International Research Journal of Pure and Applied Physics

Vol.8, No.2, pp.10-35, 2021

ISSN 2055-009X(Print),

ISSN 2055-0103(Online)

A STUDY OF ASYMMETRIC NUCLEAR MATTER WITH THE B3Y-FETAL EFFECTIVE INTERACTION

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ABSTRACT: *In this paper, the asymmetric nuclear matter is studied within the framework of Hartree-Fock approximation with the B3Y-Fetal interaction, a new M3Y-type effective nucleon-nucleon interaction based on the lowest order constrained variational approach (LOCV). This study has been carried out to ascertain and validate the viability of the new effective interaction. For comparative purposes, the famous M3Y-Paris and M3Y-Reid effective interactions are used in this study. The B3Y-Fetal has been used in our previous work in its various density-dependent versions to successfully reproduce the saturation properties of the cold symmetric nuclear matter at the saturation density,* $\rho = 0.17 fm^{-3}$*. It has been used herein in its DDM3Y1, BDM3Y0 and BDM3Y1 density dependent versions to compute the binding energy per nucleon, also called equation of state (EOS), incompressibility and pressure of asymmetric nuclear matter with results that have proven to be acceptably in agreement with previous work done with the M3Y-Paris and M3Y-Reid by other researchers.*

KEYWORDS: asymmetric nuclear, matter, B3Y-Fetal, effective interaction

INTRODUCTION

The asymmetric nuclear matter is the nuclear matter in which the number of protons is not the same as the number of neutrons (Z = N) and is represented by the fourth term of the semi-empirical mass formula [1]. The study of this nuclear matter falls under a research direction in Nuclear Physics called isospin Physics whose ultimate goal is to provide information on the isospin dependence of asymmetric nuclear matter, particularly its isospin-dependent term, the density dependence of the nuclear symmetry energy [2, 3]. Understanding the EOS of asymmetric nuclear matter gives a significant insight into nuclear ground-state properties, stability of neutron-rich nuclei, excitation energies of giant Monopole Resonances (GMRs), dynamics of heavy-ion collisions, structure of neutron stars and simulation of supernova collapse [4].

Theoretical studies of the EOS of asymmetric nuclear matter (ANM) were pioneered by

https://doi.org/10.37745/irjpap.13

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ISSN 2055-009X(Print),

ISSN 2055-0103(Online)

Brueckner and his co-researchers [5] and Siemens [6] in the late 60's. Since then, the subject has undergone much expansion with the application of different many-body theories based on various two-body and three-body forces or interaction Lagrangians. The many-body theories such as microscopic, effective field, and phenomenological approaches serve as useful tools for understanding the properties of symmetric, asymmetric and neutron nuclear matter. The phenomenological, either relativistic (relativistic mean-field models) or non-relativistic such as Skyrme or Gogny forces, are based on density-dependent forces in which parameters are adjusted to nuclear bulk properties and finite nuclei. These phenomenological approaches allow the most precise description of the properties of finite nuclei [3]. Effective field theory (EFT) approaches, based on density functional theory, are such that they lead to a systematic expansion of the EOS in powers of density [7]. The advantage of EFT is the smaller number of parameters and correspondingly higher predictive power. Both the phenomenological and EFT approaches contain approximately the same number of model parameters that are fixed by nuclear properties around the saturation density, and thus usually give excellent descriptions of the nuclear properties around or below the saturation density. Their predictions at the high-density region are, however, probably unreliable [2, 3]. Microscopic or abinitio approaches, based on high-precision free space nucleon-nucleon interaction that reproduces the scattering and bound state properties of the free two-nucleon system, naturally include isospin dependence. Many-body correlations are then built using many-body techniques which microscopically account for isospin asymmetry effects such as the difference in the Pauli blocking factors of neutrons and protons in the asymmetric nuclear matter [8], and the predictions of the nuclear EOS are parameter-free [7]. Examples of such approaches are variational calculations [9, 10, 11, 12], Brueckner-Hartree-Fock (BHF) [13], or relativistic Dirac-Brueckner-Hartree-Fock (DBHF) calculations [14] and Green function Monte-Carlo approaches [15].

In nuclear matter and finite nuclei studies, the mean field theory, employing effective interactions, is the most widely used. The mean field theory includes non-relativistic mean-field theory with effective nucleon-nucleon interactions such as Skyrme or Gogny, and the relativistic mean field theory (RMF) [16]. Non-relativistic calculations do not meet the empirical region of saturation unlike relativistic calculations that simultaneously generate a three-body force that contributes to nuclear saturation [17]. This deficiency can be solved by the inclusion of appropriate density dependence in the non-relativistic calculations. Thus, in contemporary Nuclear Physics, effective density-dependent interactions are computational tools for nuclear matter and nuclear structure studies. In this way, the nuclear properties, heavy-ion reactions and many other nuclear reactions provide a good opportunity to probe the density-dependence of the in-medium effective interaction. In such studies, the basic characters of the effective nuclear interactions are also studied and discussed via properties at and around the saturation

https://doi.org/10.37745/irjpap.13

point [18]. This is possible because there is a definite relationship between equations of state and the density dependence of the effective interaction. So, we study the equations of state of asymmetric nuclear matter based on the density dependence of the new M3Y-type effective interaction, the B3Y-Fetal [1], in this work.

It should be noted that there is no significant methodological difference between the present efforts and previous calculations. However, our aim is to develop an effective interaction for nuclear reactions, which is based on the LOCV method and to compare its performance with the ones based on the G-matrix (M3Y-Reid), and (M3Y-Paris). However, the input matrix elements used in determining our B3Y-Fetal effective interaction are quite different from those for M3Y-Reid and M3Y-Paris interactions. It is after comparing the results of effective interactions obtained in the different methods that we will apply our new effective interaction to different applications, including its mass-dependence.

Overall, the predictions of the properties of isospin asymmetric nuclear matter, especially the density dependence of the nuclear symmetry energy, are known to be model-dependent, meaning that they are different for different many-body approaches. This is a well-established common feature of computational results for both symmetric and asymmetric nuclear matter and nuclear reactions. The M3Y-Reid and M3Y-Paris are effective forces derived from the bare nucleon-nucleon (NN) interactions by fitting the G-matrix elements and those from dispersion theory, respectively, to a sum of Yukawa functions in a harmonic oscillator basis. Remarkable progress has been made lately to obtain nuclear matter saturation properties from the G-matrix-based microscopic approaches using these effective interactions. Used in their density-dependent versions, the M3Y-Paris and M3Y- Reid effective interactions have successfully reproduced the saturation properties of symmetric [19] and asymmetric [4, 20] nuclear matter within the nonrelativis- tic Hartree-Fock (HF) scheme. The success of these effective interactions gives hope that newly developed effective interactions based on other microscopic approaches could fare well in nuclear matter and nuclear reaction studies. This is our motivation for using the B3Y-Fetal effective interaction, derived from the lowest order constrained variational (LOCV) approach [11] in this study. We have used it previously, successfully in its various density-dependent versions to reproduce the saturation properties of symmetric nuclear matter (SNM) in our earlier paper [21].

Our choice in this work is the nonrelativistic study of homogenous ANM at zero temperature where protons and neutrons are spin-saturated. The B3Y-Fetal effective interation is to be used herein along with M3Y-Reid and M3Y-Paris effective interactions for the study of ANM properties. Our focus is to determine and analyze the

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ISSN 2055-009X(Print),

ISSN 2055-0103(Online)

performance of the density dependent B3Y-Fetal effective interaction as it relates to ANM binding energy (EOS), incompressibility and pressure in comparison with the other two effective interactions. The density-dependent versions of the M3Y interaction used here are the DDM3Y1, BDM3Y0 and BDM3Y1 versions [19, 20, 21]. These are known to consistently reproduce the equilibrium density and binding of the normal nuclear matter as well as the microscopic results obtained by Jeukenne, Lejeune and Mahaux (JLM) [22] for the nucleon optical potential [19, 23]. They have been used severally within the framework of the folding model to calculate the nucleon-nucleus and nucleus-nucleus optical potentials [20, 24, 25]. This paper is so organized that it presents the functional form of the B3Y-Fetal effective interaction in Section 2 and discusses its application to asymmetric nuclear matter in Section 3. Finally, result presentation and discussion are carried out in Section 4, leading to concluding remarks in Section 4.

Density-Dependent B3Y-Fetal Effective Interaction

The matrix elements of the B3Y-Fetal interaction were calculated in a harmonic oscillator basis using the lowest-order constrained variational (LOCV) method. The details of the calculation were reported in [11, 26] and in our earlier paper, in which we applied it to symmetric nuclear matter (SNM) calculations [21]. In that paper, we used only the isoscalar part of the B3Y-Fetal interaction, however, in this paper, we use both the isoscalar and isovector components of the effective interaction as shown in Equation (2).

The direct (v_D) and exchange (v_{EX}) components of the central part of the M3Y-type NN effective interaction, in terms of spin $\boldsymbol{\sigma}, \boldsymbol{\sigma}'$ and isospin $\boldsymbol{\tau}, \boldsymbol{\tau}'$ of the nucleons, are expressed as [24]:

$$v^{D(EX)}(r) = v_{00}^{D(EX)}(r)+v_{10}^{D(EX)}(r)(\boldsymbol{\sigma}\bullet\boldsymbol{\sigma})+v_{01}^{D(EX)}(r)(\boldsymbol{\tau}\bullet\boldsymbol{\tau}') +v_{11}^{D(EX)}(r)(\boldsymbol{\sigma}\bullet\boldsymbol{\sigma})(\boldsymbol{\tau}\bullet\boldsymbol{\tau}') \qquad (1)$$

where r is the inter-nucleon distance and p is the nuclear density around the interacting nucleon pair, $\boldsymbol{\alpha}, \boldsymbol{\alpha}'$ are the spins and $\boldsymbol{\tau}, \boldsymbol{\tau}'$ are the isospins of two nucleons participating in the interaction. For the SNM study, only the first term contributes, whereas the first and third terms contribute to ANM properties. Accordingly, the radial strengths (in MeV) of the isoscalar and isovector components of the B3Y-Fetal effective interaction are given in terms of three Yukawas respectively as [11]:

$$v^{D}_{00}(r) = \frac{10472.13e^{-4r}}{4r} - \frac{2203.11e^{-2.5r}}{2.5r}$$

$$v^{EX}_{00}(r) = \frac{499.63e^{-4r}}{4r} - \frac{1347.77e^{-2.5r}}{2.5r} - \frac{7.847477e^{-0.7072r}}{0.7072r}$$

https://doi.org/10.37745/irjpap.13

$$v^{D}_{01}(r)=\frac{-6197.63e^{-4r}}{4r}+\frac{1277.38e^{-25r}}{2.5r} \quad (2)$$

$$v^{EX}_{01}(r)=\frac{365.38e^{-4r}}{4r}+\frac{576.99e^{-2.5r}}{2.5r}+\frac{2.6157e^{-0.7072r}}{0.7072r}$$

We have taken the interaction strengths used for constructing the B3Y-Fetal from Table V of the work of Fiase *et al.* [11].
Since it is intended to compare the results of our calculation with previous work done with the famous M3Y-Reid and M3Y-Paris effective interactions, their explicit radial forms are shown in Equations (3) and (4) respectively. M3Y-Reid [20, 27]:
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$$v^{D}_{00}(r)=\frac{7999.00e^{-4r}}{4r}+\frac{2134.25e^{-2.5r}}{2.5r}$$

$$v^{ex}_{00}(r)=\frac{4631.375e^{-4r}}{4r}-\frac{1787.125e^{-2.5r}}{2.5r}-\frac{7.8474e^{-0.7072r}}{0.7072r}$$

$$v^{D}_{01}(r)=\frac{-4885.5e^{-4r}}{4r}+\frac{1175.5e^{-2.5r}}{2.5r} \quad (3)$$

$$v^{EX}_{01}(r)=\frac{-1517.875e^{-4r}}{4r}+\frac{828.375e^{-2.5r}}{2.5r}+\frac{2.6157e^{-0.7072r}}{0.7072r}$$

M3Y-Paris [20, 28]:

$$v^{D}_{00}(r)=\frac{11061.625e^{-4r}}{4r}-\frac{2537.5e^{-2.5r}}{2.5r}$$

$$v^{EX}_{00}(r)=\frac{-1524.25e^{-4r}}{4r}-\frac{518.75e^{-2.5r}}{2.5r}-\frac{7.8474e^{-0.7072r}}{0.7072r}$$

$$v^{D}_{01}(r)=\frac{313.625e^{-4r}}{4r}+\frac{223.5e^{-2.5r}}{2.5r}$$

$$v^{EX}_{01}(r)=\frac{-4118.0e^{-4r}}{4r}+\frac{1054.75e^{-2.5r}}{2.5r}+\frac{2.6157e^{-0.7072r}}{0.7072r}$$

For a good description of the saturation properties of nuclear matter within the

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https://doi.org/10.37745/irjpap.13

Vol.8, No.2, pp.10-35, 2021

ISSN 2055-009X(Print),

ISSN 2055-0103(Online)

nonrelativistic HF scheme [20, 23], a density dependence is introduced into the original M3Y-type interaction as shown in Equation (5). With the density dependence introduced, the isoscalar and isovector density-dependent M3Y-type interaction becomes:

$$v_{00}^{D(EX)}(\rho,r)=F_0(\rho)v_{00}^{D(EX)}(r) \quad (5)$$

and

$$v_{01}^{D(EX)}(\rho,r)=F_1(\rho)v_{01}^{D(EX)}(r) \quad (6)$$

where $F_0(\rho)$ and $F_1(\rho)$ are the isoscalar and isovector density-dependent factors respectively. The explicit forms of these density dependences are [20, 23, 24]:

For the DDM3Yn interaction (n = 1),

$$F_{0(1)}(\rho)=C_{0(1)}\,(1+\alpha e^{-\beta\rho}) \quad (7)$$

In the case of the BDM3Yn (n = 0, 1, 2, 3) interaction,

$$F_{0(1)}(\rho)=C_{0(1)}\,(1-\alpha\rho^{\beta}) \quad (8)$$

The DDM3Yn and BDM3Yn density-dependent interactions are based on the original M3Y interaction obtained from the G-matrix elements of the Paris and Reid NN potential. The underlying idea is to make the original M3Y interaction density-dependent to be able to reproduce the saturation properties of nuclear matter. With the inclusion of appropriate density dependence, the parameters of these interactions have been adjusted to the data on nuclear structure; some have been applied to nuclear reactions alongside nuclear matter and neutron stars [29]. This set of density-dependent interactions (DDM3Yn, BDM3Yn) has the density dependence included directly in the finite-range Yukawa terms with the parameters adjusted to reproduce the saturation properties of symmetric nuclear matter [23, 24]. When done this way, the exchange terms are treated approximately within the double folding model. Therefore, this class of density-dependent M3Y interactions has continued to be applied, with great success to numerous nuclear reactions, including folding analyses of nucleon-nucleus, nucleus-nucleus and charge-exchange reactions [20, 23, 30]. These density-dependent interactions have undergone several stages of development and improvement over the years. The DDM3Yn interactions were first to be used [19] followed by the BDM3Yn interactions, and the CDM3Yn interactions came into use as the latest, hybrid versions of the first two versions. The last version of the DDM3Yn interaction, dubbed DDM3Y1, gave the incompressibilities K=170 MeV and 176 MeV [16] with the M3Y-Reid and M3Y-Paris interactions respectively. However, the exponential nature of this density dependence made a further readjustment of its parameters for a higher value of K impossible. Therefore, the BDM3Yn interaction was used to compute a harder EOS. To have better and harder equations of state, the BDM3Y0, BDM3Y1, BDM3Y2 and BDM3Y3 versions of BDM3Yn interaction were subsequently used; and these

https://doi.org/10.37745/irjpap.13

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ISSN 2055-009X(Print),

ISSN 2055-0103(Online)

generated higher incompressibility values.

For our calculations in this work, DDM3Y1, BDM3Y0, and BDM3Y1 versions are specifically used. Here, the parameters Co, a, and *[* of the isoscalar density dependences are such that they reproduce the saturation properties of nuclear matter at density ρ_0 = 0.17fm^{-3} with a binding energy E/A = 16 MeV within HF calculations. The isoscalar density dependence F_0(p) and isovector density dependence F_1(p) have practically the same form. Still, they may be different concerning the scaling factors C_o and C_1, because C_1 is, if necessary, adjusted to reproduce the empirical symmetry energy to construct a realistic equation of state (EOS) for asymmetric nuclear matter (ANM) within HF scheme.

Nuclear Matter Energy in Hartree-Fock Approximation

The Hartree-Fock (HF) mean-field approximation, based on the independent particle model, has been successfully applied to explain many nuclear properties [31]. Within this approximation, nuclei and nuclear matter are represented by independent particles (nucleons) moving in a mean (central) potential. Each nucleon is, here, considered to move in a single-particle effective potential representing the average effect of the interactions with the other nucleons.

The infinite nuclear matter is considered in this study as an asymmetric and a symmetric Fermion system enclosed in an infinite volume Ω, having *Z* protons, *N* neutrons, an infinite mass number *A* = *N* + *Z*, a finite density ρ = A/Ω and a total energy *E*. The total ground-state energy of the system at the absolute zero temperature is given by the sum of kinetic energy part and potential energy part [10, 32, 33]. The mathematical representation of this energy, as shown shortly in this section, is essentially an excerpt from the nuclear matter formula of the Nuclear Physics Group, INST, Vietnam [34].

In the HF approximation, the total ground-state energy *E* of nuclear matter is [34]:

$$E = \sum_i \left\langle i \left| -\frac{\hbar^2}{2m}\nabla^2 \right| i \right\rangle + \frac{1}{2}\sum_{i\neq j}\sum_j \langle ij|G|ij\rangle = E_{kin} + E_{pot} \quad (9)$$

where *G* is a generalized two-body effective NN interaction such as the B3Y-Fetal effective interaction. The factor 1/2 in the total potential energy is meant to avoid double counting of the two-body mutual interactions.

The binding energy per nucleon of isospin asymmetric nuclear matter, otherwise known as its equation of state (EOS) [35], is obtained from Equation (17). The study of the properties of ANM in this work begins with the EOS, which gives expressions for nuclear pressure, incompressibility, and symmetry energy as functions of density at zero temperature [33]. The EOS of asymmetric nuclear matter is calculated by adding the isovector component of the M3Y-type interaction, which does not contribute to the EOS

https://doi.org/10.37745/irjpap.13

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ISSN 2055-009X(Print),

ISSN 2055-0103(Online)

of symmetric nuclear matter (SNM) to the isoscalar component [2, 20].
Many theoretical studies [8] have shown that the EOS of the ANM can be expressed as a power series in the isospin asymmetry $\delta=(\rho_n-\rho_p)/\rho$ where $\rho = \rho_n+\rho_p$ is the total baryonic density with p_n and p_p denoting the neutron and proton densities, respectively.
To the second order, it is:

$$\frac{E}{A}(\rho,\delta \neq 0) = \frac{E}{A}(\rho,\delta = 0) + S(\rho)\delta^2 \quad (10)$$

where $\frac{E}{A}(\rho,\delta \neq 0)$ is the energy per nucleon at density ρ and asymmetry δ, $\frac{E}{A}(\rho,\delta = 0)$ is the binding energy per nucleon of symmetric nuclear matter (SNM) and S(ρ) is the symmetry energy coefficient which is expressed as:

$$E_{sym}(\rho) = S(\rho)\delta^2 \quad (11)$$

Here, $E_{sym}(\rho)$ is the symmetry energy, defined as the energy per nucleon required to change the symmetric nuclear matter to asymmetric nuclear matter [2]. This symmetry energy computed at the nuclear matter saturation density, $E_{sym}(\rho_o)$ is well-known to have the numerical value:

$$\frac{E_{sym}(\rho_0)}{\delta^2} \sim 30 \pm 2\ MeV \quad (12)$$

with δ, the isospin asymmetry parameter, expressed as:

$$\delta = \frac{(\rho_n - \rho_p)}{\rho} \quad (13)$$

In terms of δ, the neutron and proton densities are respectively given by:

$$\rho_n = \frac{(1+\delta)\rho}{2} \quad (14)$$

$$\rho_p = \frac{(1-\delta)\rho}{2} \quad (15)$$

The asymmetry parameter δ can have values between -1 and +1, corresponding to pure proton matter and pure neutron matter respectively, while for SNM, it is zero. However, its values in the present calculations are 0.00, 0.35, and 0.70. The quadratic dependence of the symmetry energy upon the asymmetry parameter δ, shown in Equation (12), is crucial in testing the isospin dependence of the effective NM interaction used for asymmetric NM. When δ =1, the value of the scaling factor, C_1, of the effective interaction in Equation (5) is such that the condition in Equation (12) is satisfied by the effective NN interaction being used for the determination of the EOS of the asymmetric

https://doi.org/10.37745/irjpap.13

ISSN 2055-009X(Print),
ISSN 2055-0103(Online)

nuclear matter. Once this is achieved, an acceptable EOS can be obtained, followed by other properties such as pressure and incompressibility of asymmetric nuclear matter, at any desired value of asymmetry parameter, δ. In this paper, we determine these properties at the asymmetry parameter values, δ = 0.00, 0.35 and 0.70.

In studying the properties of asymmetric nuclear matter, the nuclear symmetry is usually the first property to determine. To determine its empirical value in Equation (19), the scaling factor, C_1, of the isovector component of the effective interaction is first determined in relation to the scaling factor, C_0, of the isoscalar component when the proton-neutron asymmetry, $\delta = 1.0$. We have done this in our earlier paper [36] where we also studied the behaviour of the symmetry energy around saturation characterized in terms of a few bulk parameters expressed as [8, 32, 37,]:

$$E_{sym}(\rho) = S_0 + L_y + \frac{1}{2}K_{sym}y^2 \tag{16}$$

where $y = \frac{\rho - \rho_0}{\rho}$ is a parameter representing the deviations of density from its value at saturation, S_0 is the symmetry energy at saturation density, L and K_{sym} are the slope and curvature parameters of symmetry energy at ρ_0 expressed respectively as:

$$L = 3\rho_0 \frac{\delta E_{sym}(\rho)}{\delta\rho}|\rho = \rho_0 \tag{17}$$

$$K_{sym} = 9\rho_0^2 \frac{\delta^2 E_{sym}(\rho)}{\delta\rho^2}|\rho = \rho_0 \tag{18}$$

where *Ksym* and L, like symmetry energy, are well-known based on analyses of terrestrial Nuclear Physics experiments and astrophysical observations. In our HF calculation of nuclear symmetry energy, we observed the M3Y-Paris forces to be more realistic as the empirical value of symmetry energy was reproduced well without the need for further renormalization of the C_1 factor; thus, $C_1 = C_0$; and the values of symmetry energy obtained with the DDM3Y1, BDM3Y0 and BDM3Y1 versions of this effective interaction at the saturation density were 31.3, 30.8 and 30.8 MeV respectively. On the contrary, the isovector components of the B3Y-Fetal and M3Y-Reid interactions were found to be too strong, so the C_1 factor had to be reduced by $C_1 = 0.62C_1$ in both cases to obtain the correct value of symmetry energy (E_{sym}) at equilibrium. Accordingly, the values of symmetry energy at the saturation density obtained with DDB3Y1-, BDB3Y0- and BDB3Y1-Fetal effective interactions were 30.5, 30.6, and 30.7 MeV respectively, whereas 30.0, 30.6, and 30.6 MeV were the

https://doi.org/10.37745/irjpap.13

Vol.8, No.2, pp.10-35, 2021

ISSN 2055-009X(Print),

ISSN 2055-0103(Online)

values of symmetry energy obtained with DDM3Y1-, BDM3Y0- and BDM3Y1-Reid effective interactions respectively. Correspondingly, the values of the slope parameter L obtained with DDM3Y1, BDM3Y0 and BDM3Y1 versions of the M3Y-Paris, M3Y-Reid and B3Y-Fetal effective interactions were 48.2, 48.2 and 48.2; 51.0, 51.0 and 51.0; and 53.0, 53.0 and 53.0 repectively. These results, forming a strong basis for the study of ANM in the present work, have been found to tally well with such empirical standards as 31±2 MeV [32, 38], 30±2 MeV [20, 39] and 31.6±2.7 MeV [40] for symmetry energy; and L = 58±16 MeV [40] for the slope parameter. These results, showing the B3Y-Fetal effective interaction to be in excellent agreement with the M3Y-Reid and M3Y-Paris effective interactions, were well illustrated in [36] and are partly reproduced in Figure 2.0.

Within the framework of HF mean-field calculation, we calculate the EOS of ANM in this study by adding the isovector component of the B3Y-Fetal effective interaction to the isoscalar component. Accordingly, with the spin-dependent terms of equation (1) averaged out, it (from equation (9)) is [20]:

$$\frac{E}{A}(\rho) = \frac{3\hbar^2[K_F^2(n)\rho_n + K_F^2(p)\rho_p]}{5m} + \varepsilon_0(\rho) + \varepsilon_1(\rho) \tag{19}$$

$\varepsilon_0(\rho)$ and $\varepsilon_1(\rho)$ are the isoscalar and isovector components of the potential energy part, which are respectively given by

$$\frac{E_{pot}^0}{A} = \frac{F_0(\rho)}{2\rho}\left[\left(\rho_n + \rho_p\right)^2 J_0^D + \int \left[\rho_n \hat{j}1\,(K_F(n)r) + \rho_p\,\hat{j}1\,(k_F(p)r)\right]^2 v_{00}^{EX}(r)d^3r\right]$$

and

$$\frac{E_{pot}^1}{A} = \frac{F_1(\rho)}{2\rho}\left[\left(\rho_n - \rho_p\right)^2 J_1^D + \int \left[\rho_n \hat{j}1\,(K_F(n)r) - \rho_p\,\hat{j}1\,(k_F(p)r)\right]^2 v_{01}^{EX}(r)d^3r\right]$$

The other properties that are directly related to the binding energy or EOS of asymmetric nuclear matter are the nuclear pressure, P and incompressibility, K, which have the expressions:

$$P(\rho) = \rho^2 \frac{\delta}{\delta\rho}\left(\frac{E}{A}\right) \tag{20}$$

and

$$K(\rho) = 9\frac{\delta}{\delta\rho}P(\rho) \tag{21}$$

When the indicated mathematical operations are performed on Equation (19), the pressure of asymmetric nuclear matter is obtained as:

$$P(\rho) = \frac{\hbar^2[k_F^2(n)\rho_n + k_F^2(p)\rho_p]}{5m} + P_0(\rho) + P_1(\rho) \tag{22}$$

https://doi.org/10.37745/irjpap.13

Vol.8, No.2, pp.10-35, 2021

ISSN 2055-009X(Print),

ISSN 2055-0103(Online)

where the isoscalar and isovector components of the potential are:

$$P_0(\rho) = a_0(\rho)\rho^2 J_0^D + \int d^3 r v_{00}^D(r)[a_0(\rho)A_0^2(r) - F_0(\rho)A_0(r)B_0(\rho)]$$

$$P_1(\rho) = a_0(\rho)\rho^2 J_1^D \delta^2 + \int d^3 r v_{01}^D(r)[a_1(\rho)A_1^2(r) - F_1(\rho)A_1(r)B_1(\rho)] \quad (23)$$

$$with\ A_0(r) = \rho_n \hat{j}1\,(k_F(n)r) + \rho_P \hat{j}1\,(k_F(p)r)\ and$$

$$A_1(r) = \rho_n \hat{j}1\,(k_F(p)r) - \rho_P \hat{j}1\,(k_F(p)r)$$

Similarly, the incompressibility of asymmetric nuclear matter is obtained as:

$$K(\rho) = \frac{3\hbar^2\left[k_F^2(n)\rho_n + k_F^2(p)\rho_p\right]}{m\rho} + K_0(\rho) + K_1(\rho) \quad (24)$$

With the isoscalar and isovector components expressed as:

$$K_0(\rho) = b_0(\rho)J_0^D + \frac{1}{\rho^2}\int d^3 r V_{00}^D(r)[b_0(\rho)A_0^2(r) - c_0(\rho)A_0(r)B_0(r) + d_0(\rho)A_0(r)D_0(r) + d_0(\rho)B_0^2(r)]$$

$$K_1(\rho) = b_1(\rho)J_1^D\delta^2 + \frac{1}{\rho^2}\int d^3 r V_{01}^D(r)[b_1(\rho)A_1^2(r) - c_1(\rho)A_1(r)B_1(r) + d_1(\rho)A_1(r)D_1(r) + d_1(\rho)B_1^2(r)] \quad (25)$$

Where,

$$B_0(r) = \rho_n\, j_2(k_F(n)r) + \rho_P\, j_2(k_F(p)r)$$

$$B_0(r) = \rho_n\, j_2(k_F(n)r) - \rho_P\, j_2(k_F(p)r)$$

$$D_0(r) = \frac{1}{3}\rho_n\, j_3(k_F(n)r)(k_F(n)r + \frac{1}{3}\rho_P\, j_3(k_F(p)r)(k_F(p)r$$

$$D_1(r) = \frac{1}{3}\rho_n\, j_3(k_F(n)r)(k_F(n)r - \frac{1}{3}\rho_P\, j_3(k_F(p)r)(k_F(p)r$$

$$a_{0(1)}(r) = 0.5\left[\rho\frac{d}{d\rho}F_{0(1)}(\rho) + F_{0(1)}(\rho)\right]$$

$$b_{0(1)}(r) = 4.5\rho^3\frac{d^2}{d\rho^2}F_{0(1)}(\rho) + 18\rho^2\frac{d}{d\rho}F_{0(1)}(\rho) + 9\rho F_{0(1)}(\rho)$$

$$c_{0(1)}(r) = 18\rho^2\frac{d}{d\rho}F_{0(1)}(\rho) + 33\rho F_{0(1)}(\rho)$$

$$d_{0(1)}(r) = 9\rho F_{0(1)}(\rho) \quad (26)$$

The DDM3Y1, BDM3Y0 and BDM3Y1 interactions used for ANM calculations here have corresponding equations of state which are dependent on the three effective interactions - M3Y-Paris, M3Y-Reid and B3Y-Fetal - employed in the calculations. Used in the DDM3Y1, BDM3Y0 and BDM3Y1 density dependent versions, the corresponding B3Y-Fetal-based density-dependent effective interactions are DDB3Y1-

https://doi.org/10.37745/irjpap.13

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Fetal, BDB3Y0-Fetal and BDB3Y1-Fetal interactions respectively. Different nuclear equations of state obtained with these interactions are shown in Section 4.

RESULTS AND DISCUSSION

The study of asymmetric nuclear matter within Hartree-Fock formalism has been exhaustively undertaken in this work, with the reproduction of the saturation properties of symmetric nuclear matter at the asymmetry parameter, $5 = 0.00$ as the starting point. To be sure of the accuracy of the computational procedure, the saturation properties of symmetric nuclear matter were computed with the M3Y-Paris and M3Y-Reid interactions first, and the results obtained with these interactions in their DDM3Y1, BDM3Y0, and BDM3Y1 density-dependent versions were compared with the results of [20] and found to be exact, leading to the substitution of the B3Y-Fetal interaction for them in the same computational procedure.

This approach was also employed for the computation of the properties of asymmetric nuclear matter. The results obtained from the numerical calculation of the nuclear matter (symmetric and asymmetric) properties obtained with these effective interactions, shown in Table 1.0 and Figures 1.0 - 8.0, show the B3Y-Fetal effective interaction to be in excellent agreement with the M3Y-Reid and M3Y-Paris interaction

https://doi.org/10.37745/irjpap.13

ISSN 2055-009X(Print),

ISSN 2055-0103(Online)

Table 1. Parameters of Density Dependent M3Y and B3Y Effective Interactions and Nuclear Incompressibilities. The results obtained by [20] with the M3Y-Reid and M3Y-Paris interactions are in brackets.

Interaction	*Co*	C_l	α	β	K_0[MeV] (δ= 0.00)	K_0[MeV] (δ= 0.35)	K_0[MeV] (δ= 0.70)
DDB3Y1-Fetal	0.2986	0.1857	3.1757	2.9605/m^3	176	150	67
DDM3Y1-Reid	0.2485	0.1764	3.6391	2.9605/m^3	171	143	63
	(0.2485)	(0.1764)	(3.6391)	(2.9605/m^3)	(171)	(146)	(65)
DDM3Y1-Paris	0.2963	0.2963	3.7231	3.7384/m^3	176	146	77
	(0.2963)	(0.2963)	(3.7231)	(3.7384/m^3)	(176)	(151)	(72)
BDB3Y0-Fetal	1.3045	0.8113	1.0810/m^2	2/3	196	156	62
BDM3Y0-Reid	1.3817	0.8290	1.1132/m^2	2/3	191	149	58
	(1.3817)	(0.8290)	(1.1132/m^2)	2/3	(190)	(155)	(62)
BDM3Y0-Paris	1.4366	1.4366	1.2627/m^2	2/3	221	200	67
	(1.4366)	(1.4366)	(1.2627/m^2)	2/3	(218)	(180)	(72)
BDB3Y1-Fetal	1.1603	0.7216	1.4626/m^3	1	235	174	79
BDM3Y1-Reid	1.2244	0.7346	1.5118/m^3	1	232	168	73
	(1.2253)	(0.7597)	(1.5124)/m^3	1	(232)	(185)	(73)
BDM3Y1 -Paris	1.2511	1.2511	(1.7445/m^3	1	270	234	86
	(1.2521)	(1.2521)	(1.7452)/m^3	1	(270)	(223)	(89)

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https://doi.org/10.37745/irjpap.13

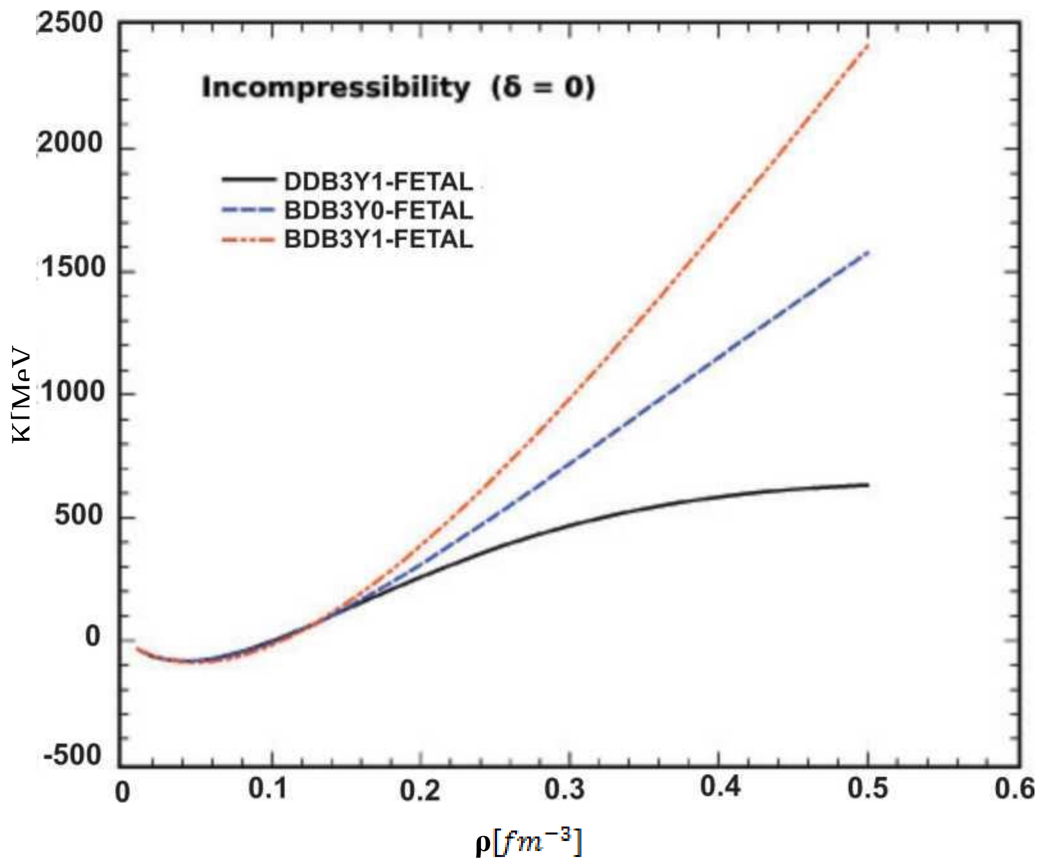

Figure 1: Nuclear Incompressibilities of Cold NM Calculated with DDB3Y1-Fetal, BDB3Y0-Fetal, and BDB3Y1-Fetal Interactions at the Asymmetry Parameter, δ = 0.00.

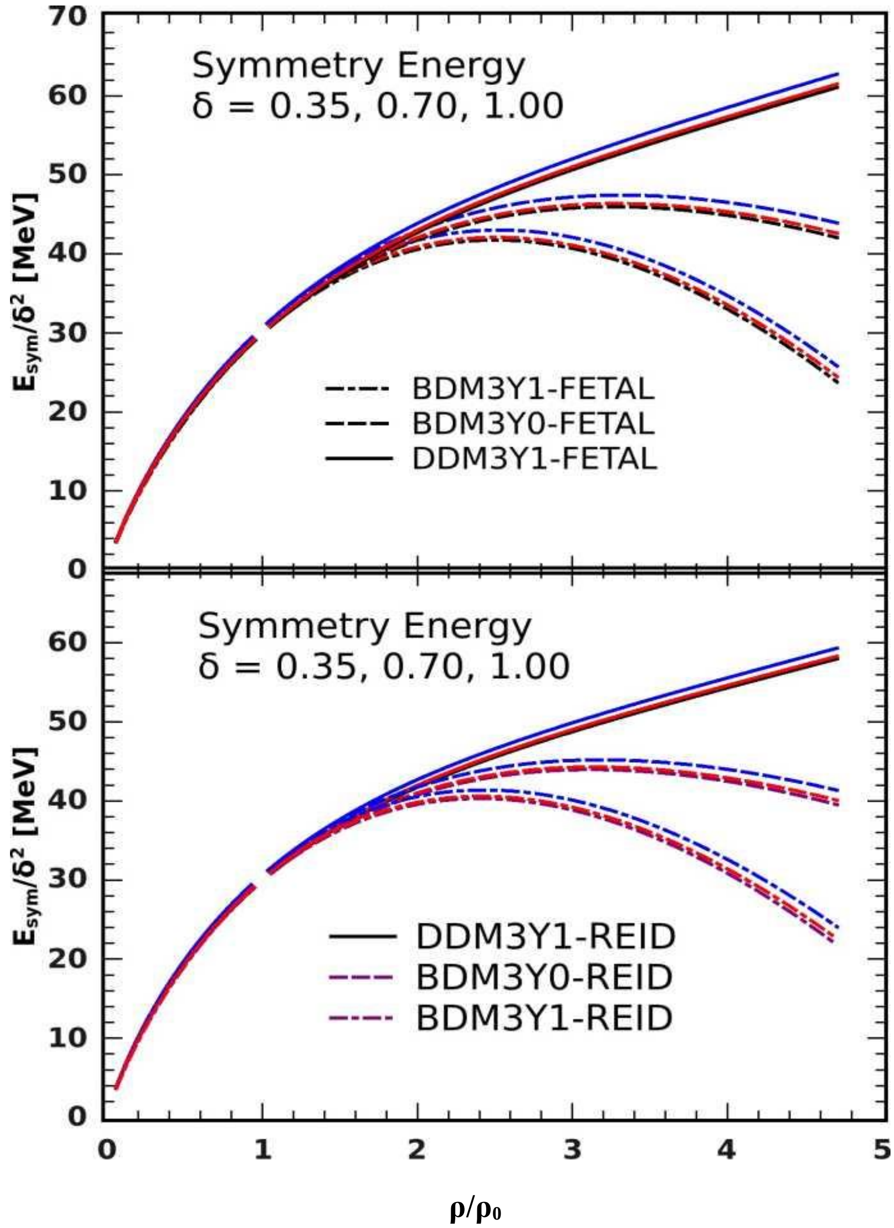

Figure 2: Symmetry Energy of Cold NM Calculated with Density.
Dependent M3Y-Reid and B3Y-Fetal Interactions at the Asymmetry Parameters (δ) between 0 and 1.00. The solid rectangle is the empirical value of symmetry energy at $\rho \sim \rho_0$

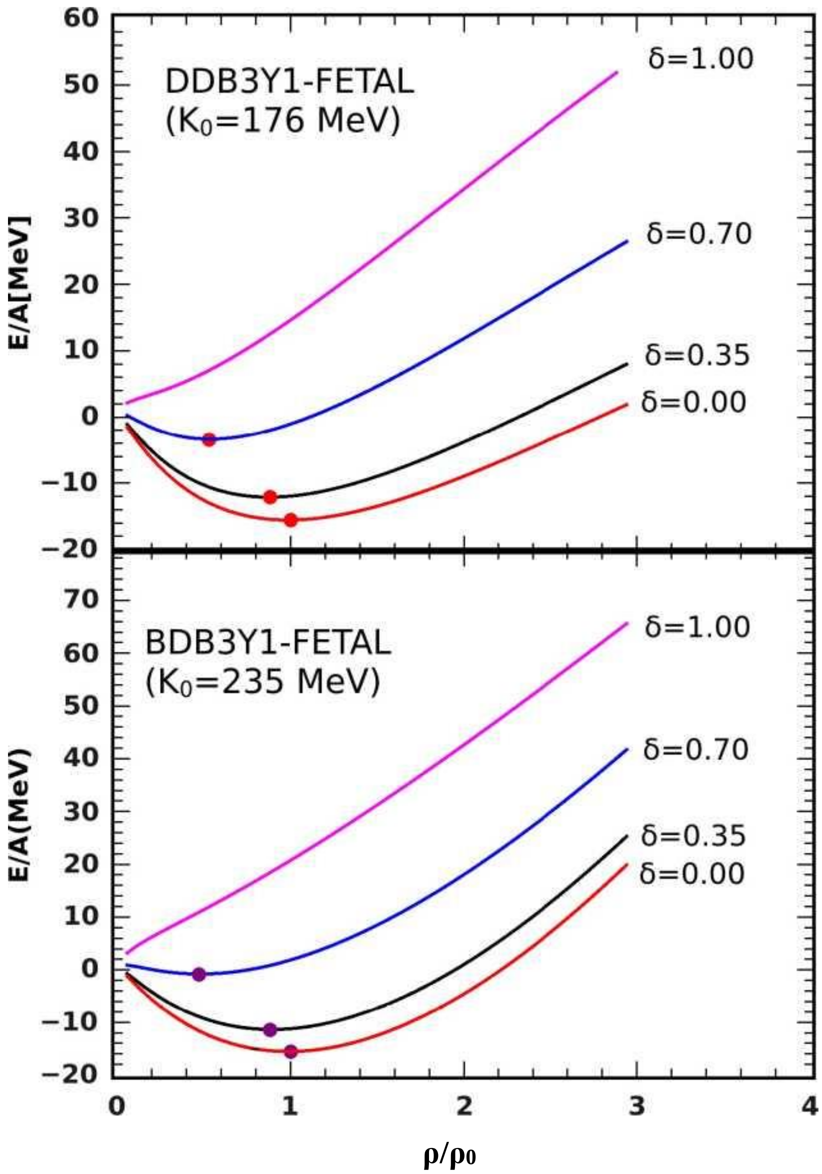

Figure 3: EOS's of Cold NM Calculated with the Density-Dependent DDB3Y1- and BDB3Y1-Fetal Interactions at the Asymmetry Parameters (δ) between 0 and 1. The solid Circles are the NM Binding Energies at their Respective Equilibrium Densities.

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ISSN 2055-0103(Online)

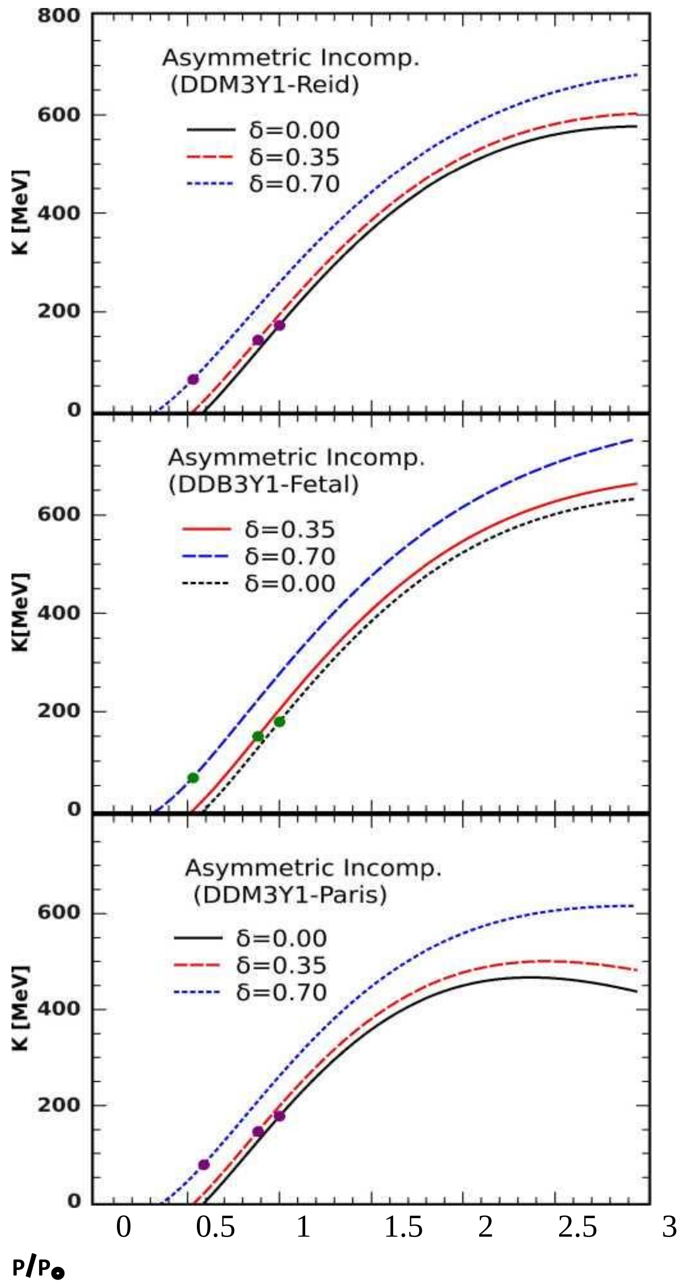

Figure 4: Nuclear Incompressibilities of Cold NM Calculated with DDM3Y1- Paris, DDB3Y1-Fetal and DDM3Y1-Reid Interactions at the Asymmetry Parameters (δ) between 0 and 0.70. The Solid Circles are the K-values at their Respective Equilibrium Densities.

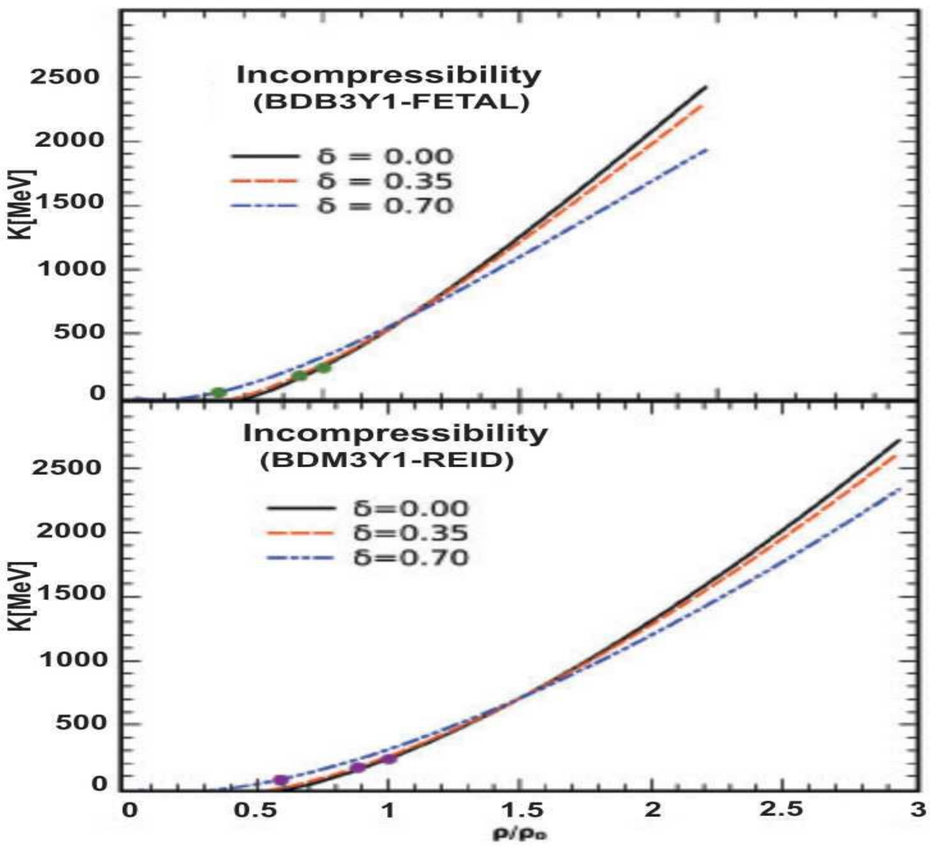

Figure 5: Nuclear Incompressibilities of Cold NM Calculated with BDB3Y1-Fetal and BDM3Y1-Reid Interactions at the Asymmetry Parameters (δ) between 0 and 0.70. The Solid Circles are the K-values at their Respective Equilibrium Densities.

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ISSN 2055-0103(Online)

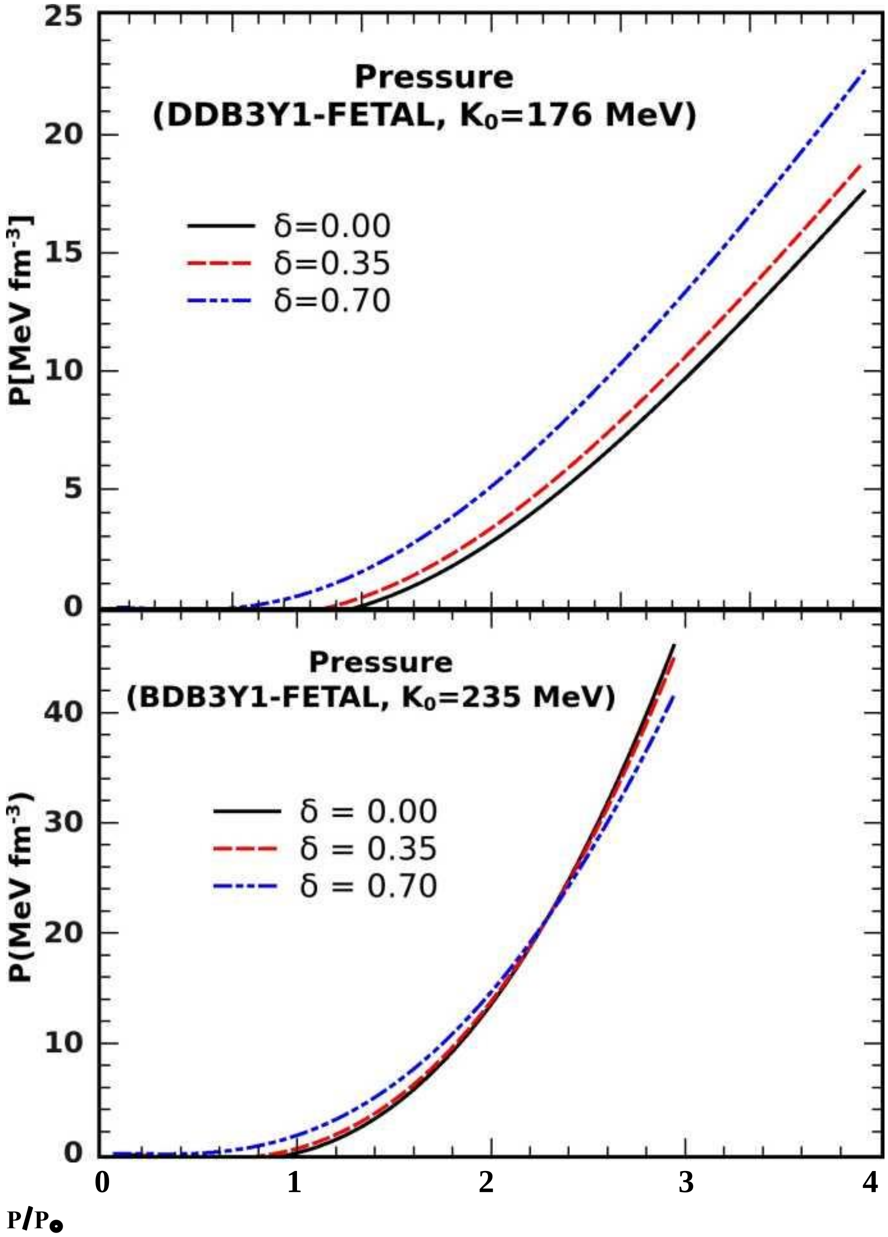
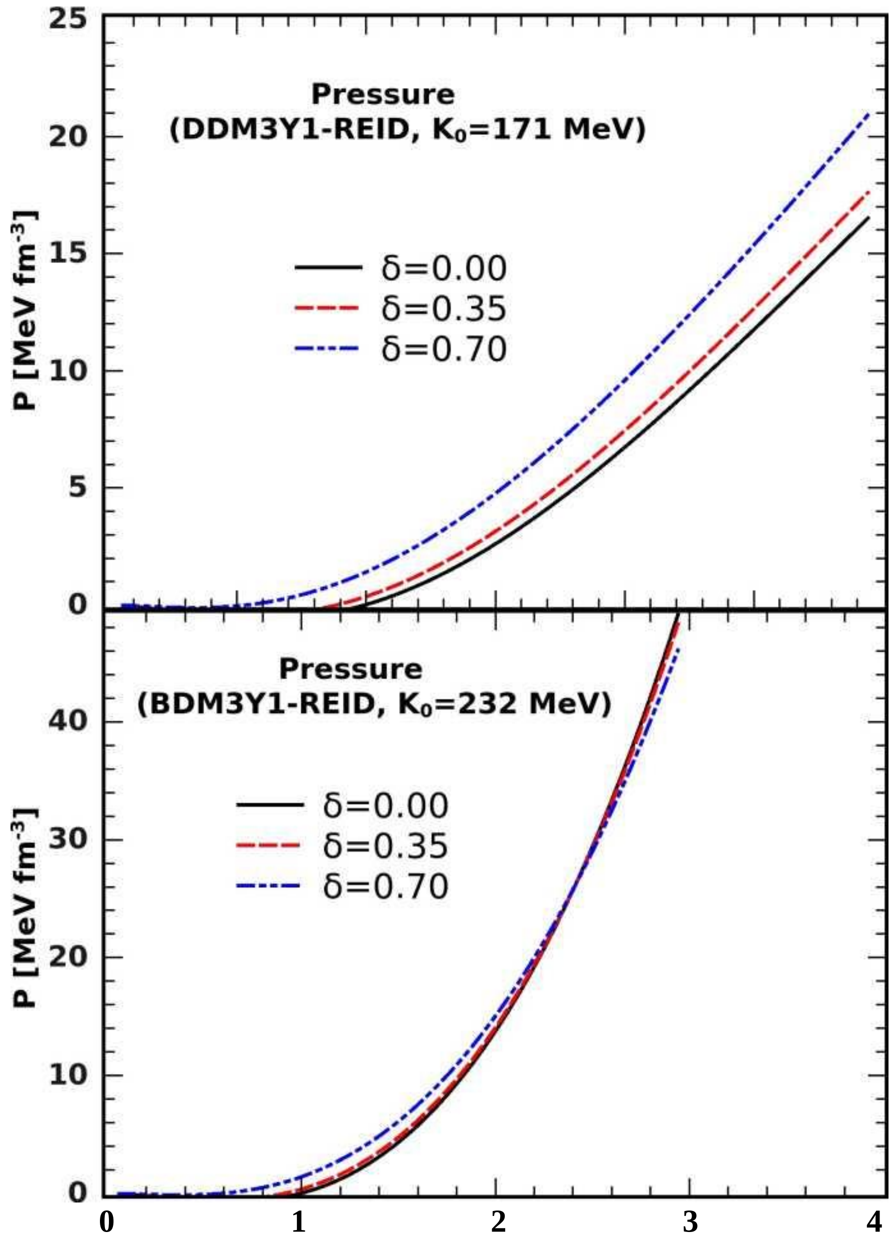


Figure 6: Pressure of Cold NM Calculated with DDB3Y1-Fetal and BDB3Y1- Fetal Interactions at the Asymmetry Parameters (δ) between 0 and 0.70.

ISSN 2055-009X(Print),

ISSN 2055-0103(Online)



P/Po

Figure 7: Pressure of Cold NM Calculated with DDM3Y1-Reid and BDM3Y1-Reid Interactions at the Asymmetry Parameters (δ) between 0 and 0.70.

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ISSN 2055-009X(Print),

ISSN 2055-0103(Online)

As a prerequisite for asymmetric nuclear matter (ANM) study, the B3Y-Fetal has been used in its DDB3Y1-Fetal, BDB3Y0-Fetal and BDB3Y1-Fetal, BDB3Y2- Fetal and BDB3Y3-Fetal versions along with the M3Y-Reid and M3Y-Paris effective interactions to reproduce the saturation properties of symmetric nuclear matter in [36], with results that have shown the B3Y-Fetal to demonstrate a behaviour in the nuclear matter that compared favourably well with the other two effective interactions. The results obtained with the DDB3Y1-Fetal, BDB3Y0-Fetal, and BDB3Y1-Fetal versions are reproduced along with the others in Table 1.0. The results obtained in this work for incompressibility of SNM ($\delta = 0$) in respect of the M3Y-Paris and M3Y-Reid in Table 1.0, respectively, are observed to be exact when compared with those obtained by [20, 23, 36], giving a good evidence that the results obtained with the B3Y-Fetal for ANM at ($\delta = 0.35$ and $\delta = 0.70$) in the same computational procedure are acceptably accurate.

For the study of asymmetric nuclear matter, we have added the isovector component of the density-dependent M3Y-type interaction to the isoscalar component. In this study, the C_1 factor for the M3Y-Paris effective interaction is seen in Table 1.0 to be the same as C_0 while $C_1 = 0.62C_0$ for both B3Y-Fetal and M3Y-Reid effective interactions to be able to obtain the correct value of symmetry energy *(E_{sym})* at equilibrium as observed in [36]. The calculated symmetry energy obtained with the DDM3Y1 and BDM3Y0 and BDM3Y1 versions of the B3Y-Fetal and M3Y-Reid interactions at different neutron-proton asymmetries (6) is displayed in Figure 2.0. The quadratic dependence of the symmetry energy upon the asymmetry parameter up to high nuclear matter densities is well illustrated in Figure 2.0, in which the curves of E_{sym}/δ^2 are portrayed to be almost independent of δ. This is known to demonstrate good agreement with relativistic Brueckner-Hartree-Fock (RBHF) calculations [20], confirming the empirical quadratic law up to the highest asymmetry parameter, 6 =1. Also evident from Figure 3.0 is a steady increase of E_{sym} with increasing nuclear density, up to about $2p_0$; but at much higher densities, the E_{sym} obtained with the BDM3Y0- as well as BDM3Y1-based effective interaction reaches a maximum and decreases smoothly as density increases in a manner typical of all BDM3Y-type effective interactions. This form of density dependence of E_{sym} is called ‘soft ’dependence [2, 8].

Figure 3.0 presents a plot of binding energies (EOS’s) of asymmetric nuclear matter at different values of asymmetry parameter, δ with those generated by the DDB3Y1-Fetal occupying the upper region while those based on BDB3Y1-Fetal occupy the lower region. The EOS curves show clearly that the saturation density becomes smaller as the neutron-proton asymmetry increases, but the neutron matter whose curve represents an asymmetry parameter of 1 is not bound at all. This result is in agreement with the behaviour of a realistic EOS for the asymmetric nuclear matter constructed phenomenologically to be used for the hydrodynamic calculation of supernova [20]. This suggests that the semi-microscopic EOS’s for ANM obtained herein could be used

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ISSN 2055-0103(Online)

for hydrodynamic simulation of supernova in which one of the values of nuclear incompressibility K_0 in Table 1.0 might prove to be the most realistic. It is also clear from Figure 3.0 that a higher incompressibility K_0 of symmetric nuclear matter implies a stiffer EOS of the associated neutron-rich nuclear matter.

Concerning nuclear matter incompressibility, the results obtained from calculations with the DDM3Y1 and BDM3Y1 versions of the M3Y-Reid, B3Y-Fetal and M3Y-Paris interactions are presented in Figures 4.0 and 5.0 respectively. The first obvious point from the figures and Table 1.0 is that the incompressibility at equilibrium decreases as the asymmetry, δ increases. This tallies well with the findings of past researchers [4, 20]; and it has been particularly observed [20] to corroborate the work of Baron, Cooperstein and Kahana in which the softening of EOS for asymmetric nuclear matter was identified as a useful requisite for a successful simulation of a prompt supernova explosion. A closer look at Figure 4.0 suggests that neutron enrichment leads to a stiffer EOS at all nuclear densities in a situation where the initial value of K_0 is less than 200 MeV such as the one produced by the DDM3Y1 effective interactions. Again, this has been found to agree with RBHF results [20]. Figure 5.0 portrays a different situation, based on a higher initial value of K_0 produced by the BDM3Y1 effective interactions (see Table 1.0), in which neutron enrichment generates a stiffer EOS than in the symmetric case up to about $1.1\rho_0$ for BDB3Y1-Fetal and $1.5\rho_0$ for the other two interactions; but above these values, the neutron-rich nuclear matter grows increasingly softer with increasing density than the symmetric nuclear matter. The processes arising from these two situations reveal the complexity with which shock energy formation takes place in the supernova core.

About nuclear matter pressure, P, Figure 6.0 shows features that are similar to those representing nuclear matter incompressibility just discussed. However, the curves of P based on the pretty soft DDM3Y1 interaction indicate that the nuclear matter pressure increases at all densities with neutron enrichment, whereas the curves involving the BDM3Y1 indicate that the pressure starts decreasing with increasing neutron enrichment (δ) as the density exceeds about 1.7 - $2.4\rho_0$. It is also obvious from Figure 7.0 that the pressure is less than zero at nuclear matter densities lower than the corresponding saturation densities, so the total pressure in the supernova matter in that region is considered to be dominated by the electronic pressure [20]. Finally, it has to be emphatically stressed based on the findings from Figures 4.0 - 7.0 that the absolute strengths of the nuclear pressure, P and incompressibility, K of asymmetric nuclear matter are observed to increase strongly with the starting (initial) EOS of symmetric nuclear matter; this makes accurate determination of the incompressibility of symmetric nuclear matter at saturation, K_0 from such sources as giant monopoles [2, 42] as well as optical model analyses of refractive nucleus-nucleus scattering for the construction of a realistic EOS for asymmetric nuclear matter essentially imperative.

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CONCLUSION

This paper investigates asymmetric nuclear matter with a new M3Y-type effective interaction called B3Y-Fetal interaction. It has been used alongside the M3Y-Paris and M3Y-Reid interactions in its density-dependent versions, DDM3Y1, BDM3Y0, and BDM3Y1, to compute the EOS, pressure, incompressibility, and other properties of asymmetric nuclear matter with results that are in good agreement with those of other researchers [4, 23]. The insight gained into the character of the B3Y-Fetal in this study is quite impressive. Since nuclear matter has remained a trusted testing ground for the viability of an effective interaction as well as the many-body technique or theory involved and the results of our computation have proven the B3Y-Fetal to have demonstrated a performance that is favourably comparable with well-known M3Y-Reid and M3Y-Paris interactions, this work serves to validate with all certainty and confidence the B3Y-Fetal as an effective interaction that can be relied on for correct explanation of nuclear matter and its associated phenomena such as supernova explosions in Nuclear and Astro-Physics.

Acknowledgements
We gratefully acknowledge Prof. Dao T. Khoa of INST, Vietnam, for his insightful work on nuclear matter that has provided a helpful guide for the successful completion of this work.

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