International Research Journal of Pure and Applied Physics
Vol.6, No.2, pp.22-33, December 2019
Published by ECRTD-UK
ISSN 2055-009X(Print), ISSN 2055-0103(Online)

A MICROSCOPIC STUDY OF NUCLEAR SYMMETRY ENERGY WITH AN EFFECTIVE INTERACTION DERIVED FROM VARIATIONAL CALCULATIONS

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ABSTRACT: *In this paper, a microscopic calculation of nuclear symmetry energy has been carried out with a new M3Y-type effective interaction derived from variational calculations. The new effective interaction, called B3Y-Fetal in this work, has been used in its DDM3Y1, BDM3Y0 and BDM3Y1 density dependent versions within the framework of Hartree-Fock approximation to obtain the values of 30.5, 30.6 and 30.7 MeV respectively for symmetry energy at the nuclear matter saturation density,* $\rho_0 = 0.17$ fm^{-3}. *When compared with an empirical symmetry energy of 31.6 ± 2.7 MeV established based on several analyses of terrestrial Nuclear Physics experiments and astronomical observations, these values of symmetry energy have been found to be in excellent agreement. The curves of symmetry energy obtained with the three density-dependent versions of the B3Y-Fetal effective interaction at different proton-neutron asymmetries in this work have also been found to demonstrate good agreement with previous work done with M3Y-Reid and M3Y-Paris effective interactions*

KEYWORDS: microscopic study, nuclear, symmetry energy, variational calculations

INTRODUCTION

In the study of asymmetric nuclear matter, the nuclear symmetry energy (NSE) is one of the most important quantities to consider. It is of crucial importance in testing a microscopic model for asymmetric nuclear matter, particularly the isospin dependence of the effective nucleon-nucleon interaction involved [7, 15]. The NSE plays a key role in the understanding of the structure of such systems as exotic nuclei, supernovae and neutron stars; and it enters as an input to heavy-ion reactions. The density dependence of NSE is a very crucial ingredient of the nuclear equation of state (EOS) needed to understand not only the important properties of isospin-rich nuclear systems such as radioactive nuclei, the reaction dynamics induced by rare isotopes and liquid-gas phase transition in asymmetric nuclear matter, but also to determine neutrino emission probability, cooling rate of proto-neutron stars which can be measured from X-ray bursts, mass and density profile of neutron stars as well as the equilibrium proton fraction [26, 27]. Experimental information on NSE below, close to and above nuclear saturation density, ρ_0 can be obtained from a number of nuclear structure and heavy-ion observables such as giant resonances, isospin diffusion measurement, isobaric analog states or meson production (pions and kaons) in heavy-ion collisions, neutron/proton differential flow [26].

Studies carried out with a number of effective interactions have shown NSE to be strongly model dependent, most especially in the high-density region. Theoretically, microscopic many-body calculations and phenomenological approaches predict various forms of the density dependence of the symmetry energy. Two different forms have been generally identified [2, 24]. In one form, which is called "stiff" dependence, the symmetry energy increases monotonically with increasing density while the other form called "soft" dependence is such that the symmetry energy increases initially up to normal nuclear matter density and then decreases at higher densities [2, 24]. The determination of the exact form of the density dependence of the NSE is important for studying the structure of neutron-rich nuclei as well as for studies relating to astrophysical phenomena such as the structure of neutron stars and dynamics of supernova collapse [5, 24, 26]. Based on past studies, a "stiff" dependence of the NSE has been predicted lead to a large neutron skin, a large neutron star radius and rapid cooling of a neutron star compared with a 'soft' density dependence.

Amongst a number of theoretical models, the NL3 relativistic model and Skyrme forces ($SLy_{230a)}$ have shown NSE to be an increasing function of density (stiff dependence) [27], whereas Gogny force have predicted a 'soft' dependence [24]. Similarly, the density dependent M3Y-Reid and M3Y-Paris interactions have predicted a 'soft' density dependence [16, 24, 25]. In the course of this work, we apply a new M3Y-type effective interaction derived from variational calculations to the study of NSE. The new effective interaction called B3Y-Fetal [19, 21] is used in its DDM3Yn and BDM3Yn density-dependent versions for this purpose. The density dependence of NSE predicted by the density-dependent B3Y-Fetal is to be determined and analyzed in comparison with the above-mentioned models. Specifically, we have, in this paper, M3Y-Reid and M3Y-Paris effective interactions as standards to compare the performance of the B3Y-Fetal with. With this goal in mind, we find it necessary to organize this paper into the following sections. Section 2.0 gives an overview of the functional form of the density-dependent B3Y-Fetal effective interaction along with the functional forms of the M3Y-Reid and M3Y-Paris effective interactions. Section 3.0 provides a succinct explanation of the theoretical layout of the nuclear symmetry energy, whereas Section 4.0 presents and discusses the results of the study.

The Variational Effective Interaction

In this work, we apply our variational effective interaction, the B3Y-Fetal interaction, whose matrix elements were calculated in a harmonic oscillator basis using the lowest-order constrained variational (LOCV) method, to the study of nuclear symmetry energy. The details of the calculation of the matrix elements were reported in [9, 10]. The direct (v_D) and exchange (v_{EX}) components of the central part of the M3Y-type nucleon-nucleon (NN) effective interaction, in terms of spin $\boldsymbol{\sigma}, \boldsymbol{\sigma}'$ and isospin $\boldsymbol{\tau}, \boldsymbol{\tau}'$ of the nucleons, are expressed as [16]:

$$v^{D(EX)}(r) = v_{00}^{D(EX)}(r)+v_{10}^{D(EX)}(r)(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')+v_{01}^{D(EX)}(r)(\boldsymbol{\tau}\cdot\boldsymbol{\tau}')+ v_{11}^{D(EX)}(r)(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')(\boldsymbol{\tau}\cdot\boldsymbol{\tau}') \quad (1)$$

where r is the inter-nucleon distance and ρ is the nuclear density around the interacting nucleon pair, $\boldsymbol{\sigma}, \boldsymbol{\sigma}'$ are the spins and $\boldsymbol{\tau}, \boldsymbol{\tau}'$ are the isospins of two nucleons participating in the interaction. The study of NSE, which is a property of ANM, involves the use of the isoscalar and isovector components of the M3Y-type effective interaction. Accordingly, the radial strengths (in MeV) of the isoscalar and isovector components of the central part of B3Y-Fetal NN interaction are given in terms of three Yukawas respectively as:
B3Y-Fetal [10]:

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$$v^{D}_{00}(r) = \frac{10472.13e^{-4r}}{4r} - \frac{2203.11e^{-25r}}{2.5r}$$

$$v^{EX}_{00}(r) = \frac{499.63e^{-4r}}{4r} - \frac{1347.77e^{-25r}}{2.5r} - \frac{7.847477e^{-0.7072r}}{0.7072r}$$

$$v^{D}_{01}(r) = \frac{-6197.63e^{-4r}}{4r} + \frac{1277.38e^{-25r}}{2.5r}$$

$$v^{EX}_{01}(r) = \frac{365.38e^{-4r}}{4r} + \frac{576.99e^{-25r}}{2.5r} + \frac{2.6157e^{-0.7072r}}{0.7072r} \quad (2)$$

Since we are interested in using M3Y-Paris, M3Y-Reid NN effective interactions as standards with which to compare the performance of the B3Y-Fetal effective interaction, the radial strengths (in MeV) of the isoscalar and isovector components of the former are given in terms of three Yukawas respectively as: M3Y-Paris [1, 15]:

$$v^{D}_{00}(r) = \frac{11061.625e^{-4r}}{4r} - \frac{2537.5e^{-25r}}{2.5r}$$

$$v^{EX}_{00}(r) = \frac{-1524.25e^{-4r}}{4r} - \frac{518.75e^{-25r}}{2.5r} - \frac{7.8474e^{-0.7072r}}{0.7072r}$$

$$v^{D}_{01}(r) = \frac{313.625e^{-4r}}{4r} + \frac{223.5e^{-25r}}{2.5r}$$

$$v^{EX}_{01}(r) = \frac{-4118.0e^{-4r}}{4r} + \frac{1054.75e^{-25r}}{2.5r} + \frac{2.6157e^{-0.7072r}}{0.7072r} \quad (3)$$

M3Y-Reid [3, 15]:

$$v^{D}_{00}(r) = \frac{7999.00e^{-4r}}{4r} + \frac{2134.25e^{-25r}}{2.5r}$$

$$v^{EX}_{00}(r) = \frac{4631.375e^{-4r}}{4r} - \frac{1787.125e^{-25r}}{2.5r} - \frac{7.8474e^{-0.7072r}}{0.7072r}$$

$$v^{D}_{01}(r) = \frac{-4885.5e^{-4r}}{4r} + \frac{1175.5e^{-25r}}{2.5r}$$

$$v^{EX}_{01}(r) = \frac{-1517.875e^{-4r}}{4r} + \frac{828.375e^{-25r}}{2.5r} + \frac{2.6157e^{-0.7072r}}{0.7072r} \quad (4)$$

For correct description of the saturation properties of nuclear matter within the non-relativistic HF scheme, it has been shown [11, 13] that the introduction of a density dependence into the original M3Y interaction is the necessary and sufficient solution. Thus, with the density dependence introduced, the isoscalar and isovector density-dependent M3Y-type interactions, respectively, become:

$v_{00}^{D(EX)}(\rho, r) = F_0(\rho)v_{00}^{D(EX)}(r)$ (5)

and

$v_{01}^{D(EX)}(\rho, r) = F_1(\rho)v_{01}^{D(EX)}(r)$, (6)

where $F_0(\rho)$ and $F_1(\rho)$ are the isoscalar and isovector density dependent factors respectively. The explicit forms of these density dependences are [12, 14, 17]:

For the DDM3Yn interaction (n = 1),

$F_{0(1)}(\rho) = C_{0(1)}(1+\alpha e^{-\beta\rho})$ (7)

In the case of the BDM3Yn (n = 0, 1, 2, 3) interaction,

$F_{0(1)}(\rho) = C_{0(1)}(1+\alpha\rho^{\beta})$ (8)

The parameters C_0, α and β of the isocalar density dependences are such that they reproduce the saturation properties of nuclear matter at density $\rho_0 = 0.17fm^{-3}$ with a binding energy E/A = 16 MeV within HF calculations. It has to be stressed here that the isoscalar density dependence, $F_0(\rho)$ and isovector density dependence, $F_1(\rho)$ have practically the same form, but may possibly be different with respect to the scaling factors C_0 and C_1, because C_1 is, if necessary, adjusted to reproduce the empirical symmetry energy so as to construct a realistic equation of state (EOS) for asymmetric nuclear matter (ANM) within HF scheme.

We have chosen to use the DDM3Yn and BDM3Yn interactions in this work following the approach of Khoa and co-workers [12, 13, 15, 17]; and the DDM3Y1, BDM3Y0 and BDM3Y1 versions are specifically used for calculations. In our earlier work [21], we applied the B3Y-Fetal effective interaction, in its DDM3Y1, BDM3Y0, BDM3Y1, BDM3Y2 and BDM3Y3 density-dependent versions, to the study of the symmetric nuclear matter and found it to reproduce the saturation properties of this nuclear matter well. Doing this, we made use of the isoscalar component of the B3Y-Fetal effective interaction and obtained the parameters of density dependence and incompressibilities in Table 1.0 along with the performance plots in Figure 1.0.

Table 1: Parameters of Density Dependence and Nuclear Incompressibility at Equilibrium for B3Y-Fetal.

Density dependent version	C	Α	β	K[MeV]
DDB3Y-Fetal	0.2986	3.1757	2.9605	176
DDB3Y-Fetal	1.3045	1.0810	2/3	196
DDB3Y-Fetal	1.1603	1.4626	1	235
DDB3Y-Fetal	1.0160	4.9169	2	351
DDB3Y-Fetal	0.9680	20.250	3	467

Each density dependent version of the B3Y-Fetal interaction in Figure 1.0 has been found to reproduce the equation of state (EOS) of the symmetric nuclear matter acceptably when compared with the one obtained with the M3Y-Reid and M3Y-Paris effective interactions [19, 21]. This success is the strong basis for the application of the new effective interaction to the calculation of the nuclear symmetry energy, which is itself a test of the suitability of an effective interaction for the study of asymmetric nuclear matter, in this paper. However, for the study of NSE, the isovector component of the B3Y-Fetal effective interaction is used herein in conjunction with the isoscalar component.

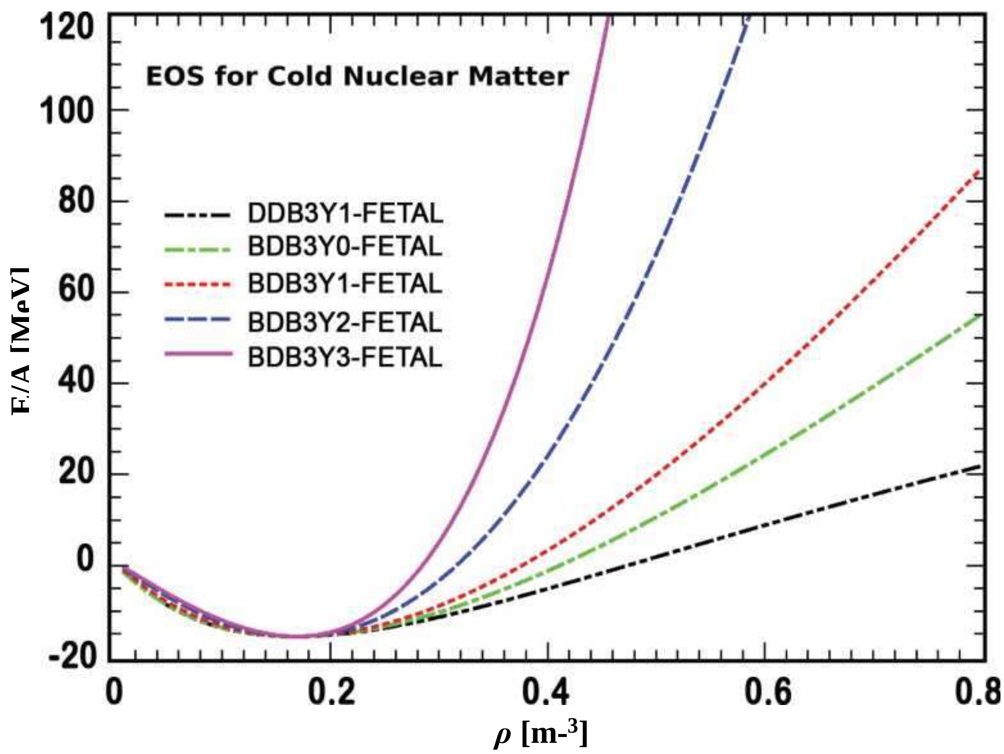

Figure 1: Equations of State of Cold NM Calculated with DDB3Y1- BDB3Y0-, BDB3Y1-, BDB3Y2- and BDB3Y3-Fetal Interactions.

The Nuclear Symmetry Energy

Many theoretical studies [27] have shown that the EOS of the ANM can be expressed as a power series of the form [23]:

$$\frac{E}{A}(\rho,\delta) = \frac{E}{A}(\rho,\delta = 0) + S(\rho)\delta^2 + O\delta^4 \ldots \quad (9)$$

where the isospin asymmetry $\delta = \frac{\rho n - \rho p}{\rho}$, $\rho = \rho_n + \rho_p$ is the total baryonic density with ρ_n and ρ_p denoting the neutron and proton densities respectively. The contribution of $O\delta^4$ and higher-order terms has been shown to be negligible [8, 15, 18]; so equation (9) becomes

$$\frac{E}{A}(\rho,\delta) = \frac{E}{A}(\rho,\delta = 0) + S(\rho)\delta^2 \qquad (10)$$

where $\frac{E}{A}(\rho,\ \delta)$ is the energy per nucleon at density ρ and asymmetry δ, $\frac{E}{A}$ $(\rho,\ \delta\ =\ 0)$ is the binding energy per nucleon of symmetric nuclear matter (SNM) and S(ρ) is the symmetry energy coefficient which is expressed as:

$$E_{sym} = S_{(\rho)}\delta^2 \ , \qquad (11)$$

with E_{sym} as the nuclear symmetry energy (NSE). Equation (10) shows that the dominant dependence of the energy per nucleon of asymmetric nuclear matter on isospin asymmetry is essentially quadratic.

Within the framework of HF mean-field calculation, the binding energy per nucleon of asymmetric nuclear matter is calculated by adding the isovector component of the M3Y interaction, which does not contribute to the symmetric nuclear matter, to the isoscalar component. Accordingly, with the spin dependent terms of equation (1) averaged out, it is [16, 23]:

$$\frac{E}{A}(\rho) = \frac{3\hbar^2 k_F^2[(1+\delta)^{\frac{3}{5}}\ (1-\delta)^{\frac{3}{5}}]}{20m} + F(\rho)\frac{\rho}{2}\Big[C_0 J_{00}^D + \delta^2 C_1 J_{01}^D + \frac{1}{4}\int [C_o v_{oo}^{EX} B_0^2 + C_1 v_{o1}^{EX} B_1^2] d^3r\Big], \qquad (12)$$

where $B_0(\delta, r) = (1+\delta)\, \mathbf{j}_1(k_{Fn}r) + (1+\delta)\, \mathbf{j}_1(k_{Fp}r)$
and $B_1(\delta, r) = (1+\delta)\, \mathbf{j}_1(k_{Fn}r) - (1+\delta)\, \mathbf{j}_1(k_{Fp}r)$

The first term of equation (12) is the kinetic energy, while $C_0 J_{00}^D = \int v_{00}^{D}(r)d^3r$ and $C_0 J_{01}^D = \int v_{01}^{D}(r)d^3r$. $\boldsymbol{j}_1(x)$ is defined in terms of the first-order spherical Bessel function as $\boldsymbol{j}_1(x) = 3j_1(x)/x$. k_{Fn}, k_{Fp} and k_F are the neutron, proton and total Fermi momenta given as $k_{Fn(p)} = (3\pi^2\rho_{n(p)})\frac{1}{3}$ and $k_F = (3\pi^2\rho/2)\frac{1}{3}$ respectively. From the knowledge of the energy per nucleon in equation (12), expressions for asymmetric NM pressure and incompressibility could be derived by taking the partial derivatives of equation (12).

Using equations (10) and (12), an expression for the symmetry energy coefficient can be written as

$$S(\rho) = \frac{3\hbar^2 k_F^2[(1-\delta)^{\frac{3}{5}}\ (1+\delta)^{\frac{3}{5}}-2]}{20m\delta^2} + F(\rho)\frac{\rho}{2}\Big[C_1 J_{01}^D + \frac{1}{\delta^2}\int (\frac{C_0}{4} v_{oo}^{EX} B_0^2 + \frac{C_1}{4} v_{o1}^{EX} B_1^2 - C_0 v_{oo}^{EX}[\boldsymbol{j}_{\mathbf{1}\,(k_{Fr})}]^2)\ d^3r\Big] \ , \qquad (13)$$

Now, with the contributions of higher-order terms neglected, one can, to a good approximation, estimate the symmetry energy from the two extreme cases of pure neutron matter and symmetric nuclear matter according to:

$$E_{sym} = \frac{E}{A}(\rho, 1) - \frac{E}{A}\ (\rho, 0) \qquad (14)$$

The symmetry energy, defined as the energy per nucleon required to change the symmetric nuclear matter to pure neutron matter (PNM), represents a penalty levied on the system as it departs from the symmetric limit of equal proton and neutron numbers [2]. This nuclear matter symmetry energy computed at the nuclear matter saturation density, $E_{sym} = S(\rho_0)$ with $\rho_0 = 0.17 fm^{-3}$ is well-known in the literature as the symmetry energy or symmetry coefficient. To determine its empirical value, the scaling factor, C_1, of the isovector component of the effective interaction is first determined in relation to the scaling factor, C_0, of the isoscalar component when the proton-neutron asymmetry, $\delta = 1.0$. In addition to the asymmetry parameter, $\delta = 1.0$, $\delta = 0.35$ and 0.70 are also employed for the study of the density dependence of NSE in this paper.
The behaviour of the symmetry energy around saturation can be characterized in terms of a few bulk parameters such as [11, 25, 27]:

$$E_{sym}(\rho) = S_0 + Lx + \frac{1}{2} K_{sym} x^2 + \ldots \quad (15)$$

where $x = \frac{\rho - \rho 0}{3\rho 0}$ is a parameter representing the deviations of density from its value at saturation, S_0 is the symmetry energy at saturation density, L and K_{sym} are the slope and curvature parameters of symmetry energy at ρ_0 expressed respectively as:

$$L = 3\rho_0 \frac{\delta E_{sym}(\rho)}{\delta \rho}\Big|_{\rho=\rho_0} \quad (16)$$

$$K_{sym} = 9\rho_0^2 \frac{\delta^2 E_{sym}(\rho)}{\delta \rho^2}\Big|_{\rho=\rho_0} \quad (17)$$

Within a given set of models, these coefficients are so strongly correlated that their determination is generally sufficient to visualize and constrain the density dependence of NSE, at least, in the density interval probed by the constraint [11]. It is, however, challenging to obtain empirical constraints from finite nuclei as various calculations of the NSE even at the sub-saturation densities are known to demonstrate rather large differences. But, heavy-ion collisions at intermediate energies are believed to provide constraints on the low-density behaviour of NSE because the degree of isospin diffusion in these reactions is affected by the stiffness of the NSE [2, 24]. The high-density behaviour is, however, largely unknown or undetermined due to non-availability of data on simultaneously measured masses and corresponding radii of neutron stars [2]. But there are indirect indications such as the neutron star cooling processes. Even though the behaviour of $E_{sym}(\rho)$ at densities above saturation is not well constrained and predictions of different models diverge, numerous many-body calculations and those from the empirical liquid-drop mass formula predict its value at saturation density to be 30 ± 2 MeV [16, 26]; but a direct experimental determination of the symmetry energy does not exist [10]. Similarly, Khoa and his co-researchers estimated the most realistic value of the symmetry energy to be about 31 ± 2 MeV from the analysis of experimental cross section data carried out with an optical potential based on the isospin dependent CDM3Y6 interaction in a charge exchange p(6He,6 Li∗) reaction [24, 25]. While the curvature parameter K_{sym} is poorly known, the values of symmetry energy and its slope parameter, L have been established based on analyses of the terrestrial Nuclear Physics experiments and astrophysical observations to be 31.6±2.7 MeV and L = 58±16 MeV, respectively, as shown in [8]. We have chosen in this work to explore the density dependence of NSE, using the density dependent B3Y-Fetal effective interaction in the hope to have a good insight into its performance strength to be compared with empirical standards. Consequently, the values of NSE and its slope parameter at saturation density are to be determined to give a quantitative picture of the viability and position of the B3Y-Fetal effective interaction in relation to M3Y-Reid and M3Y-Paris effective interactions in this study. The findings from this study will certainly determine the suitability of the B3Y-Fetal interaction for a successful and comprehensive study of asymmetric nuclear matter in our subsequent papers.

RESULTS AND DISCUSSION

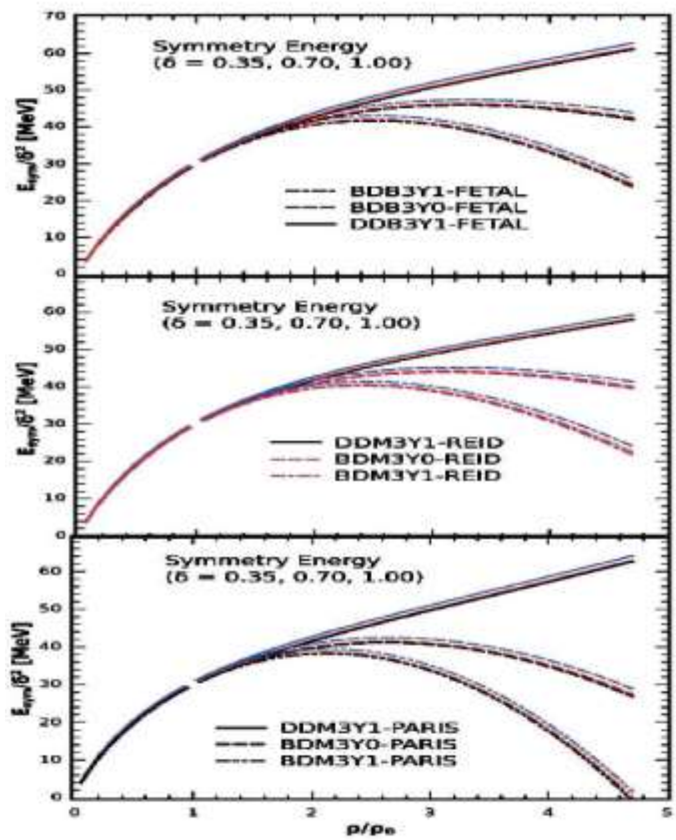

Figure 2: Symmetry Energy of Cold NM Calculated with Density Dependent M3Y-Paris, M3Y-Reid and B3Y-Fetal Interactions at the Asymmetry Parameters (δ) between 0 and 1.00. The solid rectangle is the empirical value of symmetry energy at $p \sim p_0$

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In this study, the isovector component of the density-dependent M3Y-type interaction was used in conjunction with the isoscalar component. The determination of NSE was first carried out with M3Y-Paris and M3Y-Reid effective interactions whose predictions of this nuclear matter property had been well determined previously [23]; the B3Y-Fetal effective interaction was then substituted for them in the same computational procedure with results that have demonstrated good agreement.

In the course of the HF calculation of nuclear symmetry energy, the M3Y-Paris forces have been observed to be more realistic as the empirical value of symmetry energy could be reproduced well without the need for further renormalization of the C_1 factor; thus, $C_1 = C_0$ for this effective interaction in this work. Accordingly, the values of symmetry energy obtained with the DDM3Y1, BDM3Y0 and BDM3Y1 versions of the M3Y-Paris at the saturation density are 31.3, 30.8 and 30.8 MeV respectively. On the contrary, the isovector components of the B3Y-Fetal and M3Y-Reid interactions were found to be too strong, so the C_1 factor had to be reduced by $C_1 = 0.62C_0$ in both cases to be able to obtain the correct value of symmetry energy (E_{sym}) at equilibrium. Based on this adjustment, the values of symmetry energy at the saturation density obtained with DDB3Y1-, BDB3Y0- and BDB3Y1-Fetal effective interactions are 30.5, 30.6 and 30.7 MeV respectively;

whereas 30.0, 30.6 and 30.6 MeV are the values of symmetry energy obtained with DDM3Y1-, BDM3Y0- and BDM3Y1-Reid effective interactions respectively. In addition, the values of the slope parameter, L obtained with DDM3Y1, BDM3Y0 and BDM3Y1 versions of the M3Y-Paris, M3Y-Reid and B3Y-Fetal effective interactions are 48.2, 48.2 and 48.2; 51.0, 51.0 and 51.0; and 53.0, 53.0 and 53.0 respectively. These results obtained in the present work tally well with such empirical standards as 31±2 MeV [24, 25], 30±2 MeV [16, 26] and 31.6±2.7 MeV [8] for symmetry energy; and L = 58±16 MeV for the slope parameter. Specifically, the B3Y-Fetal effective interaction appears to demonstrate excellent agreement with the M3Y-Reid and M3Y-Paris effective interactions. This is more clearly depicted in Figure 2.0.

The results of the calculated symmetry energy obtained with the DDM3Y1, BDM3Y0 and BDM3Y1 versions of the B3Y-Fetal, M3Y-Reid and M3Y-Paris effective interactions at different neutron-proton asymmetries (δ) are displayed in Figure 2.0. The first obvious thing is that the quadratic dependence of the symmetry energy upon the asymmetry parameter up to high nuclear matter densities is well illustrated in Figure 2.0 in which the curves of E_{sym}/δ^2 are portrayed to be almost independent of δ. This is known to demonstrate good agreement with relativistic Brueckner-Hartree-Fock (RBHF) calculations [16], confirming the empirical quadratic law up to the highest asymmetry parameter, $\delta = 1$. Also evident from Figure 2.0 is a steady increase of E_{sym} with increasing nuclear density, up to about $2\rho 0$; but at much higher densities, the E_{sym} obtained with the BDM3Y0- as well as BDM3Y1-based effective interaction reaches a maximum and decreases smoothly as density increases in a manner typical of all BDM3Y-type effective interactions. This form of density dependence of E_{sym} is called ‘soft’ dependence [2, 27]. The curves of E_{sym} representing the DDM3Y1-based effective interaction evidently demonstrate a continuous increase with increasing nuclear density, depicting a form of density dependence referred to as ‘stiff’ dependence [2, 27].

CONCLUSION

The goal of this work has been the microscopic study of the nuclear symmetry energy with a new M3Y-type effective interaction derived from variational calculations. The new effective interaction, called B3Y-Fetal interaction, has been used in its DDM3Y1, BDM3Y0 and BDM3Y1 density-dependent versions alongside the M3Y-Reid and M3Y-Paris effective interactions to study the density dependence and reproduce the values of symmetry energy at the saturation density, $\rho_0 = 0.17 fm^{-3}$, which have been found to fall agreeably within established empirical range. The empirical symmetry energies of 30.5, 30.7 and 30.7 obtained with the DDB3Y1-, BDB3Y0- and BDB3Y1-Fetal interactions, when compared with those obtained with their counterparts based on the M3Y-Reid and M3Y-Paris effective interactions, have been found to be in excellent agreement, validating the B3Y-Fetal as a viable tool for probing asymmetric nuclear matter.

Acknowledgements
Special thanks to Professor D. T. Khoa of INST, Vietnam and Professor W. M. Seif of University of Cairo, Egypt whose work on nuclear matter has provided helpful hints for the successful completion of this work.

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