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CHARACTERISTICS OF A SYSTEM IN ABNORMAL WEATHER CONDITIONS BY USING L APL ACE TRANSFORMS

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ABSTRACT: *The research studies the availability and reliability to a system in which each unit of two modes, (normal, total failure) in two weather conditions- normal and stormy. To calculate reliability, the differential eq. of system resolved by Laplace Transform (L .T) depend on Complex imagine roots. Suppose the system rate of failure and rates of change of weather conditions are constant and the rate for repair of every unit are exponential distribution. The distribution of repair time be based on starting state of repair and not with the change in weather. we analysis graphically to watch the impress of several system parameters in mean time failure and availability.*

KEYWORDS: Reliability, Availability, State Dependent System, MTTF, Laplace Transform, Change of Weather Conditions.

INTRODUCTION

Reliability study of the repair problem of the device is more important in our lives where it is used widely in the industrial system and the manufacturing system. Many papers have analyzed system with the concepts of normal weather, stormy (abnormal) weather using theory of Markov renewal process and semi-Markov process For example, [1] This paper deal with cost analysis of a system with preventive maintenance by using

Kolmogorov's forward equations method. [2] Present system cost analysis with three modes, (total failure, partial failure & normal) the distributions of the failure time and the time of weather conditions change are exponential distributed and the time repair is general. [3]&[4] The author presents two mathematical models to predict the performance of the man—machine systems under different weather conditions. The conditions weather Change is constant but the repair time is general. [5] investigate on reliability and availability analysis of n-unit outdoor power system subject to the adjustable repair facility.[6] He study the stochastic behavior of two unit cold standbys, considering the change of the weather condition due to the operating or failure for a weather controlled device where the failure distributions and weather repair rate are exponential where the rate of repair is general. [7] Display analysis of availability and the reliability of a standby repairable system with degradation facility, [8] investigate on availability of system of multiple degradations

This paper focus on study a system having two modes, (normal and total failure) in two weather conditions- normal and stormy resolved by L aplace Transform (L .T) depend on Complex imagine roots then we show numerical results to analyze the impress of the system parameters on reliability and system availability.

The objective of this research is summarized as part 1 shows the mathematical introduction and notation. Part 2 talks about the cubic equations roots and their cases, availability, reliability and mean time failure for every case. Part 3 present the system behavior through graphs. Finally, part4 show outlines the finely conclusions.

System Description

The system is analyzed under following practical assumptions:

The system unit contain two modes- normal (N) and total failure (F)

The failure time and rate of weather condition change are exponential distribution.

Facility of repair is available to totally failed unit but not permanently while can available whenever needed.

The failed system failure rates are the different whether the weather is normal or stormy.

After repair the failed unit will be new.

Table (1). Transition states

	So	S1	S2	S3
So		$\lambda 1$	µ3	
S1	µ1			
S2	µ4			$\lambda 2$
S3			µ2	

The system may be in one of the following states

Up states S0= (No, W) , S2 = (No,$\overline{W}$)

Down state S1 = (F rw, W) , S3 = (F r $\overline{w}$,$\overline{W}$)

A unit description:

No system in N –mode and operative

W/W̄ system in N –mode / stormy weather

Frw /Fr $\overline{w}$ System in F-mode and under repair when failed in normal / stormy weather.

Notations and system states:

λ_1 Rate of failure at normal weather from N- mode to F-mode

$\lambda 2$ Rate of failure at stormy weather from N- mode to F-mode

µ1 Rate of repair of totally failed system when the repair starts in a normal weather

µ2 Rate of repair of totally failed system when the repair starts in a stormy weather

µ3 Change of weather is Constant from normal to stormy μ4 Change of weather is Constant from stormy to normal

$p(\tau)$ Probability for j =0, 1, 2, 3 $pj*$ (s) Laplace transform (L.T) of $pj(\tau)$

A (τ): functions of availability.

R (τ): functions of reliability.

MTTF: mean time failure.

Where L aplace transforms (L.T) of $p(\tau)$ is:

$$p_j^*\ (s) = \int_0^\infty e^{-S\tau}\, p_j(\tau)$$

Mathematical model description:

This part showing the differential eq. for the system of Table (1) Transition states

$$\frac{dP_0(t)}{dt} = -\ [\lambda_1 + \mu_3]\ P_0(t) + \mu_1\ P_1(t) + \mu_4 P_2(t) \tag{1}$$

$$\frac{dP_1(t)}{dt} = -\ \mu_1\ P_1(t) + \lambda 1\ P0(t) \tag{2}$$

$$\frac{dP_2(t)}{dt} = -\ [\lambda_2 + \mu_4]\ P_2(t) + \mu_3\ P_0(t) + \mu 2\ P3(t) \tag{3}$$

$$\frac{dP_3(t)}{dt} = -\mu_2\ P_3(t) + \lambda 2\ P2(t) \tag{4}$$

Initial conditions:

$$P_j\ (0)\ = \begin{cases} 1 & where\ j = 0 \\ 0 & \text{else} \end{cases}$$

Applying Laplace transform (L .T) for (1) – (4), we obtain:

$$\mu_3 + s]\ P_0^*(s) - \mu_1\ P_1^*(s) - \mu_4 P_2^*(s) = P_0\ (0) \tag{5}$$

$$P_1^*(s) - \lambda_1\ P_0^*(s) = P_1\ (0) \tag{6}$$

$$\mu_4 + s]\ P_2^*(s) - \mu_3 P_0^*(s) - \mu_2\ P_3^*(s) = P_2\ (0) \tag{7}$$

$$[\lambda 1 + \quad P_3^*(s) - \lambda_2\ P_2^*(s) = P_3\ (0) \tag{8}$$

[μ1 + s]

[λ2 +

[μ2 + s]

Solving eq. (5-8) by crammer rule, we obtain:

$$P_0^*(s) = \frac{s^3+As^2+BS+m}{s[s^3+a_1s^2+a_2S+a_3]} \quad , \quad P_1^*(s) = \frac{\lambda_1 s^2 + (\lambda_1 \mu_4 + \lambda_2 \lambda_1 + \mu_2\lambda_1) s + \mu_4\mu_2\lambda_1}{s[s^3+a_1s^2+a_2S+a_3]}$$

$$P_2^*(s) = \frac{\mu_3 (\mu_1 + s) (\mu_2 + s)}{s[s^3+a_1s^2+a_2S+a_3]} \quad , \quad P_3^*(s) = \frac{\mu_3 \lambda_2 (\mu_1 + s)}{s[s^3+a_1s^2+a_2S+a_3]}$$

Where,

$a_3 = \mu_1\mu_2\mu_3 + \mu_1\mu_2 \mu_4 + \mu_3\mu_1 \lambda_2 + \mu_2 \mu_4\lambda_1$

$a2 = \lambda 1\lambda 2 + \mu 1\mu 2 + \mu 1 \lambda 2 + \mu 2\lambda 1 + \mu 1 \mu 4 + \mu 1\mu 3 + \mu 2 \mu 4 + \mu 2\mu 3 + \mu 3\lambda 2 + \mu 4 \lambda 1$

$a1 = \mu 1 + \mu 2 + \lambda 1 + \lambda 2 + \mu 4 + \mu 3$

$A = \mu 1 + \mu 2 + \mu 4 + \lambda 2$

$B = \mu 1 \lambda 2 + \mu 1\mu 2 + \mu 4\mu 2 + \mu 1 \mu 4$

$m = \mu 4 \mu 1\mu 2$

We know that the system contain of (2) up state and (2) down state so:

$$P^* (s) = \frac{s^3+As^2+BS+m+\mu_3 (\mu_1 + s) (\mu_2 + s)}{s[s^3+a_1s^2+a_2S+a_3]}$$

$$P^* (s) = \frac{s^3+A_1s^2+B_1S+m_1}{s[s^3+a_1s^2+a_2S+a_3]}$$

Where,

$A1 = \mu 1 + \mu 2 + \mu 4 + \lambda 2 + \mu 3$

$B1 = \mu 1 \lambda 2 + \mu 1\mu 2 + \mu 4\mu 2 + \mu 1 \mu 4 + \mu 3\mu 2 + \mu 1\mu 3$

$m1 = \mu 4 \mu 1\mu 2 + \mu 3 \mu 1\mu 2$

Cubic equations roots have are 2 cases

First case (D > 0) [1 root is real and 2 complex]

$$P^* (s) = \frac{s^3+A_1s^2+B_1S+m_1}{s(s+A_2-W)(S+A_2+w_1-i\sqrt{3}v_1)(S+A_2+w_1+i\sqrt{3}\,v_1)}$$

Where $q = \frac{3a_2 - a_1{}^2}{9}$, $r = \frac{9a_1a_2 - 2a_1{}^3 - 27a_3}{54}$

$D = q3 + r2$, $u = (r+\sqrt{D})^{\frac{1}{3}}$, $t = (r-\sqrt{D})^{\frac{1}{3}}$

$w_1 = \frac{(u+t)}{2}$, $w = (u+t)$, $v_1 = \frac{(u-t)}{2}$, $A_2 = \frac{a_1}{3}$

By using inverse of L aplace Transform (I.L.T) of eq., we obtain

$$P(\tau) = \frac{m_1}{(A_2 - w)(A_2{}^2 + A_2 w + w_1{}^2 + 3v_1{}^2)}$$

$$+ \frac{(-A_2 + w)^3 + A_1(-A_2 + w)^2 + B_1(-A_2 + w) + m_1}{(-A_2 + w)\left(9w_1{}^2 + 3v_1{}^2\right)} e^{(-A_2 + W)\tau}$$

$$+ \left\{\frac{2[PX + 3HT][\cos\sqrt{3}(v_1)t] - 2\sqrt{3}\,[(HX) - (TP)]\left(\sin\sqrt{3}(v_1)t\right)}{X^2 + 3T^2}\right\} e^{(-A_2 - w_1)\tau}$$

Where,

$P = (-A_2{}^3 - 3A_2{}^2 w_1 + 9\,A_2\,v_1{}^2 - w_1{}^3 + 9\,w_1 v_1{}^2 - \;A_2\,w_1{}^2)$

$+ A1\,(A22 + A2w - 3v12 + w12\,) + B\,(-A2 - \;w1) + m1$

$H = (6A_2 v_1 w_1 - 3v_1{}^3 + 3w_1{}^2 v_1 + 3A_2{}^2 v_1 - A_1 A_2 v - A_1\,v_1 w + Bv_1\;)$

$X = 3v2w + 6v12A2$

$T = 3wv_1 A_2 + 3w_1{}^2 v - \;6v_1{}^3$

Reliability System and availability

System availability

We obtaining the system availability from the relation

$$\boldsymbol{A(\tau)} = \frac{m_1}{(A_2 - w)(A_2{}^2 + A_2 w + w_1{}^2 + 3v_1{}^2)}$$

$$+ \frac{(-A_2 + w)^3 + A_1(-A_2 + w)^2 + B_1(-A_2 + w) + m_1}{(-A_2 + w)\left(9w_1{}^2 + 3v_1{}^2\right)} e^{(-A_2 + W)\tau}$$

$$+ \left\{\frac{2[PX + 3HT][\cos\sqrt{3}(v_1)t] - 2\sqrt{3}\,[(HX) - (TP)]\left(\sin\sqrt{3}(v_1)t\right)}{X^2 + 3T^2}\right\} e^{(-A_2 - w_1)\tau}$$

So the steady – state availability (A) from the following relation

A=**lim** (τ)

$\boldsymbol{t}\rightarrow\infty$

$$A = \frac{m_1}{(A_2 - w)(A_2{}^2 + A_2 w + w_1{}^2 + 3v_1{}^2)}$$

2.1.3. System reliability:

Supposing that at least one for failed states is absorbing state and the state transition rate equal to zero, ∴ the reliability function for this model as like that

$$\boldsymbol{R(\tau)} = \frac{m_1}{(A_2 - w)(A_2{}^2 + A_2 w + w_1{}^2 + 3v_1{}^2)}$$

$$+ \frac{(-A_2 + w)^3 + A_1(-A_2 + w)^2 + B_1(-A_2 + w) + m_1}{(-A_2 + w)\left(9w_1{}^2 + 3v_1{}^2\right)} e^{(-A_2 + W)\tau}$$

$$+ \left\{\frac{2[PX + 3HT][\cos\sqrt{3}(v_1)t] - 2\sqrt{3}\,[(HX) - (TP)](\sin\sqrt{3}(v_1)t)}{X^2 + 3T^2}\right\} e^{(-A_2 - w_1)\tau}$$

2.1.4 Mean time failure (MT T F):

$$\text{MTTF} = \int_0^{\infty} R(\tau)\,dt = \lim_{t\to\infty}\int_0^t R(\tau)\,dt$$

$$\text{MTTF} = \lim_{t\to\infty}\int_0^t R(\tau)\,dt = \lim_{s\to 0} SL\left\{\int_0^t R(\tau)\,dt\right\} = \lim_{s\to 0} S\,\frac{R^*(S)}{S}$$

$$\text{MTTF} = \lim_{s\to 0} R^*(S) \qquad , \qquad R*(S) = L\,(R(\tau))$$

$$\text{MTTF} = -\left(\frac{(-A_2 + w)^2 + A_1(-A_2 + w) + B_1}{(-A_2 + w)(9w_1{}^2 + 3v_1{}^2)}\right) - \frac{(2[PX + 3HT])(-A_2 - w_1) + 6v_1\,[HX - TP]}{(X^2 + 3T^2)[(-A_2 - w_1)^2 + 3v_1{}^2]}$$

2.2. Second case $D < 0$ [All roots are real and unequal]

$$P^*(s) = \frac{s^3 + A_1 s^2 + B_1 S + m_1}{s(s + A_2 - w_0)(S + A_2 - w_2)(S + A_2 - v_2)}$$

Where

$$s_1 = w_0 - \frac{a_1}{3} \quad , \quad s_2 = w_2 - \frac{a_1}{3} \quad , \quad s_3 = v_2 - \frac{a_1}{3} \quad , \quad \theta = \cos^{-1}\frac{r}{\sqrt{-q^3}}$$

$$w_0 = 2\sqrt{-q}\cos\left(\frac{\theta}{3}\right) \quad , \quad w_2 = 2\sqrt{-q}\cos\left(\frac{\theta}{3} + 120\right)$$

$$v_2 = 2\sqrt{-q}\cos\left(\frac{\theta}{3} + 240\right) \quad , \quad A_2 = \frac{a_1}{3}$$

By using inverse of L aplace Transform (I.L.T) of eq., we obtain

$$P(\tau) = \frac{m_1}{(A_2 - w_0)(A_2{}^2 - A_2 w_2 - A_2 v_2 + w_2 v_2)}$$

$$+ \frac{(-A_2 + w_0)^3 + A_1(-A_2 + w_0)^2 + B_1(-A_2 + w_0) + m_1}{(-A_2 + w_0)(w_0{}^2 - w_0 w_2 - w_0 v_2 + w_2 v_2)} e^{(-A_2 + w_0)\tau}$$

$$+ \frac{(-A_2 + w_2)^3 + A_1(-A_2 + w_2)^2 + B_1(-A_2 + w_2) + m_1}{(-A_2 + w_2)(w_2{}^2 - w_0 w_2 - w_2 v_2 + w_0 v_2)} e^{(-A_2 + w_2)\tau}$$

$$+ \frac{(-A_2 + v_2)^3 + A_1(-A_2 + v_2)^2 + B_1(-A_2 + v_2) + m_1}{(-A_2 + v_2)(v_2{}^2 - w_0 v_2 - w_2 v_2 + w_0 w_2)} e^{(-A_2 + v_2)\tau}$$

Reliability System and availability

System availability

$$A(\tau) = \frac{m_1}{(A_2 - w_0)(A_2{}^2 - A_2 w_2 - A_2 v_2 + w_2 v_2)}$$

$$+ \frac{(-A_2 + w_0)^3 + A_1(-A_2 + w_0)^2 + B_1(-A_2 + w_0) + m_1}{(-A_2 + w_0)(w_0{}^2 - w_0 w_2 - w_0 v_2 + w_2 v_2)} e^{(-A_2 + w_0)\tau}$$

$$+ \frac{(-A_2 + w_2)^3 + A_1(-A_2 + w_2)^2 + B_1(-A_2 + w_2) + m_1}{(-A_2 + w_2)(w_2{}^2 - w_0 w_2 - w_2 v_2 + w_0 v_2)} e^{(-A_2 + w_2)\tau}$$

$$+ \frac{(-A_2 + v_2)^3 + A_1(-A_2 + v_2)^2 + B_1(-A_2 + v_2) + m_1}{(-A_2 + v_2)(v_2{}^2 - w_0 v_2 - w_2 v_2 + w_0 w_2)} e^{(-A_2 + v_2)\tau}$$

And the steady – state availability are:

$$A = \lim_{t \to \infty} A(\tau)$$

$$A = \frac{m_1}{(A_2 - w_0)(A_2{}^2 - A_2 w_2 - A_2 v_2 + w_2 v_2)}$$

2.2.3. The mean time failure (MTTF):

$$\mathbf{MTTF} = -\left(\frac{(-\mathbf{A_2} + \mathbf{w_0})^2 + A_1(-\mathbf{A_2} + \mathbf{w_0}) + B_1}{(-\mathbf{A_2} + \mathbf{w_0})(\mathbf{w_0}^2 - \mathbf{w_0 w_2} - \mathbf{w_0 v_2} + \mathbf{w_2 v_2})}\right) - \left(\frac{(-\mathbf{A_2} + \mathbf{w_2})^2 + A_1(-\mathbf{A_2} + \mathbf{w_2}) + B_1}{(-\mathbf{A_2} + \mathbf{w_2})(\mathbf{w_2}^2 - \mathbf{w_0 w_2} - \mathbf{w_2 v_2} + \mathbf{w_0 v_2})}\right)$$

$$-\left(\frac{(-\mathbf{A_2} + \mathbf{v_2})^2 + A_1(-\mathbf{A_2} + \mathbf{v_2}) + B_1}{(-\mathbf{A_2} + \mathbf{v_2})(\mathbf{v_2}^2 - \mathbf{w_0 v_2} - \mathbf{w_2 v_2} + \mathbf{w_0 w_2})}\right)$$

The system behavior through graphs study of MTTF and availability. We plot the steady -state for the models, against λ1 keeping other parameters

For more the sensible availability and MTTF System with normal weather

$\lambda 2 = 0.25$, $\mu 2=0.3$.,$\mu 1=0.1$ $\mu 4=0.7$.,$\mu 3=0.5$ $\lambda 1= 0.2$,0.4 , 0.6 , 0.8 , 1 , 1.2

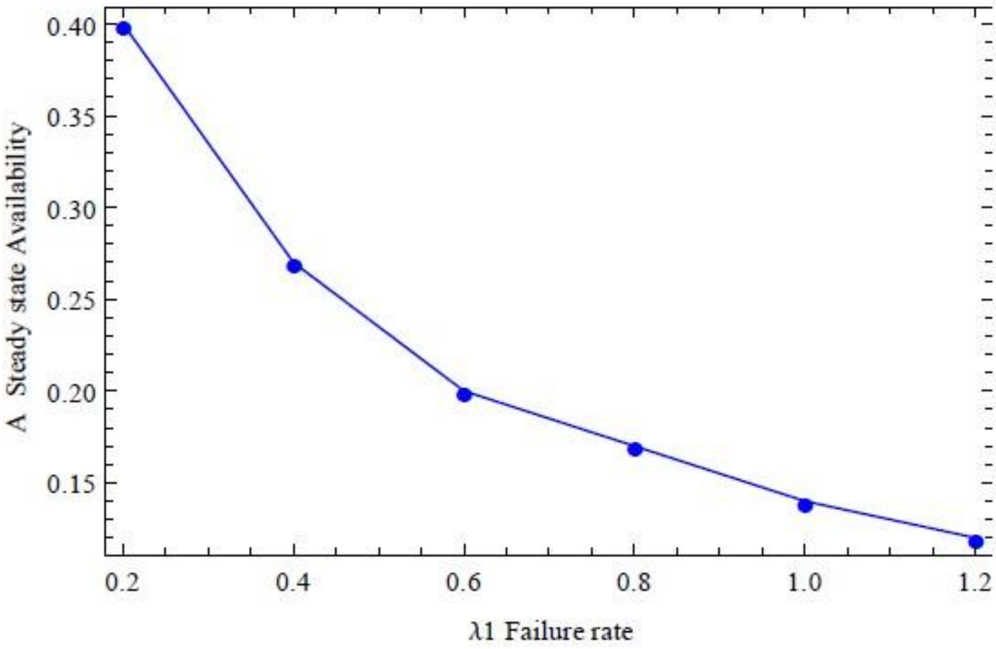

Fig. 1 The Steady state Availability w.r.t. Failure Rate $\lambda 1$

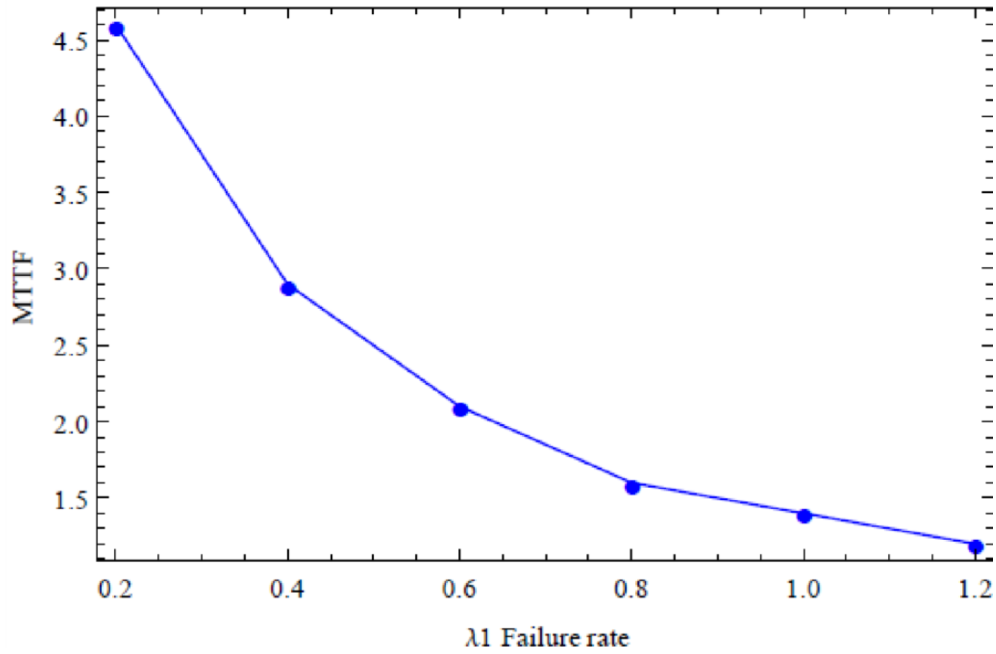

Fig. 2 The mean time failure w.r.t. F ailure Rate $\lambda 1$

System with stormy weather

$\lambda 1 = 0.25$, $\mu 2=0.3$,$\mu 1=0.1$ $\mu 4=0.7$.,$\mu 3=0.5$ $\lambda 2= 0.2$,0.4 , 0.6 , 0.8 , 1 , 1.2

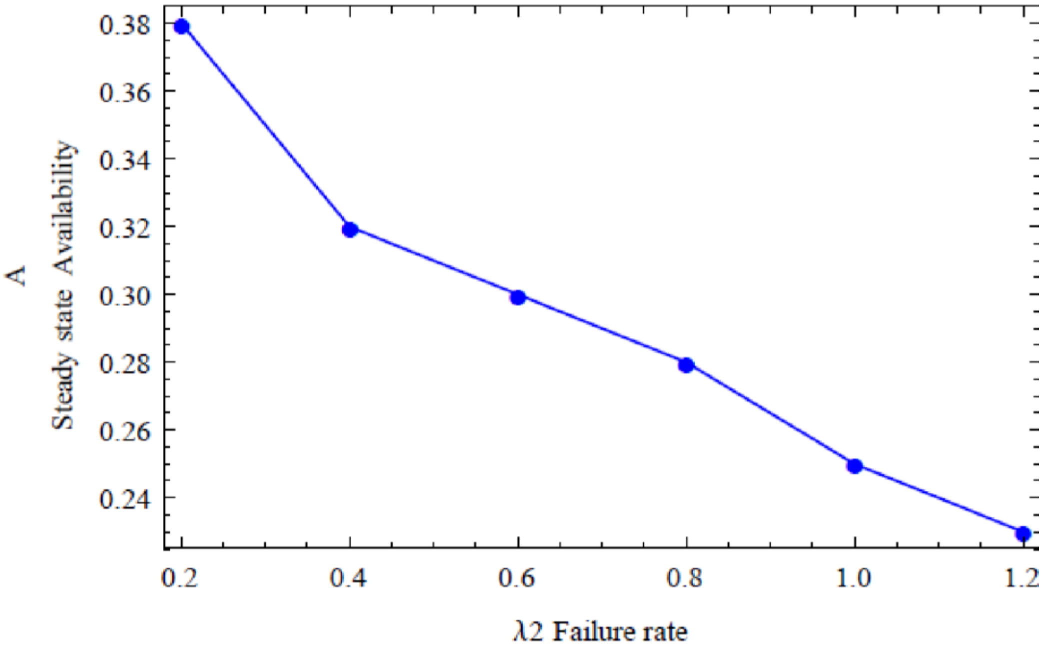

Fig. 3 The Steady state Availability w.r.t. Failure Rate $\lambda 2$

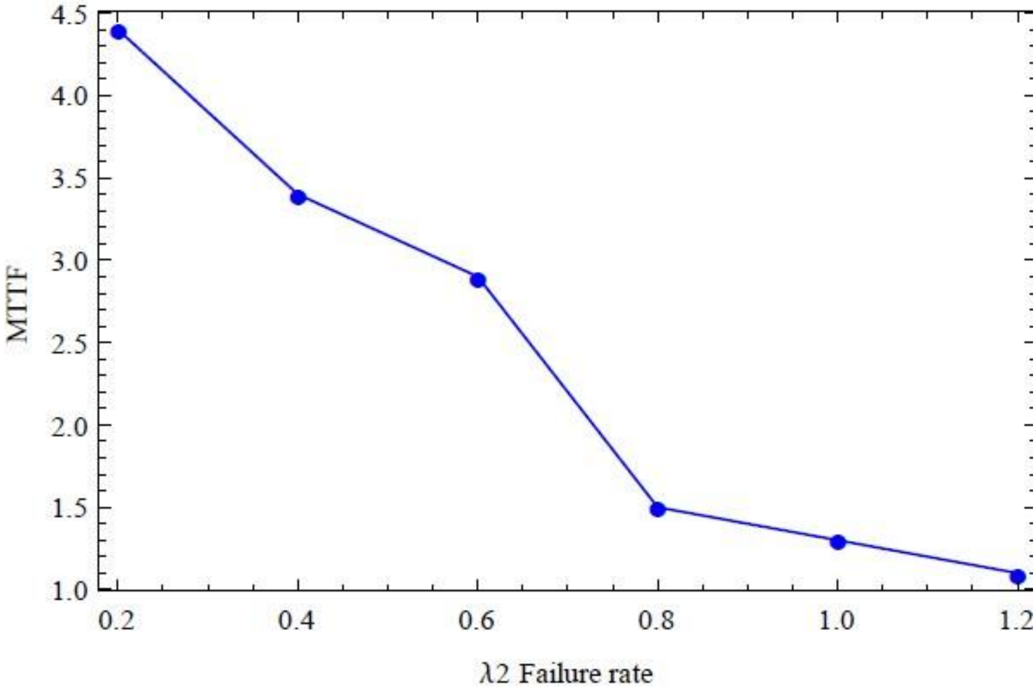

Fig.4 The mean time failure w.r.t. F ailure Rate $\lambda 2$

CONCLUSIONS

We use computer software, to plot system availability and MTTF in figure 1 and 2 respectively. It is noted that A decrease as λ increases and MTTF decrease as λ increases in normal and stormy weather

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