International Journal of Petroleum and Gas Engineering Research
Vol.4, No.1, pp.1-12, Februarys 2020
Published by ***ECRTD-UK***
Print ISSN: ISSN 2514-9253
Online ISSN: ISSN 2514-9261

A SIMPLIFIED APPROACH TO THE ANALYSIS OF OIL DISPLACEMENT BY WATER IN STRATIFIED RESERVOIRS

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ABSTRACT: *Prediction of reservoir performance during water displacement process is a routine procedure for homogeneous reservoirs but complicated in stratified heterogeneous reservoirs. Most analytical methods assume the simplistic no-crossflow condition while models that incorporate crossflow use numerical techniques which are not very amenable to routine calculations. In this study, by using the concept of pseudo-relative permeability, the effects of crossflow and stratification were incorporated into the fractional flow equation for reservoir performance evaluation and tracked using the well-known Welge analysis procedure. Application to field examples confirmed observations of previous researchers that reservoirs with favorable end-point mobility, crossflow between layers can enhance oil recovery (by more than 10%), while unfavorable end-point mobility leads to significant reduction (over 20%) in oil recovery. The study therefore confirmed that the Welge analysis front tracking method has wider scope of application than currently used and can be extended to stratified crossflow reservoirs without loss of accuracy.*

KEYWORDS: crossflow, cumulative oil produced, fractional flow, frontal velocity, heterogeneous reservoir, pseudo-relative permeability

INTRODUCTION

The use of waterflooding to enhance production in oil reservoirs is an established and dominant technique in the petroleum industry (Dystra and Parsons, 1950; Higgins and Leighton, 1963; Wasson and Schrider, 1968; Hearn, 1971; El-Khatib, 1985; Gullick and McCain, 1998; Shotton and Stephen, 2016). Factors that has enhanced the development and increasing use of waterflooding include availability, relatively low cost, high displacement efficiency, and opportunity for produced water disposal. Buckley and Leverett (1942) developed a theory of displacement based on the relative permeability concept to describe immiscible displacement in one direction. They showed that the frontal advance equation can be used to compute the saturation distribution in a linear waterflood system as a function of time.

Terwillinger et al (1951) confirmed the frontal advance theory of Buckley and Leverett, and recognized that in practical situations, the flood front will not exist as a discontinuity, but will exist as a stabilized zone of finite length with a large saturation gradient. While applying the frontal advance theory to a gravity drainage system, they found a zone at the leading edge of the front where displacing fluid saturations all moved at the same velocity. The shape of the zone was observed to be constant with respect to time and this zone was termed the *stabilized zone*. Welge (1952) simplified the graphical procedure of Buckley and Leverett by proposing a method for predicting oil recovery by either gas or water. This method requires that the initial water saturation be uniform. Several authors have proposed different methods of evaluating waterflooding in stratified reservoirs. Some of the notable works include those of Stiles (1949), Dykstra and Parsons (1950), and Warren and Price (1961).

In 1971, Hearn used the concept of pseudo relative permeability functions to simplify the numerical simulation of waterflooding in stratified reservoirs with crossflow. He observed that neglecting crossflow in some stratified reservoirs can lead to incorrect results. El-Khatib (1985) published the results of performance evaluation of stratified reservoirs using numerical techniques. He established that crossflow between layers enhances oil recovery for systems with favourable mobility ratios (mobility ratio <1), and retards oil recovery for systems with unfavourable mobility ratios. In 2014, Isehunwa and Falade considered the steam flood displacement problem in oil reservoirs 3-phse non-isothermal process that is also amenable to the Welge front tracking analysis technique. Nwaka and Isehunwa (2017) proposed a method to apply the Welge analysis procedure to heterogenous reservoirs. They adopted the stratified reservoir concept for heterogeneous reservoirs and used geometric average permeability for each layer.

METHODOLOGY

The method described by Hearn (1971) has been adopted, using pseudo relative permeability functions to approximate the effect of reservoir stratification. The following assumptions were made:

1. The reservoir is composed of layers with individual properties.
2. Layer cake model with communicating layers (crossflow).
3. Flow is linear and all assumptions in the Buckley-Leverett method apply to each layer.
4. The fluids are incompressible.
5. The water-oil displacement front in each layer is represented by a sharp interface. Ahead of the interface only oil flows at connate water saturation and behind the interface only water flows at residual oil water saturation.
6. Vertical pressure gradients are negligible compared with horizontal gradients.
7. Gravity and capillary forces are negligible relative to viscous effects.
8. Constant injection rate.
9. Negligible porosity variation.

For analysis, the layers are arranged in decreasing values of the factor $\frac{K}{\emptyset \Delta S}$; where ΔS is the change in flowing water saturation across the front in each.

Using the approach by Hearn,

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Let,

k_{wr}= relative permeability to water at residual oil saturation
k_{oc}= relative permeability to oil at connate water saturation
μ_o = oil viscosity
μ_w = water viscosity
K = absolute permeability
h = thickness
Q = flowrate
The velocity of the front in in each layer can be expressed as:

$$v_n = \frac{Q_T \Delta f_n}{W h_n \emptyset \Delta S_N}, n = 1,2,3, \ldots, N \quad (1)$$

Where,

Δf_n is the change in fractional flow and is given by:

$$\Delta f_n = \frac{\frac{k_{wr}}{\mu_w}\sum_{i=1}^{n} K_i h_i}{\frac{k_{wr}}{\mu_w}\sum_{i=1}^{n} K_i h_i + \frac{k_{oc}}{\mu_o}\sum_{i=n+1}^{N} K_i h_i} - \frac{\frac{k_{wr}}{\mu_w}\sum_{i=1}^{n-1} K_i h_i}{\frac{k_{wr}}{\mu_w}\sum_{i=1}^{n-1} K_i h_i + \frac{k_{oc}}{\mu_o}\sum_{i=n}^{N} K_i h_i} \quad (2)$$

Equation (1) is similar to the Buckley-Leverett theory and shows that the frontal velocity in each layer is a constant (for constant injection rate). It can be compared side by side with the Buckley-Leverett equation:

$$v_n = \frac{Q_T \Delta f_n}{W h_n \emptyset \Delta S_n} \quad ; \quad x = \frac{5.615 q_t t}{\emptyset A_n} \frac{\partial f_w}{\partial S_w} \quad (3)$$

Therefore,

$$\frac{\Delta f_n}{\Delta S_n} \equiv \frac{\partial f_w}{\partial S_w} \quad (4)$$

This similarity allows the development of unique pseudo relative permeability functions for the stratified system.

The pseudo relative permeability functions are obtained by calculating the average water saturation and the average water and oil flow rates at any cross section. The layers are first arranged in decreasing breakthrough of the water-oil displacement front, so that layer 1 is flooded out first, layer 2 next, etc. the proper ordering is obtained by arranging the layers in order of decreasing values of the factor $\frac{K}{\emptyset \Delta S}$.

Before breakthrough of layer 1;

$$S_{w0} = \frac{\sum_{i=1}^{N} h_i \emptyset S_{wci}}{\sum_{i=1}^{N} h_i \emptyset} \quad (5)$$

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After breakthrough of layer n;

$$S_{wn} = \frac{\sum_{i=1}^{n} h_i \emptyset (1 - S_{roi}) + \sum_{i=n+1}^{N} h_i \emptyset S_{wci}}{\sum_{i=1}^{N} h_i \emptyset} \qquad (6)$$

After breakthrough of the last layer, N;

$$S_{wN} = \frac{\sum_{i=1}^{N} h_i \emptyset (1 - S_{roi})}{\sum_{i=1}^{N} h_i \emptyset} \qquad (7)$$

Where:
$\emptyset$ = porosity
S_{ro} = residual oil saturation
S_{wc} = connate water saturation

The pseudo relative permeabilities corresponding to these saturations can be obtained by calculating the flow rates of water and oil at the outflow face using Darcy's law with a weighted average permeability, the total cross-sectional area, and pseudo relative permeabilities.

$$Q_w = -\frac{\tilde{k}_w K_{avg} A_T}{\mu_w} \left(\frac{dP}{dx}\right) \qquad (8)$$

$$Q_o = -\frac{\tilde{k}_o K_{avg} A_T}{\mu_o} \left(\frac{dP}{dx}\right) \qquad (9)$$

If the horizontal pressure gradient is assumed to be the same in all layers, we have:

$$K_{avg} = \frac{\sum_{i=1}^{N} K_i h_i}{\sum_{i=1}^{N} h_i} \qquad (10)$$

$$A_T = W \sum_{i=1}^{N} h_i \qquad (11)$$

After breakthrough of layer n, the pseudo relative permeabilities $\tilde{k}_{wn}$ and $\tilde{k}_{on}$ are obtained as:

$$\tilde{k}_{wn} = \frac{k_{wr} \sum_{i=1}^{n} K_i h_i}{\sum_{i=1}^{N} K_i h_i} \qquad (12)$$

$$\tilde{k}_{on} = \frac{k_{oc} \sum_{i=n+1}^{N} K_i h_i}{\sum_{i=1}^{N} K_i h_i} \qquad (13)$$

Therefore, in order to evaluate the pseudo relative permeabilities in equations (12) and (13), it is necessary to know the relative permeability to oil at connate water saturation, k_{oc} , and the relative permeability to water at residual oil saturation, k_{wr}. These are the endpoints of the rock permeability curves and are obtained from laboratory tests.

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Using Buckley and Leverett approach, the fractional flow equation for a horizontal bed can be expressed as:

$$f_w = \frac{1}{1+\frac{\mu_w \tilde{k}_o}{\mu_o \tilde{k}_w}} \quad (14)$$

$$\therefore f_w = \frac{1}{1+\frac{\mu_w}{\mu_o}\left\{\frac{\frac{k_{oc}\sum_{i=n+1}^{N} K_i h_i}{\sum_{i=1}^{N} K_i h_i}}{\frac{k_{wr}\sum_{i=1}^{n} K_i h_i}{\sum_{i=1}^{N} K_i h_i}}\right\}} \quad (15)$$

The fractional flow curve can then be constructed, and the reservoir system analyzed using Welge procedure.

Welge showed that the average water saturation in the reservoir at the time the saturation at the outlet is S_{wn} is given by the equation:

$$\bar{s}_w = s_{wn} + \frac{1-f_{wn}}{\left(\frac{df_w}{ds_w}\right)_n} = s_{wn} + \frac{f_{on}}{\left(\frac{df_w}{ds_w}\right)_n} \quad (16)$$

From equation (1), the displacement x_n will be:

$$x_n = \frac{Q_T \Delta f_n \times 5.615t}{\emptyset A_n \Delta S_n} \quad (17)$$

At breakthrough of any layer *n*, $x_n = L$

$$L = \frac{5.615 Q_T \Delta f_n t_{bt}}{\emptyset A_n \Delta S_n} \quad (18)$$

Therefore, the breakthrough time for each layer *n* is given as:

$$\therefore t_{btn} = \frac{\emptyset A_n L \Delta S_n}{5.615 q_t \Delta f_n} \quad (19)$$

$$\left(\frac{df_w}{ds_w}\right)_n^{-1} \equiv \left(\frac{\Delta S_n}{\Delta f_n}\right)\frac{h_n}{h} \quad (20)$$

Where;

$h_n = Layer\ thickness$

$h = Total\ thickness$

Where ΔS_n and Δf_n can therefore be expressed respectively as:

$$\Delta S_n = 1 - S_{wcn} - S_{ron} \quad (21)$$

$$\Delta f_n = \frac{\frac{k_{wr}}{\mu_w}\sum_{i=1}^{n} K_i h_i}{\frac{k_{wr}}{\mu_w}\sum_{i=1}^{n} K_i h_i + \frac{k_{oc}}{\mu_o}\sum_{i=n+1}^{N} K_i h_i} - \frac{\frac{k_{wr}}{\mu_w}\sum_{i=1}^{n-1} K_i h_i}{\frac{k_{wr}}{\mu_w}\sum_{i=1}^{n-1} K_i h_i + \frac{k_{oc}}{\mu_o}\sum_{i=n}^{N} K_i h_i} \quad (22)$$

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The average saturation behind the front as at the breakthrough of each layer can be determined:

$$\left(\frac{df_w}{ds_w}\right)_n^{-1} \equiv \left(\frac{\Delta S_n}{\Delta f_n}\right)\frac{h_n}{h} \quad (23)$$

$$\therefore \left(\frac{\Delta S_n}{\Delta f_n}\right)\frac{h_n}{h} = \frac{ds_w}{df_w} = \frac{\bar{S}_{wn} - S_{wn}}{1 - f_{wn}} \quad (24)$$

Making $\bar{S}_{wn}$ the subject,

$$\therefore \bar{S}_{wn} = \left[\left(\frac{\Delta S_n}{\Delta f_n}\right)\frac{h_n}{h}(1 - f_{wn})\right] + S_{wn} \quad (25)$$

The cumulative oil produced for each layer at breakthrough will be:

$$N_{ptbn}(STB) = \frac{7758\emptyset Ah[\bar{S}_{wn} - S_{wi}]}{B_o} \quad (26)$$

While the cumulative water injection for each layer will be:

$$W_{in} = q_t t_{btn} \quad (27)$$

Where,

$$t_{btn}(days) = \frac{7758\emptyset Ah_n \Delta S_n}{q_t \Delta f_n} \quad (28)$$

and

$$W_{in}(STB) = q_t t_{btn} = 7758\emptyset Ah_n \left(\frac{\Delta S_n}{\Delta f_n}\right) \quad (29)$$

RESULTS AND DISCUSSION

Equations 3 and 4 show that the use of pseudo relative permeability concept allows for the derivations of expressions that are similar to the Buckley and Leverett and the Welge equations. The proposed procedure in this study was applied to three different cases with varying mobility ratios and areal and vertical permeability variations. All the three cases had no initial gas saturation. For each case, the cumulative oil produced was evaluated with time to attain breakthrough and afterward. The results were compared with the situations of no crossflow between layers s presented by Nwaka and Isehunwa. The input data for the three cases are as indicated in the appendix (Tables A, B and C).

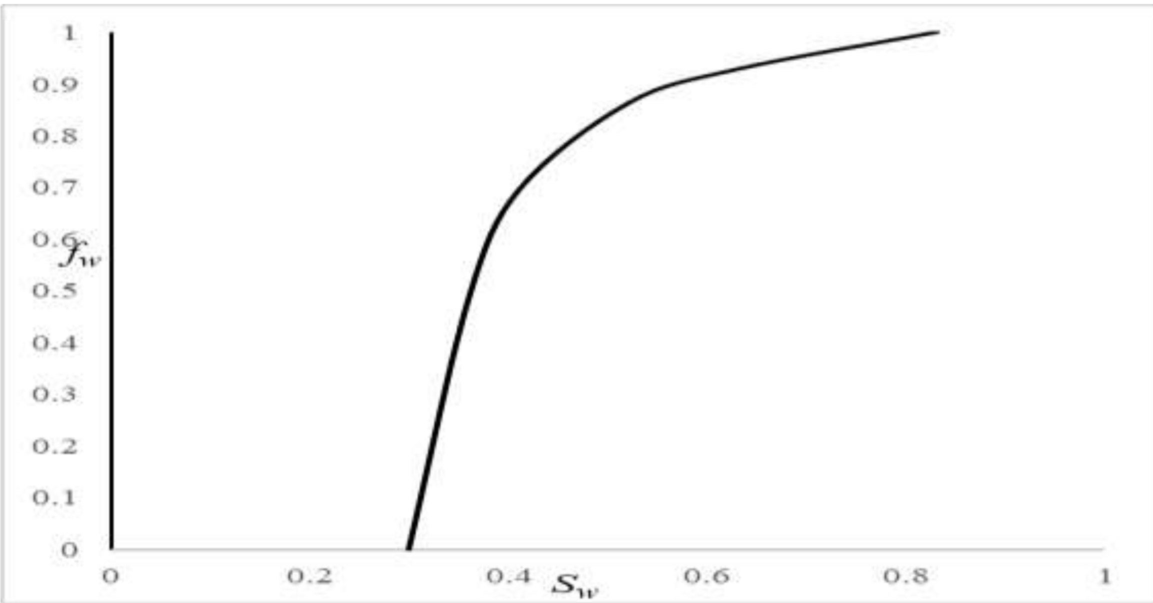

Figure 1: Fractional Flow Curve for Case 1

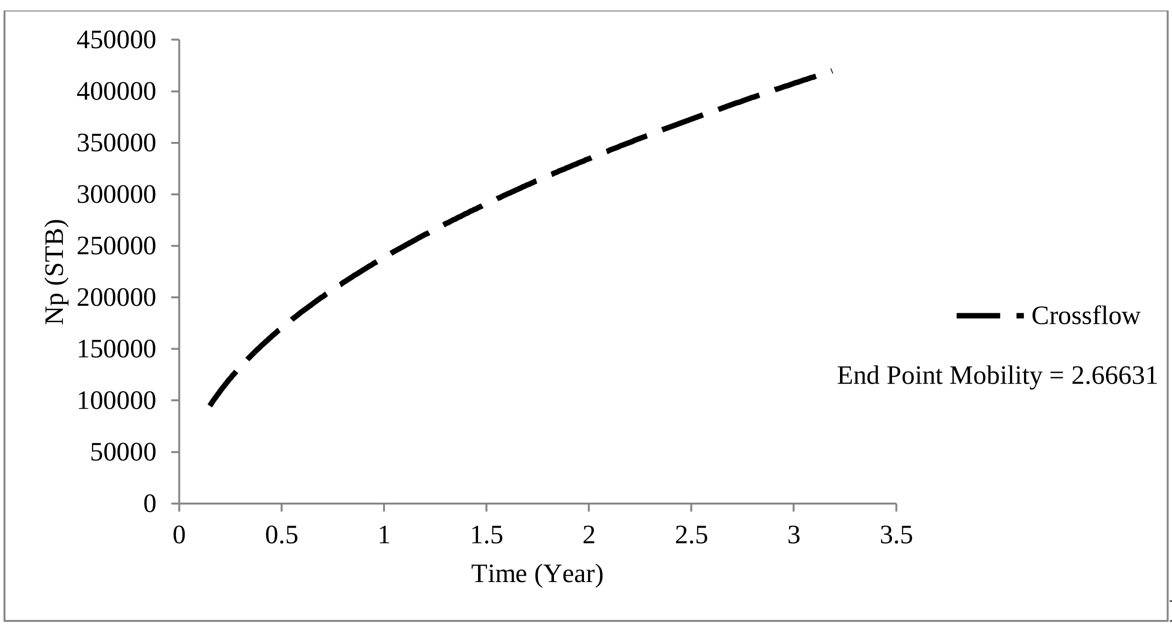

Figure 2: Cumulative Production with Time (Case 1)

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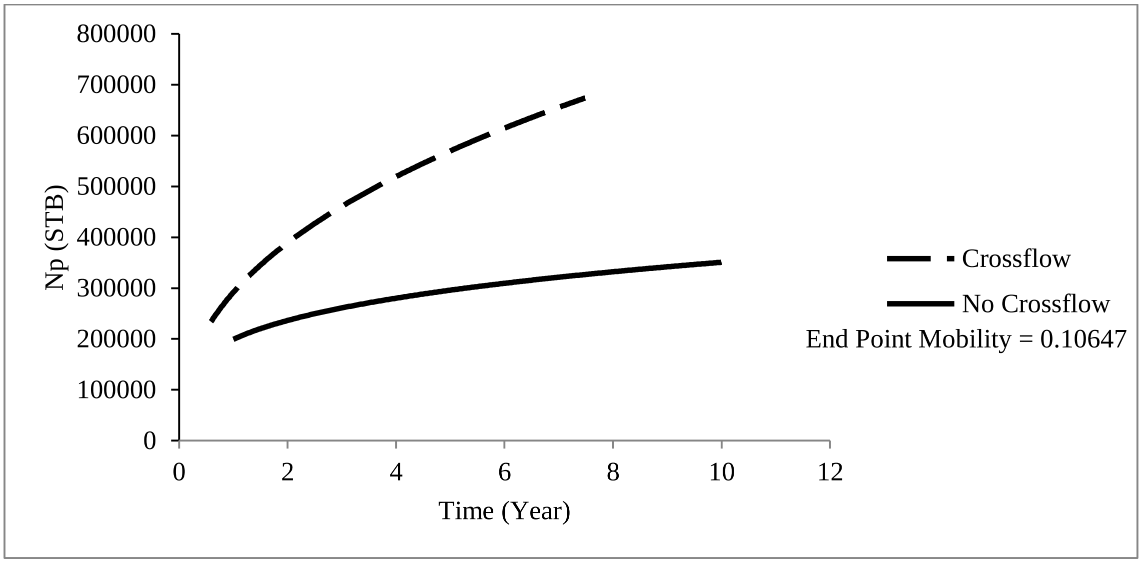

Figure 3: Cumulative Production with Time (Case 2)

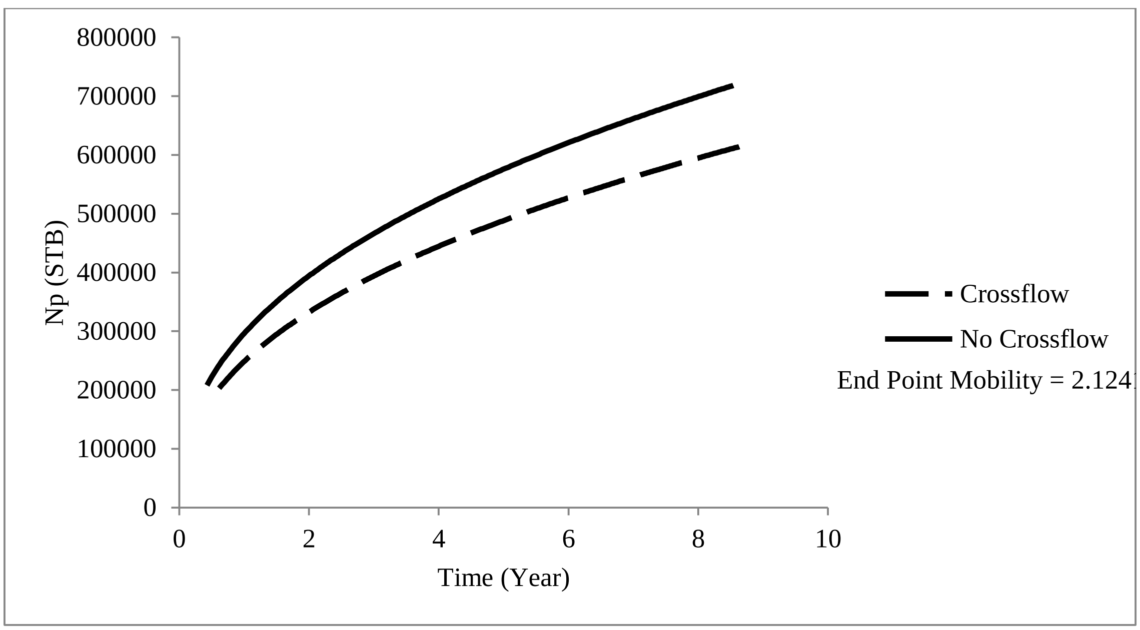

Figure 4: Cumulative Production with Time (Case 3)

It is pertinent to note that the pseudo relative permeability functions require visualizing the reservoirs as if homogeneous, but with relative permeability relationships that cause them to act exactly as stratified systems. The effects of crossflow on production can be seen clearly in the

Figures 2-4. The results show that crossflow between layers enhances the oil recovery for systems with favorable mobility ratios (mobility ratio < 1) and retards oil recovery for systems with unfavorable mobility ratios (mobility ratio > 1).

For cases 1 and 3 with mobility ratios > 1, it can be seen in Figures 2 and 4 respectively that the cumulative production for no crossflow was greater than that with crossflow. In case 2, Figure 3 shows the reverse with favorable mobility ratio < 1. Cumulative production was significantly higher for the communicating system with crossflow than for the non-communicating system without crossflow.

Furthermore, crossflow tends to make the influence of mobility ratio in flooding performance more pronounced as can be seen in the cases with unfavorable mobility ratio (Cases 1 and 3) where crossflow between layers retarded oil recovery by an average of about 21% while it improved recovery for favorable mobility ratio (Case 2) by about 11%.

CONCLUSION

This study has shown that by using pseudo relative permeability concept, it is possible to obtain a simplified approach for the analysis of waterflood performance for stratified reservoirs with crossflow between layers, using the well-known Wedge analysis procedure applying the fractional flow equation. This agrees with the observation by Isehunwa and Falade (2014) that the simple front tracking technique proposed by Welge has wider application in the analysis of petroleum reservoirs than hitherto known. The results of this study also agreed with the findings of researchers such as El-Khatib who used numerical techniques to show that in stratified reservoirs undergoing water flood, oil recovery under communicating and non-communicating layers are different.

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Vol.4, No.1, pp.1-12, Februarys 2020
Published by ***ECRTD-UK***
Print ISSN: ISSN 2514-9253
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APPENDIX

TABLE A: PERMEABILITY VARIATION DATA

Case 1						
Interval	Thickness (ft)	Well A	Well B	Well C	Well D	Well E
6783-6784	1	273	228	161	187	282
6784-6746	2	43	88	148	47	109
6786-6787	1	276	349	454	295	308
6787-6790	3	87	49	77	49	88
6790-6792	2	127	159	262	62	178
Case 2						
6783-6784	1	47	77	62	109	50
6784-6787	3	187	109	228	273	148
6787-6789	2	35	47	71	88	127
6789-6790	1	476	454	252	308	349
6790-6791	1	282	161	178	148	228
6791-6793	2	62	15	58	77	50
Case 3						
6783-6784	1	47	77	62	109	148
6784-6787	3	187	109	228	273	127
6787-6789	2	35	454	252	88	349
6789-6790	1	243	161	178	278	228
6790-6791	1	282	62	58	148	77

Table B: Reservoir Data

Reservoir Data	**Case 1**	**Case 2**	**Case 3**
Average Porosity	18	25	25
Average Connate Water	30	20	20
Recovery by Primary Depletion	140.5	140.5	140.5
B_{oi} (bbl/STB) (no initial gas-cap)	1.3	1.15	1.5
B_o (bbl/STB) @ depletion (beginning of flood)	1.3	1.15	1.5
B_w (rb/STB)	1.0	1.0	1.0
μ_o (cp)	4.50	1.39	6.50
μ_w (cp)	0.79	0.50	0.90
S_{or} (%) (after flooding)	20	25	15
i_w (RB/D)	2000	1000	1000
E_A (%)	100	100	100
Area (acres)	80	60	80

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Print ISSN: ISSN 2514-9253
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Table C: Relative Permeability Variation Data

Case 1			**Case 2**			**Case 3**		
S_w	k_{ro}	k_{rw}	S_w	k_{ro}	k_{rw}	S_w	k_{ro}	k_{rw}
0.3	0.94	0	0.3	0.94	0	0.3	0.61	0.009
0.4	08	0.04	0.4	08	0.04	0.4	0.37	0.029
0.5	0.44	0.11	0.5	0.44	0.11	0.5	0.22	0.064
0.6	0.16	0.2	0.6	0.16	0.2	0.6	0.12	0.117
0.7	0.045	0.3	0.7	0.045	0.3	0.7	0.05	0.19
0.8	0	0.44	0.75	0	0.36	0.8	0.01	0.247