Analytical Solution of two Model Equations for the Variation of Capital Market Prices

Amadi, Innocent Uchenna[1] , Uchechi, Ahana[2] and Anyamele, Bethel Azunna[3]

[1]Department of Mathematics/Statistics Captain Elechi Amadi polytechnic,Rumuola Port Harcourt.
[2and3]Department of Mathematics/Statistics, Ignatius Ajuru University of Education, Port Harcourt,Rumuolumeni.

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ABSTRACT: *This paper investigated different methods for estimation of parameters of Weibull distribution, using Mean Square Error (MSE) as a criterion for selecting the best model. The result showed that Method of Moments outstripped other methods. In the same vain, the estimated results were logically extended to form a matrix that would help in predicting different commodity price processes, and the result obtained by exploring the properties of the principal component analysis solution and results showed the level of proportion accounted by first Principal Component Analysis (PCA). However, the eigenvectors describe the direction of the stock market prices in terms of changes in short-run and long-run respectively.*

KEYWORDS: Weibull, stock market prices, principal component and matrix solution

INTRODUCTION

In two models method is the pretentious and effectual method in modeling stock prices in order to predict the future. The idea behind it is that the two models will be solved or analyzed independently using stock market prices for prediction and detailed analysis for the purpose of model fitness. The two models, we shall explore in this paper are as follows; Weibull distribution and Principal component analysis. Weibull distribution was discovered by [1].It has two ranges of parameters. The popularity of the distribution is attributable to the fact that it provides a useful description for many different kinds of data, especially in emerging areas such as Wind speed, Statistics, Mathematics of finance and Engineering etc. Statisticians makes use of probability plots, usually referred to as graphical procedure to analyze life data prior to the admit of desktop computers and reliability analysis software became available, in modeling processes of stock market prices.

However, the unstable property and other considerable factors such as liquidity on stock return, since the sudden change in share prices occur randomly and frequently. Researchers are kin to study the behavior of the unstable market variable so as to enable investors and owners of cooperation make decisions on the level of their investment in stock market exchange.

The great interest on the two model approach is to understand the dynamic nature of the effect on stock market prices. So the ability to adequately understand the key two parameter of Weibull distribution and Principal component analysis with respect to stock market prices is of great practical importance. More so, stock price modeling is very essential because it is the medium in which the stock variables are modeled for the purpose of practical findings. That is why, it is important to know the natural-nature of the problem under study. Handling such problems needs viable Mathematical models such as; Weibull distribution and Principal component analysis.

The application of Weibull distribution has been of mammoth interest to Scholars, Mathematicians and Statisticians alike. For instance, [2] workers on compartment analysis of methods of Estimating Weibull distribution. They applied three method of estimation such as maximum likelihold estimator, method of moments were selected as the best method bases on the selection criteria. In the same vein, [3] considered Weibull distribution for both analytics and numerical and results shows that the mean rank is the best method among the methods in the graphical and analytical procedures; on the numerical simulation studies the maximum likelihood estimation method (MLE) significantly outperformers other methods.

[4] after working on the method for estimating the parameters of weibull distribution, he presented both graphical and analytical method of estimating the Weibull distribution. Parameters discovered after computation of results that the method which gives the best estimates is the method of moments. Yunn-Kuangchu and Jau-chuanke examined the comparison of the two methods for Weibull parameters, one is the maximum likelihood method and the other is the least squares method. A numerical simulation study is carried out to understand the performance of the two methods. Based on sample root mean square errors they made a comparison between the two computation approaches and found out that the last square method significantly out performs the maximum likelihood when the sample size is small.

[2] compared three methods of estimating 2-parameter Weibull distribution by using the Mean Squared Error(MSE) as the test criterion. Three methods used were Maximum Likelihood Estimator, Methods of moments and the Least Square Method. They concluded that the Method of moments was the best method based on the selection test criterion. So many authors have equally used principal component Analysis under different conditions as its applications are very numerous, for instance, [5] introduced a measure of systemic risk called the absorption ratio. It is the fraction of variance absorbed by a finite number of Principal components. They reported that most global financial crises were coincident with positive shifts of the absorption ratio. [6] not only looked at the absolute value of variance explained by the first component, they also computed the change in the variance explained to capture the systemic risk. They obtained similar funds to [5] that both the absolute value and change of variance explained by principal component one increased during a financial crises. [7] applied PCA to measure diversification quantitatively and testing equal-weighted portfolios of stocks in S and P 100 index and reported that a pool of 40 randomly selected stocks approximately as diversified as only 20 truly independent components.

PCA provides us with a way to identity uncorrelated risk sources in the market and pick stocks from those different risk sources, the resulting portfolio size is more meaningful from the point of view of diversification. Market connectedness does not stay constant over time [8] did not point out the number of stocks needed to achieve a certain level of diversification was not the same in the 1963-85 period and the 1986-97 period. [9] first proposed the idea of using PCA to analyse the efficient portfolio problem. Their basic idea was based on the fact that if there were no correction among assets,the complexity in portfolio selection dramatically decreased.

Modeling of Stock market prices cannot be over emphasized due to its numerous applications in the fast growing field of science and technology:For instance, [10] studied stochastic analysis of stock market returns and Growth-rates. The precise conditions for obtaining the drifts, volatilities and Growth-rates of four different stocks were considered. Also [11]studied the stochastic analysis of stock market returns for investors. They compared the variances of four different stocks using a criteria for selection and result showed that stock (1) is the best among the stocks.

[12] considered the unstable nature of stock market forces using proposed differential equation model. In the work of Osu et al.(2009) studied stability analysis of stochastic model of price change at the floor of a stock market. In their research précised conditions are obtained which determines the equilibrium price and growth rate of stock shares.

[13]Considered stochastic analysis of the behavior of stock prices. Results reveal that the proposed model is efficient for the prediction of stock prices. In the same vain, Ofomata et al.(2017) studied the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in their research the drift and volatility coefficients for the stochastic differential equations were determined and the Euler-Maruyama method for system of SDE'S was used to stimulate the stock prices.[4],built the geometric Brownian Motion and studied the accuracy of the model with detailed analysis of simulated data.

In this paper, we considered estimating two parameter Weibull distribution using Method of Moment, Least square Method, and Maximum Likelihood estimation where Mean square error was used as a criterion for selecting the best model. Secondly, we used the estimates of these stock market prices to form a matrix that would help in predicting different commodity price processes by exploring the properties of the principal component analysis (PCA).

The aim of this project is first, present Weibull distribution and methods of estimators in determining the best method of estimating Weibull parameter via stock market prices and using principal component analysis to measure the proportion of total stock variance accounted for by the first principal component analysis to a time varying investment returns.

To the best of our knowledge this is the first study that combined Weibull distribution and PCA model to predict stock market prices. To this end we extended the works of Egwim et al. (2015) and Nwobi and Ugomma (2014) by incorporating PCA model and giving detailed analysis in predicting stock market prices..

This paper is arranged as follows; Section 2.1 presents Materials and Methods, Section 3.1 presents the Analysis and Results, Discussion of results is seen in Section 4 and concluded in Section 5.

MATERIALS AND METHODS

let S_1 S_2…, S_N be a random sample of size N from a population, ∋ $S_i \in \Box$, $S_i > 0$.

The general form of a three parameter Weibull probability density function (pdf) is given by

$$F(x) = \frac{\beta}{\propto}\left(\frac{x_1 - V}{\propto}\right)^{\beta-1} \exp\left\{\left(\frac{x_1 - u}{\alpha}\right)^{\beta}\right\} xv \geq 0; \alpha, \beta > 0 \quad (1)$$

Where; x_1 is the data vector at time; β is the shape parameter; ∝ is the scale parameter that indicates the spread of the distribution of sampled data and v is the location parameter.

The cumulative distribution function (cdf) of the weibull distribution is mathematically given as:

$$F(x_1) = 1\text{-}\exp\left\{\left(\frac{x_1 - v}{\alpha}\right)\right\} \quad (2)$$

In case of v=0, the probability distribution function in equation (1) reduces to equation (2)

$$F(x_1) = \left\{\left(\frac{\beta}{\alpha}\right)\left(\frac{x_1}{\alpha}\right)\right\}^{\beta-1} \exp\left\{-\left(\frac{x_1}{\alpha}\right)^{\beta}\right\} \; x \geq 0, \alpha, \; \beta > 0 > 0 \quad (3)$$

otherwise with a corresponding cumulative distribution function as

$$F(x_1) = \begin{cases} 1 - \exp\left\{-\left(\frac{x_2}{\alpha}\right)^{\beta}\right\}, x \geq 0 \\ 0, \text{ otherwise} \end{cases} \quad (4)$$

The mean and variance of the weibull are E(x) = αr (1 + $^1/_\beta$) and v(x) = α^2 [r(1 + $^2/_\beta$) = ⌈^{2} (1 + $^1/_\beta$)] respectively, where r(n) gamma function evaluated at n. The various analytical methods in estimating weibull parameters are:

Methods of Estimation

The methods of estimation are as follows(a)Maximum likelihood Estimator (MLE) (b) Methods of Moments (MOM) and (c)The Least Square Method (LSM)

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Maximum Likelihood Estimator (MLE)

Let $x_1,x_2\ldots\ x_n$ represent a random sample of size n drawn from a population with probability density function f (x,λ) where $\lambda = (\beta, \alpha)$is an unknown vector of parameters. The likelihood function is defined as: $L = f(\alpha, \beta) = \prod_{i=1}^{n} f(x_i, \lambda)$ (5)

The maximum likelihood of λ = (β, α),maximum L or equivalently, the logarithm of L when

$$\frac{\delta InL}{\delta\lambda} = 0 \quad (6)$$

Where solutions that are not function of the sample values.$x_1,x_2\ldots\ x_n$are not admissible. Now are apply the maximum likelihood estimator to estimate the parameters of weibull distribution, namely ∝ and β respectively where y = 0.

Consider the Weibull probability density function given in equation (5) the likelihood function will be given as:

$$L\ (x_1,x_2\ldots\ x_n: \beta, \propto) = \prod_{i=1}^{n} =1\left(\frac{\beta}{\propto}\right)\left(\frac{x_i}{\propto}\right)^{\beta-1} - e\left\{\left(\frac{x_i}{\propto}\right)^{\beta}\right\}$$

$$= \left(\frac{\beta}{\propto}\right)^n \left(\frac{1}{\propto}\right)^{\beta-1} \sum_{n-1}^{n} \propto i(\beta - 1)e - (x_i)^{\beta-1}\ e - \left(\frac{x_i}{\propto}\right)^{\beta}$$

$$= \left(\frac{\beta}{\propto}\right)^n \left(\frac{1}{\propto}\right)^{n\beta-n} \sum_{i1}^{n} \alpha i^{(\beta-1)} e - \sum_{n=1}^{n}\left(\frac{xi}{\alpha}\right)^{\beta} \quad (7)$$

Taking the algorithms of both sides and differentiating partially with respect to β and ∝ in turn and equating to zero, we obtain the estimating equations as follows:

$$\frac{\delta I_{nL}}{\delta\beta} = \frac{n}{\beta} \div \sum^{n}{}_{i=1}\sum^{\beta}{}_{i}\ I_n x_i = 0 \quad (8)$$

$$\frac{\delta I_{nL}}{\delta\propto} = \frac{n}{\propto} + \frac{1}{\alpha^2}\sum_{L=1}^{n} x_i^{\beta} = 0 \quad (9)$$

$$\frac{\delta I_{nL}}{\delta\propto} = \frac{-n}{\propto} + \frac{1}{\propto^2}\sum_{n=1}^{n} x^{\beta} = 0 \quad (10)$$

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so substituting (10) in (11) gives

$$I_n x_i \frac{-1}{\beta} + \frac{1}{n} \sum_{i=1}^{n} In x_i = 0 \quad (11)$$

$$\text{Hence } \beta MLE = \sum_{L=1}^{n} x_i \overset{n}{P} \sum_{i=1}^{n} In \frac{(xi)}{n} \quad (12)$$

∝ is now estimated using equation (10)

$$-\propto^2{}_n \div \propto \sum_{i=1}^{n} x_i \beta = 0 \quad (13)$$

So that:

$$\overset{n}{\alpha}_{MLE} = \sum_{L=1}^{n} x_i \overset{n}{P} \sum_{i=1}^{n} x_i^{\hat{\beta}} \quad (14)$$

Method Of Moments (MOM)

Let $x_1, x_2 \ldots x_n$ be a random sample and then an unbased estimator for the K^{th} moment is given by:

$$m_k = \frac{n}{\alpha} \sum_{i=1}^{n} x_i^k \; \text{i} \quad (15)$$

Where $\hat{m}_k$ denotes the estimate of K^{th} moment. In weibull the K^{th} moments follows from equation (16).

$$\mu_k = \left(\frac{1}{\propto\beta}\right)^{-x/p} - {}^k/_\beta \, \Gamma\left(1 + \frac{k}{\beta}\right) \quad (17)$$

Where Γ is a gamma function evaluated at the value of $(1 + \frac{1}{\beta})$ which

Provides the values Γ (k) at any value of k from equation (15), we can find the 1st and 2nd moments as follows.

$$\hat{M}_1 = \hat{\mu}_1 = \left(\frac{1}{\propto}\right)^{1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right) \quad (17)$$

$$\hat{M}_L = \mu_1 + \sigma^2 = \left[\frac{1}{\propto}\right]^{2/\beta} \{[\Gamma(1 + \frac{2}{\propto}) - \Gamma(1 + \frac{1}{\beta})]^2\} \quad (18)$$

Dividing m_1 by the square of m_2, we get an expression which is a function of β only .

$$\frac{\mu_i}{\sigma^2+\mu^2} = \frac{\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1+\frac{1}{\beta}\right)}{\Gamma\left(1+\frac{2}{\beta}\right)} \qquad (19)$$

Where $\hat{\mu} = \sum_{i=1}^{n} In(\frac{Si}{s_{i-1}}) = E(x_i) = \frac{1}{n}\sum_{t=1}^{n} x_i$, $\sigma^2 = E(x^2{}_i) - (E(x_i))^2$

And $Z = {}^1/_\beta$

Equation (18) is transformed in order to estimate β and ∝ repectively

$$\frac{\mu^2}{\sigma^2+\mu^2} = \frac{\Gamma\left(1+\frac{z}{\beta}\right)\Gamma\left(1+\frac{z}{\beta}\right)}{\Gamma\left(1+\frac{2z}{\beta}\right)} \qquad (20)$$

The value of the scale parameter ∝mom can be estimated thus:

$$\hat{\propto}_{Mom} = \frac{\hat{\mu}}{\Gamma\left(1+\frac{1}{\beta}\right)} \qquad (21)$$

Where $\hat{\mu}$ is the mean of the original data

Least Squares Method (LSM)

Here we assume that there is a linear relationship between two values considering.

$$Y = \propto + \beta x_1 \qquad (22)$$

$$Y = In\,[In\,(\frac{1}{1-F(T)})];\ m = \beta,\ x = Inx \text{ and } b = \beta In\propto \qquad (23)$$

Assume that a set of data pairs (y_1, y_2), (x_1,x_2) … (x_n,x_y) were obtained and plotted.

Following the least squares concept which minimizes the vertical distance between the data points and the straight line fitted to the data, the best fitting line to this data is the straight line

$$Y = \propto + \beta x$$

Such that $\sum^{n}_{t=1}(y_i - \propto + \beta_{xi})^2 = m_{In}(\propto,\beta) = \sum^{n}_{t=1}(y_i - \propto + \beta x_i)^2$ (24)

Where $\hat{\alpha}$ and $\hat{\beta}$ we let $Q = \sum^{n}_{t=1}(y_t - \propto + \beta_{xt})^2$ and differentiating Q with respect to β and equating to zero yields the following systems of equations:

$$\frac{\delta Q}{\delta\beta} = 2\sum^{n}_{t=1}(y_i - \propto + \beta x_i)^2 = 0 \qquad (25)$$

$$\frac{\delta Q}{\delta\beta} = 2\sum^{n}_{t=1}(y_i - \propto + \beta x_i)^2 x_i = 0 \qquad (26)$$

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Expanding and solving equation (24) and (25) simultaneously, we have:

$$\hat{\beta}_{Lsm} \frac{\sum_{t=1}^{n} x_i y_i - \sum_{t=1}^{n} x_i \sum_{t=1}^{n} y_i}{\sum_{t=1}^{n} x_i - \frac{(\sum_{t=1}^{n} x_t)}{n}}) \quad (27)$$

$$\text{And } \hat{\propto}_{LSM} = \frac{\sum_{t=1}^{n} y_i}{n} - \hat{\beta} \frac{\sum_{t=1}^{n} x_i}{n} = y_i - \beta x_i \quad (28)$$

Comparison of Estimation Methods

We have derived the three analytical methods for estimating Weibull distribution such as Maximum Likelihood Estimator, Method of Moment and Least Square Method.

The Mean Square Errors shall be used as a criterion for selection.

The mean squared error (MSE) criterion is given by:

$$MSE = \frac{1}{n} \sum_{i=1}^{n} [\hat{f}(x_i) - F((x_i)]^2 \quad (29)$$

Where f $((\hat{x}_i)$ is obtained by substituting the estimates of ∝ and β in (4) (for each method) while $F(x_i) = {}^{i}/_{n}$ is the empirical distribution function. The method with the mimmum mean squared error (MMMSE) becomes the best methods for the estimation of weibull parameters among the candidate methods.

Principal component Analysis of the stock variables

Definition 3.1: Suppose $\underline{X}$ has a joint distribution which has a variance matrix $\sum$ with eigenvalues $\lambda_1, \geq \lambda_2 \geq ... \geq \lambda_p \geq 0$. consider the random variables $y_1 ... y_p$ which are linear combination of the $X_1's$ ie:

$$\left.\begin{array}{c} y_1 = \underline{l}_1'\underline{X} = l_{11} X_1 + ... + l_{p_1} \lambda_p \\ \vdots \\ y_p = \underline{l}_1'\underline{X} = l_1 p X_1 + ... + l_{p_1} \lambda_p \end{array}\right\} \quad (30)$$

The y_i's will be PC if they are uncorrelated and the variances of $y_{1,} y_2$ are as large as possible.

Recall that if $y_i = l_i' X$. In order to look at the amount of information that is in y_1 .We can consider the proportion of the total population variance due to y_i

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$$\frac{\lambda_i}{\lambda_1+\lambda_2+\cdots+\lambda_p}, i=1,\ldots,p \quad (31)$$

Hopefully the proportion is large for eg=1,2, 3.

Analysis and Results

Maximum Likelihood Estimation

To obtain the estimates of $\hat{\beta}$ MLE and ${}^{\alpha}MLE$, we employ methods in subsection 2.2.1 as follows:

$\hat{\beta}$ MLE = $\sum_{i=1}^{n} 1n(xi)$ = $\frac{36.9940}{50}$ = 0.7399

${}^{\alpha}MLE$= $\frac{\sum_{i=1}^{n} X_i^{\beta}}{n}$ = $\frac{34.9596}{50}$ = 0.6992

MSE = 2.9799 x 10^{-05}

Methods Of Moments

Applying the methods in subsection 2.2.2 gives the following results

$E(X_i)$ = $\hat{\mu}$ = $\frac{36.9940}{50}$ = 0.7399

σ^2 = $\frac{\sum_{i=1}^{n}(X_i - \widehat{X_i})^2}{n-1}$ = $\frac{95.027639}{49}$ = 1.9393

$F(X_i)$ = 1 + 2 + 3 + …….. + 50

To estimate the 2- parameter weibull distribution, we use the two computers $\hat{\mu}$ and σ^2

$\hat{\beta}$ MOM

$$\frac{\mu^2}{\sigma^2+\mu^2} = \frac{\Gamma(1+Z)\Gamma(1+Z)}{\Gamma(1+2Z)}$$

$$\frac{0.7399^2}{1.9393+(0.7399)^2} = \frac{\Gamma(1+Z)\Gamma(1+Z)}{\Gamma(1+2Z)}$$

$$\frac{0.5475}{2.4868} = 0.2202$$

$\frac{\Gamma(1+Z)\Gamma(1+Z)}{\Gamma(1+2Z)} = 0.2202$

Following Gamma function table, (0.86 <Z< 0.87). So using the concept, we obtain Z as follows:

0.8 <Z< 0.87
0.3051 + 0.3106
= 0.6157
= 0.6157 – 0.2202
= 0.3955

Since $Z = \frac{1}{\beta} = 2.5284$

Therefore $\hat{\beta}$ MOM = 2.5284

To obtain the estimate of $^{\alpha}MOM$, we use :

$\hat{\alpha}$ MoM $= \frac{\mu}{\Gamma(1+\frac{1}{\beta})}$

$= \frac{0.7399}{\Gamma(1+\frac{1}{2.5284})} = \frac{0.7399}{\Gamma(1.3955)}$

From the value of the gamma function Γ(n) table

Γ(0.5302) = 1.6740

$^{\alpha}MOM = \frac{0.7399}{1.6740} = 0.4420$

Hence, the estimates for the method of moments are as follows:

$^{\alpha}MOM$ = 0.4420, $\hat{\beta}$ MOM = 2.5284

MSE = 2.1849×10^{-6}

The least square method to estimate parameters

$\hat{\beta}$ = 0.4371

$\hat{\alpha}$ = -0.3268

Hence, our estimate for the LSM are as follows;

$\hat{\alpha}$LSM = 0.4371

$\hat{\beta}$ LSM = -0.3268

MSE = 2.9799×10^{-5}

Table 1: Comparison of stocks Estimates

Method of Estimation	$\hat{\alpha}$	$\hat{\beta}$	MSE
MLE	0.6992	0.7399	2.9799×10^{-5}

MOM	0.4420	2.5284	2.1849×10^{-6}
LSM	0.4371	-0.3268	2.9799×10^{-5}

The model questions are solved analytically for Weibull distribution and Principal component analysis model for stock market variables.

The estimates of the parameters based on theoretical procedures described in Section 3 are presented on Table3.1.The shape parameter lies within the interval $(0,3)$ which implies, as indicated in Section 3, that the function decreases exponentially. We ranked the performance of the methods based on the least MSE criterion. The Method of Moments (MOM) showed the least values when compared to MLE and LSM respective see Table

Table 2: The ordered stock price data with Rank (i)

Rank(i)	Data	Rank(i)	Data	Rank(i)	Data	Rank(i)	Data	Rank(i)	Data
1	0.7700	11.25	0.9900	21.25	1.460	31	3.0000	41	4.8900
2	0.9100	11.25	0.9900	21.25	1.8300	32	3.1200	42	4.0400
3	0.9200	11.25	0.9900	21.25	2.3100	33	3.1400	43	4.1200
4	0.9300	11.25	0.9900	21.25	2.4900	34.5	3.1800	44	4.1300
5	0.9400	15.5	1.0300	25.5	2.4900	34.5	3.7500	45	4.1900
6	0.9500	15.5	1.0300	25.5	2.5200	36	3.7600	46	4.2000
7.5	0.9600	17	1.0500	27	2.7000	37	3.7700	47	4.2500
7.5	0.9600	18	1.0600	28	2.7300	38	3.7900	48	4.5500
9	0.9700	19	1.1500	29	2.7600	39	3.8400	49.5	4.7000
10	0.9800	20	1.2700	30	2.8200	40	3.8800	49.5	4.7000
Total		150		479.5		355		455	

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Table 3 ;The ranked stock price data using Benard's median rank approximation

I	Ti	F(ti)	I	Ti	F(ti)	I	Ti	F(ti)	I	Ti	F(ti)	I	Ti	F(ti)
1.	0.7700	0.01389	11	0.9900	0.2123	21	1.460	0.4107	31	3.0000	0.6091	41.	4.8900	0.8075
2.	0.9100	0.03373	12	0.9900	0.3321	22	1.8300	0.4300	32	3.1200	0.6290	42	4.0400	0.8274
3.	0.9200	0.05357	13	0.9900	0.2520	23	2.3100	0.4504	33	3.1400	0.6488	43	4.1200	0.8472
4.	0.9300	0.07341	14	0.9900	0.2718	24	2.4900	0.4702	34	3.1800	0.6687	44	4.1300	0.8671
5.	0.9400	0.09325	15	1.0300	0.2917	25	2.4900	0.4901	35	3.7500	0.6885	45	4.1900	0.8869
6.	0.9500	0.11310	16	1.0300	0.3115	26	2.5200	0.5099	36	3.7600	0.7083	46	4.2000	0.9067
7.	0.9600	0.1329	17	1.0500	0.3313	27	2.7000	0.5298	37	3.7700	0.7282	47	4.2500	0.9266
8.	0.9600	0.1528	18.	1.0600	0.3512	28	2.7300	0.5496	38	3.7900	0.7480	48	4.5500	0.9464
9.	0.9700	0.1726	19	1.1500	0.3710	29	2.7600	0.5694	39	3.8400	0.7679	49	4.7000	0.9663
10.	0.9800	0.1528	20	1.2700	0.3909	30	2.8200	0.5893	40	3.8800	0.7877	50	4.7000	0.9861

We took the stock price estimates of MLE, and LSM to form a complete 2×2 matrix as seen below:

$$\sum = \begin{pmatrix} 0.6992 & 0.7399 \\ 0.4371 & -0.3268 \end{pmatrix} \quad (32)$$

The detailed solving of this problem is seen in appendix 1
Solving the above matrix gives eigenvalues and eigenvectors respectively

$$\lambda_1 = 0.9521,\ \lambda_2 = -0.5797,\ K_1 = \begin{pmatrix} 1.0001 \\ 0.3418 \end{pmatrix},\ K_2 = \begin{pmatrix} -3.5847 \\ -1.7285 \end{pmatrix}$$

To obtain a normalized eigenvectors for stock market prices

$$e_1 = \begin{pmatrix} \dfrac{1.0001}{\sqrt{1.11702725}} \\ \dfrac{0.3418}{\sqrt{1.11702725}} \end{pmatrix} = \begin{pmatrix} 0.9463 \\ 0.3234 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} \dfrac{-3.5847}{\sqrt{15.83778634}} \\ \dfrac{-1.7285}{\sqrt{15.83778634}} \end{pmatrix} = \begin{pmatrix} -0.9008 \\ -0.4343 \end{pmatrix}$$

First and second principal component

$$Y_1 = e_1'K = 0.9463K_1 + 0.3234K_2 \quad (33)$$

$$Y_2 = e_1'K = -0.9008K_1 - 0.4343K_2 \quad (34)$$

To calculate principal component accounted for $\lambda_1 = 0.9521,\ \lambda_2 = -0.5797,\ \dfrac{\lambda_1}{\lambda_1 + \lambda_2} = 2.5567$

The proportion of total stock variance accounted for by the first principal component: 255.7%

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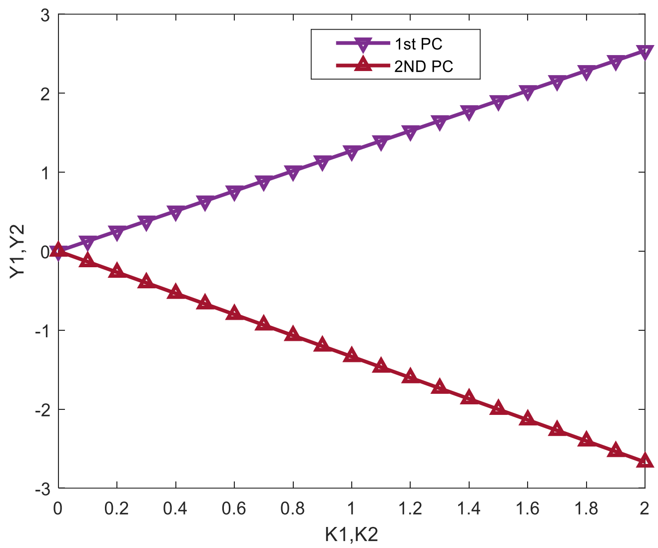

Figure 1: 1st and 2nd Principal component analysis of stock market variation

DISCUSSION OF RESULTS

The model questions are solved analytically for Weibull distribution and Principal component analysis model for stock market variables.

The estimates of the parameters based on theoretical procedures described in section 4.1. are presented on Table 1.The shape parameter lies within the interval (O,R) which implies, as indicated in section 2.2, that the function decreases exponentially. We ranked the performance of the methods based on the least MSE criterion. The Method of Moments (MOM) showed the least values when compared to MLE and LSM respective see Table 1.

It is clear that that in Table 3 the original stock prices are ranked according to ascending order of magnitude to enable us achieve our purpose of the study; such as estimating stock price changes.

In Table 3 we used the Benard's median rank approximation to generate the stock prices. It is otherwise called the median rank estimate of stock prices. Hence, it measures the percentage of stock prices that will change over time when n=50.

we formed the matrix from the estimates of MLE and LSM which information of the NSE will not necessarily yield higher returns since the price formation is assumed to be stochastic process.

Two eigenvalues represents the total amount of stock variance that can be explained by the principal component. The $\lambda_1 = 0.9521$ is greater than zero which is a good sign of high level of investment return in the side of the investment whose aim and passion is to maximize profit. So $\lambda_2 = -0.5797$ represents the levels of losses made all through the trading treading days by an investor.

However, the eigenvectors determines the direction of the stock market prices in terms of changes in short-run and long-run respectively.

CONCLUSION

The increasing prominence of the stock market in Nigeria is one of the most striking features of financial development over the last decade. The Nigerian stock exchange (NSE) plays an essential role in raising capital funds and also as a link between firm and the investing public. Empirical studies of the NSE revealed that results can only be derived when the dynamics of the stock pairs are known. If the market is fair the price fluctuation can be defined based in the sequence of the past price changes.

This paper, examined the estimation of two parameter Weibull distribution and principal component analysis of stock market prices. The analytical problem is derived in detail. The numerical solution has been presented as follows:

i) In estimation of weibull distribution, Method of no moments has the least error compared to Maximum likelihood Estimation and the Least square method in modeling stock prices .

ii) The proportion of total stock variance accounted for by the first principal component and analyze.

iii) The positive eigenvalues implies good profit margin in the long-run while the negative eigenvalues stipulates security crashes in short period of time. This will help investors, economist, policy makers and opinion leaders who are working assiduously in making sure of maximizing profit.

Applying growth rate model will be an interesting area to explore in the future studies.

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