VOTING POWERS AND A FUNCTIONAL REPRESENTATION OF THE VOTING CLOUT OF UNITED NATIONS SECURITY COUNCIL MEMBERS WITH VETO POWER RELAXATION

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ABSTRACT: *An independent mathematical structure was developed for the computation of the voting powers of United Nations Security Council members using the Shapley value concept, cooperative games and deft constructions of coalition sets. The results, obtained through Microsoft Excel implementations show that each permanent member has more than ten times as much voting clout as all ten nonpermanent members put together. A sensitivity analysis-based theorem was formulated to address this unacceptable unwholesome lopsidedness in voting powers while preserving the veto status of permanent members.Finally the paper exploited the Shapley value concept to obtain mathematical formulations and representations of voting powers of the members subject to any resolution passing threshold of votes supported by at least a partial coalition of Permanent representatives. The solution expressions can be used to obtain various levels of voting powers by appropriate adjustments of the parameters, thus giving prescriptions for more equitable distribution of voting powers.*

KEYWORDS: Coalition, Council, Power, Shapley, Security, Voting.

INTRODUCTION

In Winston (1994), it was pointed out that Shapley value could be used as a measure of the power of individual members of a political or business organization; it was indicated that using a 0-1 characteristic function, it could be shown that 98.15% of the power in the Security Council resided with the permanent members. Kerby and Gobeler (1996) validated the assertion in Winston (1994) and much more using a functional set-theoretic cardinality approach that was rather involved. See also O'Neill (1996) for more information on Power and Satisfaction in the United Nations Security Council.

The aim of this paper is to obtain independent and much less esoteric proofs of the relevant results in Kerby and Gobeler (1996) by constructing and deploying a cardinality-based listing structure of combined winning coalitions from the sets of permanent and nonpermanent members; this approach holds a lot of promise for enhanced appreciation and extensions of the results to more general coalitional structures.

THEORETICAL UNDERPINNING

The preliminary definitions and requisite terminology needed for this investigation are considered.

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Preliminaries

Let $N = \{1, 2, \cdots, n\}$

The following definitions will be found relevant and appropriate:

Characteristic function

For each subset S of N, the characteristic function v gives the amount $v(S)$ those members of S can be sure of receiving if they act together and form a coalition.

Sequential coalition

A sequential coalition is one in which the players are listed in the order in which they entered the coalition.

Pivotal player

A pivotal player is the player in a sequential coalition who changes the coalition from a non-winning to a winning one.

Critical player

A critical player is one whose desertion of a winning coalition turns that coalition into a non-winning one.

Dictator

A dictator is a player who has enough votes to pass any motion or resolution single-handedly.

Veto power

A player that is not a dictator but can single-handedly prevent any group of players from passing a motion or resolution is said to have veto power.

Dummy

A dummy is a player with no power.

Quota

The quota is the minimum number of votes needed to pass a motion.

Shapley Value Theorem

Given an *n*-person game with characteristic function *v*, there is a unique reward vector $x = (x_1, x_2, \cdots, x_n)$ satisfying axioms 1-4 stated below. The reward to the ith player (x_i) is given by

$$x_i = \sum_{S \subset N : i \notin S} P_n \left[v(S \cup \{i\}) - v(S) \right], \text{ where } p_n(S) = \frac{|S|!(n-1-|S|)!}{n!}$$

and $|S|$ is the number of players in the coalition *S*.

Axiom 1: Relabeling of players interchanges the players' rewards.

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Axiom 2: $\sum_{i=1}^{n} x_i = v(N)$

Axiom 3: If $v(S-\{i\}) = v(S)$ holds for all coalitions *S*, then the Shapley value has $x_i = 0$. If player i adds no value to any coalition, player i receives a reward of zero from the Shapley value.

Axiom 4: Let *x* be the Shapley value vector for game *v* and *y* the Shapley value vector of game $\bar{v}$. Then the Shapley value vector for game $(v+\bar{v})$ is the vector *x* + *y*.

See Winston (1994) for the above theorem.

Preliminary analysis

The coalitional structure in this problem is cardinality-based and sequential. For player *i* to add value to the passing of a resolution, there must exist at least one coalition of players for which player *i* is pivotal, (by definition 2.3 and axiom 3). Therefore the Shapley value to player i is obtained by listing/noting all coalitions that satisfy the above property (inability to pass a resolution until the arrival of the pivotal player *i* and totalling the values added to all such coalitions following the arrival of player *i*. The prospect for a solution is enhanced by choosing *i* from the set of permanent members of the Security Council, which in turn implies that the cardinality of all subcoalitions from the permanent membership less *i* must be the same as that of its full membership decremented by 1. Hence by the probability structure of the arrival process, all permanent members must have the same Shapley value.

Since $v(N)=1$, for $N=\{1, 2, \cdots, n\}$ the Shapley value to each nonpermanent member equals $\frac{1}{\text{cardinality of nonpermanent membership}}\left(\text{Sum of the Shapley values to the nonpermanent members}\right)$.

The subcoalitions from the nonpermanent membership that would satisfy the stated property need to be identified, not by listing, but by cardinality, otherwise the computations may become unwieldy, prohibitive or intractable. Following this approach, it is easy to note all coalitions of both subcoalition groups {nonpermanent, permanent} that would be necessary for adding value to player *i*. These coalitions become sufficient for resolution passing following their augmenting with *i*.

Above analysis motivates the following methodology exposed shortly for obtaining the Shapley value to each member of the UN Security Council. The computations of the Shapley values will hinge greatly on constructing a set of sets S_3 of nonpermanent and permanent members that will exploit the probability structure of the arrival process of coalition and noncoalition members to appropriate the Shapley value theorem.

METHODOLOGY

Permanent Representatives

Let $N_2 = \{1, 2, 3, 4, 5\}$ represent the set of permanent members of the UN Security Council. Then $N_2 = \{$CHINA, FRANCE, BRITAIN, USA, RUSSIA$\}$. Each member has a veto power; so any resolution not supported by at least one member is doomed. A necessary condition for resolution

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passing is that it must be supported by all five permanent members. For a resolution to pass, it must be supported by at least nine votes. Each permanent member has one vote, for a total of five votes for permanent members. The remaining minimum of four votes must be obtained from nonpermanent members. Note that each permanent member is a critical player.

Nonpermanent representatives

The minimum coalition size necessary for resolution passing is 4, that is, the minimum "YES" votes required for resolution passing from this category is 4.

Let S_1 be a singleton (set of cardinality 1) whose sole element is the number of "YES" votes garnered for passing a resolution from nonpermanent members. This value is the cardinality of the coalition. Hence $S_1 \in \{\{1\},\{2\},\cdots\{10\}\}$. But in coalition with N_2, the set of sets $\{\{1\},\{2\},\{3\}\}$ is resolution passing infeasible (by the standing hypothesis). Therefore, $S_1 \in \{\{4\},\{5\},\cdots,\{10\}\}$. Therefore are $\binom{10}{j}$ possible ways of selection the set $\{j\} \in S_1$ from the group of ten nonpermanent members for the purpose of passing a resolution; for each $j \in \{4,5,\cdots,10\}$, $\binom{10}{j}$ must be incorporated into the Shapley value to determine the voting powers of all members. Let S_2 be a subset of the set N_2 of permanent members and let $S_3 = \{S_1, S_2\}$, where $|S_3| = j + |S_2|$, for $S_1 = \{j\}$. Thus $|S_3|$ is a function of j for fixed $|S_2|$.

Assigning the value 1 to any coalition that could pass a resolution and 0, otherwise triggers the following property for *v*: $v(S_1) = v(S_2) = 0$: No group can pass a resolution all by itself.

$v(\{\}) = 0$: A resolution cannot pass without a meeting or casting of votes.

$$v(S_3) = v(S_1, S_2) = \begin{cases} 1, & \text{if } 4 \le j \le 10 \text{ and } |S_2| = 5 \\ 0 & \text{otherwise} \end{cases}$$

A resolution can only sail through with all permanent members and at least four nonpermanent members supporting it; consequently we have the following tabular organization of coalitions for the computation of voting powers of permanent members.

$S_1 = \{j\}$	S_2	$P_n(S_3)$	$v(S_3 \cup \{1\} - v(S_3))$	$\binom{10}{j}$
{4}	{2, 3, 4, 5}	8!6!/15!	1	210
{5}	{2, 3, 4, 5}	9!5!/15!	1	252
{6}	{2, 3, 4, 5}	10!4!/15!	1	210
{7}	{2, 3, 4, 5}	11!3!/15!	1	120
{8}	{2, 3, 4, 5}	12!2!/15!	1	45
{9}	{2, 3, 4, 5}	13!1!/15!	1	10
{10)	{2, 3, 4, 5}	14!/15!	1	1

Table 1: Tabular organization of coalitions for computation of voting powers excluding a pivotal veto member.

Therefore the voting power of each permanent member is given by

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$$x_i = \sum_{j=4}^{10} \binom{10}{j} p_n\left(S_3(j)\right) = \sum_{j=4}^{10} \binom{10}{j} \frac{|S_3(j)|\,!(n-1-|S_3(j)|)\,!}{n!}$$
$$= \sum_{j=4}^{10} \binom{10}{j} \frac{\left(j+|S_2|\right)!\left(n-1-[j+|S_2|]\right)!}{n!} = \sum_{j=4}^{10} \binom{10}{j} \frac{(j+4)\,!(15-1-[j+4])\,!}{15!}$$
$$= \frac{\sum_{j=4}^{10} \binom{10}{j} (j+4)!(10-j)!}{15!}$$

for each i in N_2, while that of each non-permanent member is $\frac{1}{10}\left(1-5x_1\right)$.

The Excel format for implementing the above voting power formula to each permanent member is shown as in the worksheet sheet below.

Step 1: Type the following Excel code segment in cell reference B4 in the embedded Excel object:

=fact (4+$a4)*fact (10-$a4)*combin (10, $a4)/fact (15) and press the Enter key.

Step 2: Click on cell C4 and position the cursor on the right boundary of the cell until a crosshair appears. Then drag the crosshair vertically down to terminate in cell B10 to generate the components C_j of the Shapley value shown in contiguous cell locations B4:B10. The sum 0.196270396 which appears in cell reference C11 is the Shapley value to player 1 (CHINA) and hence to each of the remaining permanent members of the Security Council.

$\boxed{\lvert S_1 \rvert = j}$	$\boxed{C_j}$
4	0.004662005
5	0.008391608
6	0.013986014
7	0.021978022
8	0.032967033
9	0.047619048
10	0.066666667
SUM =	**0.196270396**

Table 2: Computation of voting powers of veto members of UN Security Council

It is clear that $x_i = \frac{\sum_{j=4}^{10} 10!(j+4)!(10-j)!}{15!(10-j)!\,j!} = \frac{10!}{15!}\sum_{j=4}^{10}\left(\frac{(j+4)!}{j!}\right)$. This simplied form will prove instructive for subsequent results.

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REMARKS AND RECOMMENDATIONS

The five permanent members of the UN Security Council have a total voting power of about 98.1352 %, leaving all ten nonpermanent members with a meager 1.8648% voting power. Thus each permanent member of the Council has 105.25 times as much voting power (105.25 times as much clout) as a nonpermanent member. Put more succinctly, each permanent representative has more than ten times as much clout/ voting power as all ten nonpermanent representatives put together. As part of the effort to address this unwholesome situation we will first formulate and prove a result in which the number of permanent and nonpermanent representatives and the quota are parameters. Finally, we develop and prove the most general result on membership of UN Security Council and resolution passing. These will be reflected in the ensuing theorems, with implementation on the platform of Microsoft Excel.

RESULTS

Theorem 1

Consider an n-member UN Security Council with k permanent representatives, who must obtain the cooperation of at least j nonpermanent representatives to pass a resolution. All $n - k$ nonpermanent members put together cannot pass a resolution without the support of all permanent members. Then the voting power of each permanent representative $i \in \{1, 2, \ldots, k\}$ is

$$x_i = x = \frac{(n-k)!}{n!} \sum_{t=j}^{n-k} \frac{(t+k-1)!}{t!}, \text{ or } X_i = X = 100\,x\,\%,$$

while that of each nonpermanent member $i \in \{k+1, k+2 \ldots, n\}$ is given by

$$x_i = y = \frac{1}{n-k}(1 - k\,x), \quad \text{or } X_i = Y = 100\,y\,\%.$$

Proof

The permanent members have the same status and hence must have the same rewards. For ease of exposition refer to the permanent members as players 1, 2, … k; and the nonpermanent representatives as players k + 1, k + 2,… n. Let C_t be a subcoalition of t "YES" votes from nonpermanent members and let S_1 be the coalition of "YES" votes from permanent members awaiting a "YES" vote from player 1 to guarantee the passing of a resolution. Then $S_1 = \{2, 3, \ldots, k\}, |S_1| = k - 1$. Let S be the set of all pairs of coalitions of "YES" votes from permanent and nonpermanent members. Then

$S = \{\{C_j, S_1\}, \{C_{j+1}, S_1\}, \ldots, \{C_{n-k}, S_1\}\}$. Let $\tilde{S}_{t+k-1} = (C_t, S_1);\ t \in \{j, j+1, \cdots, n-k\}$. Then

$$x_i = x_1 = \sum_{t=j}^{n-k} \binom{n-k}{t} P_n\left(\tilde{S}_{t+k-1}\right) = \sum_{t=j}^{n-k} \frac{(n-k)!}{(n-k-t)!\,t!} \frac{(t+k-1)!(n-t-k)!}{n!}$$

$$= \frac{(n-k)!}{n!} \sum_{t=j}^{n-k} \frac{(t+k-1)!}{t!}, \forall i \in \{2, 3, \ldots, k\}.$$ Now, set $X_i = X = 100\,x_1\,\%$.

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Clearly, $x_i = y = \frac{1}{n-k}(1-kx_1),\ \forall i \in \{k+1,\ k+2,\dots,n\}$. Set $X_i = Y = 100\,y$ %, $\forall i \in \{k,\ k+1,\ \dots,n\}$. This completes the proof.

Excel Implementation:

	A	B	C
1	Voting powers of UN Security Council members		
2	n	k	
3	"t"		
4	j		
5	j+1		
⋮	⋮		

Table 3: Data layout for sensitivity analysis of powers of UN Security Council Members

(i) Type the parameters n and k in the fixed cell references $A$2 and $B$2 and the identifier "t" in A3, as shown.

(ii) Type the t variable values j, j + 1 , …, n – k in the contiguous cell references $A4: $A[n – k – j + 4]

(iii) Type the following code into $B4, < Enter > and use the crosshair – dragging routine to secure the values down to cell $B[n-k-j+4] corresponding to the t – values $A4 : $A[n – k – j + k]:

=fact($A$2-$B$2) / fact($A$2) * fact($A4+$B$2-1) / fact($A4)

(iv) Obtain x_1 by typing the code =sum ($B$4: B[n – k – j + 4]), then < Enter> in cell reference B[n – k – j + 4] or any other preferred cell location.

(v) Obtain y by typing the code = (1 - $B$2*B[n – k – j + 6]) / ($A$2 - $B$2) in B [n – k – j + 8]. Done

Implementation examples on an embedded Excel worksheet

A	B	C	D	E
15	5			
t	j = 4	j = 5	j = 6	j = 7
4	0.0047			
5	0.0084	0.0084		
6	0.0140	0.0140	0.0140	
7	0.0220	0.0220	0.0220	0.0220
8	0.0330	0.0330	0.0330	0.0330
9	0.0476	0.0476	0.0476	0.0476
10	0.0667	0.0667	0.0667	0.0667
$x_1 =$	0.1963	0.1916	0.1832	0.1692
$y =$	0.0019	0.0042	0.0084	0.0154
x / y	105.25	45.67	21.83	11.00

Table 4: Voting power implementation example on embedded Microsoft Excel worksheet

In the above worksheet, n =15 and k =5. The four values 4, 5 6 and 7 for the parameter j yield the indicated x_1 and y values.

As can be gleaned from the worksheet, the voting power of permanent members is a decreasing function of j ; n kept fixed. In terms of size the decrease is marginal. However the decrease is quite dramatic in relative terms or proportions, from 105 times as much as that of a nonpermanent member to mere 11 times, for j ranging from 4 to 7.

In the next result the above theorem will be extended to give specific recommendations on how to obtain a more equitable allocation of voting powers between both categories of representatives. The theorem is quite encompassing and can be used to perform sensitivity analysis on the parameters, as a guide to achieving desired levels of voting power for both categories of representatives.

Theorem 2

Consider an n - member UN Security Council with k permanent representatives, and $n - k$ nonpermanent members with the requirements for passing a resolution listed as follows:

- m votes are required to pass a resolution
- A resolution must be supported by at least k_1 permanent representatives, where $k_1 < k < m < n$.

Let x_i be the voting power of the i^{th} representative, $i \in \{1, 2, \ldots, n\}$.

Then the voting power of each permanent representative $i \in \{1, 2, \ldots, k\}$ is given by

$$x_i = x = \sum_{t_2 = m-k_1}^{n-k} \binom{k-1}{k_1 - 1}\binom{n-k}{t_2}(t_2 + k_1)^{-1}\binom{n}{t_2 + k_1}^{-1} + \sum_{t_1 = k_1}^{k-1}\binom{k-1}{t_1}\binom{n-k}{m - t_1 - 1}(m)^{-1}\binom{n}{m}^{-1}, \text{ or } X_i = X = 100\,x\,\%,$$

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while that of each nonpermanent member $i \in \{k+1, k+2\ldots, n\}$ is given by $x_i = y = \frac{1}{n-k}(1-k\,x)$, or $X_i = Y = 100\,y$ %. Hence there is no dummy in the cooperative game; needless to say, there is no dictator.

Proof

Suppose that a resolution is supported by t_1 permanent members, $k_1 \le t_1 \le k$; then the resolution is passed if it is supported by at least $m - t_1$ nonpermanent members. Let PM_{t_1} be the set of all permanent members of cardinality t_1 from the set $\{1, 2, \ldots, k\}$. Let NPM_{t_2} be the set of all nonpermanent members of cardinality t_2 from the set $\{k+1, k+2, \ldots, n\}$.

Let $S_t = \{PM_{t_1}, NPM_{t_2}\} = S_{t_1+t_2}$, a set of cardinality $t = t_1 + t_2$.

PM_{t_1} can be selected in any of $\binom{k}{t_1}$ ways, while NPM_{t_2} can be chosen in $\binom{n-k}{t_2}$ ways.

Then S_t can be selected in $\binom{k}{t_1} * \binom{n-k}{t_2}$ ways, by independence of categorical selections.

Now, $P_n(S_t) = \frac{t!(n-t-1)!}{n!} = \frac{(t_1+t_2)!(n-t_1-t_2-1)!}{n!}$.

Suppose S_t already arrived, that is, already voted "YES" in support of a resolution. Then the resolution passes if $v(S_t) = 1$. Pick a member i from the set $\{1, 2, \ldots, k\}$: $i \notin PM_{t_1}$. Without loss of generality let $i = 1$. Fix t_1 at $k_1 - 1$. Then PM_{k_1-1} can be chosen in $\binom{k-1}{k_1-1}$ ways. The corresponding NPM_{t_2} sets for which the arrival of player 1 will add value (leading to the passing of a resolution) are NPM_{t_2} for $m - k_1 \le t_2 \le n-k$ yielding the set $\{PM_{k_1}, NPM_{t_2} : m-k_1 \le t_2 \le n-k\}$. Note that t_2 cannot exceed $n-k$. Consequently, we have the following component of x_1:

$$\sum_{t_2 = m-k_1}^{n-k} \binom{k-1}{k_1-1}\binom{n-k}{t_2} P_n\left(S_{t_2+k_1-1}\right).$$

Now fix t_1 at k_1 with player 1 not in PM_{k_1} and player 1 being the pivotal player. PM_{k_1} can be chosen in $\binom{k-1}{k_1}$ ways. Note that player 1's arrival will add value only if we choose

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NPM_{t_2} such that $t_2 = m-\left(k_1+1\right)$. Then for $k_1 \le t_1 \le k-1$, we must choose only $NPM_{m-(t_1+1)}$, leading to the following component of x_1: $\sum_{t_1=k_1}^{k-1}\binom{k-1}{t_1}\binom{n-k}{m-t_1-1}P_n\left(S_{m-1}\right)$. Hence, for $i \in \{1,2,\ldots,k\}$,

$$x_i = x_1 = \sum_{t_2=m-k_1}^{n-k}\binom{k-1}{k_1-1}\binom{n-k}{t_2}P_n\left(S_{t_2+k_1-1}\right) + \sum_{t_1=k_1}^{k-1}\binom{k-1}{t_1}\binom{n-k}{m-t_1-1}P_n\left(S_{m-1}\right).$$

The reader can verify that $\binom{n}{t+1}^{-1} = (t+1)^{-1}P_n\left(S_t\right)$, so that $P_n\left(S_t\right) = \frac{1}{t+1}\binom{n}{t+1}^{-1}$

Hence, $x_i = x_1 = \sum_{t_2=m-k_1}^{n-k}\binom{k-1}{k_1-1}\binom{n-k}{t_2}\left(t_2+k_1\right)^{-1}\binom{n}{t_2+k_1}^{-1} + \sum_{t_1=k_1}^{k-1}\binom{k-1}{t_1}\binom{n-k}{m-t_1-1}(m)^{-1}\binom{n}{m}^{-1}$

or $X_i = X = 100x\%$, as desired.

The reward to each nonpermanent member $i \in \{k+1, k+2\ldots,n\}$ is given by

$$x_i = y = \frac{1}{n-k}\left(1-kx\right), \quad \text{or } X_i = Y = 100y\ \%.$$

For the case $k_1 = k$, the second summation in the rewards to permanent members is set to 0, and the results coincide with those obtained for the current existing UN Security Council. That there is no dummy follows from the fact $x < \frac{1}{k}$, for otherwise $x = \frac{1}{k}$, contradicting the hypothesis that banding together by permanent members is necessary but not sufficient for resolution passing.

Excel Implementation

n, k, k_1, and m are the parameters of the game. Type given values of these parameters in \$A\$2:\$D\$2 of an Excel worksheet.

	A	B	C	E
1	Proposed Voting powers for UN Security Council members.			
2	n	k	k_1	m
3	"t_2"	t_1		
4	j			
5	$m-k_1$	k_1		
⋮	⋮			

Table 5: Data table for power configurations for the Membership of UN Security Council

(i) Type the identifiers as in A3 and A4; type the t_2 values from m – k to n – k into cells

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$A5:$A [n + k $_1$ + 5 – k – m] using the cell references instead of the actual values and type the t $_1$ values k $_1$ to k – 1 into cells $B5 : $B[k – k $_1$ + 4] accordingly

(ii) Type the following code into cell $C5
= Combin ($B$2 – 1, $C$2)*combin ($A$2-$B$2, $A5)*combin ($A5+$C$2)^(- 1)*
(combin ($A$2, $A5 + $C$2)) ^ (- 1), < Enter > and use the crosshair dragging routine to secure the coded values down to cell $C [n + k $_1$ + 5 – k – m]; graphically denoted by $C5

$C [n – k $_1$ + 5 – k – m]

(iii) Type the code = Combin ($B$2 – 1, $B5*combin ($A$2-$B$2, $D$2 - $B5 – 1) *($D$2) ^ (-1) * (combin ($A$2, $D$2)) ^ (- 1), then < Enter >.

(iv) Secure the reward $x_i = x;\ i \in \{1, 2, \ldots, k\}$, to each permanent member i by typing the following code into cell reference E2:
= Sum ($C$5 : C[n + k $_1$ + 5 – k – m]) + sumif (“ $B$2 > $C$2 ” , $D$5 : D[k – k $_1$ + 4], then < Enter >.

(v) Secure the reward $x_i = x;\ i \in \{1, 2, \ldots, k\}$, to each nonpermanent member i by typing the following code into cell reference F2:
= (1 - $B$2 * $E$2) / ($A$2 - $B$2) and < Enter >.

DISCUSSION

Sensitivity Analysis on the parameter vector (n, k, k$_1$, m)

The parameter vector (15, 5, 3, 9) yielded the reward x = 0.112354 to each permanent member and y = 0.043823 to each nonpermanent member. Thus, the revised voting power of each permanent member is 11.235 %, while that of a nonpermanent member is 4.382 %. Hence each permanent member now has about 2.56 times as much clout / voting power as a nonpermanent member. Contrast this with the prevailing ratio of 105 to 1.

A	B	C	D	E	F	G
n	k	k $_1$	m	X =	Y =	X/Y =
15	5	4	9	0.1683	0.0159	10.618
t $_2$	t $_1$					
5	4	0.0223776	0.004662			
6		0.027972				
7		0.031968				
8		0.032967				
9		0.029304				

Table 6: Excel Implementation for a chosen vector of parameter values (n, k, k_1, m)

Results corresponding to other feasible parameter values are easily obtainable by adding or removing rows below as appropriately determined by those parameter values. The variable

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values of t_2 and t_1 and hence the results will adjust automatically with the completion of the necessary dragging routine.

IMPLICATIONS TO RESEARCH AND PRACTICE

The established results have wide-ranging implications for redressing the gross violation of democratic norms, values and ethics in the current power configuration in the composition of United Nations Security Council; a situation where the rest of the non-veto- power- wielding world is considerably consigned to mere onlookers on critical decisions that affect them is simply unacceptable. The UN Security Council needs to be "rebranded" and democratized.

CONCLUSION

This paper appropriated cardinality dependent sequential coalitional structures to derive the voting powers of United Nations Security Council members for various quota configurations, based on the Shapley Value measure. The solution expressions can be used to obtain various levels of voting powers by appropriate adjustments of the problem parameters, thus giving prescriptions for more equitable distribution of voting powers. In a follow- up paper the method developed here will be exploited to derive reward structures for players in more general cooperative game settings.

FUTURE RESEARCH

In the immediate future, research effort will centre on in-depth analyses of the reward structures of n - person cooperative games with specified winning coalitions involving at least one major player. Thereafter further research interest and extensions of this article will include the following investigations:

(i) Reward structure of finite - person cooperative games with a lone player category and two other categories of players subject to specified winning coalitions.

(ii) Reward structure of finite - person cooperative games with three broad categories of players subject to specified winning coalitions.

(iii) Analyses of three - category, finite - person cooperative games with relaxation in winning coalitions.

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