International Journal of Mathematics and Statistics Studies
Vol.9, No.3, pp.31-48, 2021
Print ISSN: 2053-2229 (Print),
Online ISSN: 2053-2210 (Online)

A BI-CRITERIA OPTIMIZATION MODEL AND METHODOLOGY FOR RAIL CONSTRUCTION USING REAL DATA FROM GHANA

1. **Douglas Yenwon Kparib,**
Department of Mathematics, Statistics and Actual Science
Takoradi Technical University, Ghana.

2. **Stephen Boakye Twum, 3. Douglas Kwasi Boah**.
Department of Mathematics
Faculty of Mathematical Sciences
CK Tedam University of Technology and Applied Sciences
Post Office Box 24 – Navrongo, Ghana

ABSTRACT: *Rail construction to link major towns or cities is capital intensive, affecting directly or indirectly the tax-payer and owners of property and businesses situated along the path of the rail network. It is in this regard especially that actions that can mitigate or reduce the impact to victims and extraneous cost to the project become important. Two ways are to identify and use the shortest path linking the source and the destination and to factor into the project the cost of demolition and or displacement of property, persons, and businesses. In a previous work of the authors, a bi-criteria optimization model and methodology was developed for the problem in which both the distance and the costs of compensations referred to as Social Cost by the authors were minimized concurrently. Unlike the previous works however, the current paper is premised on testing the model and methodology on real data obtained in connection with a potential case of rail construction to link a town and a major City in Ghana. Furthermore, unlike in the previous works, a model is developed for the social cost criterion and the resulting bi-criteria optimization model run in Matlab using the real data. The results produced four non-dominated paths with the same distance and social cost values for decision-making. Therefore, there can be costs benefits in approaching rail construction in Ghana on the basis of the model.*

KEYWORDS: rail construction, shortest distance, social cost, model, bi-criteria optimization

INTRODUCTION

Rail transportation is one of the world's major, fastest, and efficient means of movement of both humans and goods from one location to another. The advent of rails is attributed to the Greeks who first constructed the stone rails in the sixth century and iron rails in 1768 (Jedwab & Moradi, 2012). In Ghana, rails were built by the British in 1898 (Jedwab & Moradi, 2012). Even though the 21st century world can boost of a very advanced and sophisticated rail transport system, this development is largely found in the developed world; the developing world of which Ghana is a part is, however, beginning to see some modest developments in provision of modern rail infrastructure to serve as a major means of transportation. In Ghana, there are intentions by the government to develop the rail infrastructure given the socio-economic advantages that it presents to the Nation and its people.

Rail construction however is a capital-intensive endeavor with consequent disruptions and cost to individuals, businesses and property caught along the pathway of the rail line or network. It is in this regard especially that this research work is conceived and important. Apart from the actual construction cost (which is outside the scope of this work), one may consider reducing the extraneous cost which has to do with compensations that has to be paid to displaced individuals, families and businesses. For instance, an amount of about Gh¢ 8.3 million (1 US Dollar: Gh¢ 5.4) was paid to people whose property were affected during the construction of the Awoshie-Pokuase road in Accra (ADB - African Development Fund, 2009). Also, the Ghana Gas Company paid to compensation to displaced individuals during the Atuobo Gas Project in the Western Region of Ghana (Morrison, 2015, TV3 News). The compensations which are paid to the people whose property are demolished, and any other monetary cost that is incurred during construction of rails outside the actual cost are referred to as Social Cost in this work. Additionally, linking the source and destination along their shortest path (which for two points, is the straight-line joining them) can be a huge cost saving to the rail construction project.

The problem described above belongs to the class of multi-objective shortest path problems (Caramia and Dell'Olmo, 2008), which unlike the classical shortest path problem (which finds the single-objective shortest path from a source to a destination in a network), requires the simultaneous minimization of two or more objective functions in a network. This is more realistic (though not simple) since it affords the management of competing interests and different points of view and therefore, provides solutions which yield higher levels of equity (Caramia and Dell'Olmo, 2008). In a previous work (see Kparib et al, 2019) of the authors, the problem was viewed as a bi-objective optimization and a model and methodology developed for the purpose. The Shortest Path and Social Cost criteria were optimized simultaneously using hypothetical problems and data. The problems were run using an improved Ant Colony algorithm of the authors (see Kparib et al, 2018). The current work applies the model and methodology to real data collected in Ghana from various state agencies and institutions in a considered case of a potential rail project to link the city of Kumasi to the town of Nyinahin, both in the Ashanti region of Ghana. The Social Cost criterion, this time round, was modeled and used on the real data to estimate the Social Cost values along the various paths of the network of towns connecting Kumasi and Nyinahin.

A number of cost estimation models are available in the literature. For instance, Montana Department of Transportation (2006) in designing a cost estimate model for the department and contractors, suggested nine procedures that can be used (funding splits, estimate form, quantities, unit prices, inflation factors, quantity descriptions, lump-sum items, contingencies and construction engineering, and approval and distribution). Sodikov (2005) developed a cost estimation model for highways projects for developing countries. A study by Özgen (2010) on comparative regression analysis, neural networks and case-based reasoning for early range cost estimation of mass housing projects classified the various cost estimation models into unit cost, factor, probabilistic modelling and simulation, and parametric estimation. Schmiedel et al (2012) researched on social cost and private cost of different payment instruments with the participation of thirteen (13) European national central banks. Cook and Ludwig (2006) did a study on the effect of household gun prevalence on homicide rates and inferred the marginal external cost of handgun ownership (IAWG, 2010). Technical Support Document on Social Cost of Carbon for Regulatory Impact Analysis defined social cost of carbon as an estimate of the monetary value for damages associated with an increase in carbon emission in a given year. Therefore, in the field of economics, social cost is made-up of the private costs and any other costs (external costs) that are incurred in the production of goods which are not catered for by the free market (DR ECON, 2002). Private costs then, become the direct cost that is incurred in the production of a commodity. For instance, cost of labor, cost of land and cost of raw materials. On the other hand, external cost has no direct link to the production of a commodity; for instance, cost that is incurred as a result of (the activities of the firm) pollution from a factory. Johnstone and Lonergan (2006) developed the Depreciated Optimized Replacement Cost (DORCt) model for assets valuation, employed by the Australian Competition and Consumer Commission (ACCC) and other Australian regulators. This particular model by Johnstone and Lonergan (2006) with it assets valuation feature provides basis for its incorporation into the development of the Social Cost model in this work.

In the next section, reformulations of both the bi-criteria optimization model (as in Kparib et al, 2019) and the Social Cost model are presented. Subsequently, an overview of the Ant Colony algorithm is presented. Application of the model to real data follows next together with discussion of the results. The paper is concluded with recommendations in the final section.

MODELS FORMULATIONS

Bi-Criteria Optimization Model

The optimization model as in Kparib et al (2019) is reformulated here together with the model for social cost.

Let $G(V, E)$ be an undirected graph consisting of an indexed set of nodes V with $n = |V|$ and a spanning set of edges (arcs) E with $m = |E|$. Let each arc be denoted by (i, j) and be associated with two numbers d_{ij} and A_{ij} which respectively denote the distance and social cost values along an arc. Suppose that it is conceived to link the source node $s \in V$ and the destination node $t \in V$ using a number of intervening nodes (i, j) and edges such that the total distance and aggregate social cost values are minimum. This is a bi-objective optimization problem and modelled as:

$$min\left[\sum_{(i,j)\in E} d_{ij}x_{ij}\,,\ \sum_{(i,j)\in E} A_{ij}x_{ij}\ \right]$$

$$\text{Subject to } x_{ij} \in \{0, 1\},\ \forall(i, j) \in E,\ i = 1, 2, \dots, \text{n} - 1,\ j = 1,2, \dots, n. \qquad (1)$$

where x_{ij} are the decision variables. The model in (1) describes a Multi-objective Optimization Integer Problem (MOIP) which has linear objective functions and prescribes integrality constraints on the variables x_{ij}. As a MOIP, a variety of techniques and methods are available for solving the problem. The scalar notion of optimality becomes untenable in this case; rather Pareto optimality becomes an appropriate optimality concept. A decision vector $\bar{x} \in X$ is Pareto optimal if $f_i(\bar{x}) \leq f_i(x)\ \forall\ i = 1,2, \dots, n$ and $f_j(\bar{x}) < f_j(x)$ for some $j \in [1,2, \dots, n]$, where f_i is the i^{th} objective function and is the feasible set . Generally, there is no unique minimum to (1), but rather a set of equally good solutions (Miettinen, 1998; Ehrgott, 2005). In Kparib et al (2019), the weighted sum scalarization technique which converts the vector optimization problem as in (1) into a scalar one was used under an improved Ant Colony algorithm to generate Pareto optimal solutions. Subsequently, a post-optimality ratio min-max strategy (see Kparib et al, 2019) was employed as an aid in the process of identifying a best compromise solution for the decision maker.

Social Cost Model

The Johnstone and Lonergan (2006) model for assets valuation seeks to obtain an estimate of the value of an asset and this fits well with the problem of compensation. It is, therefore, incorporated into the model development with slight modifications. The Johnstone and Lonergan (2006) model for assets valuation at time t, C_t, is defined as:

$$C_t = C_0(1+k)^t \left\{\frac{T-t}{T}\right\} \tag{2}$$

where k is the inflation rate, T is the asset expiry date, t is the age of the asset/facility and C_0 is the nominal opening asset value. Clearly, the younger the age of an asset the bigger the asset's value. This means in effect that the affected individual or group (with old asset ages) could be paid less compensation (or even no compensation) than the actual value of the asset in most practical situations. To avoid this in the model, the Johnstone and Lonergan (2006) model is modified to exclude the asset expiry date as follows:

$$C_t = C_0(1+k)^t \tag{3}$$

where tis time. Even so, the model (3) alone would not be sufficient (or fair) for estimating the compensation to be paid for a property affected in a rail lines construction taking the face value of the asset alone. Other factors need to be considered as well. It is the belief of this study that factors such as: inconvenience allowances, compensation for those who earn their living directly or indirectly from the asset, and moving expenses need to be taken into account as well. Therefore, the proposed mathematical model for quantifying Social Cost along each (i, j) in a network is formulated as:

$$C_{r_{ij}} = C^0_{r_{ij}}(1+k)^{t_{r_{ij}}} + I^c_{r_{ij}} + M_{r_{ij}} + T^c_{r_{ij}} \tag{4}$$

where r represents the r^{th} facility, k is the interest rate, while $C_{r_{ij}}$, $C^0_{r_{ij}}$, $t_{r_{ij}}$, $I^c_{r_{ij}}$, $M_{r_{ij}}$ and $T^c_{r_{ij}}$ are the actual due compensation, the present value, the time needed to replace the r^{th} facility, the inconvenience allowance, compensation for businesses, and moving expenses of persons in the r^{th} facility, respectively along $(i,\ j)$. The inconvenience allowance is modeled as:

$$I^c_{r_{ij}} = I_{r_{ij}}(1+K)^{T-\tau} + E_{r_{ij}} + R_{r_{ij}}\ , \qquad I_{r_{ij}} \geq 0,\ R_{r_{ij}} \geq 0,\ \tau \leq T,\ (T-\tau) \leq \delta \tag{5}$$

where $I_{r_{ij}}$ is the annual transportation cost to a person from a new location to the place of work displaced from the r^{th} facility along $(i,\ j)$, K is the average annual percentage increase in transport fares, T is the compulsory retirement age, τ is the present age of a person, $E_{r_{ij}}$is the cost incurred in transferring a displaced person's children of the r^{th} facility along $(i,\ j)$ to a new school, and $R_{r_{ij}}$ is the cost of a temporal lodging place for a person displaced in the r^{th} facility along $(i,\ j)\ and\ \delta$ is a fixed constant. The model for compensation for businesses is formulated as:

$$M_{r_{ij}} = \sum_{s=1}^{p} h_{sr_{ij}} B_{sr_{ij}} \tag{6}$$

where $B_{sr_{ij}}$ is the monthly average income earned by the s^{th} business type in the r^{th} facility along (i,j), $h_{sr_{ij}}\left(0 < h_{sr_{ij}} \leq h_{s_{r_{ij}max}}\right)$ is the period that the s^{th} business type in the r^{th} facility will not be in operation due to the displacement, $h_{s_{r_{ij}max}}$ is an upper bound on $h_{sr_{ij}}$ and p is the number of businesses.The model for moving expenses is formulated as:

$$T^c_{r_{ij}} = Q_{r_{ij}} + F_{r_{ij}} + D_{r_{ij}} \tag{7}$$

where $Q_{r_{ij}}$ is the total cost of transportation of goods in the r^{th} facility to the new place, $F_{r_{ij}}$ is the total lorry fares for persons in the r^{th} facility, and $D_{r_{ij}}$ is the total cost of damaged property in the r^{th} facility. It follows that for m edges $(i,\ j)$ of a network, the Social Cost function is given by:

$$A_{ij} = \sum_{r=1}^{m} \mu_{r_{ij}} C_{r_{ij}} \quad \forall\ i,j \tag{8}$$

where $\mu_{r_{ij}}$ is the number of facilities along each $(i,\ j)$.

ANT COLONY SYSTEM AND IMPROVEMENTS

This section reviews briefly the Ant Colony System algorithm and draw attention to the specific modifications made by the authors towards its improvement. The choice of this algorithm for the bi-objective optimization problem given in (1) is premised on the fact that it was originally designed for network problems and has demonstrated its flexibility and ability to obtain very good solutions for large and combinatorial optimization problems (Yacin and Erginel, 2011; Janina, 2005) of which the current problem is a type.

The Ant Colony System Algorithm

Ant Colony Optimization (ACO) is a meta-heuristic algorithm inspired by the socialist lifestyle exhibited by ants in search of food and other provisions (Dhiman & Hooda, 2014; Deneubourg, et al, 1990). Individual ants are considered un-intelligent and are practically blind and crappy in sight (Tewani, 2017). However, with their social structure, they are able to determine the shortest path to a food source after discovering it (Pradhananga et al, 2009). Ants usually move randomly in search of food and drop a chemical called pheromone in their trail anytime they come across one, to attract other ants to the location (Gupta et al, 2012). Therefore, the ants that move along the shortest path accumulate more amount of pheromone per unit length (Wong et al., 2012). The amount of pheromone that an ant deposits is inversely proportional to the distance travelled. That is the higher the density of pheromone the shorter the path (Zaninudin & Paputungan, 2013). There are three main types of ACO algorithms: (i) Ant System (AS) by Dorigo (1992); (ii) Max-Min Ant System (MMAS) by Hoos and Stutzle (1996); and (iii) Ant Colony System (ACS) by Dorigo and Gambardella (1997). The three algorithms differ by the way they update the pheromone trail values. The improvements to the ACO algorithm by the authors (Kparib et al, 2018) are in connection with the ACS.

The ACS is characterized by seven main steps: (i) setting parameters, (ii) initializing pheromone trails, (iii) calculating the heuristic Information, (iv) building the ant solution using a stochastic state transition rule, (v) carrying out local pheromone update. (vi) applying local search to improve solution constructed by an ant, (vii) updating the global pheromone information. An initial pheromone plays a vital role in the initial construction of solutions by ants. It is deposited along a route (i, j) at the beginning of the search and is usually a small positive constant which is defined (see Oto et al, 2014) as:

$$\tau_{ij}(0) = \frac{1}{n} \tag{9}$$

where n is the number of nodes. The heuristic information expresses how attractive a route is; it assists the ants in the construction of a shortest path (Mavrovouniotis, 2013). In a single objective problem, it is usually defined as a reciprocal of the distance between any two nodes. Lopez-Ibanez and Stutzle (2011) solved a bi-objective problem using the linear combination approach to combine the heuristics information as:

$$\eta_{ij} = w_1\eta_{ij}^0 + w_2\eta_{ij}^1 \tag{10}$$

where η_{ij}^0 and η_{ij}^1 are the respective heuristics information of the two objectives. $w_1$and w_2 are the weights assigned to the objectives. The heuristic information are always calculated based on the current node (i) that the ant is at to the next node (j). However, the approach does not provide the ant with information on the nature of the path from source to the current node i.

The pheromone along a trail is updated after all the ants have constructed their solutions. Three considerations in pheromone update are: the amount of pheromone to be laid, the amount to be evaporated, and the ants that are permitted to lay their pheromone (Nada, 2009). Pheromone updates are in two categories: local pheromone update and global pheromone update. The local pheromone updating rule is carried out on the edges whilst constructing the solutions, in order to reduce the quantity of pheromone on them (Nada, 2009). Since its introduction by Dorigo and Gambardella (1997), a number of methods for calculating the local pheromone update have been proposed; that due to Chen and Ting (2009) is the best, and given by the formula:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \rho\tau_{ij}(0)\,, \quad if\ \{\,(i,\,j) \in R_k \subset E\} \tag{11}$$

where ρ is the rate of evaporation of the pheromone along (i, j) and R_k is the set of edges not yet visited by the ants. The formula (11) is not, however, diversifying enough. The global update is performed by all the ants that have completed their schedule. The update is intended to increase the content of pheromone on a path to facilitate convergence of the algorithm. Yu et al (2009) proposed a formula for calculating the incremental pheromone trail update as:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \frac{1}{\rho}\sum_{k=1}^{m} \Delta\tau_{ij}{}^{k} \tag{12}$$

where $\Delta\tau_{ij}{}^{k} = \begin{cases} \frac{Q}{K\times L} \times \frac{D^k - d_{ij}}{m^k \times D^k}, & \text{if link } (i,\,j) \text{ is on the } k\text{th route} \\ 0 & \text{otherwise} \end{cases}$

and Q is a constant value, D^k is length of the k^{th} path (solution); L is the sum of the lengths of all the paths generated i.e. $\sum_k D^k$, k is the name of the path, m^k is the number of nodes in the k^{th} path $(m^k > 0)$, K is the number of paths generated $(K > 0)$.

In constructing a feasible solution, the ants have to decide to move from one node to the next based on a stochastic probability decision rule. The probabilistic decision rule by Majid and Mohammad (2011) is given as:

$$P_{ij}^{k}(t) = \begin{cases} 1 & if\ \{q \le q_0\ \ and\ j = j^* \\ 0 & if\ \{q \le q_0\ \ and\ j \ne j^* \\ \dfrac{\left(\tau_{ij}(t)\right)^{\alpha}\left(\eta_{ij}(t)\right)^{\beta}}{\sum_{r\in N_i^k}(\tau_{ir}(t))^{\alpha}(\eta_{ir}(t))^{\beta}} & otherwise \end{cases} \tag{13}$$

where j^* yields arg $max_{r\in N_i^k}\left(\left(\tau_{ij}(t)\right)^{\alpha}\left(\eta_{ij}(t)\right)^{\beta}\right)$ and it is used to identify the unvisited node in N_i that maximizes $P_{ij}^k(t)$ (Majid and Mohammad., 2011). $N_i^k \in N_i$ is the set of nodes which are neighbors of node i and have not yet been visited by ant k (nodes in N_i^k are obtained from those in N_i by the use of ant $k's$ private memory H_i^k (which stores nodes already visited by the ant)). N_i is the set of nodes which are directly linked to node i by an edge (i.e. the neighbours of node i); $\tau_{ij}(t)$ is the quantity of pheromone trail laid along the edge linking node i and node j; $\eta_{ij}(t)$ is the heuristic information for the ant visibility measure; α is the parameter to control the influence of τ_{ij}; β is the parameter to control the influence of η_{ij}; q_0 is the pre-defined parameter $(0 \le q_0 \le 1)$; q is the uniformly distributed random number to determine the relative importance of exploitation versus exploration, $q \in [0, 1]$.

Modified and Improved Ant Colony System Algorithm

The modifications made to the ACS algorithm by the authors (see Kaprib et al, 2018) fall under initial pheromone trail, heuristic information, local pheromone update and global pheromone update (Otoo et al, 2014; Lopez-Ibanez and Stutzle, 2010). The initial pheromone trail plays a vital role in the initial stage of the construction of solutions by ants. However, the uniform values used as initial pheromone trails as in Otoo et al (2014) and Blum (2005) do not fairly discriminate among the routes for the ants to make an informed choice, resulting in many ants generating solutions that are far from the best solution. Therefore, the initial pheromone trail is reformulated as:

$$\tau_{ij}(0) = \frac{1}{(n-1)\bar{L}_{ij}} \tag{14}$$

where n is the number of nodes (i.e. the size of the problem) and $\bar{L}_{ij}$ is the ratio of the of the weighted sum of the distance and Social Cost along (i, j) to the overall distance and Social Cost.

The heuristic information tells how attractive a route is and assists the ants in the construction of a shortest path. The heuristic information is typically calculated on the basis of the current node, i, where an ant is and the next node, j. However, the approach does not provide the ant with information on the nature of the path from the source to the current node i. It is to address this short-coming that the formula (15) modifying the heuristic information as in (10) is construed. In this case, the heuristic information is not dependent only on the distance from i to j but also on Social Cost from i to j and from the source to i for both. This is given as:

$$\eta_{ij} = w_1{\eta_{0j}}^1 + w_2{\eta_{0j}}^2 = w_1\frac{1}{(d_{0i}+d_{ij})} + w_2\frac{1}{(A_{0i}+A_{ij})}, w_1, w_2 \in w,\ 0 \le w \le 1 \tag{15}$$

where d_{0i} and A_{0i} are respectively distance and Social Cost values from the source node to the present node i; w_1 and w_2 weighting factors.

The aim of the local pheromone update is to reduce the content of the pheromone along the routes to encourage other ants to generate new paths. However, care has to be taken so that paths which are far from the optimal are not created. Therefore, in the modified approach, the reduction in the pheromone levels during the construction of paths is done in such a way that smaller amounts are taken from shorter routes compared to those taken from longer routes. This departs from the convention (Chen and Ting, 2009) where a constant (uniform) amount is deducted along all the routes. The modified local pheromone trail update formula is, therefore, given as:

$$\tau_{ij}(t+1) = (1-\rho)\frac{\tau_{ij}(t)}{L_{ij}} + \rho\tau_{ij}(0) \tag{16}$$

where $L_{ij} = \sum \tau_{ij}(t)$, and ρ is rate of evaporation of the pheromone. The global pheromone trail update (as in (12)) is modified as:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \frac{1}{\rho}\sum_{k=1}^{m}\Delta\tau_{ij}^{k} \tag{17}$$

where $\Delta\tau_{ij}^{k} = \begin{cases} \frac{1}{\bar{L}}.\frac{L^k - L_{ij}}{L^k}, & if(i,j) \in U_{ij} or\ I_{ij} \\ 0, & otherwise \end{cases}$, U_{ij} and I_{ij} are respectively the global best solution and
the iteration best solution; L^k is the weighted sum of distance and Social Cost of the k^{th} solution generated by ant k; L_{ij} is the weighted sum of distance and Social Cost of the edge (i,j); $\bar{L}$ is the ratio of weighted sum of distance and Social Cost of the solutions.

The probabilistic decision rule is the same as the already stated one proposed by Majid and Mohammad (2011) with the proposed initial pheromone trail, heuristic information, local pheromone trail update and global pheromone trail update embedded in it.

The Ant Colony System Optimization algorithm was coded using MATLAB R2015a to find the Pareto optimal paths. The parameters $\alpha = 1$, $\beta = 5$ and $\rho = 0.5$ (Dorigo, 1992), $q_0 = 0.9$ (Majid & Mohammad, 2011) and 100 ants (Blum, 2005) were used to arrive Pareto optimal solutions. The specifications of the computer used are: Intel(R)Core (TM)i3-2348M CPU(e)2.30GHz, RAM:6.00GB, System Type:64-bit operating system, X64-based processor.

REAL LIFE APPLICATION

Background and Data

Given the potentially high economic benefits to be derived in linking Kumasi and Nyinahin by train (they, together with their surrounding towns, are major producers of gold, bauxite, cocoa, and other food items in the country), the government of Ghana stands to reap huge economic benefits by constructing rail lines to link both towns of the Ashanti Region of Ghana in the for seeable future. Therefore, the authors chose these two towns as source and destination. Eight other major towns were selected, on the basis of their economic significance, to connect Kumasi and Nyinahin; they, together with Kumasi and Nyinahin, have been assigned numerical representations as follows: Kumasi (1), Nwineso (2), Nkawie (3), Ahewerewa (4), Mpatoam (5), Adiemmra (6), Kotokuom (7), Bakoniaba (8), Akantansu (9) and Nyinahin (10). The data on distances linking the ten towns, together with infrastructure that were likely to be affected in a potential rail lines construction were obtained from the Kumasi city's Town and Country Planning Department. Other data sets obtained from a number of private and state agencies were concerned with charges for goods and services, cost of loading of goods onto trucks and transportation fares, the value of each identified facility, the costs of plots of land, the types of businesses, the average monthly earnings and number of persons working in the businesses; the time needed to set up each type of business; and the average cost of plots of lands.

The facilities likely to be affected in rail construction along all routes in the network linking the ten (10) towns (see Figure 1) are categorized as block houses (Bh), mud houses (Mh), security structures (Se), primary schools (Sc), hospitals (Ho), stores in storey buildings (St), kiosks (Ki), cocoa farms (Fa), and a research centre (Rc). Table 1 provides the number of each category of facilities (from Bh to Rc) on each route (from 1, 2 to 9, 10) that are likely to be affected (demolished or displaced) in a rail lines construction project. Table 2 gives the corresponding number of persons to be affected, a facility type and route combination. Table 3 provides the estimated cost (in Ghana Cedis) and time for replacing a facility.

Table 1: Routes and the Estimated Number of Facilities that would be affected

	1,2	**1,3**	**1,7**	**2,3**	**2,4**	**3,4**	**3,6**	**3,7**	**4,5**	**4,6**	**4,7**	**5,6**	**5,10**	**6,8**	**6,9**	**6,10**	**7,6**	**7,9**	**8,9**	**8,10**	**9,10**
Bh	72	105	99	28	9	6	40	25	8	16	5	20	47	16	12	32	7	8	8	42	30
Mh	20	15	25	45	52	31	22	38	45	45	50	35	22	32	47	26	59	51	44	34	21
Se	0	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sc	3	1	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ho	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
St	11	5	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ki	5	3	6	10	4	4	3	8	5	6	9	4	3	4	8	6	3	6	12	5	24
Rc	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 2: Estimated Number of Persons to be displaced in a Facility

	1,2	**1,3**	**1,7**	**2,3**	**2,4**	**3,4**	**3,6**	**3,7**	**4,5**	**4,6**	**4,7**	**5,6**	**5,10**	**6,8**	**6,9**	**6,10**	**7,6**	**7,9**	**8,9**	**8,10**	**9,10**
Bh	288	420	406	102	36	24	160	100	32	64	20	80	168	64	48	128	28	64	64	168	120
Mh	80	60	100	180	108	64	88	152	180	180	200	140	88	128	208	108	186	204	176	136	84
Se	0	3316	3316	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sc	120	30	150	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ho	0	340	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
St	550	250	700	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ki	15	9	18	30	12	16	9	24	15	18	27	12	9	12	24	18	9	18	12	15	24
Fa	1	2	2	1	1	1	2	3	1	1	2	1	3	1	1	1	2	3	1	1	2
Rc	0	302	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Print ISSN: 2053-2229 (Print),
Online ISSN: 2053-2210 (Online)

Table 3: Estimated Cost and Time to build the Various Facilities

Facility	Bh	Mh	Se	Sc	Ho	St	Ki	Fa	Rc
Cost (Ghs)	151	30	6,100	336	7,200	500	1.5	2	10,000
Time (years)	1	0.5	2	1	2	1.5	0.167	4	2

The data for arriving at estimates for inconvenience allowance, compensation for businesses, and moving expences as presented in Tables 4 (a and b), 5, and 6, were obtained following the collection and estimation of the relevant quantities for average annual transportation cost per person displaced, average cost to a person loding at a temporal place due to displacement, transportation costs, cost of transferring children of displaced persons to a new school etc.. Two main location assumptions were used, which are: those within the Kumasi area and those outside the Kumasi area.

Table 4a: Estimated Inconvenience Allowance in Kumasi area (in Thousand Ghana Cedis)

Age group	$I_{r_{ij}}$	K	T	τ	$E_{r_{ij}}$	$R_{r_{ij}}$	$I^c_{r_{ij}}$
18 – 30	1.578	10%	60	27	0.000	0.000	36.649
31 – 40	1.578	10%	60	35	3.600	1.020	22.417
41 – 50	1.578	10%	60	45	3.600	1.300	22.417
51 – 60	1.578	10%	60	56	1.200	2.300	7.410

Furthermore, workers ages in the facilities were grouped as 18 – 30, 31 – 40, 41 – 50, and 51 – 60 years and the mid-points of the age groups (t) used as the estimated age of a person in a facility in an age group. The retirement age (T) of a worker was taken as 60 years and average annual increase in transport fares (K) as 10%.

Table 4b: Estimated Inconvenience Allowance outside Kumasi Area (in Thousand Ghana Cedis)

Age group	$\boldsymbol{I_{r_{ij}}}$	$\boldsymbol{K}$	$\boldsymbol{T}$	$\boldsymbol{\tau}$	$\boldsymbol{E_{r_{ij}}}$	$\boldsymbol{R_{r_{ij}}}$	$\boldsymbol{I^c_{r_{ij}}}$
18 – 30	0.4734	10%	60	27	0.000	0	1.095
31 – 40	0.4734	10%	60	35	1.300	1.020	12.749
41 – 50	0.4734	10%	60	45	1.300	1.300	11.378
51 – 60	0.4734	10%	60	56	1.300	2.300	5.793

Table 5: Estimated Compensation for Business types (s) within a Facility (in Thousand Ghana Cedis)

s	$B_{s_{r_{ij}}}$	$h_{sr_{ij}}$	$h_{srij}B_{s_{r_{ij}}}$
1	8	5	40
2	6	3	18
3	7.5	5	37.5
			$\sum_{s=1}^{3} h_{sr_{ij}} B_{s_{r_{ij}}} = 95.5$

The transportation cost that people in the r^{th} facility would incur in moving to their new places (Table 6) were estimated on the basis of information offered by the Local Transport Union in Kumasi which were as follows: The charge of 1kilogram goods per km 0.10, the lorry fare per person per kilometer 0.30, and fee for loading and off-loading of goods per kilogram 0.10. An estimated weight of 550 kg of goods was used per adult. For those within Kumasi, a fare per kilometer was estimated as 30 Ghana Cedis and that of those outside Kumasi as 9 Ghana Cedis.

Table 6: Estimated Transportation Cost for Moving People and their Goods in r^{th} Facility to a New Place

	$Q_{r_{ij}}$	$F_{r_{ij}}$	$D_{r_{ij}}$	$T^{c}_{r_{ij}}$
In Kumasi	2.640	0.030	2.500	5.170
Outside Kumasi	0.385	0.009	1.200	1.594

RESULTS AND DISCUSSIONS

Table 7 gives the estimated actual cost of compensation for a facility which is to be demolished for construction of rail lines. The facilities are: Block house (Bh) (this was categorized as Bh_1 and Bh_2 respectively referring to block house in the city and block house in the village), Mud house (Mh), Security Zones (Se), School (Sc), Hospital (Ho), Storey Building Stores (St), Kiosk (Ki), Cocoa Farm (Fa), and a research centre (Rc). This takes into account transportation expenses, inconvenience allowance and those who do business within the facility.

Table 7: Estimated Actual Compensation Cost $\left(C_{r_{ij}}\right)$ for the r^{th} Facility

r	$C^0_{r_{ij}}$	k	$t_{r_{ij}}$	$M_{r_{ij}}$	$T^c_{r_{ij}}$	$I^c_{r_{ij}}$	$C_{r_{ij}}$
Bh_1	151	0.179	1	0	5.17	84.99	268.24
Bh_2	151	0.179	1	0	1.59	22.52	202.17
Mh	30	0.179	0.5	0	1.59	22.52	56.68
Se	6100	0.179	2	95.5	5.17	84.99	8686.99
Sc	336	0.179	1	95.5	5.17	0	495.79
Ho	7200	0.179	2	95.5	5.17	84.99	10199.05
St	500	0.179	1.5	0	5.17	0	645.50
Ki	1.5	0.179	0.167	0	5.17	0	6.71
Fa	2	0.179	4	0	1.59	0	5.45
Rc	10000	0.179	2	95.5	2.30	84.99	14093.15

Tables 8a and 8b summarize the estimated total compensations along the routes and therefore the social costs (in millions of Ghana Cedis) along the routes. The social cost along a route is obtained by multiply the estimated total cost of compensation for a facility by the number of that particular facility along the route (see Table 1) and summing up the resulting total cost of compensations of all the different facilities along the route.

Table 8a: Estimate of Compensations along the routes 1, 2 to 5, 7 (in million Ghana Cedi)

Route	1, 2	1, 3	1, 7	2, 3	2, 4	3, 7	3, 6	3, 4	4, 5	4, 7
$\boldsymbol{A_{ij}} = \sum_{r=1}^{m} \mu_{r_{ij}} C_{r_{ij}}$	23.00	74.32	57.17	6.64	3.44	6.00	8.00	2.10	3.01	2.68

Table 8b: Estimate of Compensation along the routes 4, 6 to 9, 10 (in million Ghana Cedi)

Route	4, 6	5, 6	5, 10	6, 10	6, 8	6, 9	7, 9	7, 6	8, 10	8, 9	9, 10
$\boldsymbol{A_{ij}} = \sum_{r=1}^{m} \mu_{r_{ij}} C_{r_{ij}}$	9.00	4.82	9.17	6.68	30.00	3.82	3.23	3.30	8.74	3.02	6.27

Figure 1 shows a network of the various towns from 1 to 10 with twenty-one edges. Each edge is associated with distance (d_{ij}) and social cost (A_{ij}) values. The aim is to find a route from town 1 to town 10 which minimizes both total distance and total social cost simultaneously. The results and discussions are presented next.

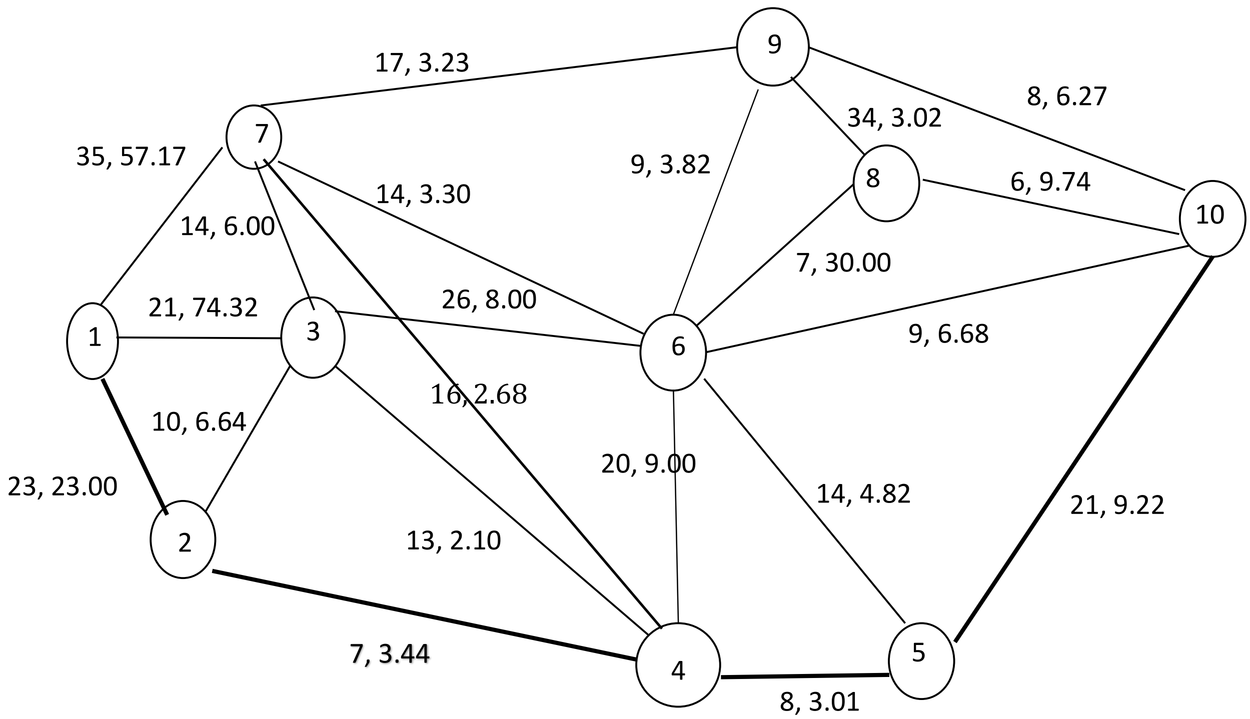

Figure 1: A network of the various towns with the distances and social cost values

The results of the optimization are presented in Table 9. The first four columns present the Pareto optimal paths or routes, the weight sets, and the corresponding objective functions values. The next two columns which are concerned with post-optimality assessment of the Pareto optimal objective functions values as described in Kparib et al (2019) show the Min-Max ratio and the Margin of Error values. The last two columns show the number of iterations required for convergence of both the existing (E) and the improved (I) ACS algorithm (as described in section 3). The last row gives the unique optimal values (i.e. the Ideal values) of the objective functions.

The first row of Table 9 shows that rail lines connecting the towns 1–2–4–7–9–10 will yield a minimum distance of 71 kilometers and a minimum Social Cost of 38.62 million Ghana Cedis. The min-max ratio values for distance and Social Cost are approximately 1.27 and 1.00 respectively, with margins of errors of 0.27 and 0.00 respectively. The margin of error values indicate that whereas the Pareto optimal distance value was approximately 27% short of the unique optimal value for distance, the Pareto optimal Social Cost value was coincident with the unique minimum value for Social Cost. The margin of error information is vital for decision making about which of the solutions is preferred, since it provides a measure of how close to the unique minimum value, or otherwise, a Pareto optimal objective function value is. Similar interpretations apply to the other rows of the table. The number of iterations required for convergence by the algorithms differed with the sets of weights, indicating that the optimization model was sensitive to the weights; their impact on the solution was not as striking, however. Furthermore, as observed also in Kparib et al (2018, 2019), the modified ACS algorithm converges faster than the existing one;

Vol.9, No.3, pp.31-48, 2021
Print ISSN: 2053-2229 (Print),
Online ISSN: 2053-2210 (Online)

the numbers of iterations required for its convergence to a Pareto optimal solution were fewer in all the iterations. Also, the paths P_2 to P_9, and P_{11}, P_{12}, P_{14}, P_{16}, P_{17} yield the same solution (i.e. route) of (1 – 2 – 4 – 5 –10) and the same objective function values (i.e. distance and Social Cost values) in spite of the varied weight sets . Similar comments apply to the set of paths $\{P_{13}, P_{15}, P_{18}\}$.

Table 9: Results for the Real-Life Problem

Path	Route	Weight Set	Min. Dist. & Social Cost	Min-Max Ratio values	Margin of Error	E	I
P_1	1 - 2 - 4- 7- 9-10	(0.05, 0.95)	(71, 38.67)	1.2679, 1.0000	0.2679, 0.0000	169	162
P_2	1 - 2 - 4 - 5 – 10	(0.10, 0.90)	(59, 38.67)	1.0536, 0.0000	0.0536, 0.0000	185	181
P_3	1 - 2 - 4 - 5 – 10	(0.15, 0.85)	(59, 38.67)	1.0536, 1.0000	0.0536, 0.0000	260	255
P_4	1 - 2 - 4 - 5 – 10	(0.20, 0.80)	(59, 38.67)	1.0536, 0.0000	0.0536, 0.0000	194	190
P_5	1 - 2 - 4 - 5 – 10	(0.25, 0.75)	(59, 38.67)	1.0536, 0.0000	0.0536, 0.0000	435	422
P_6	1 – 2 – 4 – 5 – 10	(0.30, 0.70)	(59, 38.67)	1.0536, 0.0000	0.0536, 0.0000	213	209
P_7	1 – 2 – 4 – 5 -10	(0.35, 0.65)	(59, 38.67)	1.0536, 1.0000	0.0536, 0.0000	192	180
P_8	1 – 2 – 4 – 5 – 10	(0.40, 0.60)	(59, 38.67)	1.0536, 1.0000	0.0536, 0.0000	333	320
P_9	1 – 2 – 4 – 5 – 10	(0.45, 0.65)	(59, 38.67)	1.0536, 1.0000	0.0536, 0.0000	210	201
P_{10}	1 - 2 - 4 - 6 – 10	(0.50, 0.50)	(59, 42.12)	1.1026, 1.0892	0.1026, 0.0892	250	240
P_{11}	1 – 2 – 4 – 5 – 10	(0.55, 0.45)	(59, 38.67)	1.0536, 1.0000	0.0536, 0.0000	253	242
P_{12}	1 – 2 – 4 – 5 – 10	(0.60, 0.40)	(59, 38.67)	1.0536, 1.0000	0.0536, 0.0000	265	256
P_{13}	1–2– 4 – 5–6– 10	(0.65, 0.35)	(61, 40.95)	1.0893, 1.0590	0.0893, 0.0590	154	150
P_{14}	1 - 2 - 4 - 5 – 10	(0.70, 0.30)	(59, 38.67)	1.0536, 1.0000	0.0536, 0.0000	350	330
P_{15}	1 – 2 – 4 – 6 – 10	(0.75, 0.25)	(59, 40.95)	1.0536, 1.1026	0.0536, 0.1026	203	258
P_{16}	1 – 2 – 4 – 5 – 10	(0.80, 0.20)	(59, 38.67)	1.0536, 1.0000	0.0536, 0.0000	160	152
P_{17}	1 – 2 – 4 – 5 – 10	(0.85, 0.15)	(59, 38.67)	1.0536, 1.0000	0.0536, 0.0000	430	420
P_{18}	1–2–4–5 – 6 – 10	(0.90, 0.10)	(61, 40.95)	1.0893, 1.0590	0.0893, 0.0590	200	192
P_{19}	1 – 7 - 9 – 10	(0.95, 0.05)	(60, 66.69)	1.0714, 1.6923	0.0714, 0.6923	148	146
Unique Distance value (56), Unique Social Cost Value (38.67)							

Table 10: Summary of the Pareto optimal solutions

Paths	**Routes**	**Weight Set**	**Min Dist. & Social Cost**	**Margin of Error**	**E**	**I**
P_1	1-2-4 –7–9 –10	(0.05, 0.95)	(71, 38.67)	(0.2679, 0.0000)	169	162
P_2 to P_9, P_{11}, P_{12}, P_{14}, P_{16}, P_{17}	1–2– 4 – 5 –10	(0.10, 0.90) to (0.45, 0.55), (0.55, 0.45), (0.60, 0.40), (0.70, 0.30), (0.80, 0.20), (0.85, 0.15)	(59, 38.67)	(0.0536, 0.0000)	185	181
P_{10}	1-2-4-6-10	(0.50, 0.50)	(59, 42.12)	(0.1026, 0.0892)	250	240
P_{13}, P_{15}, P_{18}	1- 2- 4- 5–6–10	(0.65,0.35), (0.75, 0.25), (0.90, 0.10)	(61, 40.95)	(0.0893, 0.0590)	200	182
P_{19}	1 – 7 – 9 – 10	(0.95, 0.05)	(60, 66.69)	(0.0714, 0.6923)	148	146

Table 10 summarizes the results in Table 9 and depicts five non-dominated solutions. The Pareto optimal objective functions value (71, 38.67) has a higher distance value than (59, 38.67), but the same social cost value. Thus, the point (59, 38.67) weakly dominates the point (71, 38.67) since it is not strictly better than (71, 38.67). Comparing (59, 42.12) with (71, 38.12) it is observed that (59, 42.12) is better than (71, 38.67) as far as distance is concerned, but (71, 38.67) is better than (59, 42.12) as far as Social Cost is concerned. Hence, the two points are non-dominated. Similar observations can be made with the other pairs of solutions. Any of the five Pareto optimal solutions can be adopted by a decision maker. Even so, on the basis of the information provided by the margins of errors, clearly the solution 1–2– 4 – 5 –10 with objective function values of (59, 38.67) yields the least margin of error of (0.0536, 0.0000). Therefore, a decision maker may select this solution for implementation; which means rail lines constructed to link the towns of Kumasi, Nwineso, Ahewerewa, Mpotoam, and Nyinahin would give the least distance of 59 Kilometers and least Social Cost of 38.67 Million Ghana Cedis (the route bolded in Figure 1 depicts this solution).

CONCLUSIONS

A bi-objective optimization model to simultaneously minimize distance and compensations costs (called Social Cost) as a result of displacements and dislocations of people, businesses and infrastructure such as schools work places and farms among others in rail lines construction, has been demonstrated using a real-life situation. Ten towns within the Ashanti region of Ghana with notable local and national economic significance were selected to form a network connecting Kumasi as source and Nyinahin as destination. Particularly in this work, the Social Cost criterion was modeled taking into account compensation for businesses, inconvenience allowance to displaced persons as well as moving expenses due to relocation of facilities and persons. The necessary data obtained from identifiable local and state organizations were implemented in the Social Cost model to find estimates along the routes of the network. The optimization model was run using both the existing and improved Ant Colony System algorithms. The results indicated five Pareto optimal solutions with their margins of errors which provided basis for the selection of a best compromise solution, which is the rail link through Kumasi, Nwineso, Ahewerewa, Mpotoam, and Nyinahin which gives the least distance of 59 Kilometers and least Social Cost of 38.67 Million Ghana Cedis.

Future Work

It is important to note that the data used were only estimates and therefore future work can focus on building variability into the Social Cost model in order to capture variance information in the data for a better and more reliable results, to increase confidence in the implementation of the results in a practical situation. Furthermore, decision makers could for some justifiable reasons desire that the rail lines pass through some specific towns by all means. This scenario is interesting and useful future work; also, an additional criterion such as the cost of construction of the rail lines can be included in the model in a future work to increase the equity in the solutions.

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