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USING GRAPHICAL MODEL TO IMPROVE STUDENTS' PERFORMANCE IN COMPOSITION AND INVERSE OF FUNCTIONS AT OFFINSO COLLEGE OF EDUCATION

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ABSTRACT: *The concept of function is one of the central concepts in Mathematics; as such it has received attention in Mathematics Education Research. However, the research focused explicitly on the two subtopics of a function: Composition and Inverse. This study examined the use of graphical model to improve the performance of first year Diploma in Basic Education (DBE) students of Offinso College of Education, in Ashanti Region of Ghana. The study also investigated how the textbooks treated this particular topic and how teachers taught these two subtopics of a function. A total of fifty (50) students participated in the study. The data were collected through semi-structured interview and rating scale questionnaire. In addition to these, the students wrote pre-test and post-test. The data were analyzed in detail using quantitative and qualitative techniques. The results of the study suggested that, the performance of the students in the post-test was far better than the pre-test indicating that the intervention worked. Finally, the students' interest was motivated and also developed confidence in solving composition and inverse of functions using graphical model. It is recommended that, constructivist theory of learning should be encouraged in all spheres of Mathematics classroom environments so as to give learners the opportunity to find meaning to their own learning in order to explore new ideas and findings concerning their future.*

KEYWORDS: Diploma in Basic Education (DBE), graphical model, Composition, Inverse Function

INTRODUCTION

Mathematics is a core subject in the Colleges of Education, Senior High Schools and the Basic Schools. As a result, teacher trainees who will be going to teach the Basic schools must be well equipped in mathematics so that they can impart well the knowledge they have received to the younger generations. Ball and Bass (2000) are of the view that prescriptive methods are insensible to classroom context and stifles teachers' constructive pedagogies. They observed that the knowledge required for teaching is underspecified. This triggered further questions as to what kind of Mathematics knowledge is required for teaching function concepts in order to help students acquire the concepts of function values. Most of the teacher trainees lack the basic concepts in mathematics most especially inverse and composite functions which first year Diploma in Basic Education (DBE) students at Offinso College of Education is no exception. According to Mathematics Chief Examiners' report (2017) on Number and Basic Algebra, many candidates failed to select the correct option. Students' inability to choose the correct answer could be attributed to their misinterpretation of gof; putting f(1) into the function g. the report continuous to state that students showed weaknesses in certain mathematics content areas, and inverse of a function was one of them.

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Again, mid-semester internal examination used as School-Based Assessment involved questions on inverse and composite functions and students exhibited poor performance on this particular topic after scoring the marks. Some trainees interviewed were of the view that the Mathematics they learnt at the Senior High School was for the purpose of passing examination and their aims were not to continue any Mathematics thereafter. Another group of trainees also said that they studied Mathematics by themselves since there were no Mathematics tutors in their various schools. Another group of students asserted that their subject tutors' teaching approaches did not make them understand some mathematics concepts very well, inverse and composite of a function was one of them. Such teachers do not know that in developing and understanding a concept, axioms come last. The view that Mathematics is in essence derived from axioms is backward. In such classrooms, students are made to learn rules, and spent many hours practicing their use. Students in these classrooms often come away with the impression that doing Mathematics is a rote and mechanical activity. For that matter, many students turn away from Mathematics early; they often see it as a discipline understandable by a select few and choose not to pursue higher Mathematics. Now, the question is what can be done to erase such problem first year students at Offinso College of Education (OFCE) faced in composition and inverse of functions? It is against this background that this research was designed to investigate, and to find a suitable approach to improving the performance of the first year students of OFCE through the use of graphical model in learning composition and inverse of function concepts.

Purpose and Objective of the Study

The purpose of this study is to investigate the problems that hinder first year DBE students at Offinso College of Education from solving composition and inverse of function problems and design and implement graphical model as an intervention to address the situation. It is also to serve as a spring board for studying Mathematics and to establish a strong national Mathematics culture for industrialization. A major focus of the research is the role of manipulative materials in facilitating the acquisition and use of graphical model concepts as the student understands moves from concrete to abstract.

Research Questions

Based on this purpose of the study, the following research questions were stated.

1. To what extent will the use of the graphical model to develop the concept of composition and inverse of functions improve students' performance in Mathematics as a whole?

2. To what extent will the use of the graphical model motivate students' interest in the topic?

REVIEW OF RELATED LITERATURE

The Theory of Function in Mathematics

The theory of functions compared to other mathematical domains such as geometry, arithmetic and algebra is a relatively new field in mathematics, but it represents a central piece in modern mathematics and in mathematics education, through what is called Calculus and mathematical

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analysis. Gardiner (1982) noted that For many students today, Calculus is the climax of their encounter with elementary mathematics: it exploits almost all the mathematics they have previously learnt, (especially algebra, trigonometry, graphs and coordinate geometry), and it opens the way to the analysis of a whole host of interesting problems (in elementary geometry, in differential geometry, in statics and dynamics, in physics –in fact, in any subject where may reasonably assume that one quantity varies more or less smoothly with respect to some other quantity. The most basic question is "What is a function?" Hermann Weyl answered it as follows: nobody can explain what function is, but this is what really matters in mathematics: A function f is given whenever with real number "a" there is associate a number "b". "b" is said to be the value of the function f for the argument "a" (Gardiner, 1982). This answer is little simplistic, but in scholarly argument on the topic, there is no universally accepted definition for the concept of function. Mathematicians are still struggling with defining mathematical functions in a rigorous way. Indeed, one need not even be able to express the relationship through mathematical operations. It doesn't matter if one thinks of this so that different parts are given by different laws or designates it entirely lawless? (Davis & Hersh, 1981, p.264). A function is a special relation. "A function is a set of ordered pairs in which for every x, there is only one y" (Knill et al., 1998). This definition is a simpler variant of the following definition that uses more sophisticated mathematical language. "A function f is an application of set A in the set B, in other words it is a set of ordered pairs (x,y) $\in$f, with x $\in$X $\subsetneq$ A, y $\in Y \subsetneq B$, with the property that for every x $\in$X there is exactly one correspondent y $\in$Y. The set X is called the domain of the function f, and the set Y is called the co-domain of the function "(Gellert, Kustner, Hellwich, &Kustner, 1975).

The last given example for the definition of the function concept can be called "a modern" definition. With this definition though, the controversy over the concept of functions did not cease to exist. The set of functions was divided into two main subsets: functions of a real variable and functions of a complex variable. Weirstrass ordered and systematized Cauchy's work regarding analytical functions of a complex variable, and gave a definition for the functions of a complex variable: an analytic function f(x) is the totality of elements obtained from a given one by means of successive direct continuations. This definition was clarified and completed later on by Volterra and Poincare. On the other hand, Weierstrass's definition seems for some to be too restrictive. Schnitzer and Stillwell compare the actual controversy over the definition of the function concept with the situation existent two or three centuries ago: "It thus became clear that, in our own time, the controversy about the vibrating string has been renewed in another light and with a different content" (Schnitzer & Stillwell, 2002).

A function describes the way one quantity depends on or varies with another. For instance, pressure is a function of temperature or population is a function of time etc, (Flanders & Price, 1975). To them, function lurks everywhere; they are the basic idea in almost every application of Mathematics. Therefore, a great deal of study is devoted to their nature and properties. According to Alan, Tussy and Gustafson (2002), a function is a rule that assigns to each input value a single output value. The notation $y = f(x)$ denotes that y is a function of x. they stated that the letter f used in the notation y = f(x) represents the word function. However, other letters can be used to represent functions. For example, y = g(x) and y = h(x) also denote functions involving the variable x.

In conclusion, there still are new pages to be added to the history of the function concept. As Kleiner (1993) observes "Textbook definitions or descriptions of function have varied with time, context and level of presentation" (p.183). Apparently, these definitions and descriptions

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are still changing in presents, at least in the circle of advanced mathematics. The concept of function is central to both today's mathematics and mathematics education, and following the history of this concept, the past and present struggle for defining this concept, it seems that all the difficulties encountered by students and teachers who have to learn and teach this topic have a natural root into this mathematical concept.

Brief descriptions of composite and inverse functions and how they are related

Functions under discussion over here are not functions in general, but the researcher's scope of the study is limited to composition and inverse of functions. The researcher treated these two aspects of functions together, because the researcher saw that they are interlined, and one being a subtopic of the other. To be exact, the operation of composition would lead itself to a natural extension of the concept of inverses. The composition of function is treated quite extensively, from a computational point of view in most of the textbooks. But the graphical aspect of the topic was partially treated or not at all. The property of composition of function of not being commutative is presented in the textbook as an observation after an example. The emphasis is on the fact that sometimes the order of composition does not matter, but for computation to be possible the domain and the range must be checked.

The inverse function of a function is treated as a different topic than the composition of functions, and not as a subsequent topic: "Inverse functions are a special class of functions that undo each other" (Knill et al., 1999, p.261). The fundamental relation for the inverse functions (when composing inverse functions, the result is always the identity function) is mentioned in the text as a computational exercise of determining if two functions are inverse for each other or not.

Given how functions are defined in textbooks and the definition of injective, surjective and bijective functions, the composition of function starts with a discussion or analysis of the case when two functions can be composed, according to their domain and range. The inverse function of a function is introduced by definition, and the fundamental relation between the function and its inverse is given as a special consequence of the definition. In defining the inverse function of a function, some books start with the condition that the initial function has to be bijective (one-to-one): "If function f: A →B is a bijective function, and x is an element of the domain A, and y = f(x) is an element of the co-domain B, we define a function f^{-1} with domain B and Co-domain A, such as $f^{-1}(y) = x$. This new function is called the inverse function of function f, and it is bijective function" (Micic, 1989, p.56).

The first examples use arrow diagrams with the intent of stressing the importance of the domains and ranges of the two functions. Related to this topic, the curriculum mentions two properties of the composition of functions: the composition of functions is associative and non-commutative. The inverse function is introduced in the following way: "A function f: A →B has an inverse if there is a function g: B →A, such that fog = 1_B and gof = 1_A, where 1_A and 1_B and gof = 1_A, where 1_A and 1_B are the identity functions defined on the sets A and B" (Nastasescu, Nita & Rizescu, 1993, p.64). The lesson related to the inverse function ends with the discussion on the relation between the graphs of inverse functions. "Composite functions are not interchangeable" (Lee, 1999b, p.105). The inverse function is treated as a separate lesson by the text, but it is a follow up of the lesson on composition of functions. The fact that the inverse function exists only if the initial function is one-to-one is clearly stated: "f^{-1} exist as a function only when f is a one-to-one function" (Lee, 1999b, p.109). The fundamental

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relation f inverse function is stated on the next page, as the starting point for finding the inverse of a function. The topic of inverse functions ends with the discussion the graphs of inverse.

Stevens (1994) said that a function can be manipulated in such a way that new function is formed. This is termed as a composition of functions, for instance, if f and g are functions, then f composed with g, denoted fog, is formed by using the output of g as the input of f. that is [fog](x) = f[g(x)]. Again, g composed with f, denoted gof, is formed by using the output of f as the input of g. thus the composite function gof is defined by [gof](x) = g[f(x)] to make sense, the value of f(x) must be acceptable inputs for the function g. since the domain of gof is the set of all inputs x in the domain f such that f(x) is in the domain of g. likewise, the domain of fog is the set of all inputs in the domain of g such that g(x) is the domain of f.

Similarly, if g is function whose values lie in the domain of a second function f, then the composite fog of f and g is defined by the formula [fog] (x) = f[g(x)}, (Flanders & Price, 1975). Think of substituting one function into the other, or replacing the variable of f by the function g.

Hornsby & Lial (1996) affirmed that if f is a one-to-one function with domain A and range B. then its inverse function f^{-1} has domain B and range A and is defined by $f^{-1}(y) = x \rightarrow f(x) = y$. they continued that the inverse of a function is a rule that acts on the output of the function and produces the corresponding input. Therefore, inverse "undoes" or reverse what the function has done. This means that the inverse of any one-to-one function f can be found by exchanging the components of the ordered pairs of f. The rule for the inverse of a function defined by y=f(x) also is found by exchanging x and y. Stevens (1994) stated that there exists an inverse function for a given function f only if f is one-to-one function. This is because each output of a one-to-one function comes from just one input. Thus, when we reverse the roles, we can assign to each output the input from which it came. The inverse function of the function f is denoted by using the notation f^{-1} (read "f inverse) and writes as $f[f^{-1}(x)] = f^{-1}[f(x)] = x$

According to him, there is a link between composition functions and inverse functions. As we can use one-to-one function to determine there exist an inverse for a certain function so as composition of function? For example, cubing function f defined by $f(x) = x^3$ and the cube root function g defined by $g(x) = \sqrt[3]{x}$. The composite function gof represents cubing a number and then taking the cube root of the result, whereas the composite function fog represents taking the cube root of a number and then cubing the result. Thus $[gof](x) = g[f(x)] \rightarrow g(x^3) = \sqrt[3]{x^3} = x$ and $[fog](x) = f(g(x)] \rightarrow f(\sqrt[3]{x}) = (\sqrt[3]{x}) = \left(\sqrt[3]{x}^3\right) = x.$

In short, composing f with g in either order is the identity function, the function that assigns each input to itself. Since one function "undoes" the other, we say that the cubing function and the cube root function are inverse functions. This implies that the functions f and g are inverses of each other if f [g(x)] = x for all x in the domain of g and g [f(x)] = x for all x in the domain of f. To him, in view of the fact that inverse functions deal with the composition of functions, it is essential to check the domains and ranges to be certain that the domain of f equals to range of g and the domain of g equals the range of f.

Theoretical Framework

This study was based on the constructivist theory of teaching and learning. Constructivism is a theory of knowledge that argues that humans generate knowledge and meaning from and

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interaction between their experiences and their ideas. During infancy, it is an interaction between their experiences and their reflexes or behavior –patterns (EIkipedia, 2011).

It is important to note that constructivism is not a particular pedagogy. In fact, constructivism is a theory describing how learning happens, regardless of whether learners are using their experiences to understand a lecture or following the instructions for building a model airplane (Wikipedia, 2011). In both cases, the theory of constructivism suggests that learners construct knowledge out of their experiences. However, constructivism is often associated with pedagogic approaches that promote active learning, or learning by doing. Social Constructivism views each learner as a unique individual with unique needs and backgrounds. The learner is also seen as complex and multidimensional. Social constructivism not only acknowledges the uniqueness and complexity of the learner, but actually encourages, utilizes and rewards it as an integral part of the learning process (Wertsch, 1997).

Several educators have also questioned the effectiveness of this approach towards instructional design, especially as it applies to the development of instruction for novices (Mayer, 2004; Kirschner, Sweller, and Clark, 2006). While some constructivists argue that "Learning by doing" enhance learning, critics of this instructional strategy argue that little empirical evidence exists to support this statement given novice learners (Mayer, 2004; Kirschner, Sweller, and Clark, 2006). Sweller and his colleagues argues that novices do not possess the underlying mental models, or "schemas" necessary for "learning by doing" (e.g. Sweller, 1988). Indeed, Mayer (2004) reviewed the literature and found that fifty years of empirical data do not support using the constructivist teaching technique of pure discovery; in those situations requiring discovery, he argues for the use of guided discovery instead.

Mayer (2004) argues that not all teaching techniques based on constructivism are efficient or effective for all learners, suggesting many educators misapply constructivism to use teaching techniques that require learners to be behaviourally active. He describes this inappropriate use of constructivism as the "constructivist teaching fallacy". "I refer to this interpretation as the constructivist teaching fallacy because it equates active learning with active teaching" (Mayer, 2004, p.15). Instead Mayer proposes learners should be "cognitively active" during learning and that instructors use "guided practice". Despite all the critics about this learning theory the researcher deems it important because learners construct knowledge for themselves- each learners individually (and socially) constructs meaning –as he or she learns. Various approaches in pedagogy derive from constructivist theory. They usually suggest that learning is accomplished best using a hands-on approach. Learners learn by experimentation, and not by being told what will happen, and are left to make their own inferences, discoveries and conclusions.

According to the social constructivist approach, instructors have to adapt to the role of facilitators and not teachers (Bauersfelf, 1995). Whereas a teacher gives a didactic lecture that covers the subject matter, a facilitator helps the learner to get to this or her own understanding of the content. In the former scenario the learner plays a passive role and in the latter scenario the learner plays an active role in the learning process. The emphasis thus turns away from the instructor and the content, and towards the learner (Gamoran, Secada, & Marrett, 1998). This dramatic change of role implies that a facilitator needs to display a totally different set of skills than a teacher (Brownstein 2001). A teacher tells, a facilitator asks; a teacher lecturers from the front, a facilitator supports from the back; a teacher gives answers according to a set curriculum, a facilitator provides guidelines and creates the environment for the learner to arrive at his or her own conclusions; a teacher mostly gives a monologue, a facilitator is in

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continuous dialogue with the leaners (Rhodes and Bellamy, 1999). A facilitator should also be able to adapt the learning experience "in mid-air' by taking the initiative to steer the learning experience to where the learners want to create value.

With this theory at the back of the researcher's mind, the researcher acted as a facilitator during the intervention stage to help the learners to build their own understanding of the concept of composite and inverse of functions using graphical model. The researcher structured learning experience just enough to make sure that students get clear guidance and parameters within which to achieve learning objectives, yet the learning experience should be open and free enough to allow for the learners to discover, enjoy, interact and arrive at their own truth. Students' participation in the instructional period is very vital in their education. Intrinsic motivated students are more willing to learn, and I believe such students become active participants in the instructional time. As students become more actively involved in their learning, they develop interest and enthusiasm for the subject. Through this study, the researcher found that students visualized a function well through its graph. And this has helped them to acquire new knowledge, develop new concepts and demonstrate better understanding.

Usefulness of Graphical Model

According to Schaaf (1969), a function is a relation between variables, may be described or specified in several different ways. To him, the most commonly used methods, at least in elementary mathematics are by means of: a) a table b) a graph c) a formula d) a verbal statement. He stated that the graph is a common device for indicating a functional relation. It may be a curve drawn automatically by recording instrument such as a barograph or a thermograph, which represent respectively the atmospheric pressure and temperature at a fixed place as a function of the time. Or the graph may be a curve drawn to fit a given table of paired values, where each ordered pair in the table is represented by a point in the lattice or in the plane, and the points are then "connected" by a broken line, a smooth curve or a series of disconnected line segments. However, he cautioned not to think of the formula as a way of expressing functional relation because of its symbolic statements.

The graph of an equation in two variables is the set of points in the plane whose coordinates (ordered pairs) are solutions of the equation. Thus, the graph of an equation is a picture of its solutions. Since a typical equation has infinitely many solutions, its graph has infinitely many points (Hugerford, 1999). He continued to state that the graph of a function f(x) is defined to be the graph of the equation y = f(x). For example, the graph of f(x) = $x^3 + x + 5$is the graph of the equation $y = x^3 + x + 5$. Thus, the graph consists of the points in the Cartesian plane whose coordinates are of the form$(x, x^3 + x + 5)$, that is, all points [x f(x)]. The same thing is true in the general case. Each point on the graph of a function f is an ordered pair whose first coordinate is an input number from the domain of f and who's second coordinate is the corresponding output number. Here (Barnett, Ziegler & Byleen, 1999) recalled that Cartesian coordinate system is two real number lines, one horizontal and one vertical which meet at their origins. These two number lines are called the horizontal axis and the vertical axis, in other words, coordinate axes. The horizontal axis is usually referred to as the x-axis and the vertical axis as the y-axis. To add to this, they gave two numbers written as the ordered pair (a, b) from the coordinates of the point p. The first coordinate, 'a' is called the abscissa of P; the second coordinate, 'b' is also called the ordinate of P. This means that there is a one-to-one correspondence between the points in a plane and the elements in the set of all ordered pairs of real numbers. And this is basis of analytic geometry. To them, this fundamental theorem of analytic geometry allows us to look at algebraic forms geometrically and also to look at

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geometric forms algebraically. In conclusion, the graph of an equation is the graph of all the ordered pairs in its solution set. To sketch the graph of an equation, we plot enough points from its solution set in a Cartesian coordinate system so that the total graph is apparent and then connect these points with a smooth curve.

Function can be represented in four different ways: i) Verbal (using words), ii) Visual (using a graph), iii) Algebraically (using formula) and iv) Numerical (using a table of values), (Stewart, Redlin & Watson, 2004). To them, a single function may be represented in all the four ways, and it is often useful to go from one representation to another to gain an insight into the function. However, certain functions are described more naturally by one method than by the others. But the most useful way to visualize a function is through its graph. If f is a function with domain A, then the graph of f is the set of ordered pairs [x, f(x)/x∈A]. In other words, the graph of f is the set of all points (x,y) such that y = f(x); that is the graph of f is the graph of the equation y = f(x). Hence, the graph of a function f gives the picture of the behavior or "life history" of the function. The values of a function are represented by the height of its graph above the x-axis. So we can read off the values of a function from its graph.

Sullivan and Sullivan III (1998) also said the graph of the function is the graph of the equation, that is, the set of points (x,y) in the xy-plane that satisfies the equation. For a function, each number x in the domain has one and only one image y. thus, the function cannot contain two points with the same x-coordinate and different y-coordinates. Therefore, the graph of a function must satisfy the vertical line test which states a set of points in the xy-plane is the graph of a function if and only if a vertical line intersects the graph in at most one point. In other words, if any vertical line intersects a graph at more than one point, the graph is not the graph of a function.

Again, the graph of a function f is just the graph of equation $y = f(x)$. it consist of those points in the Cartesian plane whose coordinates (x,y) are pairs of input-output values for f. thus, (x,y) lies on the graph of f provided x is in the domain of f and $y = f(x)$, (Adams & Essex, 2010). They avowed that drawing the graph of a function f sometimes involves making a table of coordinate pair [x, f(x)] for various values of x in the domain of f, then plotting these points connecting them with a "smooth curve". The graph of the function y = f(x) is the set of all points [x, f(x)], x in the domain of f. we match domain values along the x-axis with their range values along the y-axis to get the ordered pairs that yield the graph of $y = f(x)$, (Demana, Warts, Foley& Kennedy, 2007). They emphasized that a graph [set of points (x, y)] in the xy-plane defines y as a function of x if and only if no vertical line intersects the graph in more than one point.

Stevens (1994) made a contribution here that for every point (a,b) on the graph of g there corresponds a point (b, a) on the graph of g^{-1}. Thus, if we were to fold the coordinate plane along the dotted line $y = x$, the graph of g and g^{-1} would coincide. In other words, the graphs of g and g^{-1} are reflections of one another in the line $y = x$. He proceeded that this special relationship between the graphs of g and g^{-1} is true for any function and its inverse. Hornsby and Lial (1996) added to what they have stated earlier on about inverse functions. They claimed that a function is one-to-one if and only if no horizontal line intersects its graph more than once. The graph of $f^{-1}$is obtained by reflecting the graph of f in the line $y = x$.

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METHODOLOGY

It was an action research that employed both quantitative and qualitative designs to describe the situation. Purposive sampling was employed due to the inadequacy of logistic, financial constraints and ease of accessibility. The researcher again, used purposive sampling techniques because the researcher already knew student-teachers who have problem and this called for immediate intervention. The sample consisted of fifty (50) first years Diploma in Basic Education students, and this constituted one-sixth of the entire population of First year students of the College. A particular class, 1E (Students who offer Catering as their Elective Subject) was chosen for the study because it was here that the problem was identified. The class is made of fifteen (15) males and thirty-five (35) females. Although not the main aim of the study, it will be interesting to find out whether there will be any grades differences in the students' understanding of functions. This was possible after the mid-semester internal examination used as their School Based Assessment has been conducted. Students in this class exhibited poor performance in this particular topic after proper assessment was carried out. Interviews, pre-test and post-test and rating scale were also used to find out the effect of the graphical model in improving students' computational skills as well as students' achievement in mathematics. The pre-test and post-test were analyzed quantitatively. Scores obtained by students (participants) in the pre-test and post-test were organized into frequency distribution tables whereas the interview and rating scale were discussed and presented with pie charts. The means and standard deviations were calculated and used to test hypothesis as to whether there was significant improvement in the performance of students in the post-test or not.

Reliability and Validity

To ensure internal validity, "peer review" was employed. This was done by asking self-governing auditors such as Master of Education students majoring in Mathematics Education to comment on the results during the period of the data analysis. Then again, the data collected and interpretations were sent back to the students in order to ask if results were reasonable.

Intervention Design and Implementation

This sub-section of the study describes the various intervention strategies that were put in place to eradicate the problem the students were having in composition of functions and inverse of functions. It showed how these mechanisms were used to bring the situation under control. The action plan was divided into three sub-sections: pre-intervention, intervention (implementation) and post –intervention stage.

- **Pre –Intervention Analysis**

After analyzing the data collected through the interview and pre-test, it became evident that the poor performance of the students in the concepts of composition and inverse of functions had been caused by the following factors:

- Inadequate of qualified Mathematics tutors in pre-tertiary to handle the subject.
- Inadequate of teaching and learning materials in Mathematics classrooms.
- General perceptions about Mathematics being a difficult subject.
- Lack of interest and motivation in the topic.

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- Frequent refusal of qualified Mathematics tutors to teach in the respective schools.
- Poor methods of teaching Mathematics in classrooms.

Looking at the above data gathered as to why students performed poorly in functions, only two of these factors, lack of interest and motivation in the topic and the issue of poor methods of teaching Mathematics were relevant and therefore became the focus of this study.

- **Pre-Intervention Stage**

Data was collected on the students with regards to their poor performance in functions through interview and pre-test. Based on the data gathered, these mechanisms were put in place to solve the problem:

- Students were given a series of activities to develop their concept in composition and inverse of functions.
- Extra tuition was given to the participation students after classes' hours.
- Career guidance and counseling services were organized for students to keep them well informed of the prospects in the study of functions and mathematics in general.
- Students were well motivated to develop "I can do spirit" within themselves. \

- **Intervention (Implementation) Stage**

The interview administered by the researcher. Students were interviewed using six semi-structured interview questions. Responses from the interview sessions were analyzed question by question. Five (5) pre-test questions were written, typed and printed for students. Students were given thirty minutes to answer the questions. The researcher and the other mathematics tutors invigilated the students after which the scripts were collected, marked and the scores recorded. The scores were grouped into three categories. The first category was for those who scored between 0 and 5 whiles those who scored between 5 and 10 were placed in the second category and the 10 to 15 were also placed in the third category. The intervention design was implemented after the pre-test was administered using a day to day activity. The following were the subtopics that were treated; one week on administering of interview and pre-test and also distributing of course outlines to the participants, second week on using graphical model to develop the concept of composition of functions and inverse of functions and the third week on combining the graphical model and algebraic algorithm (formulae) to emphasize the concept of composite and inverse of functions and also administering pre-test. Planned visits of three days pre-week in three weeks intervention was implemented after the pre-test.

In order to sustain and arouse the interest of students in composition and inverse of functions, graphical model was used to develop the composition and inverse of function concepts. The strategy to teaching using graphical model involves demonstration, problem solving, discussion and the principles of multiple embodiment of representing functions concepts since the theoretical framework of the study was based on the constructivist theory of learning. In these sense, the researcher involved the students with a lot of activities on the composition and inverse of functions to arouse and sustain their interest in class. Students developed self confidence in the use of the graphical model in learning composition and inverse of functions. As part of the implementation design, students (participants) were also educated to use

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graphing calculators to support the learning where necessary. Personal calculators, mathematical instruments, work book and other logistics to speed up the implementation design were adopted.

Motivation was given to hardworking, regular and punctual students to class. Provoking questions were designed to keep students on their toes in order not to relax which could throw the purpose of the study overboard. This plan was used to inculcate in students the spirit of self-confidence in learning composition and inverse of functions and mathematics in general. Individual differences were taken care of in the study, and students were also given course outline and guidelines regarding the day to day activity in class work and discussion. Extra tuition was given to slow learners after the normal contact hours. Students constructed their own learning was the hallmark of students' understanding of composition and inverse of function concepts in this study. During the extra tuition hours, students who had already grasped the concept were given more challenging tasks as a sort of motivation.

Based on the fact that the class was heterogeneous in nature, that is students come from different socio-economic backgrounds and diverse learning environments, the researcher used mixed ability grouping based on interest, competence, gender, attitudes and social economic background to carry out the activity. Finally, as part of the implementation design the entire team of Mathematics tutors in the department in consultation with Vice Principal Academic educated the students on some prospects that awaited students who offered Mathematics at the higher levels in a forum. This was done in order to inculcate "interest and love" for the learning of Mathematics in the students.

❖ Post Intervention Stage

After the intervention, five (5) post-test questions were administered to the students to find out whether students computational skills had improved using the graphical model. The same time of thirty minutes was given to the students as it was allotted for them in the pre-test. The researcher and other mathematics tutors invigilated the students after which the scripts were collected, marked and the scores recorded. It was realized that students' performance in composition and inverse functions has been improved tremendously. This was made manifest during the end of first semester Diploma in Basic Education Examination from Institute of Education, University of Cape Coast. When result were published, the fifty students who participated in the study, 10 of them got 'A'; 15 had 'B^+'; 12 got 'B'; 8 had 'C^+' and the remaining 5 also scored 'C'. Based on the above facts and figures produced from the just ended semester exams, the Head of Department, the Vice Principal Academic and other Mathematics tutors in the College were convinced and testified that students' performance in Mathematics has improved tremendously.

❖ Rubrics for Scoring Pre-test and Post-test

The marks awarding system served the purpose of assigning the learners into various. Each correct answer to the 5-item test was assigned 3 points. Hence, each student's scored ranged from 0-15 marks. The percentage score was calculated for each student and question analysis of students' pre-test and post-test was done using SPSS paired –test.

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ANALYSIS AND DISCUSSION

The results were presented under the following themes:

- The use of the graphical model to develop the concept of composition and inverse of functions to improve students' performance in Mathematics as a whole.
- The use of the graphical model to motivate students' interest in composition and inverse of functions

Research Question 1. ***To what extent will the use of the graphical model to develop the concept of composition and inverse of functions improve students' performance in Mathematics as a whole?***

The first research question raised for the study was to find out the impact of the graphical model to develop the concept of composition and inverse of functions to improve OFCE level 100 students' performance in mathematics. In order to accomplish this, OFCE Level 100 students were taking through as series of activities in the intervention as described in the previous chapter. Here pre-test and the post-test were used as an instrument to analyse this question. The number of students who had application and knowledge questions correct in the pre-test and post-test were presented in Tables 1 and 2

Table 1. Number of students who had Application and Knowledge questions correctly in the Pre-test and Post-test

Students marks	Pre-test Results		Post-test Results	
	N	%	N	%
0	32	64	7	14
Above 3	18	36	43	86

Field study, 2018

From table 1, 32 students constituted (64%) scored zero (0) in the application questions in pre-test whereas in the post-test 7 students (14%) scored zero(0) but in the case of the knowledge questions, 18 students (36%) and marks above 3 in the pre-test while 43 students (86%) also had marks above 3. The above statistics indicates that majority of the students had difficulty in solving the application questions in the pre-test but did better in the post-test.

Table 2. Number of students who used wrong computational skills for each item in the Pre-test and Post-test

Students marks	Pre-test		Post-test	
Test Items	N	%	N	%
Item 1	30	60	3	6
Item 2	19	38	2	4
Item 3	16	32	5	10
Item 4	9	18	7	14
Item 5	16	32	8	16

Field study, 2018

From Table 2, further analysis indicates that students' total errors in both the pre-test and post-test were 114. Out of this number of errors, students committed more errors in the pre-test than

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in the post-test. In the pre-test, the total errors were ninety (90) with application of concept having the highest number whilst knowledge questions were on the minimum. In the post-test, the total errors were twenty-five (25) with application of concept still having the highest number whiles knowledge questions were again on the minimum. From the errors made in the pre-test and post-test in Table 2, students' difficulties reduced immensely after they have been taken through the intervention. Table 2, indicates the descriptive statistics on students' group pre-test and post-test scores differences with respect to the mean and standard deviation with the latter better than the former (i.e. with pre-test mean and SD 1.44 and 0.577 and post-test mean and SD 2.68 and 0.513). This above statistics has shown that the standard deviation value of the pre-test (0.577) is larger than the post-test (0.513) which indicated that the pre-test was more spread in the data set in relation to the mean than the post-test with the least standard deviation.

Table 3. Descriptive statistics on students' group pre-test and post –test scores

	N	**Mean**	**Std. Deviation**
Group pre-test	50	1.44	0.577
Group post-test	50	2.68	0.513

Field study, 2018

Table 3, indicates that there was a difference in the performance between the male and female in the pre-test scores with respect to the minimum, maximum, mean and standard deviation with the male (i.e. with pre-test mean and SD 5.80 and 4.004) performed better than the female (i.e. with pre-test mean and SD 3.51 and 2.571).

Table 4. Descriptive statistics showing the differences in the performance between the male and female in the pre-test scores.

	N	Mean	Std Deviation	Minimum	Maximum
Male	15	5.80	4.004	0	12
Female	35	3.51	2.571	0	9

Field study, 2018

Further analysis also showed that, there was significant difference in the performance between the male and female in the post-test with respect to the minimum, maximum, mean and standard deviation with the male (i.e. with post-test mean and SD 12.00 and 3.586) again performed better than the female (i.e. with post-test mean and SD 11.57 and 2.638).

Table 5. Descriptive statistics showing the difference in the performance between the male and female in the post-test scores.

	N	Mean	Std Deviation	Minimum	Maximum
Male	15	12.00	3.586	3	15
Female	35	11.57	2.638	6	15

Field study, 2018

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In both Table 4, and Table 5, the performances of the male students were comparatively better than their female counterparts. The situation was little bit improved in the case of the female in the post-test since the difference in mean was 0.43 as against mean difference of 2.29 in the pre-test. Table 6, indicates that there was a difference in pre-test and post-test scores with respect to the minimum, maximum, mean and standard deviation with the latter being better than the former (i.e. with pre-test mean and SD 4.20 and 3.21 and post-test mean and SD 11.70 and 2.92).

Table 6. Descriptive statistics on students' pre-test and post-test scores

	N	Mean	Std. Deviation	Minimum	Maximum
Male	50	4.2	3.207	0	12
Female	50	11.7	2.922	3	15

Field study, 2018

Further analysis was conducted to find out if the differences in mean were statistically significant. The results of the t-test (49df, t-55.871, and p = 0.00) also indicated that the difference in means was significant at p=0.00. Hence the null hypothesis (H_0) that there is no significant difference between students pre-test and post-test is rejected in favour of the alternative hypothesis (H_a). This is so because p=0.00<0.05 suggest that H is unlikely to be true. And since the students post-test (M=11.70, SD = 2.922) was higher than their pre-test, (M=4.20, SD =3.207), it can be argued that there was significant difference between the post-test. Again, smaller the standard deviation the better indication of variation in the data set.

Table 7: Overall students' performance in the pre-tests and post-test

Students marks	Pre-test Results		Post-test Results	
	N	%	N	%
0-5	30	60	1	2
6-15	20	40	49	98

Field study, 2018

The analysis of the results from Table 7 indicates that students' general performance was better in the post-test than in the pre-test. Students' performance in the pre-test was very weak as majority of the students had below the mark of 5. The post-test results also indicated that the students' performance was excellent since most of the students had above the marks of 6. Comparatively, the performance of students in the post-test was far better than the performance of students in the pre-test.

Research Question 2. ***To what extent will the use of the graphical model motivate students' interest in the topic?***

The last research question raised for the study was to find out the rate at which the graphical model has motivated the students' interest in composition and inverse of functions. In order to achieve this, OFCE Level 100 students were taken through a series of activities in which "self-confident" and "I can do spirit" were inculcated in the students during the intervention as described in the previous chapter. Here rating scale questionnaire was used as an instrument to

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analyze this question. Frequency table and pie chart were used to analyze each of the four items in questionnaire. Item 1 which states that students have been using graphical model was presented in Table 8.

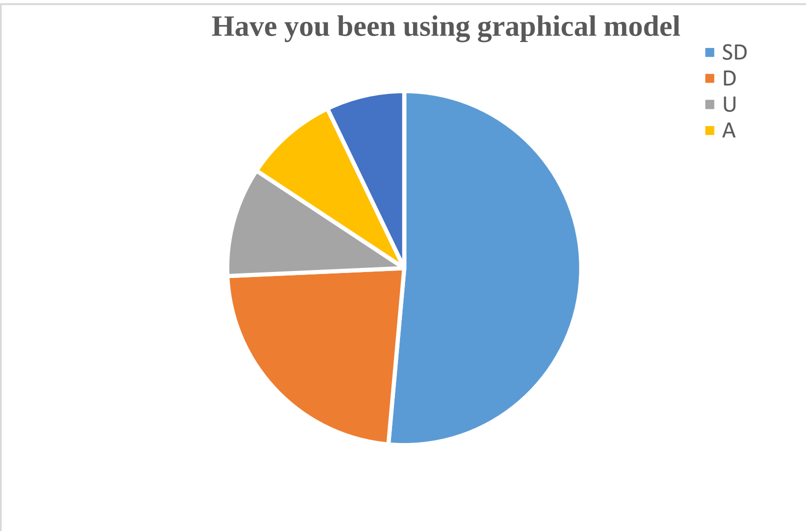
Table 8. Number of Students who rated that they have been using graphical model

Rating	Frequency	Percent
SD	25	50
D	4	8
U	3	6
A	5	10
SA	13	26
Total	50	100

Field study, 2018

From table 8 above, 25 out of the 50 students which constituted 50% said they strongly disagree on item 1. This implied that they have not been using graphical model for solving composition and inverse of functions. 4 of the students (8%) also disagree to the same item 1, 3 of them (6%) rated undecided, 5 of the students (10%) agreed and 13 of them (26%) said they strongly agreed that they have been using graphical model. In a whole majority of the students have not been using graphs for solving composition and inverse of functions.

This is only a different representation of the same data using pie chart. The pie chart also revealed similar results. The pie chart was presented below.



Field study, 2018

From the pie chart above, one could testify that the students who said they strongly disagreed (SD) have not been using the graphical model and they formed half of the portion of the pie chart as compared to the students who rated strongly agreed (SA). In short, most of the students have not been using graphs when it comes to solving composition and inverse of functions.

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Table 9. Students' Responses on Item 2 which shows their first experience in the use of graphical model

Rating	Frequency	Percent
SD	5	10
D	2	4
U	4	8
A	7	14
SA	32	64
Total	50	100

Field study, 2018

The analysis of the result from Table 9 indicates those students' responses on Item 2 which states that this is my first time of applying graphical model was presented in Table 9. From the table, 5 out of the 50 students (10%) said they strongly disagree on item 2. This implied that this was not their first time of applying graphical model for solving composition and inverse of functions. 2 of the students (4%) also disagree the same item 2, 4 of them (8%) rated undecided, 7 of the students (14%) agreed and 32 of them which constituted 64% said they strongly agreed that this was their first time of applying graphical model for solving composition and inverse of functions. In totality, majority of the students testified that this was their first time of applying graphs for solving composition and inverse of functions.

Table 10, Students' responses on Items 3 which states that the use of graphical model has helped to build their self confidence in solving composition and inverse of functions

Rating	Frequency	Percent
SD	5	10
D	7	14
U	5	10
A	13	26
SA	20	40
Total	50	100

Field study, 2018

Table 10, indicates those students' responses on item 3 in the rating scale questionnaire which states that the use of graphical model has made them confident in solving composition and inverse of functions was presented beneath. From the table, 5 out of the 50 students (10%) said they strongly disagreed on item 3. This implied that they have not acquired any confident after they have used graphical model for solving composition and inverse of functions. 7 of the students (14%) also disagree to the same item 3, 5 of them (10%) rated undecided, 13 of the students (26%) agreed and 20 of them which constituted 40% said they strongly agreed that the use of graphical model has made them confident in solving composition and inverse of functions. In a whole majority of the students did testify that the use of graphical model has made them confident in solving composition and inverse of functions. Again, to buttress the above analysis pie chart was used. The pie chart below also revealed similar results.

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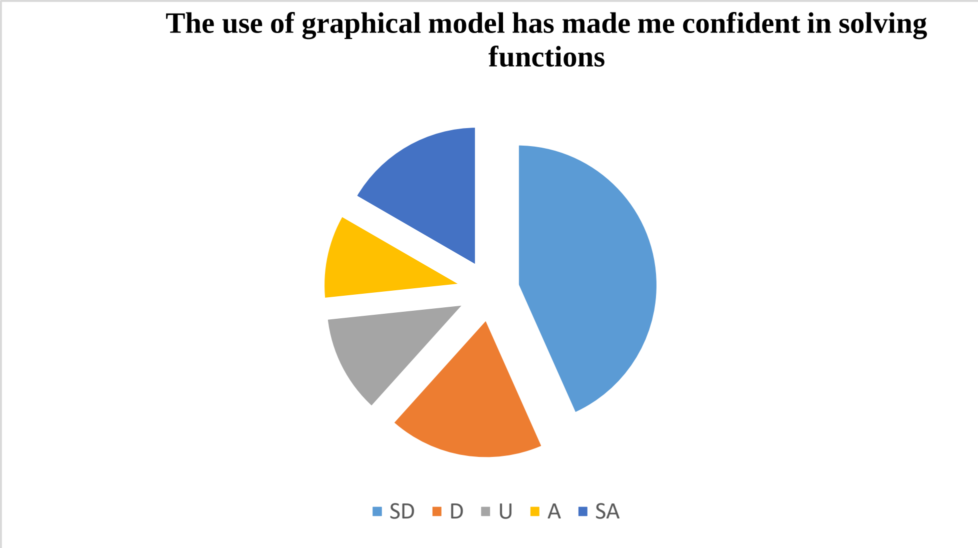

Field study, 2018

The Pie chart above revealed that, a greater number of the students said they strongly agreed (SA) that the use of graphical model for solving composition and inverse of functions has made them confident and this formed greater number of the respondents and followed by the students who rated agreed (A) and the rest in that order to the students who rated strongly disagreed (SD). In brief, majority of the students showed confidence when it comes to solving composition and inverse of functions using graphs.

Table 11. Number of students who rated that they were no longer afraid of the composition and inverse of functions.

Rating	Frequency	Percent
SD	4	8
D	8	16
U	1	2
A	26	52
SA	11	22
Total	50	100

Field study, 2018

The responses showed that, Students were no longer afraid of the composition and inverse of functions. From table 11, 4 out of the 50 students which constituted 8% said they strongly disagree for the fact that they were no longer afraid of the composition and inverse of functions. This implied that they afraid when solving composition and inverse of functions. 8 of the students (16%) also disagree to the same item 4, 1 of them (2%) rated undecided, 26 of the students represented 52% agreed and 11 of them (22%) said they strongly agreed that they were no longer afraid of the composition and inverse of functions. In short, greater parts of the students testified that they were no longer afraid of solving composition and inverse of functions. Further analysis was done on the same vein using pie chart. The pie chart also showed similar results.

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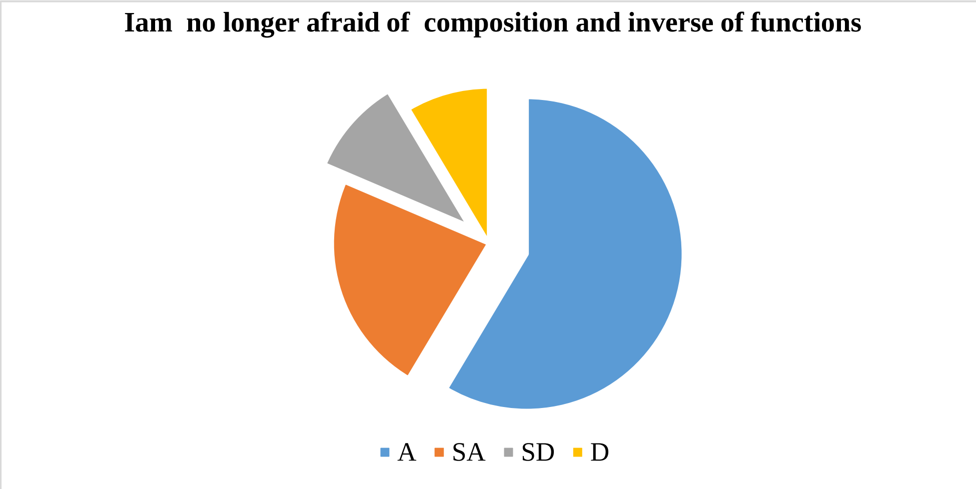

Field study, 2018

From the pie chart above, a greater number of the students said they agreed (A) for the fact that they were no longer afraid of solving composition and inverse of functions followed by strongly agreed (SA) and this formed greater number of the students as compared to the those who rated strongly disagreed (SD) and disagreed (D). Finally, majority of the students confirmed that they no longer afraid to solve mathematical problems involving composition and inverse of functions.

CONCLUSION

On the basis of the findings of the study, the following conclusions were drawn:

Mathematics topics in the syllabus are more abstract and theoretical, and heavily founded on the basis of deductions. The way the syllabi and textbooks are written seems to be a fairly accurate reflection of how functions are taught in the various levels, and as an extension, how mathematics is related in these levels, mathematics is treated more as a very exact science, taught for its own sake, using its specific language. The research literature that looks at the students' understanding or knowledge of the topics composition and inverse of functions is very limited. The present study contributes to the field of mathematics education in studying the content knowledge of students regarding the two above mathematical topics.

Another contribution that this study brings to the field of mathematics education consists in the methods of collecting the data. To collect data for investigating the students' concept development of composition and inverse of functions, the researcher used semi-structure interview and tests. The reasons for using this combination of methods for data collection were presented extensively in the analysis. In addition to what is contained there, the researcher considered rating scale questionnaire to find out the rate at which the students' interest in composition and inverse of functions were motivated.

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RECOMMENDATIONS

The following recommendations are made for the improvement of the mathematics curriculum in Ghana:

- Curriculum developers and policy makers should emphasize the use of visual manipulative such as graphs to teach certain topics in Mathematics especially composition and inverse of functions.

- Teachers are advised to treat composition of functions before inverse of functions since they are interrelated and composition of a function acts as a sort of catalyst to speed up the learning of inverse of functions.

- Constructivist theory of learning should be encouraged in all spheres of Mathematics classroom environments so as to give learners the opportunity to find meaning to their own learning in order to explore new ideas and findings concerning their future. It also promotes the doing of Mathematics as a creative, sense making activity that entails interpretation, hypothesizing, effort and exploration.

REFERENCES

Adams, R.A & Essex, C.(2010). *Calculus: A Complete Course*, (7th Ed.)

Ball, D.L. &Bass, H. (2000). *Interweaving Content and Pedagogy in teaching and learning to teach: Knowing and using mathematics*. In J Boaler (Ed.), multiple perspectives on the teaching and learning of mathematics. Westport.CT:Alex.

Barnett, R.A., Ziegler, M.R & Byleen, K.E. (1999). *College Mathematics For Business, Economics, Life Sciences And Social Sciences*, (8th Ed.)

Bauersfeld, H.(1995). *The Structuring of the Structures: Development and Function of Mathematizing as a Social Practice*. Inc: P. Steffe&J. Gale (Eds.). Constructivism in Education. Hillsdale, New Jersey: Lawrence Eribaum Associates Publishers.

Brownstein, J. (2001). *Constructivism Learning Theory.* Hardcover Beaun Blankets. *Constructivism (learning theory)* (n.d). Retrieved July 28, 2010, from http://www.en.wikipedia.org/wiki/

Davis P.J. & Hersh, R.(1981). *The Mathematical Experience.* Boston: Birkhauser.

Demana, F.D., Warts, B.K., Foley, G.D. & Kennedy, D. (2007). *Precalculus Graphical, Numerical, Algebraic,* (7th Ed.). Boston: Pearson Education, Inc.

Flanders, H. And Price, J.J.(1975). *Algebra and Trigonometry by Academic Press*, Inc.

Gardiner, M. (1982). *"An Attribute Determinance:* A Function of Past Memory and External Factors" in advance in Consumer Research, A. Mitchell(Ed.).

Gellert, W., Kustner, H., Hellwich, M. & Kastner, H.(1975). *The Function constructs a Roman numeral representative from its positive.* Van Nostrand Reinhold Company, New York.

Hornsby, Jr. E.J. & Lial, M.L. (1996). *A Graphical Approach to College Algebra.* http://www.lab.brown.edu.

Hungerford, L. (1999). *Mathematics with Applications*, (7th Ed.) by Addison –Wesley Publishers, Inc.

Print ISSN: 2053-2229 (Print), Online ISSN: 2053-2210 (Online)

Institute of Education Chief Examiners' report (2010) on mathematics, FDC 112: Number and Basic Algebra dated February 24 – March 4, Diploma in Basic Education Examination from institute of Education, University of Cape Coast.

Kirschner, P.A., Sweller, J., and Clark, R.E. (2006) Why minimal guidance during instruction does not work: and analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry –base teaching. Education Psychologist 41(2) 75-86.

Kleiner, I. (1993). *Functions: historical and pedagogical aspects*. Science and Education, Vol. 2, no. 2, p. 183-209. The Netherlands: Kluwer Academic Publishers.

Knill, G., Baxter, R., Dottori, D., Faweetl, G., Forest, M.L., Hamilton, M. & Pasko, S.(1998). Mathpower 10. Canada: McGraw –Hill Ryerson Ltd.

Knill, G., Baxter, R., Dottori, D., Faweetl, G., Forest, M.L., Hamilton, M. & Pasko, S.(1999). Mathpower 11. Canada: McGraw-Hill Ryerson Ltd.

Lee, P. Y. et al. (1999b). *Pure Mathematics*. Singapore, Shing Lee Publishers Pte.

Mayer, R. (2004). "Should there be a three –strikes rule against pure discovery learning? The case for guided methods of instruction" *American Psychologist* 59 (1): 14-19.

Nastasescu, C., Nita, C. And Rizescu, G. (1993). *Matematica. Algebra* IX [Mathematics, Algebra, Grade 9], Bucharest, Romania: Editura Didactica si Pedagogica.

Schaaf, W.L. (1969). *Basic Concepts of ELEMENTARY MATHEMATICS*, (3rd Ed.)

Schnitzer, A. & Stillwell, J.(2002). *Mathematics Evolutions:* USA: The Mathematical Association of America.

Stevens, E.D.(1994). *College Algebra & Trigonometry.*

Stewart, J., Redlin, L. & Watson, S. (2004). *College Algebra*, (4th Ed.)

Sullivan, M. & Sullivan III, M.(1998). *College Algebra:* Graphing and Data Analysis

Sweller, J. (1988). *"Cognitive load during problem solving:* Effects on learning". *Cognitive Science* 12(1): 257-28

Weisstein, E. (2005a). Bijection. MathWorld. *Retrieved February 5th 2005 from the World Wide Web:* http://mathworld.wolfam.com/Bijection.html

Weisstein, E.(2005b). Injection. MathWorld. *Retrieved February 5th 2005 from the Worldwide Web:* http://mathworld.wolfam.com/One-to-one.html

wertsch, J.V(1997) "Vygotsky and the formation of the mind" Cambridge

Wilson, M.R. & Krapfl, C.M. (1994). The impact of graphics Calculators on students' understanding of function. *Journal of Computers and Science Teaching*, 13(3), 252-264.

Print ISSN: 2053-2229 (Print), Online ISSN: 2053-2210 (Online)