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ANALYSIS OF HOMOTOPY PERTURBATION METHOD (HPM) AND ITS APPLICATION FOR SOLVING INFECTIOUS DISEASE MODELS

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ABSTRACT: *This article appraised analysis of homotopy perturation method (HPM) and it's application for solving infectious disease models, we described basic concepts, assumptions of homotopy perturbation method and steps involve in using homotopy perturbation method for solving equations. We used homotopy perturbation method to solve nonlinear partial defferential equations with boundary conditions. The obtained results as compared with those in existing literature are very accurate. In particular, we also applied homotopy perturbation method to obtain a convergent series solution of HIV/AIDS model where the model is categorised into four compartments namely, Susceptible individuals (S), infected individuals not on drugs (I), infected individual on drugs (T), individuals infected with AIDS (A). It is discovered in this article, that homotopy perturbation method is accurate, flexible and can be used to solve numerous problems.*

KEYWORDS: homotopy perturbation method, infectious disease model, partial differential equation, convergent, series solution

INTRODUCTION

The Concept of homotopy forms a formalisation of the intuitive knowledge of a smooth deformation of one curve into another curve. Homotopy perturbation method was first proposed by a Chinese mathematician named Ji-Huan He [1]. The method, enables analytical solutions of many problems in various fields as it uses the idea of homotopy from topological space to generate a suitable convergent series solution for nonlinear differential equations. Benefits of HPM which distinguished it from other analytical methods include, HPM is a series expansion method that indirectly dependent on small or large physical parameters. Therefore, it can be used to solve both varieties problems. It gives convergent series solutions and the perturbation equation is easy to construct. He used HPM to solve schrodinger equation [13], initial value problems[16].

Infectious diseases continue to gain much attentions daily because of its high rate of transmission and the lethality nature. Infectious diseases include HIV/AIDS, influenza, measles, cholera,

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COVID-19, free- living amebic infection, Hantavirus pulmonary syndrome (HPS), Hendra virus infection, kuru, legionellosis, norovirus, nocardiosis, paragonimiasis, staphylococcal infection, etc [14]. Mathematical modelling enables us to predict possible disease control strategies by considering transmission dynamics of the disease, determining the threshold parameters such as the basic reproduction number and carrying out endemic equilibrium, stability analysis and sensitivity analysis. Many researchers have used mathematical modelling to analyse infectious diseases, see [2, 4, 8, 15, 17, 19, 22]. Others include, [18] considered basic reproduction numbers of infectious disease models where analysed basic reproduction numbers of many infectious disesases. [20] studied analysis of a SEIV epidemic model with nonlinear incidence rate.

MATERIALS AND METHODS

Basic Concept of Homotopy Perturbation Method

Consider the following functions:

$$A(u) - f(r) = 0 \quad where \ r \in \Omega \quad (2.1)$$

Subject to boundary conditions:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0 \quad with\, r \in D \qquad (2.2)$$

Where $A, B, f(r)$ $and\ D$ stand for a general differential operator, a boundary operator, a known analytic function and a boundary of the domain (**Ω**) respectively. Where the operator A is divided into two parts; the Linear part (L) and the nonlinear part (N), now we can rewrite equation (1) as:

$$L(u) + N(u) - f(r) = 0 \qquad (2.3)$$

By homotopy techniques, we construct a Homotopy defined by:

$V(r, p): \Omega \times [0,1] \to R$ such that $V(x, 0) = v(x)$ and $V(x, 1) = g(x)$ which satisfies:

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0$$

By expanding $H(v, p)$ we obtain:

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \qquad (2.4)$$

Where $p \in [0,1]$ is an embedding parameter, while u_0 is an initial approximation of equation (2.1) which satisfies the boundary condition. Obviously when

$p = 0\ and\ p = 1$, equation (4) becomes:

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$H(v,0) = L(v) - L(u_0) = 0$ (2.5)

$H(v,1) = A(v) - f(r) = 0$ (2.6)

The changing process of p from zero to one is just a change of $V(r,p)$ from u_o $to\, u(r)$. In topology the process is called $Deformation$, and $L(v) - L(u_0)$ and $A(v) - f(r)$ are known as homotopy [16]

Assumptions of Homotopy Perturbation Method

According to Homotopy perturbation method, we first use the embedding parameter "P" as small parameter and assume that the solution of the equation (2.4) can be written as a power series in "p" as

$V = v_0 + pv_1 + p^2v_2 + \cdots$ (3.1)

Setting p=1 result in the approximate solution of equation (2.1) that is:

$U = \lim_{P\to 1} V = v_0 + v_1 + v_2 + \cdots$ (3.2)

Now the combination of the perturbation method and the Homotopy is called the homotopy perturbation method (HPM).This method vanishes the setback of traditional perturbation method while keeping all its advantages. The series in equation (3.1) is convergent for most cases. However, the convergence rate depends on the nonlinear parameter $A(v)$, Moreover the following suggestion is to be considered:

(i) The second derivative of $N(v)$with respect to V must be small because the parameter may be relatively large as$p \to 1$.

(ii) The norm of $L^{-1}\frac{\partial N}{\partial V}$ must be smaller than one for the series:

$V = v_0 + pv_1 + p^2v_2$ converges [16].

RESULTS AND DISCUSSION

Application of Homotopy Perturbation Method to Nonlinear Problems

$Example$

$$\frac{\partial^2 u}{\partial x^2} + u\frac{\partial u}{\partial x} = \ln t + x \qquad t > 0$$

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With initial conditions:

$u_x(o,t) = 1$

$u(0,t) = \ln t$,

Solution

$\frac{\partial^2 u}{\partial x^2} + u\frac{\partial u}{\partial x} = x + \ln t\,, t > 0$ (4.1)

Constructing a homotopy

H(v,p) for equation (4.1):

$$H(v,p) = (1-p)\left[\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v_o}{\partial x^2}\right] + p\left[\frac{\partial^2 v}{\partial x^2} + v\frac{\partial v}{\partial x}\right] = x + \ln t \qquad (4.2)$$

Expanding (19) we have:

$$\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v_o}{\partial x^2} - p\frac{\partial^2 v}{\partial x^2} + p\frac{\partial^2 v_o}{\partial x^2} + p\frac{\partial^2 v}{\partial x^2} + pv\frac{\partial v}{\partial x} = x + \ln t$$

By simplifying we obtain:

$$\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v_o}{\partial x^2} - p\frac{\partial^2 v_0}{\partial x^2} + pv\frac{\partial v}{\partial x} = x + \ln t \quad (4.3)$$

The initial approximation is given by:

$$v_0(0,t) = \ln t\,, \mathrm{v_{0x}(0,t)} = 1 \qquad (4.4)$$

The solution of equation (4.2) satisfies the series of the form:

$$\mathrm{V} = \mathrm{v}_0 + \mathrm{pv}_1 + \mathrm{p}^2\mathrm{v}_2 + \cdots + \mathrm{p}^\mathrm{n}\mathrm{v}_\mathrm{n} \quad (4.5)$$

Substituting equation (4.5) into equation (4.3) we obtain:

$$\frac{\partial^2(\mathrm{v}_0+\mathrm{pv}_1+\mathrm{p}^2\mathrm{v}_2)}{\partial x^2} - \frac{\partial^2 v_o}{\partial x^2} + p\frac{\partial^2 v_o}{\partial x^2} + p(\mathrm{v}_0 + \mathrm{pv}_1 + \mathrm{p}^2\mathrm{v}_2)\frac{\partial(\mathrm{v}_0+\mathrm{pv}_1+\mathrm{p}^2\mathrm{v}_2)}{\partial x} = x + \ln t \quad (4.6)$$

Equation (4.6) can be written in a simpler form as:

$$\frac{\partial^2(\mathrm{v}_0+\mathrm{pv}_1+\mathrm{p}^2\mathrm{v}_2)}{\partial x^2} - \frac{\partial^2 v_o}{\partial x^2} + p\frac{\partial^2 v_o}{\partial x^2} + p(\mathrm{v}_0 + \mathrm{pv}_1 + \mathrm{p}^2\mathrm{v}_2)(\mathrm{v}_{0x} + \mathrm{pv}_{1x} + \mathrm{p}^2\mathrm{v}_{2x}) = x + \ln t$$
(4.7)

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Expanding equation (4.7) we obtain:

$$\frac{\partial^2 v_0}{\partial x^2} + p\frac{\partial^2 v_1}{\partial x^2} + p^2\frac{\partial^2 v_2}{\partial x^2} - \frac{\partial^2 v_0}{\partial x^2} + p\frac{\partial^2 v_0}{\partial x^2} + pv_0v_{0x} + p^2v_0v_{1x} + p^2v_1v_{0x}$$

$$+p^3v_0v_{2x} + p^3v_1v_{1x} + p^3v_2v_{0x} + p^4v_1v_{2x} + p^4v_2v_{1x} + p^5v_2v_{2x} = x + \ln t \quad (4.8)$$

Writing out terms containing the power of P i.e. P^0, P^1 and P^2, we have:

$$p^0: \frac{\partial^2 v_0}{\partial x^2} = x + \ln t$$

Satisfying the initial condition:

$$v_o(0,t) = \ln t \,, v_{0x}(0,t) = 1$$

Therefore:

$$v_o(x,t) = x + \ln t \quad (4.9)$$

$$p^1: \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_0}{\partial x^2} + v_0v_{ox} = 0$$

satisfying the initial condition:

$$v_1(0,t) = 0 \,, v_{1x}(0,t) = 0$$

Therefore:

$$v_1(x,t) = 0 \qquad (4.10)$$

$$p^2: \frac{\partial^2 v_2}{\partial x^2} + v_0v_{1x} + v_1v_{ox} = 0$$

Satisfying the initial conditions:

$$v_2(0,t) = 0, v_{2x}(0,t) = 0$$

Therefore: $v_2(x,t) = 0$ (4.11)

Continuing this process up to p^n we obtain:

$$v_m(x,t) = 0 \; for \; m \geq 3.$$

The solution $u(x,t)$ satisfies the series given as:

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$$V(x,t) = v_0(x,t) + pv_1(x,t) + p^2 v_2(x,t) + \cdots + p^n(x,t)v_n \qquad (4.12)$$

$$U(x,t) =$$

$$\lim_{p \to 1} V(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t) + \cdots + v_n(x,t)$$

$$U(x,t) = x + \ln t$$

The solution exists as $p \to 1$ as: $U(x,t) = x + \ln t$.

Analytical Solution of Infectious Disease (HIV/AIDS) Model Using Homotopy Perturbation Method (HPM)

Model Equations

$$\frac{ds}{dt} = B - \left(\mu_1 + \frac{\gamma c}{N}(I + T + A)\right)S \qquad (5.1)$$

$$\frac{dI}{dt} = \left(\mu_1 + \frac{\gamma c}{N}(I + T + A)\right)S - (\beta + \theta + \mu_1 + \mu_2)I \qquad (5.2)$$

$$\frac{dT}{dt} = \beta I - (\mu_1 + \mu_2 + \omega)T \qquad (5.3)$$

$$\frac{dA}{dt} = \theta \mathrm{I} + \omega T - (\mu_1 + \mu_2)A \qquad (5.4)$$

With the following initial conditions

$S(0) = S_0, I(0) = I_0, T(0) = T_0 \; and \; A(0) = A_0$

Where

S	Susceptible individuals
I	Infected individuals not on drugs
T	Infected individual on drugs
A	Individuals with AIDS
B	Constant recruitment rate
μ_1	Natural death rate

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μ_2 Death due to HIV infection

μ_3 Death due to AIDS infection

θ Rate at which HIV infected individuals develop AIDS

ω Rate at which HIV infected individuals on drugdevelop AIDS due to lack of treatments

β Treatment rate

Let

$S = \omega_0 + h\omega_1 + h^2\omega_2 + \cdots$ (5.5)

$I = x_0 + hx_1 + h^2x_2 + \cdots$ (5.6)

$T = y_0 + hy_1 + h^2y_2 + \cdots$ (5.7)

$A = z_0 + hz_1 + h^2z_2 + \cdots$ (5.8)

From Equation (5.1)

$$\frac{ds}{dt} + \left(\mu_1 + \frac{\gamma c}{N}(I + T + A)\right)S - B = 0 \quad (5.9)$$

Applying HPM to equation (5.9) we have

$$(1-h)\frac{ds}{dt} + h\left[\frac{ds}{dt} + \left(\mu_1 + \frac{\gamma c}{N}(I + T + A)\right)S - B\right] = 0 \quad (5.10)$$

Substituting equations (5.5), (5.6) ,(5.7) and (5.8) into equation (5.10)

$$(1-h)(\omega_0^1 + h\omega_1^1 + h^2\omega_2^1 + \cdots) + h\Big[(\omega_0^1 + h\omega_1^1 + h^2\omega_2^1 + \cdots) + (\mu_1 + \frac{\gamma c}{N}\big((x_0 + hx_1 + h^2x_2 + \cdots)(y_0 + hy_1 + h^2y_2 + \cdots) + (z_0 + hz_1 + h^2z_2 + \cdots)\big)(\omega_0 + h\omega_1 + h^2\omega_2 + \cdots) - B\Big] = 0 \quad (5.11)$$

Expanding and collecting the coefficients of the powers of h

$$h^0 : \omega_0^1 = 0 \quad (5.12)$$

$$h^1 : \omega_1^1 + \mu_1\omega_0 + \frac{\gamma c}{N}(\omega_0 x_0 + \omega_0 y_0 + \omega_0 z_0) - B = 0 \quad (5.13)$$

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$$h^2: \omega_2^1 + \frac{\gamma c}{N}(\omega_1 x_0 + \omega_0 x_1 + \omega_1 y_0 + \omega_0 y_1 + \omega_1 z_0 + \omega_0 z_1) = 0 \qquad (5.14)$$

From equation (5.2)

$$\frac{dI}{dt} + (\beta + \theta + \mu_1 + \mu_2)I - \left(\frac{\gamma c}{N}(I + T + A)\right)S = 0 \qquad (5.15)$$

Applying HPM to (5.15)

$$(1-h)\frac{dI}{dt} + h\left[\frac{dI}{dt} + ((\beta + \theta + \mu_1 + \mu_2)I - \left(\frac{\gamma c}{N}(I + T + A)\right)S\right] = 0 \qquad (5.16)$$

Substituting equations (5.5), (5.6) ,(5.7) and (5.8) into equation (5.16)

$$(1-h)(x_2^1 + hx_1^1 + h^2x_2^1 + \cdots) + h\left[(x_0^1 + hx_1^1 + h^2x_2^1 + \cdots) + (\beta + \theta + \mu_1 + \mu_2)(x_0 + hx_1 + h^2x_2 + \cdots) - \left(\frac{\gamma c}{N}(x_0 + hx_1 + h^2x_1 + \cdots) + (y_0 + hy_1 + h^2y_2 + \cdots) + (z_0 + hz_1 + h^2z_2 + \cdots)\right)(\omega_0 + h\omega_1 + h^2\omega_2 + \cdots)\right] = 0 \qquad (5.17)$$

Expanding and collecting the powers of h

$$h^0: x_0^1 = 0 \qquad (5.18)$$

$$h^1: x_1^1 + (\beta + \theta + \mu_1 + \mu_2)\, x_0 - \frac{\gamma c}{N}(x_0\omega_0 + y_0\omega_0 + \omega_0 z_0) - B = 0 \qquad (5.19)$$

$$h^2: x_2^1 + (\beta + \theta + \mu_1 + \mu_2)x_1 - \frac{\gamma c}{N}(x_0\omega_1 + x_1\omega_0 + y_0\omega_1 + y_1\omega_0 + z_0\omega_1 + z_1\omega_0) = 0$$

(5.20)

From equation (5.3)

$$\frac{dT}{dt} + (\mu_1 + \mu_2 + \omega) - \beta I = 0 \qquad (5.21)$$

Applying HPM to Equation (5.21)

$$(1-h) + h\left[\frac{dT}{dt} + (\mu_1 + \mu_2 + \omega)T - \beta I\right] = 0$$
(5.22)

Substituting equations (5.6) and (5.7) into equation (5.22)

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$(1-h)(y_0^1+hy_1^1+h^2y_2^1+\cdots)+h[(y_0^1+hy_1^1+h^2y_2^1+\cdots)+(\mu_1+\mu_2+\omega)(y_0+hy_1+h^2y_2+\cdots)-\beta(x_0+hx_1+h^2x_2+\cdots)]=0$ (5.23)

$(1-h)(y_0^1+hy_1^1+h^2y_2^1+\cdots)+h[(y_0^1+hy_1^1+h^2y_2^1+\cdots)+(\mu_1+\mu_2+\omega)(y_0+hy_1+h^2y_2+\cdots)-\beta(x_0+hx_1+h^2x_2+\cdots)]=0$ (5.24)

Expanding and collecting the co –efficient of the powers of h

$h^0: y_0^1=0$ (5.25)

$h^1: y_1^1+(\mu_1+\mu_2+\omega)\,y_0-\beta x_0=0$ (5.26)

$h^2: y_2^1+(\mu_1+\mu_2+\omega)\,y_1-\beta x_1=0$ (5.27)

From equation (5.8)

$\frac{dA}{dt}+(\mu_1+\mu_3+)A-\theta I-\omega T=0$ (5.28)

Applying HPM to equation (5.28)

$(1-h)\frac{dA}{dt}+h\left[\frac{dA}{dt}+(\mu_1+\mu_3+)\,A-\theta I-\omega T)\right]=0$ (5.29)

Substituting equations (5.6), (5.7) and (5.8) into equation (5.29)

$(1-h)(z_0^1+hz_1^1+h^2z_2^1+\cdots)+h[(z_0^1+hz_1^1+h^2z_2^1+\cdots)+(\mu_1+\mu_3)(z_0+hz_1+h^2z_2+\cdots)-\theta(x_0+hx_1+h^2x_2+\cdots)-\omega(y_0+hy_1+h^2y_2+\cdots)]=0$ (5.30)

Expanding and collecting the co –efficient of the powers of h

$h^0: z_0^1=0$ (5.31)

$h^1: z_1^1+(\mu_1+\mu_3)\,z_0-\theta x_0-\omega y_0=0$ (5.32)

$h^2: z_2^1+(\mu_1+\mu_3)\,z_1-\theta x_1-\omega y_1=0$ (5.33)

Solving equations (5.12), (5.13) and (5.14), we have;

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$$S(t) = S_0 + \left(B - \frac{\gamma c}{N}(S_0 I_0 + S_0 T_0 + S_0 A_0) - \mu_1 S_0\right) t - \frac{\gamma c}{N}\Big[(I_0 + T_0 + A_0)\left(B - \frac{\gamma c}{N}(S_0 I_0 + S_0 T_0 + S_0 A_0) - \mu_1 S_0\right) + S_0(\frac{\gamma c}{N}(S_0 I_0 + S_0 T_0 + S_0 A_0) - (\beta + \theta + \mu_1 + \mu_2) I_0 + (\beta I_0 - (\mu_1 + \mu_2 + \omega) T_0) + (\theta I_0 + \omega T_0 - (\mu_1 + \mu_3) A_0))\Big]\frac{t^2}{2}$$

Solving equations 5.19), (5.20) and (5.21), we have;

$$I(t) = I_0 + \left(\frac{\gamma c}{N}(S_0 I_0 + S_0 T_0 + S_0 A_0) - (\beta + \theta + \mu_1 + \mu_2) I_0\right) t + \Big[\frac{\gamma c}{N}(I_0 + T_0 + A_0)\left(B - \frac{\gamma c}{N}(S_0 I_0 + S_0 T_0 + S_0 A_0) - \mu_1 S_0\right) + S_0\left(\frac{\gamma c}{N}(S_0 I_0 + S_0 T_0 + S_0 A_0) - (\beta + \theta + \mu_1 + \mu_2) I_0 + (\beta I_0 - (\mu_1 + \mu_2 + \omega) T_0) + (\theta I_0 + \omega T_0 - (\mu_1 + \mu_3) A_0)(\beta + \theta + \mu_1 + \mu_2)(B - \frac{\gamma c}{N}(S_0 I_0 + S_0 T_0 + S_0 A_0) - \mu_1 S_0\right)\Big]\frac{t^2}{2}$$

S]olving equations (5.25), (5.26) and (5.27), we have;

$$T(t) = T_0 + (\beta I_0 - (\mu_1 + \mu_2 + \omega) T_0) t + \Big[\beta\left(B - \frac{\gamma c}{N}(S_0 I_0 + S_0 T_0 + S_0 A_0) - \mu_1 S_0\right) + (\mu_1 + \mu_2 + \omega)(\beta I_0 - (\mu_1 + \mu_2 + \omega) T_0)\Big]\frac{t^2}{2}$$

Solving equations (5.31), (5.32) and (5.33), we have;

$$A(t) = A_0 + (\theta I_0 + \omega T_0 - (\mu_1 + \mu_3) A_0) t + \Big[\theta\left(\frac{\gamma c}{N}(S_0 I_0 + S_0 T_0 + S_0 A_0) - (\beta + \theta + \mu_1 + \mu_2) I_0\right) + \omega(\beta I_0 - (\mu_1 + \mu_2 + \omega) T_0) - (\mu_1 + \mu_3)(\theta I_0 + \omega T_0 - (\mu_1 + \mu_3)) A_0\Big]\frac{t^2}{2}$$

CONCLUDING REMARKS AND OBSERVATIONS

In this article, we have studied introduction to homotopy perturbation method and infectious disease model, basic concepts of homotopy perturbation method, assumptions of homotopy perturbation method, application of homotopy perturbation method for solving nonlinear partial differential equations and analytical solution of infectious disease model. It is discovered that homotopy perturbation method gives a series solution that is mostly convergent for both linear and nonlinear differential equations. Studying the method, it is found that the method is a strong analytical approach and a tool which can be applied to numerious problems in scientific and engineering fields. Another interesting thing about the method is that, it is very easy to understand and apply following the discussion in this article.

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Conflict of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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