	0.0595502
$\hat{\bar{Y}}_{P29}$	0.4003231	0.0819486	0.205606	0.3821067	0.7738009	0.2194578	1.7699853
$\hat{\bar{Y}}_{P30}$	0.3980795	0.0819486	0.2040742	0.3812642	0.7677365	0.2181946	1.7576068
$\hat{\bar{Y}}_{P31}$	0.3958555	0.0809923	0.20256	0.3802802	0.7612285	0.2170243	1.7490102
$\hat{\bar{Y}}_{P32}$	0.3936507	0.0809923	0.20256	0.3789234	0.7546124	0.2161605	1.7408356
$\hat{\bar{Y}}_{P33}$	0.391465	0.0809923	0.2010632	0.3776476	0.7508014	0.2149562	1.7352347
$\hat{\bar{Y}}_{P34}$	0.3892981	0.0800541	0.1995835	0.3766689	0.7452972	0.2139211	1.7271065
$\hat{\bar{Y}}_{P35}$	0.3871499	0.0800541	0.1995835	0.3749162	0.7416476	0.212772	1.7184187
$\hat{\bar{Y}}_{P36}$	0.38502	0.0791335	0.1981206	0.3715013	0.7377642	0.2112499	1.7032901
$\hat{\bar{Y}}_{P3s7}$	0.3808148	0.0791335	0.1966744	0.3653545	0.7331869	0.209319	1.6839023
$\hat{\bar{Y}}_{P38}$	0.3666468	0.0773435	0.1896835	0.3481861	0.7280106	0.1995599	1.6073888

Table 15: MSE of the different estimators for each distribution at n = 39

	MSE						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi – Square**
$\hat{\bar{Y}}_{P1}$	0.1463253	0.0187867	0.0714706	0.2026561	0.4153652	0.0725849	0.8490558
$\hat{\bar{Y}}_{P2}$	0.1381493	0.0187867	0.0683812	0.1782484	0.3792226	0.0699205	0.8098049
$\hat{\bar{Y}}_{P3}$	0.1306811	0.0167862	0.0654998	0.1549333	0.3451033	0.0675671	0.7839171
$\hat{\bar{Y}}_{P4}$	0.1238412	0.0167862	0.0654998	0.1296167	0.3146107	0.065897	0.7602818
$\hat{\bar{Y}}_{P5}$	0.1175611	0.0167862	0.062808	0.1110867	0.2987105	0.0636591	0.7446152

$\hat{\bar{Y}}_{P6}$	0.1117813	0.0151647	0.0602897	0.0994495	0.277632	0.0618155	0.7226094
$\hat{\bar{Y}}_{P7}$	0.1064502	0.0151647	0.0602897	0.082756	0.2647653	0.0598509	0.7000015
$\hat{\bar{Y}}_{P8}$	0.1015223	0.013832	0.0579302	0.0604345	0.2519531	0.057374	0.6627488
$\hat{\bar{Y}}_{P9}$	0.0927226	0.013832	0.0557164	0.038183	0.2379134	0.0544231	0.6186212
$\hat{\bar{Y}}_{P10}$	0.0700797	0.0117916	0.0464601	**0.0161503**	0.2232892	0.0422129	0.476827
$\hat{\bar{Y}}_{P11}$	0.2427968	0.0489148	0.133048	0.0315607	0.3981207	0.1416368	1.0918242
$\hat{\bar{Y}}_{P12}$	0.2438555	0.0489148	0.1332658	0.0668727	0.4274971	0.1418252	1.0942472
$\hat{\bar{Y}}_{P13}$	0.2447077	0.0494799	0.1334528	0.0972343	0.4408137	0.1419816	1.0955283
$\hat{\bar{Y}}_{P14}$	0.2454084	0.0494799	0.1334528	0.1252776	0.4480894	0.1420874	1.0965338
$\hat{\bar{Y}}_{P15}$	0.2459946	0.0494799	0.1336152	0.1429772	0.4509223	0.142223	1.0971294
$\hat{\bar{Y}}_{P16}$	0.2464924	0.0499108	0.1337576	0.1530861	0.4540024	0.1423297	1.0978856
$\hat{\bar{Y}}_{P17}$	0.2469204	0.0499108	0.1337576	0.1664752	0.4555924	0.1424388	1.0985792
$\hat{\bar{Y}}_{P18}$	0.2472922	0.0502502	0.1338835	0.1828168	0.4570023	0.1425701	1.0995718
$\hat{\bar{Y}}_{P19}$	0.2479064	0.0502502	0.1339955	0.1981093	0.4583819	0.1427182	1.1005568
$\hat{\bar{Y}}_{P20}$	0.2492585	0.0507506	0.1344106	0.2139288	0.4596651	0.1432531	1.1028206
$\hat{\bar{Y}}_{P21}$	0.0102325	0.0046697	0.0046801	0.026148	0.0082843	0.0037309	0.0125399
$\hat{\bar{Y}}_{P22}$	0.0366487	0.0156216	0.0186119	0.0394558	0.0218243	0.0208153	0.0526967
$\hat{\bar{Y}}_{P23}$	0.0101862	0.004626	0.004665	0.0258408	0.0082493	0.0037169	0.0125155
$\hat{\bar{Y}}_{P24}$	0.0360157	0.015206	0.018304	0.039033	0.021685	0.0205166	0.0523521
$\hat{\bar{Y}}_{P25}$	0.1312704	0.0301364	0.0395134	0.2017449	0.3136785	0.0443777	0.4505683
$\hat{\bar{Y}}_{P26}$	0.227991	0.04708	0.1071228	0.2041061	0.3813447	0.120419	0.9158547
$\hat{\bar{Y}}_{P27}$	**0.0089394**	**0.003965**	**0.0041944**	0.0474226	**0.0045871**	**0.0032669**	**0.0114146**
$\hat{\bar{Y}}_{P28}$	0.014044	0.0045594	0.005247	0.0665484	0.005489	0.0082468	0.035899
$\hat{\bar{Y}}_{P29}$	0.2413295	0.0494017	0.1239469	0.230348	0.4664757	0.1322972	1.0670124
$\hat{\bar{Y}}_{P30}$	0.239977	0.0494017	0.1230234	0.2298401	0.4628199	0.1315357	1.0595502
$\hat{\bar{Y}}_{P31}$	0.2386363	0.0488252	0.1221106	0.2292469	0.4588966	0.1308303	1.0543678
$\hat{\bar{Y}}_{P32}$	0.2373072	0.0488252	0.1221106	0.228429	0.4549082	0.1303095	1.0494399
$\hat{\bar{Y}}_{P33}$	0.2359895	0.0488252	0.1212083	0.2276599	0.4526108	0.1295835	1.0460635
$\hat{\bar{Y}}_{P34}$	0.2346833	0.0482596	0.1203163	0.2270699	0.4492927	0.1289595	1.0411635
$\hat{\bar{Y}}_{P35}$	0.2333882	0.0482596	0.1203163	0.2260133	0.4470925	0.1282668	1.0359261
$\hat{\bar{Y}}_{P36}$	0.2321043	0.0477046	0.1194344	0.2239547	0.4447515	0.1273492	1.0268061
$\hat{\bar{Y}}_{P3s7}$	0.2295692	0.0477046	0.1185626	0.2202492	0.4419921	0.1261852	1.0151184
$\hat{\bar{Y}}_{P38}$	0.2210282	0.0466255	0.1143482	0.2098994	0.4388716	0.1203021	0.9689933

Table 16: MSE of the different estimators for each distribution at n = 50

	MSE						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi – Square**
$\hat{\bar{Y}}_{P1}$	0.1084529	0.0139243	0.0529723	0.150204	0.3078589	0.0537982	0.6293002
$\hat{\bar{Y}}_{P2}$	0.102393	0.0139243	0.0506825	0.1321135	0.2810709	0.0518234	0.6002084
$\hat{\bar{Y}}_{P3}$	0.0968577	0.0124416	0.0485469	0.1148329	0.2557824	0.0500791	0.5810209
$\hat{\bar{Y}}_{P4}$	0.0917882	0.0124416	0.0485469	0.0960689	0.233182	0.0488413	0.5635029
$\hat{\bar{Y}}_{P5}$	0.0871335	0.0124416	0.0465518	0.0823349	0.2213972	0.0471826	0.5518912
$\hat{\bar{Y}}_{P6}$	0.0828497	0.0112397	0.0446853	0.0737096	0.2057743	0.0458162	0.5355811
$\hat{\bar{Y}}_{P7}$	0.0788984	0.0112397	0.0446853	0.0613368	0.1962378	0.0443601	0.5188247
$\hat{\bar{Y}}_{P8}$	0.0752459	0.010252	0.0429365	0.0447926	0.1867417	0.0425242	0.4912138
$\hat{\bar{Y}}_{P9}$	0.0687238	0.010252	0.0412957	0.0283003	0.1763358	0.0403371	0.4585075
$\hat{\bar{Y}}_{P10}$	0.0519414	0.0087397	0.0344351	**0.0119702**	0.1654967	0.0312872	0.353413
$\hat{\bar{Y}}_{P11}$	0.1799553	0.0362545	0.0986121	0.023392	0.2950777	0.1049778	0.8092344
$\hat{\bar{Y}}_{P12}$	0.18074	0.0362545	0.0987734	0.0495645	0.3168508	0.1051175	0.8110303
$\hat{\bar{Y}}_{P13}$	0.1813716	0.0366733	0.0989121	0.0720678	0.3267208	0.1052334	0.8119798
$\hat{\bar{Y}}_{P14}$	0.1818909	0.0366733	0.0989121	0.0928528	0.3321133	0.1053119	0.812725
$\hat{\bar{Y}}_{P15}$	0.1823254	0.0366733	0.0990325	0.1059713	0.334213	0.1054123	0.8131665
$\hat{\bar{Y}}_{P16}$	0.1826944	0.0369927	0.099138	0.1134638	0.3364959	0.1054914	0.813727
$\hat{\bar{Y}}_{P17}$	0.1830116	0.0369927	0.099138	0.1233875	0.3376743	0.1055723	0.8142411
$\hat{\bar{Y}}_{P18}$	0.1832871	0.0372442	0.0992313	0.1354995	0.3387194	0.1056696	0.8149767
$\hat{\bar{Y}}_{P19}$	0.1837424	0.0372442	0.0993143	0.1468339	0.3397418	0.1057794	0.8157068
$\hat{\bar{Y}}_{P20}$	0.1847445	0.0376151	0.099622	0.158559	0.340693	0.1061758	0.8173847
$\hat{\bar{Y}}_{P21}$	0.0075841	0.0034611	0.0034688	0.0193803	0.0061402	0.0027653	0.0092942
$\hat{\bar{Y}}_{P22}$	0.0271632	0.0115783	0.0137947	0.0292437	0.0161757	0.0154278	0.0390576
$\hat{\bar{Y}}_{P23}$	0.0075498	0.0034287	0.0034576	0.0191526	0.0061142	0.0027549	0.0092762
$\hat{\bar{Y}}_{P24}$	0.026694	0.0112703	0.0135665	0.0289303	0.0160724	0.0152064	0.0388021
$\hat{\bar{Y}}_{P25}$	0.0972946	0.0223364	0.0292864	0.1495286	0.2324911	0.0328917	0.3339506
$\hat{\bar{Y}}_{P26}$	0.1689815	0.0348946	0.0793969	0.1512787	0.2826437	0.0892518	0.67881
$\hat{\bar{Y}}_{P27}$	**0.0066257**	**0.0029387**	**0.0031088**	0.0351485	**0.0033998**	**0.0024213**	**0.0084602**
$\hat{\bar{Y}}_{P28}$	0.0104091	0.0033793	0.003889	0.0493241	0.0040683	0.0061124	0.0266075
$\hat{\bar{Y}}_{P29}$	0.1788678	0.0366153	0.0918665	0.1707285	0.3457408	0.0980556	0.7908445
$\hat{\bar{Y}}_{P30}$	0.1778653	0.0366153	0.0911821	0.1703521	0.3430312	0.0974912	0.7853137
$\hat{\bar{Y}}_{P31}$	0.1768716	0.0361881	0.0905055	0.1699124	0.3401234	0.0969683	0.7814726
$\hat{\bar{Y}}_{P32}$	0.1758865	0.0361881	0.0905055	0.1693062	0.3371672	0.0965823	0.7778202

$\hat{\bar{Y}}_{P33}$	0.1749099	0.0361881	0.0898367	0.1687362	0.3354645	0.0960443	0.7753176
$\hat{\bar{Y}}_{P34}$	0.1739417	0.0357689	0.0891756	0.1682989	0.3330052	0.0955817	0.7716859
$\hat{\bar{Y}}_{P35}$	0.1729819	0.0357689	0.0891756	0.1675157	0.3313745	0.0950683	0.7678041
$\hat{\bar{Y}}_{P36}$	0.1720302	0.0353575	0.088522	0.1659899	0.3296393	0.0943883	0.7610445
$\hat{\bar{Y}}_{P3s7}$	0.1701513	0.0353575	0.0878758	0.1632435	0.3275942	0.0935255	0.7523819
$\hat{\bar{Y}}_{P38}$	0.1638209	0.0345577	0.0847522	0.1555725	0.3252813	0.0891651	0.718195

Table 17: MSE of the different estimators for each distribution at n = 100

	MSE						
	Poisson	**Hypergeometric**	**Binomial**	**Exponential**	**Uniform**	**Normal**	**Chi - Square**
$\hat{\bar{Y}}_{P1}$	0.0413154	0.0053045	0.0201799	0.0572206	0.1172796	0.0204946	0.2397334
$\hat{\bar{Y}}_{P2}$	0.0390069	0.0053045	0.0193076	0.050329	0.1070746	0.0197422	0.2286508
$\hat{\bar{Y}}_{P3}$	0.0368982	0.0047396	0.0184941	0.0437459	0.0974409	0.0190778	0.2213413
$\hat{\bar{Y}}_{P4}$	0.0349669	0.0047396	0.0184941	0.0365977	0.0888312	0.0186062	0.2146678
$\hat{\bar{Y}}_{P5}$	0.0331937	0.0047396	0.017734	0.0313657	0.0843418	0.0179743	0.2102443
$\hat{\bar{Y}}_{P6}$	0.0315618	0.0042818	0.017023	0.0280799	0.0783902	0.0174538	0.2040309
$\hat{\bar{Y}}_{P7}$	0.0300565	0.0042818	0.017023	0.0233664	0.0747573	0.0168991	0.1976475
$\hat{\bar{Y}}_{P8}$	0.0286651	0.0039055	0.0163568	0.0170639	0.0711397	0.0161997	0.1871291
$\hat{\bar{Y}}_{P9}$	0.0261805	0.0039055	0.0157317	0.0107811	0.0671756	0.0153665	0.1746695
$\hat{\bar{Y}}_{P10}$	0.0197872	0.0033294	0.0131181	**0.0045601**	0.0630464	0.0119189	0.1346335
$\hat{\bar{Y}}_{P11}$	0.0685544	0.0138112	0.0375665	0.0089112	0.1124105	0.0399916	0.3082798
$\hat{\bar{Y}}_{P12}$	0.0688533	0.0138112	0.037628	0.0188817	0.1207051	0.0400448	0.3089639
$\hat{\bar{Y}}_{P13}$	0.0690939	0.0139708	0.0376808	0.0274544	0.1244651	0.0400889	0.3093256
$\hat{\bar{Y}}_{P14}$	0.0692918	0.0139708	0.0376808	0.0353725	0.1265194	0.0401188	0.3096095
$\hat{\bar{Y}}_{P15}$	0.0694573	0.0139708	0.0377267	0.04037	0.1273192	0.0401571	0.3097777
$\hat{\bar{Y}}_{P16}$	0.0695979	0.0140925	0.0377669	0.0432243	0.1281889	0.0401872	0.3099912
$\hat{\bar{Y}}_{P17}$	0.0697187	0.0140925	0.0377669	0.0470048	0.1286378	0.040218	0.3101871
$\hat{\bar{Y}}_{P18}$	0.0698237	0.0141883	0.0378024	0.0516189	0.1290359	0.0402551	0.3104673
$\hat{\bar{Y}}_{P19}$	0.0699971	0.0141883	0.037834	0.0559367	0.1294255	0.0402969	0.3107455
$\hat{\bar{Y}}_{P20}$	0.0703789	0.0143296	0.0379512	0.0604034	0.1297878	0.0404479	0.3113846
$\hat{\bar{Y}}_{P21}$	0.0028892	0.0013185	0.0013214	0.007383	0.0023391	0.0010534	0.0035407
$\hat{\bar{Y}}_{P22}$	0.0103479	0.0044108	0.0052551	0.0111405	0.0061622	0.0058772	0.0148791
$\hat{\bar{Y}}_{P23}$	0.0028761	0.0013062	0.0013172	0.0072962	0.0023292	0.0010495	0.0035338
$\hat{\bar{Y}}_{P24}$	0.0101691	0.0042935	0.0051682	0.0110211	0.0061228	0.0057929	0.0147818
$\hat{\bar{Y}}_{P25}$	0.0370646	0.0085091	0.0111567	0.0569633	0.088568	0.0125302	0.1272193
$\hat{\bar{Y}}_{P26}$	0.0643739	0.0132932	0.0302464	0.05763	0.1076738	0.0340007	0.2585943

$\hat{\bar{Y}}_{P27}$	**0.0025241**	**0.0011195**	**0.0011843**	0.0133899	**0.0012952**	**0.0009224**	**0.0032229**
$\hat{\bar{Y}}_{P28}$	0.0039654	0.0012874	0.0014815	0.0187901	0.0015498	0.0023285	0.0101362
$\hat{\bar{Y}}_{P29}$	0.0681401	0.0139487	0.0349968	0.0650394	0.1317108	0.0373545	0.3012741
$\hat{\bar{Y}}_{P30}$	0.0677582	0.0139487	0.034736	0.064896	0.1306786	0.0371395	0.2991671
$\hat{\bar{Y}}_{P31}$	0.0673797	0.0137859	0.0344783	0.0647285	0.1295708	0.0369403	0.2977039
$\hat{\bar{Y}}_{P32}$	0.0670044	0.0137859	0.0344783	0.0644976	0.1284447	0.0367933	0.2963124
$\hat{\bar{Y}}_{P33}$	0.0666323	0.0137859	0.0342235	0.0642804	0.127796	0.0365883	0.2953591
$\hat{\bar{Y}}_{P34}$	0.0662635	0.0136262	0.0339717	0.0641139	0.1268591	0.0364121	0.2939756
$\hat{\bar{Y}}_{P35}$	0.0658978	0.0136262	0.0339717	0.0638155	0.1262379	0.0362165	0.2924968
$\hat{\bar{Y}}_{P36}$	0.0655353	0.0134695	0.0337227	0.0632343	0.1255769	0.0359574	0.2899217
$\hat{\bar{Y}}_{P3s7}$	0.0648195	0.0134695	0.0334765	0.062188	0.1247978	0.0356288	0.2866217
$\hat{\bar{Y}}_{P38}$	0.062408	0.0131649	0.0322866	0.0592657	0.1239167	0.0339676	0.2735981

Having summarized the results of the analysis in Table 2 – Table 17 above, the estimators were ranked based on their biases and mean squared errors. The tables below show the ranks based on Bias and MSE:

Table 18: Ranks of estimators based on bias

	Ranks based on bias														
	Poisson					**Hypergeometric**					**Binomial**				
	$\mathbf{n_1}$	$\mathbf{n_2}$	$\mathbf{n_3}$	$\mathbf{n_4}$	$\mathbf{n_5}$	$\mathbf{n_1}$	$\mathbf{n_2}$	$\mathbf{n_3}$	$\mathbf{n_4}$	$\mathbf{n_5}$	$\mathbf{n_1}$	$\mathbf{n_2}$	$\mathbf{n_3}$	$\mathbf{n_4}$	$\mathbf{n_5}$
$\hat{\bar{Y}}_{P1}$	17	17	17	17	17	15.5	15.5	15.5	15.5	15.5	17	17	17	17	17
$\hat{\bar{Y}}_{P2}$	16	16	16	16	16	15.5	15.5	15.5	15.5	15.5	16	16	16	16	16
$\hat{\bar{Y}}_{P3}$	14	14	14	14	14	13	13	13	13	13	14.5	14.5	14.5	14.5	14.5
$\hat{\bar{Y}}_{P4}$	13	13	13	13	13	13	13	13	13	13	14.5	14.5	14.5	14.5	14.5
$\hat{\bar{Y}}_{P5}$	12	12	12	12	12	13	13	13	13	13	13	13	13	13	13
$\hat{\bar{Y}}_{P6}$	11	11	11	11	11	8.5	8.5	8.5	8.5	8.5	11.5	11.5	11.5	11.5	11.5
$\hat{\bar{Y}}_{P7}$	10	10	10	10	10	8.5	8.5	8.5	8.5	8.5	11.5	11.5	11.5	11.5	11.5
$\hat{\bar{Y}}_{P8}$	9	9	9	9	9	6.5	6.5	6.5	6.5	6.5	10	10	10	10	10
$\hat{\bar{Y}}_{P9}$	8	8	8	8	8	6.5	6.5	6.5	6.5	6.5	9	9	9	9	9
$\hat{\bar{Y}}_{P10}$	7	7	7	7	7	5	5	5	5	5	8	8	8	8	8
$\hat{\bar{Y}}_{P11}$	29	29	29	29	29	27.5	27.5	27.5	27.5	26	29	29	29	29	29
$\hat{\bar{Y}}_{P12}$	30	30	30	30	30	27.5	27.5	27.5	27.5	26	30	30	30	30	30
$\hat{\bar{Y}}_{P13}$	31	31	31	31	31	32	32	32	32	32	31.5	31.5	31.5	31.5	31.5
$\hat{\bar{Y}}_{P14}$	32	32	32	32	32	32	32	32	32	32	31.5	31.5	31.5	31.5	31.5
$\hat{\bar{Y}}_{P15}$	33	33	33	33	33	32	32	32	32	32	33	33	33	33	33
$\hat{\bar{Y}}_{P16}$	34	34	34	34	34	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5
$\hat{\bar{Y}}_{P17}$	35	35	35	35	35	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5

$\hat{\bar{Y}}_{P18}$	36	36	36	36	36	36.5	36.5	36.5	36.5	36.5	36	36	36	36	36
$\hat{\bar{Y}}_{P19}$	37	37	37	37	37	36.5	36.5	36.5	36.5	36.5	37	37	37	37	37
$\hat{\bar{Y}}_{P20}$	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38
$\hat{\bar{Y}}_{P21}$	3	3	3	3	3	4	4	4	4	4	3	3	3	3	3
$\hat{\bar{Y}}_{P22}$	6	6	6	6	6	11	11	11	11	11	6	6	6	6	6
$\hat{\bar{Y}}_{P23}$	2	2	2	2	2	3	3	3	3	3	2	2	2	2	2
$\hat{\bar{Y}}_{P24}$	5	5	5	5	5	10	10	10	10	10	5	5	5	5	5
$\hat{\bar{Y}}_{P25}$	15	15	15	15	15	17	17	17	17	17	7	7	7	7	7
$\hat{\bar{Y}}_{P26}$	19	19	19	19	19	19	19	19	19	19	18	18	18	18	18
$\hat{\bar{Y}}_{P27}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\hat{\bar{Y}}_{P28}$	4	4	4	4	4	2	2	2	2	2	4	4	4	4	4
$\hat{\bar{Y}}_{P29}$	28	28	28	28	28	29.5	29.5	29.5	29.5	29.5	28	28	28	28	28
$\hat{\bar{Y}}_{P30}$	27	27	27	27	27	29.5	29.5	29.5	29.5	29.5	27	27	27	27	27
$\hat{\bar{Y}}_{P31}$	26	26	26	26	26	25	25	25	25	26	25.5	25.5	25.5	25.5	25.5
$\hat{\bar{Y}}_{P32}$	25	25	25	25	25	25	25	25	25	26	25.5	25.5	25.5	25.5	25.5
$\hat{\bar{Y}}_{P33}$	24	24	24	24	24	25	25	25	25	26	24	24	24	24	24
$\hat{\bar{Y}}_{P34}$	23	23	23	23	23	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5
$\hat{\bar{Y}}_{P35}$	22	22	22	22	22	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5
$\hat{\bar{Y}}_{P36}$	21	21	21	21	21	20.5	20.5	20.5	20.5	20.5	21	21	21	21	21
$\hat{\bar{Y}}_{P3s7}$	20	20	20	20	20	20.5	20.5	20.5	20.5	20.5	20	20	20	20	20
$\hat{\bar{Y}}_{P38}$	18	18	18	18	18	18	18	18	18	18	19	19	19	19	19

Table 19: Ranks of estimators based on bias (Continue)

	Ranks based on bias																			
	Exponential					**Uniform**					**Normal**					**Chi - Square**				
	n_1	**n_2**	**n_3**	**n_4**	**n_5**	**n_1**	**n_2**	**n_3**	**n_4**	**n_5**	**n_1**	**n_2**	**n_3**	**n_4**	**n_5**	**n_1**	**n_2**	**n_3**	**n_4**	**n_5**
$\hat{\bar{Y}}_{P1}$	26	26	26	26	26	19	19	19	19	19	17	17	17	17	17	17	17	17	17	17
$\hat{\bar{Y}}_{P2}$	22	22	22	22	22	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
$\hat{\bar{Y}}_{P3}$	20	20	20	20	20	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
$\hat{\bar{Y}}_{P4}$	17	17	17	17	17	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
$\hat{\bar{Y}}_{P5}$	15	15	15	15	15	12	12	12	12	12	13	13	13	13	13	13	13	13	13	13
$\hat{\bar{Y}}_{P6}$	14	14	14	14	14	11	11	11	11	11	12	12	12	12	12	12	12	12	12	12
$\hat{\bar{Y}}_{P7}$	12	12	12	12	12	10	10	10	10	10	11	11	11	11	11	11	11	11	11	11
$\hat{\bar{Y}}_{P8}$	9	9	9	9	9	9	9	9	9	9	10	10	10	10	10	10	10	10	10	10

$\hat{\bar{Y}}_{P9}$	5	5	5	5	5	8	8	8	8	8	9	9	9	9	9	9	9	9	9	9
$\hat{\bar{Y}}_{P10}$	1	1	1	1	1	7	7	7	7	7	7	7	7	7	7	8	8	8	8	8
$\hat{\bar{Y}}_{P11}$	4	4	4	4	4	18	18	18	18	18	29	29	29	29	29	29	29	29	29	29
$\hat{\bar{Y}}_{P12}$	11	11	11	11	11	20	20	20	20	20	30	30	30	30	30	30	30	30	30	30
$\hat{\bar{Y}}_{P13}$	13	13	13	13	13	22	22	22	22	22	31	31	31	31	31	31	31	31	31	31
$\hat{\bar{Y}}_{P14}$	16	16	16	16	16	26	26	26	26	26	32	32	32	32	32	32	32	32	32	32
$\hat{\bar{Y}}_{P15}$	18	18	18	18	18	28	28	28	28	28	33	33	33	33	33	33	33	33	33	33
$\hat{\bar{Y}}_{P16}$	19	19	19	19	19	30	30	30	30	30	34	34	34	34	34	34	34	34	34	34
$\hat{\bar{Y}}_{P17}$	21	21	21	21	21	32	32	32	32	32	35	35	35	35	35	35	35	35	35	35
$\hat{\bar{Y}}_{P18}$	23	23	23	23	23	33	33	33	33	33	36	36	36	36	36	36	36	36	36	36
$\hat{\bar{Y}}_{P19}$	24	24	24	24	24	34	34	34	34	34	37	37	37	37	37	37	37	37	37	37
$\hat{\bar{Y}}_{P20}$	29	29	29	29	29	36	36	36	36	36	38	38	38	38	38	38	38	38	38	38
$\hat{\bar{Y}}_{P21}$	3	3	3	3	3	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3
$\hat{\bar{Y}}_{P22}$	7	7	7	7	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
$\hat{\bar{Y}}_{P23}$	2	2	2	2	2	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2
$\hat{\bar{Y}}_{P24}$	6	6	6	6	6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
$\hat{\bar{Y}}_{P25}$	25	25	25	25	25	13	13	13	13	13	8	8	8	8	8	7	7	7	7	7
$\hat{\bar{Y}}_{P26}$	27	27	27	27	27	17	17	17	17	17	19	19	19	19	19	18	18	18	18	18
$\hat{\bar{Y}}_{P27}$	8	8	8	8	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\hat{\bar{Y}}_{P28}$	10	10	10	10	10	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4
$\hat{\bar{Y}}_{P29}$	38	38	38	38	38	38	38	38	38	38	28	28	28	28	28	28	28	28	28	28
$\hat{\bar{Y}}_{P30}$	37	37	37	37	37	37	37	37	37	37	27	27	27	27	27	27	27	27	27	27
$\hat{\bar{Y}}_{P31}$	36	36	36	36	36	35	35	35	35	35	26	26	26	26	26	26	26	26	26	26
$\hat{\bar{Y}}_{P32}$	35	35	35	35	35	31	31	31	31	31	25	25	25	25	25	25	25	25	25	25
$\hat{\bar{Y}}_{P33}$	34	34	34	34	34	29	29	29	29	29	24	24	24	24	24	24	24	24	24	24
$\hat{\bar{Y}}_{P34}$	33	33	33	33	33	27	27	27	27	27	23	23	23	23	23	23	23	23	23	23
$\hat{\bar{Y}}_{P35}$	32	32	32	32	32	25	25	25	25	25	22	22	22	22	22	22	22	22	22	22
$\hat{\bar{Y}}_{P36}$	31	31	31	31	31	24	24	24	24	24	21	21	21	21	21	21	21	21	21	21
$\hat{\bar{Y}}_{P3s7}$	30	30	30	30	30	23	23	23	23	23	20	20	20	20	20	20	20	20	20	20
$\hat{\bar{Y}}_{P38}$	28	28	28	28	28	21	21	21	21	21	18	18	18	18	18	19	19	19	19	19

Table 20: Ranks of estimators based on MSE

	Ranks														
	Poisson					**Hypergeometric**					**Binomial**				
	$\mathbf{n_1}$	$\mathbf{n_2}$	$\mathbf{n_3}$	$\mathbf{n_4}$	$\mathbf{n_5}$	$\mathbf{n_1}$	$\mathbf{n_2}$	$\mathbf{n_3}$	$\mathbf{n_4}$	$\mathbf{n_5}$	$\mathbf{n_1}$	$\mathbf{n_2}$	$\mathbf{n_3}$	$\mathbf{n_4}$	$\mathbf{n_5}$
$\hat{\bar{Y}}_{P1}$	17	17	17	17	17	15.5	15.5	15.5	15.5	15.5	17	17	17	17	17
$\hat{\bar{Y}}_{P2}$	16	16	16	16	16	15.5	15.5	15.5	15.5	15.5	16	16	16	16	16
$\hat{\bar{Y}}_{P3}$	14	14	14	14	14	13	13	13	13	13	14.5	14.5	14.5	14.5	14.5
$\hat{\bar{Y}}_{P4}$	13	13	13	13	13	13	13	13	13	13	14.5	14.5	14.5	14.5	14.5
$\hat{\bar{Y}}_{P5}$	12	12	12	12	12	13	13	13	13	13	13	13	13	13	13
$\hat{\bar{Y}}_{P6}$	11	11	11	11	11	8.5	8.5	8.5	8.5	8.5	11.5	11.5	11.5	11.5	11.5
$\hat{\bar{Y}}_{P7}$	10	10	10	10	10	8.5	8.5	8.5	8.5	8.5	11.5	11.5	11.5	11.5	11.5
$\hat{\bar{Y}}_{P8}$	9	9	9	9	9	6.5	6.5	6.5	6.5	6.5	10	10	10	10	10
$\hat{\bar{Y}}_{P9}$	8	8	8	8	8	6.5	6.5	6.5	6.5	6.5	9	9	9	9	9
$\hat{\bar{Y}}_{P10}$	7	7	7	7	7	5	5	5	5	5	8	8	8	8	8
$\hat{\bar{Y}}_{P11}$	29	29	29	29	29	27.5	27.5	27.5	27.5	27.5	29	29	29	29	29
$\hat{\bar{Y}}_{P12}$	30	30	30	30	30	27.5	27.5	27.5	27.5	27.5	30	30	30	30	30
$\hat{\bar{Y}}_{P13}$	31	31	31	31	31	32	32	32	32	32	31.5	31.5	31.5	31.5	31.5
$\hat{\bar{Y}}_{P14}$	32	32	32	32	32	32	32	32	32	32	31.5	31.5	31.5	31.5	31.5
$\hat{\bar{Y}}_{P15}$	33	33	33	33	33	32	32	32	32	32	33	33	33	33	33
$\hat{\bar{Y}}_{P16}$	34	34	34	34	34	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5
$\hat{\bar{Y}}_{P17}$	35	35	35	35	35	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5	34.5
$\hat{\bar{Y}}_{P18}$	36	36	36	36	36	36.5	36.5	36.5	36.5	36.5	36	36	36	36	36
$\hat{\bar{Y}}_{P19}$	37	37	37	37	37	36.5	36.5	36.5	36.5	36.5	37	37	37	37	37
$\hat{\bar{Y}}_{P20}$	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38
$\hat{\bar{Y}}_{P21}$	3	3	3	3	3	4	4	4	4	4	3	3	3	3	3
$\hat{\bar{Y}}_{P22}$	6	6	6	6	6	11	11	11	11	11	6	6	6	6	6
$\hat{\bar{Y}}_{P23}$	2	2	2	2	2	3	3	3	3	3	2	2	2	2	2
$\hat{\bar{Y}}_{P24}$	5	5	5	5	5	10	10	10	10	10	5	5	5	5	5
$\hat{\bar{Y}}_{P25}$	15	15	15	15	15	17	17	17	17	17	7	7	7	7	7
$\hat{\bar{Y}}_{P26}$	19	19	19	19	19	19	19	19	19	19	18	18	18	18	18
$\hat{\bar{Y}}_{P27}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\hat{\bar{Y}}_{P28}$	4	4	4	4	4	2	2	2	2	2	4	4	4	4	4
$\hat{\bar{Y}}_{P29}$	28	28	28	28	28	29.5	29.5	29.5	29.5	29.5	28	28	28	28	28
$\hat{\bar{Y}}_{P30}$	27	27	27	27	27	29.5	29.5	29.5	29.5	29.5	27	27	27	27	27
$\hat{\bar{Y}}_{P31}$	26	26	26	26	26	25	25	25	25	25	25.5	25.5	25.5	25.5	25.5
$\hat{\bar{Y}}_{P32}$	25	25	25	25	25	25	25	25	25	25	25.5	25.5	25.5	25.5	25.5

$\hat{\bar{Y}}_{P33}$	24	24	24	24	24	25	25	25	25	25	24	24	24	24	24
$\hat{\bar{Y}}_{P34}$	23	23	23	23	23	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5
$\hat{\bar{Y}}_{P35}$	22	22	22	22	22	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5
$\hat{\bar{Y}}_{P36}$	21	21	21	21	21	20.5	20.5	20.5	20.5	20.5	21	21	21	21	21
$\hat{\bar{Y}}_{P37}$	20	20	20	20	20	20.5	20.5	20.5	20.5	20.5	20	20	20	20	20
$\hat{\bar{Y}}_{P38}$	18	18	18	18	18	18	18	18	18	18	19	19	19	19	19

Table 21: Ranks of estimators based on MSE

	MSE																			
	Exponential					**Uniform**					**Normal**					**Chi - Square**				
	$\mathbf{n_1}$	$\mathbf{n_2}$	$\mathbf{n_3}$	$\mathbf{n_4}$	$\mathbf{n_5}$	$\mathbf{n_1}$	$\mathbf{n_2}$	$\mathbf{n_3}$	$\mathbf{n_4}$	$\mathbf{n_5}$	$\mathbf{n_1}$	$\mathbf{n_2}$	$\mathbf{n_3}$	$\mathbf{n_4}$	$\mathbf{n_5}$	$\mathbf{n_1}$	$\mathbf{n_2}$	$\mathbf{n_3}$	$\mathbf{n_4}$	$\mathbf{n_5}$
$\hat{\bar{Y}}_{P1}$	26	26	26	26	26	19	19	19	19	19	17	17	17	17	17	17	17	17	17	17
$\hat{\bar{Y}}_{P2}$	22	22	22	22	22	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
$\hat{\bar{Y}}_{P3}$	20	20	20	20	20	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
$\hat{\bar{Y}}_{P4}$	17	17	17	17	17	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
$\hat{\bar{Y}}_{P5}$	15	15	15	15	15	12	12	12	12	12	13	13	13	13	13	13	13	13	13	13
$\hat{\bar{Y}}_{P6}$	14	14	14	14	14	11	11	11	11	11	12	12	12	12	12	12	12	12	12	12
$\hat{\bar{Y}}_{P7}$	12	12	12	12	12	10	10	10	10	10	11	11	11	11	11	11	11	11	11	11
$\hat{\bar{Y}}_{P8}$	9	9	9	9	9	9	9	9	9	9	10	10	10	10	10	10	10	10	10	10
$\hat{\bar{Y}}_{P9}$	5	5	5	5	5	8	8	8	8	8	9	9	9	9	9	9	9	9	9	9
$\hat{\bar{Y}}_{P10}$	1	1	1	1	1	7	7	7	7	7	7	7	7	7	7	8	8	8	8	8
$\hat{\bar{Y}}_{P11}$	4	4	4	4	4	18	18	18	18	18	29	29	29	29	29	29	29	29	29	29
$\hat{\bar{Y}}_{P12}$	11	11	11	11	11	20	20	20	20	20	30	30	30	30	30	30	30	30	30	30
$\hat{\bar{Y}}_{P13}$	13	13	13	13	13	22	22	22	22	22	31	31	31	31	31	31	31	31	31	31
$\hat{\bar{Y}}_{P14}$	16	16	16	16	16	26	26	26	26	26	32	32	32	32	32	32	32	32	32	32
$\hat{\bar{Y}}_{P15}$	18	18	18	18	18	28	28	28	28	28	33	33	33	33	33	33	33	33	33	33
$\hat{\bar{Y}}_{P16}$	19	19	19	19	19	30	30	30	30	30	34	34	34	34	34	34	34	34	34	34
$\hat{\bar{Y}}_{P17}$	21	21	21	21	21	32	32	32	32	32	35	35	35	35	35	35	35	35	35	35
$\hat{\bar{Y}}_{P18}$	23	23	23	23	23	33	33	33	33	33	36	36	36	36	36	36	36	36	36	36
$\hat{\bar{Y}}_{P19}$	24	24	24	24	24	34	34	34	34	34	37	37	37	37	37	37	37	37	37	37
$\hat{\bar{Y}}_{P20}$	29	29	29	29	29	36	36	36	36	36	38	38	38	38	38	38	38	38	38	38
$\hat{\bar{Y}}_{P21}$	3	3	3	3	3	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3
$\hat{\bar{Y}}_{P22}$	7	7	7	7	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
$\hat{\bar{Y}}_{P23}$	2	2	2	2	2	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2
$\hat{\bar{Y}}_{P24}$	6	6	6	6	6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
$\hat{\bar{Y}}_{P25}$	25	25	25	25	25	13	13	13	13	13	8	8	8	8	8	7	7	7	7	7

$\hat{\bar{Y}}_{P26}$	27	27	27	27	27	17	17	17	17	17	19	19	19	19	19	18	18	18	18	18
$\hat{\bar{Y}}_{P27}$	8	8	8	8	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\hat{\bar{Y}}_{P28}$	10	10	10	10	10	2	2	2	2	2	4	4	4	4	4	4	4	4	4	4
$\hat{\bar{Y}}_{P29}$	38	38	38	38	38	38	38	38	38	38	28	28	28	28	28	28	28	28	28	28
$\hat{\bar{Y}}_{P30}$	37	37	37	37	37	37	37	37	37	37	27	27	27	27	27	27	27	27	27	27
$\hat{\bar{Y}}_{P31}$	36	36	36	36	36	35	35	35	35	35	26	26	26	26	26	26	26	26	26	26
$\hat{\bar{Y}}_{P32}$	35	35	35	35	35	31	31	31	31	31	25	25	25	25	25	25	25	25	25	25
$\hat{\bar{Y}}_{P33}$	34	34	34	34	34	29	29	29	29	29	24	24	24	24	24	24	24	24	24	24
$\hat{\bar{Y}}_{P34}$	33	33	33	33	33	27	27	27	27	27	23	23	23	23	23	23	23	23	23	23
$\hat{\bar{Y}}_{P35}$	32	32	32	32	32	25	25	25	25	25	22	22	22	22	22	22	22	22	22	22
$\hat{\bar{Y}}_{P36}$	31	31	31	31	31	24	24	24	24	24	21	21	21	21	21	21	21	21	21	21
$\hat{\bar{Y}}_{P37}$	30	30	30	30	30	23	23	23	23	23	20	20	20	20	20	20	20	20	20	20
$\hat{\bar{Y}}_{P38}$	28	28	28	28	28	21	21	21	21	21	18	18	18	18	18	19	19	19	19	19

Table 4.22: Summary of ranks

Estimators	Ranking based on bias							Ranking based on MSE						
	Poisson	Hyper Geometric	Binomial	Exponential	Uniform	Normal	Chi – Square	Poisson	Hyper Geometric	Binomial	Exponential	Uniform	Normal	Chi – Square
$\hat{\bar{Y}}_{P1}$	17	15.5	17	26	19	17	17	17	15.5	17	26	19	17	17
$\hat{\bar{Y}}_{P2}$	16	15.5	16	22	16	16	16	16	15.5	16	22	16	16	16
$\hat{\bar{Y}}_{P3}$	14	13	14.5	20	15	15	15	14	13	14.5	20	15	15	15
$\hat{\bar{Y}}_{P4}$	13	13	14.5	17	14	14	14	13	13	14.5	17	14	14	14
$\hat{\bar{Y}}_{P5}$	12	13	13	15	12	13	13	12	13	13	15	12	13	13
$\hat{\bar{Y}}_{P6}$	11	8.5	11.5	14	11	12	12	11	8.5	11.5	14	11	12	12
$\hat{\bar{Y}}_{P7}$	10	8.5	11.5	12	10	11	11	10	8.5	11.5	12	10	11	11
$\hat{\bar{Y}}_{P8}$	9	6.5	10	9	9	10	10	9	6.5	10	9	9	10	10
$\hat{\bar{Y}}_{P9}$	8	6.5	9	5	8	9	9	8	6.5	9	5	8	9	9
$\hat{\bar{Y}}_{P10}$	7	5	8	1	7	7	8	7	5	8	1	7	7	8
$\hat{\bar{Y}}_{P11}$	29	27.5	29	4	18	29	29	29	27.5	29	4	18	29	29
$\hat{\bar{Y}}_{P12}$	30	27.5	30	11	20	30	30	30	27.5	30	11	20	30	30
$\hat{\bar{Y}}_{P13}$	31	32	31.5	13	22	31	31	31	32	31.5	13	22	31	31
$\hat{\bar{Y}}_{P14}$	32	32	31.5	16	26	32	32	32	32	31.5	16	26	32	32
$\hat{\bar{Y}}_{P15}$	33	32	33	18	28	33	33	33	32	33	18	28	33	33
$\hat{\bar{Y}}_{P16}$	34	34.5	34.5	19	30	34	34	34	34.5	34.5	19	30	34	34

$\hat{\bar{Y}}_{P17}$	35	34.5	34.5	21	32	35	35	35	34.5	34.5	21	32	35	35
$\hat{\bar{Y}}_{P18}$	36	36.5	36	23	33	36	36	36	36.5	36	23	33	36	36
$\hat{\bar{Y}}_{P19}$	37	36.5	37	24	34	37	37	37	36.5	37	24	34	37	37
$\hat{\bar{Y}}_{P20}$	38	38	38	29	36	38	38	38	38	38	29	36	38	38
$\hat{\bar{Y}}_{P21}$	3	4	3	3	4	3	3	3	4	3	3	4	3	3
$\hat{\bar{Y}}_{P22}$	6	11	6	7	6	6	6	6	11	6	7	6	6	6
$\hat{\bar{Y}}_{P23}$	2	3	2	2	3	2	2	2	3	2	2	3	2	2
$\hat{\bar{Y}}_{P24}$	5	10	5	6	5	5	5	5	10	5	6	5	5	5
$\hat{\bar{Y}}_{P25}$	15	17	7	25	13	8	7	15	17	7	25	13	8	7
$\hat{\bar{Y}}_{P26}$	19	19	18	27	17	19	18	19	19	18	27	17	19	18
$\hat{\bar{Y}}_{P27}$	1	1	1	8	1	1	1	1	1	1	8	1	1	1
$\hat{\bar{Y}}_{P28}$	4	2	4	10	2	4	4	4	2	4	10	2	4	4
$\hat{\bar{Y}}_{P29}$	28	29.5	28	38	38	28	28	28	29.5	28	38	38	28	28
$\hat{\bar{Y}}_{P30}$	27	29.5	27	37	37	27	27	27	29.5	27	37	37	27	27
$\hat{\bar{Y}}_{P31}$	26	25	25.5	36	35	26	26	26	25	25.5	36	35	26	26
$\hat{\bar{Y}}_{P32}$	25	25	25.5	35	31	25	25	25	25	25.5	35	31	25	25
$\hat{\bar{Y}}_{P33}$	24	25	24	34	29	24	24	24	25	24	34	29	24	24
$\hat{\bar{Y}}_{P34}$	23	22.5	22.5	33	27	23	23	23	22.5	22.5	33	27	23	23
$\hat{\bar{Y}}_{P35}$	22	22.5	22.5	32	25	22	22	22	22.5	22.5	32	25	22	22
$\hat{\bar{Y}}_{P36}$	21	20.5	21	31	24	21	21	21	20.5	21	31	24	21	21
$\hat{\bar{Y}}_{P37}$	20	20.5	20	30	23	20	20	20	20.5	20	30	23	20	20
$\hat{\bar{Y}}_{P38}$	18	18	19	28	21	18	19	18	18	19	28	21	18	19

CONCLUSION:

Concluding on the ranked based on bias and Mean Squared Error, the result showed that the twenty-seventh estimator came first in all the distributions samples considered except in Hyper Geometric distribution which has the tenth estimator as the best, while the twentieth estimator came last in all the distributions and samples except in Hyper Geometric distribution.

More specifically, the analysis, results, the following conclusions are drawn:
The best of the thirty-eight ratio estimators is the twenty-seventh estimator given by,

$$\hat{\bar{Y}}_{P^{27}} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n\beta_1 + \phi_1)} \ (\bar{X}_N\beta_1 + \phi_1)$$

because it has the least mean squared error (MSE) as well as bias. This estimator performs better than others in six of the seven distributions under considerations, hence it can be said to be distribution free. The twenty-seventh estimator remains the best ratio estimator among the class of estimators under consideration irrespective of the sample size. The estimators perform better with increased sample size.

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