

## A State-Space Method of Balancing Chemical Reactions by Using Partial-Reversible Equations Approach

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**Citation:** Okafor U L., Oladejo M.O., Uwa C.O., Chinyio D.T. and, Omale P. (2022) A State-Space Method of Balancing Chemical Reactions by Using Partial-Reversible Equations Approach, Global Journal of Pure and Applied Chemistry Research Vol.10, No.2, pp.69-76

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**ABSTRACT:** *This paper describes with examples a state-space method of balancing chemical reactions using the concept of reversible equations. The method also employed the state-space analysis to show how the rows and columns of a matrix can be obtained from any given compound in a state- vector form. The proposed method employs the principle of reversible equations to demonstrate the law of conservation of mass that in any chemical reaction, atoms are not destroyed in moving from one state to another state. Considerable time and efforts are saved by the method of reversible equations when compared with the traditional echelon matrix reduction method, or the Gaussian method.*

**KEYWORDS:** *chemical reaction, products, reactants, reversible equations, state-space, state-vector.*

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### INTRODUCTION

The difference between a chemical equation and a mathematical equation has been stated by [1], and authors noting that a chemical can have a representation for a reversible and an irreversible reaction in one line. The subject of balancing chemical equations by the inspection method has been of great challenge to most students [2], equally the algebraic method has not given relive to the less gifted in mathematics as most students find it heard solving a set of simultaneous [3,4] equations. It is reported that students are at difficulty in balancing the chemical equation, while the teachers find it difficult in teaching and students find challenging to understand balancing a chemical equation [5]. The authors in [5] recommended the use of the mathematical software ..MATLAB, to circumvent the difficulty encountered in reducing a matrix to an echelon form. Andres

### Propositions

In order to implement the partial reversible equations approach, we propose as follows;

- (i) **If  $x = 3y$ ,  $x=3$  and  $y=1$  , ( 3,1)**
- (ii) **If  $3x = 4y$ ,  $x=4$ , and  $y= 3$  , (4,3)**
- (iii) **For a simultaneous equation such as,**  
 **$X = 3y$ , and**  
 **$3x = 4y$**

Then  $x=3, y=1$   $(x, y) = (3, 1)$  (\*)

and,

$3x = 4y$ , that means by the law of partial-reversible equations,  $x = 4$  and  $y = 3$  (\*\*)

The correct values for the first pair  $(3, 1)$ , will be obtained by multiplying the first coordinate (\*) by the value of  $x$  in (\*\*) to get  $4(3, 1) = (12, 4)$

## METHODOLOGY

In this section we will show how to use the space-state method to obtain the rows and columns of a matrix, and also represent the elements of a compound and the reactants and products of any chemical reaction in vector-space form. Next, it will be illustrated how the principle of reversible equations works. Represent the compound  $x_1(\text{NaOH}) + x_2(\text{H}_2\text{SO}_4) = x_3(\text{Na}_2\text{SO}_4) + x_4(\text{H}_2\text{O})$ , in a state-space form.

, checking the number of elements involved in the chemical reaction, to get the number of rows and columns that may be needed, and here, there are 4 elements: Na, H, O, and S.

3. Example 1 Balance the chemical reaction below using the method of reversible fractions;



Table 1. State-Space Representation for  $x_1(\text{NaOH}) + x_2(\text{H}_2\text{SO}_4)$

Na	$x_1$	+ 0
O	$x_1$	+4 $x_2$
H	$x_1$	+2 $x_2$
S	0	+ $x_2$

Next represent the right hand side as shown in Fig.2. below.

Table 2. Table 1. State-space and state- vector Representation of,  $x_3(\text{Na}_2\text{SO}_4) + x_4(\text{H}_2\text{O})$

Na	Chemical reaction below;
	$2x_3 + 0$
O	$4x_3$
	+ $x_4$
H	0
	+ $x_4$
S	$x_3$
	+ 0

The next concept to be explained is that of reversible fraction or mirror image, which say that, if

3y = 21x Type equation here. y=21/3

Table 3 State – Space State Vector Representation for  $X_1$  (NaOH) +  $X_2$ ( $H_2SO_4$ ) =  $X_3$  ( $Na_2SO_4$ ) +  $X_4$ ( $H_2O$ )

Na	$X_1$	+ 0		$2X_3$	+0	
O	$X_1$	+4 $X_2$		$4X_3$	+ $X_4$	
H	$X_1$	+ 2 $X_2$		0	+ 2 $X_4$	
S	0	+ $X_2$			$X_3$	

For Na,  $X_1 = 2X_3$  (1)

Using the principle of reversible fractions,  $X_1 = 2$

For S,  $X_2 = X_3$

For H,  $X_1 + 4X_2 = 4X_3 + X_4$  (2)

Put  $X_2 = X_3$  in (2), to get,  
 $X_1 = X_4$ , since  $x_1 = 2$ , therefore  $x_4 = 2$

Hence the balanced equation gives,  
 $2(NaOH) + (H_2SO_4) = (Na_2SO_4) + 2(H_2O)$

Worked Example2

The first 4 examples are taken from [1]  
 Example 1. Balance the chemical equation



Here there are two elements Fe and O

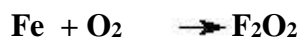


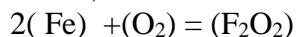
Table 3.State-Space Representation for : $X_1$ ( Fe) + $X_2$ ( $O_2$ ) =  $X_3$ ( $Fe_2O_3$ ) (1)

	$X_1 + 0$		$2 X_3$
Fe			
O	$0 + 2 X_2$		$2 X_3$

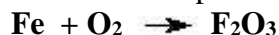
From row 1, where we have the element F,  $X_1 = 2 X_3$ ,by using the principle of reversible fractions or mirror image,  $x_1 = 2$

From row 2, where we have the element O,  $2 X_2 = 2 X_3$ , therefore,  $X_2 = X_3 = 1$

Hence,the balanced equation now becomes;



Example 2. Write a balanced equation for the chemical reaction below using the reversible equations and state-space method.



Let  $X_1$ ( Fe) + $X_2$ ( $O_2$ ) =  $X_3$ ( $Fe_2O_3$ ) (1)

Table 4.State-Space and vector- space representation for  $X_1$ ( Fe) + $X_2$ ( $O$ ) =  $X_3$ ( $Fe_2O_3$ )

Fe	$X_1 + 0$		$2 X_3$
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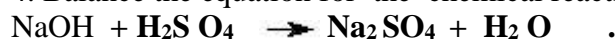
O	0+2 X <sub>2</sub>	3X <sub>3</sub>
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From row 1, where we have the element F,  $X_1 = 2 X_3$ , using the principle of reversible fractions or mirror image, the correct value for  $x_1 = 2$ , and for the element O,  $2 X_2 = 3 X_3$ , here,  $x_2 = 3$ ,  $x_3 = 2$ . Since  $x_1 = 2x_3$ , the correct value for  $x_1$  will then be giving by  $2 \cdot 2 = 4$

The balanced equation becomes as shown in (2)



4. Balance the equation for the chemical reaction using the method of reversible fractions,



Here, there are 4 elements Na, O, H, and S, as such four rows are needed as shown in Table 3.

Table 3

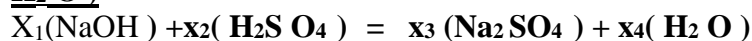
Na
O
H
S

The balanced equation is represented by (5)



Where  $X_1$ ,  $X_2$ ,  $X_3$  and  $X_4$ , are constants to be determined.

**Table 4 .State-Space and vector space for  $X_1(\text{NaOH}) + x_2(\text{H}_2\text{S O}_4) = x_3(\text{Na}_2\text{SO}_4) + x_4(\text{H}_2\text{O})$**



Na	$X_1 + 0$	$2x_3 + 0$
O	$X_1 + 4 X_2$	$4 x_3 + x_4$
H	$X_1 + 2 X_2$	$0 + 2 x_4$
S	$0 + X_2$	$X_3 + 0$

From row 1, considering Na element,  $X_1 = 2X_3$ , and using the principle of reversible fractions,  $X_1 = 2$ ,  $X_3 = 1$ .

From 4, considering the element S,  $X_2 = X_3 = 1$

For the element O in row 2,

$$X_1 + 4 X_2 = 4 x_3 + x_4 \quad (6)$$

Putting  $X_2 = X_3$  in (6), we get (7)

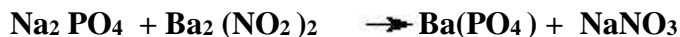
$$X_1 + 4 X_3 = 4 x_3 + x_4 \quad (7)$$

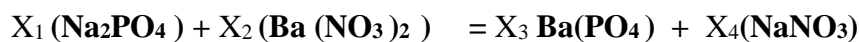
Hence,  $X_2 = X_4 = X_3 = 1$

Therefore, the balanced equation becomes;



**Obtain a balanced equation for the chemical reaction below using the method of reversible fraction.**





Where,  $X_1$   $X_2$   $X_3$  and  $X_4$ , are constants to be determined

**Table 5 .State-Space and vector space for  $X_1(\text{Na}_2\text{PO}_4) + X_2(\text{Ba}(\text{NO}_3)_2) = X_3\text{Ba}(\text{PO}_4) + X_4(\text{NaNO}_3)$**

Na	$2X_1 + 0$		$0 + X_4$
P	$X_1 + 0$		$X_3 + 0$
O	$4X_1 + 4X_2$		$4X_3 + 2X_4$
Ba	$0 + X_2$		$X_3$
			$+ 0$
N	$0 + 2x_2$		$0 + X_4$

Using the principle of reversible fractions,

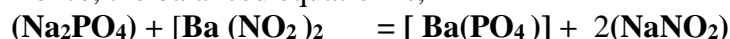
From Na :  $2x_1 = x_4$ ,  $x_1 = 1, x_4 = 2$ ,  $(x_1, x_4) = (1, 2)$

From Ba :  $x_2 = x_3$ ,  $x_3 = 1$

,  $x_2 = 1$ ,  $(x_2, x_3) = (1, 1)$

variables	$X_1$	$X_2$	$X_3$	$X_4$
value	2	2	2	4

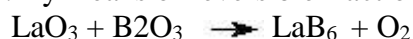
Hence, the balanced equation is,



## CONCLUSION

In this paper, we have shown how to balance chemical equations using the method of reversible fractions instead of 3 century old reduction to echelon for of the Gaussian method.,or the unnecessary use of MATALAB. This new method is less time consuming and will save students the trouble of how to use any software in balancing chemical equations.

Example 6. By means of reversible fractions, balance the equation for the chemical reaction:



Solution.

Let  $x_1[\text{LaO}_3] + x_2[\text{B}_2\text{O}_3] = x_3[\text{LaB}_6] + x_4[\text{O}_2]$

For ,La :  $x_1 = x_3$  (1)

For O:  $3X_1 + 3X_2 = 2X_4$  (2)

From B:  $2X_2 = 6X_3$ , (3)

From (1)  $x_1 = x_3$

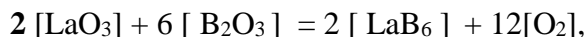
From (3)  $x_3 = 2, x_2 = 6$

Hence, equation (1), now gives  $x_1 = x_3 = 2$

Using (2),  $3X_1 + 3X_2 = 2X_4$ , therefore,

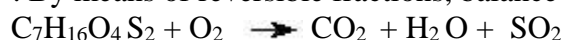
$$\begin{aligned} 3(2) + 3(6) &= 2X_4 \\ 24 &= 2X_4 \\ X_4 &= 12 \end{aligned}$$

Hence, the balanced equation gives,



Or,  $[\text{LaO}_3] + 3 [\text{B}_2\text{O}_3] = [\text{LaB}_6] + 6[\text{O}_2],$

Example 7 . By means of reversible fractions, balance the equation for the chemical reaction:



Solution.

Suppose,  $x_1[\text{C}_7\text{H}_{16}\text{O}_4\text{S}_2] + x_2[\text{O}_2] = x_3[\text{CO}_2] + x_4[\text{H}_2\text{O}] + x_5[\text{SO}_2]$

Considering the elements and their coefficients, we get the equations below:

For C,  $7X_1 = X_3, X_3 = 7, X_1 = 1$

For H,  $16X_1 = 2X_4, X_4 = 8, X_1 = 1$

Considering the element S,  $2X_1 = X_5, X_5 = 2, x_1 = 1$

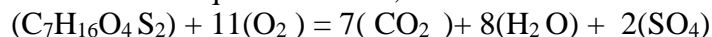
From the element O,  $4X_1 + 2X_2 = 2X_3 + X_4 + 2X_5$

Therefore,  $4 + 2x_2 = 2*7 + 8 + 2*2$

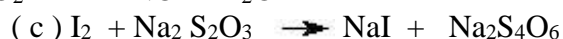
$$4 + 2x_2 = 14 + 8 + 4$$

$$2x_2 = 22, \text{hence, } x_2 = 11$$

The balanced equation is now,



Example 8 . By means of reversible fractions, balance the equation for the following chemical reactions:



Solutions.

(a) Let,  $x_1(\text{H}_3\text{BCO}) + x_2(\text{H}_2\text{O}) = x_3(\text{B(OH)}) + x_4(\text{CO}) + x_5(\text{H}_2)$

For H:  $3X_1 + 2X_2 = X_3 + 2X_5$  (1)

For B:  $X_1 = X_3$  (2)

For C:  $X_1 = X_4$  (3)

For O:  $X_1 + X_2 = X_3 + X_4$  (4)

Using  $X_1 = X_4$  in (4)

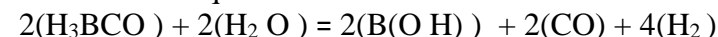
We have  $x_2 = x_3 = x_1$

To get  $x_5$  from (1)

$$3x_1 + 2x_1 = x_1 + 2x_5, \text{remembering that } x_2 = x_3 = x_1$$

$$4x_1 = 2x_5, \text{then } x_1 = 2, x_5 = 4$$

The balanced equation now becomes:



Or,  $(\text{H}_3\text{BCO}) + (\text{H}_2\text{O}) = (\text{B(OH)}) + (\text{CO}) + 2(\text{H}_2)$

(b) Let,  $X_1(\text{NH}_3) + X_2(\text{O}_2) = X_3(\text{NO}) + X_4(\text{H}_2\text{O})$

Comparing coefficients for the elements, we obtain the following equations:

$$\text{For N : } X_1 = X_3 \quad (1)$$

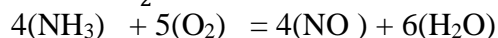
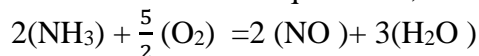
$$\text{For H, } 3X_1 = 2X_4 \quad (2), x_4 = 3, X_1 = 2, x_3 = 2$$

$$\text{For O } X_2 = X_3 + X_4 \quad (3)$$

Using  $X_3 = 2$ , and  $X_4 = 3$  in (3) we get

$$2X_2 = 2 + 3, \text{ therefore } X_2 = \frac{5}{2}$$

Hence the balanced equation is;



(c) Let,  $X_1(\text{I}_2) + X_2(\text{Na}_2\text{S}_2\text{O}_3) = X_3(\text{NaI}) + X_4(\text{Na}_2\text{S}_4\text{O}_6)$

$$\text{For I: } 2X_1 = X_3, X_3 = 2, X_1 = 1$$

$$\text{For Na: } 2X_2 = X_3 + X_4$$

$$\text{For S; } 2X_2 = 4X_4, X_4 = 1, X_2 = 2$$

$$\text{For O : } 3X_2 = 6X_4$$

$$\text{But } X_4 = 1, \text{ therefore } 3X_2 = 6, X_2 = 2$$

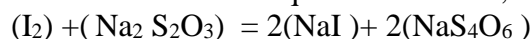
For Na :

$$2X_2 = X_3 + X_4$$

$$4 = X_3 + 1,$$

$$\text{Or, } X_3 = 3$$

Hence the balanced equation becomes;



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