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THE NON- BAYESIAN ESTIMATORS METHODS FOR PARAMETERS OF EXPONENTIATED WEIBULL (EW) DISTRIBUTION

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ABSTRACT: *In this paper we derivative and find the statistical formula for the estimater of three unknown parameters α, β, λ for Exponentiated Weibull (EW) by three Non- Bayesian methods, maximun likelihood estimator method(MLEM), rank set sampling estimator method(RSSEM) and ordinary least squares estimator method(OLS), the all equations in this paper for all using methods are solved by Newton-Raphson method, Monte Carlo simulation procedure are used in this paper to generated many sample sizes and compare between them by using Mean squares error measure.*

KEYWORDS: Exponentiated Weibull distribution, Non- Bayesian estimators methods, Monte Carlo simulation technique, Mean squares error measure.

INTRODUCTION

In (1993)[7] Mudholkar and Srivastava introduced the Exponentiated Weibull family distribution (EW) as the extension of weibull family. In (1995)[8] Mudholkar introduced the applications of exponentiated weibull (EW) distribution. Mudholkar and Hutson in (1996)[9] studies the reliability and survival about (EW) distribution. Nassar and Eissa in (2003)[12] studied in more detail the properties of (EW) distribution. Pal and Woo in (2006)[13]compared the exponentiated weibull with two-parameter weibull and gamma distributions with respect to failure rate. Raqab and Madi in (2009)[10]studied Bayesian estimation and perdiction for the (EW) distribution using informative and non-informative priors has been considered. Salem and Abo-kasem in (2011)[14] studied Bayes and non-Bayes estimator for (EW) distribution have been obtained when sample is progressive hybrid censoring scheme. Debanshee and Datta in (2013)[15] fitted a weibull distribution and (EW) distribution for wind speed data and find the mean and variance of wind speed data, then estimated the parameter by using MLE method. The Exponentiated Weibull family distribution (EW) contains distributions with bathtub-shaped and unimodal failure rates desides a broader class of monotone failure rates.
The probability density function for Exponentiated Weibull distribution (EW) is:

$$f(t;\alpha,\beta,\lambda) = \alpha\,\beta\,\lambda^{\beta}\,t^{\beta-1}\,e^{-(\lambda t)^{\beta}}\left(1-e^{-(\lambda t)^{\beta}}\right)^{\alpha-1} \quad ,t>0 \quad \ldots.(1)$$

$$\acute{\Omega} = \{(\alpha,\beta,\lambda);\alpha>0,\beta>0,\lambda>0\}$$

where the shape parameters are α and β, the scale parameter is λ
The cumulative distribution function is;

$$F(t;\alpha,\beta,\lambda) = \left(1-e^{-(\lambda t)^{\beta}}\right)^{\alpha} \quad ,t>0 \quad \ldots\ldots..(2)$$

In (2007)[1] and (2012)[3] Ashourb and Eisa introduced and study the hazard and the survival functions .
The hazard rate function is given by;

$$h(t) = \frac{f(t)}{[1-F(t)]} \qquad t>0\ldots\ldots.(3)$$

The survival function is given by;

$$S(x) = 1-\left(1-e^{-(\lambda t)^{\beta}}\right)^{\alpha} \qquad t>0\ldots\ldots.(4)$$

Because the EW family is very flexible family we can used for modeling several types of skewed lifetime data (2009)[10]
We have amany distributions which are a special case of EW family as follows;
(α=1,β=1) lead to exponential distribution, if (α=1) lead that to weibull distribution, if (α=1,β=2) lead to Rayleigh distribution, if (β=1) lead to (GE) generalized exponential, if (β=2) lead to (GR) generalized Rayleigh distribution.

Maximun Likelihood Estimator Method(MLEM)
Suppose a complete sample t=(t_1,t_2, ……,t_n) this sample are obtained, to estimate the parameters for the distribution, and recovded from life test of n individuals[4], where the life times have distributed as (EW) distribution.
The Likelihood function of exponentiated weibull (EW)distribution is:-

$$L = \prod_{i=1}^{n} f(t_i;\alpha,\beta,\lambda)$$

$$L = \prod_{i=1}^{n}\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1} \prod_{i=1}^{n}(t_i)^{\beta-1}\ e^{-(\lambda\sum_{i=1}^{n} t_i)^{\beta}}\ \lambda^{n\beta}\ \beta^{n}\ \alpha^{n}\ \ldots\ldots(5)$$

Taking the natural logarithm for the likelihood function, so we get;

$$\ln L = (\alpha-1)\sum_{i=1}^{n}\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right) + (\beta-1)\sum_{i=1}^{n}\ln t_i - \left(\lambda\sum_{i=1}^{n} t_i\right)^{\beta} + n\beta\ln\lambda + n\ln\beta + n\ln\alpha\ ..(6)$$

The partial derivatives for the log-likelihood function with respect to unknown parameters α,β and λ are:

$$\frac{\partial \ln L}{\partial\alpha} = \sum_{i=1}^{n}\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right) + \frac{n}{\alpha}\ldots..(7)$$

$$\frac{\partial \ln L}{\partial\beta} = (\alpha-1)\sum_{i=1}^{n}\frac{-e^{-(\lambda t_i)^{\beta}}.-(\lambda t_i)^{\beta}\ \ln\lambda t_i\ .1}{1-e^{-(\lambda t_i)^{\beta}}} + \sum_{i=1}^{n}\ln t_i - \left(\lambda\sum_{i=1}^{n} t_i\right)^{\beta}\ln\lambda\sum_{i=1}^{n} t_i + n\ln\lambda + \frac{n}{\beta}\ldots(8)$$

$$\frac{\partial \ln L}{\partial\lambda} = (\alpha-1)\sum_{i=1}^{n}\frac{-e^{-(\lambda t_i)^{\beta}}.-\beta(\lambda t_i)^{\beta-1}\ t_i}{1-e^{-(\lambda t_i)^{\beta}}} - \beta\left(\lambda\sum_{i=1}^{n} t_i\right)^{\beta-1}\sum_{i=1}^{n} t_i + \frac{n\beta}{\lambda}\ldots.(9)$$

For the above equations(7,8,9)we cannot separate the estimator of any parameters from the other two parameters then the three function $f(\alpha), g(\beta)$ and $z(\lambda)$ are the first derivative of natural logarithm likelihood function with respect to unknown parameters α, β, and λ respectively.

$$f(\alpha) = \sum_{i=1}^{n}\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right) + \frac{n}{\alpha}\ldots..(10)$$

$$g(\beta) = (\alpha-1)\sum_{i=1}^{n}\frac{-e^{-(\lambda t_i)^{\beta}}.-(\lambda t_i)^{\beta}\ \ln\lambda t_i\ .1}{1-e^{-(\lambda t_i)^{\beta}}} + \sum_{i=1}^{n}\ln t_i - \left(\lambda\sum_{i=1}^{n} t_i\right)^{\beta}\ln\left(\lambda\sum_{i=1}^{n} t_i\right) + n\ln\lambda + \frac{n}{\beta}..(11)$$

$$z(\lambda)=(\alpha-1)\sum_{i=1}^{n}\frac{-e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1}\,t_i}{1-e^{-(\lambda t_i)^\beta}}-\beta\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta-1}\sum_{i=1}^{n}t_i+\frac{n\beta}{\lambda}\ldots\ldots(12)$$

this three-nonlinear equation are hardness to solve, then using Newaton-Raphson method[5] to estimate the parameters of (EW) distribution.

Now, we need to find the Jacobean matrix J which are the first derivatives for each equations $f(\alpha), g(\beta)$ and $z(\lambda)$ with respect to α, β, and λ.

$$J=\begin{bmatrix}\frac{\partial f(\alpha)}{\partial\alpha} & \frac{\partial f(\alpha)}{\partial\beta} & \frac{\partial f(\alpha)}{\partial\lambda}\\ \frac{\partial g(\beta)}{\partial\alpha} & \frac{\partial g(\beta)}{\partial\beta} & \frac{\partial g(\beta)}{\partial\lambda}\\ \frac{\partial z(\lambda)}{\partial\alpha} & \frac{\partial z(\lambda)}{\partial\beta} & \frac{\partial z(\lambda)}{\partial\lambda}\end{bmatrix}=\begin{bmatrix}\frac{\partial^2\ln L(t_i;\alpha,\beta,\lambda)}{\partial\alpha^2} & \frac{\partial^2\ln L(t_i;\alpha,\beta,\lambda)}{\partial\alpha\partial\beta} & \frac{\partial^2\ln L(t_i;\alpha,\beta,\lambda)}{\partial\alpha\partial\lambda}\\ \frac{\partial^2\ln L(t_i;\alpha,\beta,\lambda)}{\partial\beta\partial\alpha} & \frac{\partial^2\ln L(t_i;\alpha,\beta,\lambda)}{\partial\beta^2} & \frac{\partial^2\ln L(t_i;\alpha,\beta,\lambda)}{\partial\beta\partial\lambda}\\ \frac{\partial^2\ln L(t_i;\alpha,\beta,\lambda)}{\partial\lambda\partial\alpha} & \frac{\partial^2\ln L(t_i;\alpha,\beta,\lambda)}{\partial\lambda\partial\beta} & \frac{\partial^2\ln L(t_i;\alpha,\beta,\lambda)}{\partial\lambda^2}\end{bmatrix}\ldots(13)$$

$$\frac{\partial f(\alpha)}{\partial\alpha}=-\frac{n}{\alpha^2}\ldots.(14)$$

$$\frac{\partial f(\alpha)}{\partial\beta}=\sum_{i=1}^{n}\frac{-e^{-(\lambda t_i)^\beta}.-(\lambda t_i)^\beta\ \ln\lambda t_i\ .1}{1-e^{-(\lambda t_i)^\beta}}=\sum_{i=1}^{n}\frac{e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta\ \ln\lambda t_i}{1-e^{-(\lambda t_i)^\beta}}\ldots\ldots(15)$$

$$\frac{\partial f(\alpha)}{\partial\lambda}=\sum_{i=1}^{n}\frac{-e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1}\,t_i}{1-e^{-(\lambda t_i)^\beta}}=\sum_{i=1}^{n}\frac{e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{\beta-1}\,t_i}{1-e^{-(\lambda t_i)^\beta}}\ldots\ldots\ldots\ldots(16)$$

$$\frac{\partial g(\beta)}{\partial\alpha}=\sum_{i=1}^{n}\frac{-e^{-(\lambda t_i)^\beta}.-(\lambda t_i)^\beta\ \ln\lambda t_i\ .1}{1-e^{-(\lambda t_i)^\beta}}=\sum_{i=1}^{n}\frac{e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta\ \ln\lambda t_i}{1-e^{-(\lambda t_i)^\beta}}\ldots\ldots(17)$$

$$\frac{\partial g(\beta)}{\partial\beta}=(\alpha-1)\sum_{i=1}^{n}\frac{e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta\,(\ln(\lambda t_i))^2\ [1-e^{-(\lambda t_i)^\beta}-(\lambda t_i)^\beta\,]}{\left(1-e^{-(\lambda t_i)^\beta}\right)^2}-\left(\lambda\sum_{i=1}^{n}t_i\right)^\beta\left(\ln(\lambda\sum_{i=1}^{n}t_i)\right)^2-\frac{n}{\beta^2}\ldots(18)$$

$$\frac{\partial g(\beta)}{\partial\lambda}=-\left[\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta-1}\sum_{i=1}^{n}t_i+\left(\ln\lambda\sum_{i=1}^{n}t_i\right)\beta\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta-1}\sum_{i=1}^{n}t_i\right]+\frac{n}{\lambda}$$
$$+(\alpha-1)\sum_{i=1}^{n}\frac{\left(1-e^{-(\lambda t_i)^\beta}\right)\left[-e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{2\beta-1}t_i\ \ln\lambda t_i+e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{\beta-1}\,t_i\ln\lambda t_i+e^{-(\lambda t_i)^\beta}(\lambda t_i)^{\beta-1}t_i\right]}{\left(1-e^{-(\lambda t_i)^\beta}\right)^2}$$
$$-(\alpha-1)\sum_{i=1}^{n}\frac{[e^{-2(\lambda t_i)^\beta}\beta(\lambda t_i)^{2\beta-1}t_i\ \ln\lambda t_i]}{\left(1-e^{-(\lambda t_i)^\beta}\right)^2}\ldots\ldots(19)$$

$$\frac{\partial z(\lambda)}{\partial\alpha}=\sum_{i=1}^{n}\frac{-e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1}\,t_i}{1-e^{-(\lambda t_i)^\beta}}=\sum_{i=1}^{n}\frac{e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{\beta-1}\,t_i}{1-e^{-(\lambda t_i)^\beta}}\ldots..(20)$$

$$\frac{\partial z(\lambda)}{\partial\beta}=-\left[\beta\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta-1}\sum_{i=1}^{n}t_i\,(\ln\lambda\sum_{i=1}^{n}t_i)+\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta-1}\sum_{i=1}^{n}t_i\right]+\frac{n}{\lambda}$$
$$+(\alpha-1)\sum_{i=1}^{n}\frac{\left(1-e^{-(\lambda t_i)^\beta}\right)\left[-e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{2\beta-1}.\ln\lambda t_i\ .t_i+e^{-(\lambda t_i)^\beta}(\lambda t_i)^{\beta-1}\,t_i+e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{\beta-1}\ln(\lambda t_i)\,t_i\right]}{\left(1-e^{-(\lambda t_i)^\beta}\right)^2}$$

$$-(\alpha-1)\sum_{i=1}^{n}\frac{\left[e^{-2(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{2\beta-1}.t_i\ln\lambda t_i\right]}{\left(1-e^{-(\lambda t_i)^{\beta}}\right)^2}\dots\dots\dots(21)$$

$$\frac{\partial z(\lambda)}{\partial\lambda}=-\beta(\beta-1)\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta-2}\left(\sum_{i=1}^{n}t_i\right)^2-\frac{n\beta}{\lambda^2}$$

$$(\alpha-1)\sum_{i=1}^{n}\frac{\left(1-e^{-(\lambda t_i)^{\beta}}\right)\left[e^{-(\lambda t_i)^{\beta}}\beta(\beta-1)(\lambda t_i)^{\beta-2}t_i^2-t_i^2\beta^2(\lambda t_i)^{2(\beta-1)}e^{-(\lambda t_i)^{\beta}}\right]}{\left(1-e^{-(\lambda t_i)^{\beta}}\right)^2}$$

$$-(\alpha-1)\sum_{i=1}^{n}\frac{e^{-2(\lambda t_i)^{\beta}}\beta^2(\lambda t_i)^{2\beta-2}.t_i^2}{\left(1-e^{-(\lambda t_i)^{\beta}}\right)^2}\dots\dots(22)$$

$$\begin{pmatrix}\alpha_{i+1}\\ \beta_{i+1}\\ \lambda_{i+1}\end{pmatrix}=\begin{pmatrix}\alpha_i\\ \beta_i\\ \lambda_i\end{pmatrix}-J^{-1}\begin{pmatrix}f(\alpha)\\ g(\beta)\\ z(\lambda)\end{pmatrix}\dots\dots\dots(23$$

Rank Set Sampling Estimator Method(RSSEM)

The seconed non-Bayesian Rank Set Sampling Estimator Method(RSSEM) disucss here, this method is one of sampling techniques which benefits from order statistics[2,11],the first step is to find the probability density function of y_i which is an order statistic formulated as follows;

$$g(y_i;n)=\frac{n!}{(i-1)!\,(n-i)!}[F(y_i)]^{i-1}[1-F(y_i)]^{n-i}f(y_i)\quad a<y_i<b\dots(24)$$

$$g(y_i;n)=\frac{n!}{(i-1)!\,(n-i)!}\left[\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{i-1}\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{n-i}\left[\alpha\,\beta\,\lambda^{\beta}t_i^{\beta-1}\,e^{-(\lambda t_i)^{\beta}}\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}\right]\dots(25)$$

let $k=\frac{n!}{(i-1)!(n-i)!}$

The second step of this method is to apply (MLE) for parameters by using (RSSEM) for $g(y_i)$.

The likelihood function of sample $t_{(1)},t_{(2)},\dots,t_{(n)}$ are:

$$L\left(t_{(1)},t_{(2)},\dots,t_{(n)};\alpha,\beta,\lambda\right)=k^n\prod_{i=1}^{n}\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha(i-1)}\prod_{i=1}^{n}\left(1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right)^{n-i}\left(\alpha^n\beta^n\lambda^{n\beta}\prod_{i=1}^{n}t_i^{\beta-1}\,e^{-\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta}}\right.$$

$$\prod_{i=1}^{n}\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}\dots\dots\dots..(26)$$

$$\ln L=n\ln k+n\ln\alpha+n\ln\beta+n\beta\ln\lambda+(\beta-1)\sum_{i=1}^{n}\ln t_i-\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta}+(\alpha i-1)\sum_{i=1}^{n}\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right)+(n-i)\sum_{i=1}^{n}\ln(1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\dots(27)$$

Finding the partial derivatives of the natural logarithm for likelihood function:

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} + i\sum_{i=1}^{n} \ln\left(1 - e^{-(\lambda t_i)^\beta}\right) + (n-i)\sum_{i=1}^{n} \frac{-\left(1 - e^{-(\lambda t_i)^\beta}\right)^\alpha \ln\left(1 - e^{-(\lambda t_i)^\beta}\right).1}{\left(1 - \left(1 - e^{-(\lambda t_i)^\beta}\right)^\alpha\right)} \dots\dots (28)$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} + n\ln\lambda + \sum_{i=1}^{n} \ln t_i - \left(\lambda \sum_{i=1}^{n} t_i\right)^\beta \ln\left(\lambda\sum_{i=1}^{n} t_i\right) + (\alpha i - 1)\sum_{i=1}^{n} \frac{-e^{-(\lambda t_i)^\beta}.-(\lambda t_i)^\beta.\ln(\lambda t_i)}{\left(1 - e^{-(\lambda t_i)^\beta}\right)}$$

$$+(n-i)\sum_{i=1}^{n} \frac{-\alpha\left(1 - e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}.-e^{-(\lambda t_i)^\beta}.-(\lambda t_i)^\beta \ln(\lambda t_i)}{\left(1 - \left(1 - e^{-(\lambda t_i)^\beta}\right)^\alpha\right)} \dots.(29)$$

$$\frac{\partial \ln L}{\partial \lambda} = \frac{n\beta}{\lambda} + \beta\left(\lambda\sum_{i=1}^{n} t_i\right)^{\beta-1} \sum_{i=1}^{n} t_i + (\alpha i - 1)\sum_{i=1}^{n} \frac{-e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1}.t_i}{\left(1 - e^{-(\lambda t_i)^\beta}\right)}$$

$$+(n-i)\sum_{i=1}^{n} \frac{-\alpha\left(1 - e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}.-e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1} t_i}{\left(1 - \left(1 - e^{-(\lambda t_i)^\beta}\right)^\alpha\right)} \dots\dots (30)$$

From the first derivative of log-likelihood function we get three non-linear equations as follows;

$$f(\alpha) = \frac{n}{\alpha} + i\sum_{i=1}^{n} \ln\left(1 - e^{-(\lambda t_i)^\beta}\right) - (n-i)\sum_{i=1}^{n} \frac{\left(1 - e^{-(\lambda t_i)^\beta}\right)^\alpha \ln\left(1 - e^{-(\lambda t_i)^\beta}\right)}{\left(1 - \left(1 - e^{-(\lambda t_i)^\beta}\right)^\alpha\right)} \dots.(31)$$

$$g(\beta) = \frac{n}{\beta} + n\ln\lambda + \sum_{i=1}^{n} \ln t_i - \left(\lambda \sum_{i=1}^{n} t_i\right)^\beta \ln\left(\lambda\sum_{i=1}^{n} t_i\right) + (\alpha i - 1)\sum_{i=1}^{n} \frac{e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta \ln(\lambda t_i)}{\left(1 - e^{-(\lambda t_i)^\beta}\right)}$$

$$-(n-i)\sum_{i=1}^{n} \frac{\alpha\left(1 - e^{-(\lambda t_i)^\beta}\right)^{\alpha-1} e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta \ln(\lambda t_i)}{\left(1 - \left(1 - e^{-(\lambda t_i)^\beta}\right)^\alpha\right)} \dots..(32)$$

$$z(\lambda) = \frac{n\beta}{\lambda} + \beta\left(\lambda\sum_{i=1}^{n} t_i\right)^{\beta-1} \sum_{i=1}^{n} t_i + (\alpha i - 1)\sum_{i=1}^{n} \frac{e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{\beta-1} t_i}{\left(1 - e^{-(\lambda t_i)^\beta}\right)}$$

$$-(n-i)\sum_{i=1}^{n} \frac{\alpha\left(1 - e^{-(\lambda t_i)^\beta}\right)^{\alpha-1} e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{\beta-1} t_i}{\left(1 - \left(1 - e^{-(\lambda t_i)^\beta}\right)^\alpha\right)} \dots..(33)$$

This system in not easy to solve, so using the Newton–Raphson method:

$$\frac{\partial f(\alpha)}{\partial \alpha} = \frac{-n}{\alpha^2} - (n-i)\sum_{i=1}^{n} \frac{\left(\ln\left(1 - e^{-(\lambda t_i)^\beta}\right)\right)^2 \left(1 - e^{-(\lambda t_i)^\beta}\right)^\alpha}{\left[1 - \left(1 - e^{-(\lambda t_i)^\beta}\right)^\alpha\right]}$$

$$-(n-i)\sum_{i=1}^{n}\frac{\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{2\alpha}\left(\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right)\right)^{2}}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{2}} \ldots\ldots (34)$$

$$\frac{\partial f(\alpha)}{\partial \beta}=i\sum_{i=1}^{n}\frac{e^{-(\lambda t_i)^{\beta}}(\lambda t_i)^{\beta}\ln(\lambda t_i)}{(1-e^{-(\lambda t_i)^{\beta}})}$$

$$-(n-i)\sum_{i=1}^{n}\frac{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right][\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}(\lambda t_i)^{\beta}\ln(\lambda t_i)}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{2}}$$

$$+\frac{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right)\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}(\lambda t_i)^{\beta}\ln(\lambda t_i)]}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{2}}$$

$$+\frac{\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right)\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}(\lambda t_i)^{\beta}\ln(\lambda t_i)}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{2}} \ldots\ldots (35)$$

$$\frac{\partial f(\alpha)}{\partial \lambda}=i\sum_{i=1}^{n}\frac{e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}t_i}{(1-e^{-(\lambda t_i)^{\beta}})}$$

$$-(n-i)\sum_{i=1}^{n}\frac{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right][\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}t_i}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{2}}$$

$$+\frac{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right)\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}t_i]}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{2}}$$

$$+\frac{\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right)\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}t_i}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{2}} \ldots\ldots (36)$$

$$\frac{\partial g(\beta)}{\partial \alpha}=i\sum_{i=1}^{n}\frac{e^{-(\lambda t_i)^{\beta}}(\lambda t_i)^{\beta}\ln(\lambda t_i)}{(1-e^{-(\lambda t_i)^{\beta}})}$$

$$-(n-i)\sum_{i=1}^{n}\frac{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right][e^{-(\lambda t_i)^{\beta}}(\lambda t_i)^{\beta}\ln(\lambda t_i)\,\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right)}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{2}}$$

$$+\frac{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}(\lambda t_i)^{\beta}\ln(\lambda t_i)]}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{2}}$$

$$+\frac{\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{2\alpha-1}e^{-(\lambda t_i)^{\beta}}(\lambda t_i)^{\beta}\ln(\lambda t_i)\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right)}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^{2}} \ldots\ldots (37)$$

$$\frac{\partial g(\beta)}{\partial \beta}=\frac{-n}{\beta^{2}}-\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta}\ln\left(\lambda\sum_{i=1}^{n}t_i\right)\ln\left(\lambda\sum_{i=1}^{n}t_i\right)$$

$$+(\alpha i - 1)\sum_{i=1}^{n}\frac{\left(1-e^{-(\lambda t_i)^\beta}\right)\left[e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta(\ln(\lambda t_i))^2+(\lambda t_i)^\beta\ln(\lambda t_i)\,e^{-(\lambda t_i)^\beta}.-(\lambda t_i)^\beta\ln(\lambda t_i)\right]}{\left(1-e^{-(\lambda t_i)^\beta}\right)^2}$$

$$-\frac{e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta\ln(\lambda t_i).-e^{-(\lambda t_i)^\beta}.-(\lambda t_i)^\beta\ln(\lambda t_i)}{\left(1-e^{-(\lambda t_i)^\beta}\right)^2}$$

$$-(n-i)\sum_{i=1}^{n}\frac{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]\left[\alpha(\alpha-1)\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-2}.-e^{-(\lambda t_i)^\beta}.-(\lambda t_i)^\beta\ln(\lambda t_i)\,e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta\ln(\lambda t}{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]^2}$$

$$+\frac{\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}e^{-(\lambda t_i)^\beta}.-(\lambda t_i)^\beta\ln(\lambda t_i)\,(\lambda t_i)^\beta\ln(\lambda t_i)+\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta(\ln(\lambda t_i)}{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]^2}$$

$$-\frac{\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta\ln(\lambda t_i).-\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}.-e^{-(\lambda t_i)^\beta}.-(\lambda t_i)^\beta\ln(\lambda t_i)}{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]^2}\ \ldots(38)$$

$$\frac{\partial g(\beta)}{\partial\lambda}=\frac{n}{\lambda}-\left(\lambda\sum_{i=1}^{n}t_i\right)^\beta\frac{\sum_{i=1}^{n}t_i}{\lambda\sum_{i=1}^{n}t_i}+\ln\left(\lambda\sum_{i=1}^{n}t_i\right)\beta\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta-1}\sum_{i=1}^{n}t_i$$

$$+(\alpha i-1)\sum_{i=1}^{n}\frac{\left(1-e^{-(\lambda t_i)^\beta}\right)[e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1}t_i(\lambda t_i)^\beta\ln(\lambda t_i)+e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{\beta-1}t_i\ln(\lambda t_i)}{\left(1-e^{-(\lambda t_i)^\beta}\right)^2}$$

$$\frac{+e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta\frac{t_i}{\lambda t_i}]-e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta\ln(\lambda t_i).-e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1}t_i}{\left(1-e^{-(\lambda t_i)^\beta}\right)^2}$$

$$-(n-i)\sum_{i=1}^{n}\frac{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]\left[\alpha(\alpha-1)\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-2}.-e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1}t_i e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta\ln(\lambda t_i)\right.}{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]^2}$$

$$+\frac{\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1}t_i(\lambda t_i)^\beta\ln(\lambda t_i)+\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{\beta-1}t_i\ln(\lambda t}{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]^2}$$

$$+\frac{\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta\frac{t_i}{\lambda t_i}]}{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]^2}$$

$$-\frac{\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}e^{-(\lambda t_i)^\beta}(\lambda t_i)^\beta\ln(\lambda t_i).-\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}.-e^{-(\lambda t_i)^\beta}.\beta(\lambda t_i)^{\beta-1}t_i}{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]^2}\ \ldots.(39)$$

$$\frac{\partial z(\lambda)}{\partial\alpha}=i\sum_{i=1}^{n}\frac{e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{\beta-1}t_i}{(1-e^{-(\lambda t_i)^\beta})}$$

$$-(n-i)\sum_{i=1}^{n}\frac{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right)e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}t_i}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^2}$$

$$+\frac{[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}]\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}t_i}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^2}$$

$$+\frac{\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{2\alpha-1}\ln\left(1-e^{-(\lambda t_i)^{\beta}}\right)e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}t_i}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^2}\dots\dots..(40)$$

$$\frac{\partial z(\lambda)}{\partial\beta}=\frac{n}{\lambda}-\beta\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta-1}\sum_{i=1}^{n}t_i\ln\left(\lambda\sum_{i=1}^{n}t_i\right)-\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta-1}\sum_{i=1}^{n}t_i$$

$$+(\alpha i-1)\sum_{i=1}^{n}\frac{\left(1-e^{-(\lambda t_i)^{\beta}}\right)[e^{-(\lambda t_i)^{\beta}}-(\lambda t_i)^{\beta}\ln(\lambda t_i)\,\beta(\lambda t_i)^{\beta-1}t_i+e^{-(\lambda t_i)^{\beta}}.1.(\lambda t_i)^{\beta-1}t_i}{\left(1-e^{-(\lambda t_i)^{\beta}}\right)^2}$$

$$\frac{+e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}\ln(\lambda t_i)\,t_i]-e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}t_i.-e^{-(\lambda t_i)^{\beta}}.-(\lambda t_i)^{\beta-1}\ln(\lambda t_i)}{\left(1-e^{-(\lambda t_i)^{\beta}}\right)^2}$$

$$-(n-i)\sum_{i=1}^{n}\frac{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right][\alpha(\alpha-1)\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-2}.-e^{-(\lambda t_i)^{\beta}}-(\lambda t_i)^{\beta}\ln(\lambda t_i)\,e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}t_i}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^2}$$

$$+\frac{\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}.-(\lambda t_i)^{\beta}\ln(\lambda t_i)\,\beta(\lambda t_i)^{\beta-1}t_i+\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}.1.(\lambda t_i)^{\beta-1}t_i}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^2}$$

$$+\frac{\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}\ln(\lambda t_i).t_i]}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^2}$$

$$-\frac{\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}t_i.-\alpha\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha-1}.-e^{-(\lambda t_i)^{\beta}}.-(\lambda t_i)^{\beta}\ln(\lambda t_i)}{\left[1-\left(1-e^{-(\lambda t_i)^{\beta}}\right)^{\alpha}\right]^2}\dots.(41)$$

$$\frac{\partial z(\lambda)}{\partial\lambda}=\frac{-n\beta}{\lambda^2}-\beta(\beta-1)\left(\lambda\sum_{i=1}^{n}t_i\right)^{\beta-2}\left(\sum_{i=1}^{n}t_i\right)^2$$

$$+(\alpha i-1)\sum_{i=1}^{n}\frac{\left(1-e^{-(\lambda t_i)^{\beta}}\right)\left[e^{-(\lambda t_i)^{\beta}}\beta(\beta-1)(\lambda t_i)^{\beta-2}t_i^2+\beta(\lambda t_i)^{\beta-1}t_ie^{-(\lambda t_i)^{\beta}}.-\beta(\lambda t_i)^{\beta-1}t_i\right]}{\left[1-e^{-(\lambda t_i)^{\beta}}\right]^2}$$

$$-\frac{\left[e^{-(\lambda t_i)^{\beta}}\beta(\lambda t_i)^{\beta-1}t_i\right]\left[-e^{-(\lambda t_i)^{\beta}}.-\beta(\lambda t_i)^{\beta-1}t_i\right]}{\left[1-e^{-(\lambda t_i)^{\beta}}\right]^2}$$

$$-(n-i)\sum_{i=1}^{n}\frac{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]\left[\alpha(\alpha-1)\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-2}.-e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1}t_i e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{\beta-1}t_i\right.}{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]^2}$$

$$+\frac{\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1}t_i\beta(\lambda t_i)^{\beta-1}t_i+\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}e^{-(\lambda t_i)^\beta}\beta(\beta-1)(\lambda t_i)^{\beta-2}t}{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]^2}$$

$$-\frac{\left[\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}e^{-(\lambda t_i)^\beta}\beta(\lambda t_i)^{\beta-1}t_i\right]\left[\alpha\left(1-e^{-(\lambda t_i)^\beta}\right)^{\alpha-1}.-e^{-(\lambda t_i)^\beta}.-\beta(\lambda t_i)^{\beta-1}t_i\right]}{\left[1-\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha\right]^2}\dots\dots(42)$$

Now, by applying equation(13) we get the estimators of three-parameters of (EW)distribution by using Rank set sampling method.

Ordinary Least Squares Estimator Method(OLSEM) The third method of estimation for (EW) distribution is ordinary least squares estimator method(OLSEM), this method is applied in many phenomenas in life such as Medical, Economics and mathematic problems and other[6], The idea of this method is to minimizes the sum squares differences between observed sample values and the expected estimate values by lineacted estimate values by linear approximation[2] as follows;

$$\sum_{i=1}^{n}\epsilon_i^2=\sum_{i=1}^{n}[y_i-E(\hat{y}_i)]^2\dots\dots(43)$$

We use the CDF of three parameters (EW) distribution as follows;

$$F(t_i)=\left(1-e^{-(\lambda t_i)^\beta}\right)^\alpha$$

$$\left(F(t_i)\right)^{-\alpha}=1-e^{-(\lambda t_i)^\beta}$$

$$1-\left(F(t_i)\right)^{-\alpha}=e^{-(\lambda t_i)^\beta}\dots\dots..(44)$$

Now, we taking the natural logarithm

$$\ln\left[1-\left(F(t_i)\right)^{-\alpha}\right]=-\lambda^\beta t_i^\beta\dots\dots.(45)$$

comparing that with simple linear model

$$y=\beta_0+\beta_1 t+\epsilon\dots\dots.(46)$$

$$y=\ln\left[1-\left(F(t_i)\right)^{-\alpha}\right],\beta_0=0,\beta_1=-\lambda^\beta,t=t_i^\beta\dots\dots.(47)$$

$$\epsilon=y-\beta_0-\beta_1 t\dots\dots.(48)$$

$$\epsilon_i=\ln\left[1-\left(F(t_i)\right)^{-\alpha}\right]+(\lambda t_i)^\beta\dots\dots(49)$$

$$\sum_{i=1}^{n}\epsilon_i^2=\sum_{i=1}^{n}\left[\ln\left[1-\left(F(t_i)\right)^{-\alpha}\right]+(\lambda t_i)^\beta\right]^2\dots\dots.(50)$$

$$\sum_{i=1}^{n}\epsilon_i^2=\sum_{i=1}^{n}\ln\left[1-\left(F(t_i)\right)^{-\alpha}\right]^2+\sum_{i=1}^{n}2(\lambda t_i)^\beta\ln\left[1-\left(F(t_i)\right)^{-\alpha}\right]+\sum_{i=1}^{n}(\lambda t_i)^{2\beta}\dots\dots.(51)$$

$$\frac{\partial\sum_{i=1}^{n}\epsilon_i^2}{\partial\alpha}=2\sum_{i=1}^{n}\left[\ln\left[1-\left(F(t_i)\right)^{-\alpha}\right]\frac{-\left(F(t_i)\right)^{-\alpha}\ln(F(t_i))}{1-\left(F(t_i)\right)^{-\alpha}}+2\sum_{i=1}^{n}(\lambda t_i)^\beta\frac{-\left(F(t_i)\right)^{-\alpha}\ln(F(t_i))}{1-\left(F(t_i)\right)^{-\alpha}}\right.$$

$$\frac{\partial \sum_{i=1}^{n} \epsilon_i^2}{\partial \beta} = 2\sum_{i=1}^{n} (\lambda t_i)^{\beta} \ln(\lambda t_i) \ln\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right] + 2\sum_{i=1}^{n} (\lambda t_i)^{2\beta} \ln(\lambda t_i)$$

$$\frac{\partial \sum_{i=1}^{n} \epsilon_i^2}{\partial \lambda} = 2\sum_{i=1}^{n} \beta(\lambda t_i)^{\beta-1} t_i \ln\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right] + 2\sum_{i=1}^{n} \beta(\lambda t_i)^{2\beta-1} t_i$$

These three equations are a nonlinear equation solving by system Newton-Raphson method.

$$f(\alpha) = -2\sum_{i=1}^{n} [\ln\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right] \frac{\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha} \ln\left(F(t_i)\right)}{\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right]} - 2\sum_{i=1}^{n} (\lambda t_i)^{\beta} \frac{\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha} \ln\left(F(t_i)\right)}{\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right]} \ldots (52)$$

$$g(\beta) = 2\sum_{i=1}^{n} (\lambda t_i)^{\beta} \ln(\lambda t_i) \ln\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right] + 2\sum_{i=1}^{n} (\lambda t_i)^{2\beta} \ln(\lambda t_i) \ldots .(53)$$

$$z(\lambda) = 2\sum_{i=1}^{n} \beta(\lambda t_i)^{\beta-1} t_i \ln\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right] + 2\sum_{i=1}^{n} \beta(\lambda t_i)^{2\beta-1} t_i \ldots ..(54)$$

$$\frac{\partial f(\alpha)}{\partial \alpha} = 2\sum_{i=1}^{n} [\ln\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right] \frac{\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right]\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha} [\ln\left(F(t_i)\right)]^2 + \left(\mathrm{F}(\mathrm{t_i})\right)^{-2\alpha} [\ln\left(F(t_i)\right)]^2}{\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right]^2} - \frac{\left(\mathrm{F}(\mathrm{t_i})\right)^{-2\alpha} [\ln\left(F(t_i)\right)]^2}{\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right]^2}$$

$$+2\sum_{i=1}^{n} (\lambda t_i)^{\beta} \frac{\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right]\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha} [\ln\left(F(t_i)\right)]^2 + \left(\mathrm{F}(\mathrm{t_i})\right)^{-2\alpha} [\ln\left(F(t_i)\right)]^2}{\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right]^2} \ldots .(55)$$

$$\frac{\partial f(\alpha)}{\partial \beta} = -2\sum_{i=1}^{n} (\lambda t_i)^{\beta} \ln(\lambda t_i) \frac{\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha} \ln\left(F(t_i)\right)}{\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right]} \ldots \ldots .(56)$$

$$\frac{\partial f(\alpha)}{\partial \lambda} = -2\sum_{i=1}^{n} \beta(\lambda t_i)^{\beta-1} t_i \frac{\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha} \ln\left(F(t_i)\right)}{\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right]} \ldots \ldots .(57)$$

$$\frac{\partial g(\beta)}{\partial \alpha} = -2\sum_{i=1}^{n} (\lambda t_i)^{\beta} \ln(\lambda t_i) \frac{\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha} \ln\left(F(t_i)\right)}{\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right]} \ldots .(58)$$

$$\frac{\partial g(\beta)}{\partial \beta} = 2\sum_{i=1}^{n} (\lambda t_i)^{\beta} [\ln(\lambda t_i)]^2 \ln\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right] + 4\sum_{i=1}^{n} (\lambda t_i)^{2\beta} [\ln(\lambda t_i)]^2 \ldots \ldots .(59)$$

$$\frac{\partial g(\beta)}{\partial \lambda} = 2\sum_{i=1}^{n} (\lambda t_i)^{\beta-1} t_i \ln\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right] + \ln(\lambda t_i)\beta(\lambda t_i)^{\beta-1} t_i \ln\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right] + \sum_{i=1}^{n} 2(\lambda t_i)^{2\beta-1} t_i + 4\ln(\lambda t_i)\beta(\lambda t_i)^{2\beta-1} t_i \ldots \ldots \ldots (60)$$

$$\frac{\partial z(\lambda)}{\partial \alpha} = -2\sum_{i=1}^{n} \beta(\lambda t_i)^{\beta-1} t_i \frac{\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha} \ln\left(F(t_i)\right)}{\left[1-\left(\mathrm{F}(\mathrm{t_i})\right)^{-\alpha}\right]} \ldots \ldots (61)$$

$$\frac{\partial z(\lambda)}{\partial \beta} = 2\sum_{i=1}^{n} \beta(\lambda t_i)^{\beta-1} t_i \ln(\lambda t_i) \ln\left[1-\left(\mathrm{F(t_i)}\right)^{-\alpha}\right] + (\lambda t_i)^{\beta-1} t_i \ln\left[1-\left(\mathrm{F(t_i)}\right)^{-\alpha}\right]$$
$$+\sum_{i=1}^{n} 4\,\beta(\lambda t_i)^{2\beta-1} t_i \ln(\lambda t_i) + 2(\lambda t_i)^{2\beta-1}\, t_i \quad \ldots\ldots..(62)$$
$$\frac{\partial z(\lambda)}{\partial \lambda} = \sum_{i=1}^{n} 2\beta t_i^2\ (\beta-1)(\lambda t_i)^{\beta-2} \ln\left[1-\left(\mathrm{F(t_i)}\right)^{-\alpha}\right] + \sum_{i=1}^{n} 2\beta t_i^2 (2\beta - 1)(\lambda t_i)^{2\beta-2} \quad \ldots\ldots(63)$$

Simulation

Simulation procedure have been performed using visual basic programming language for illustrating the theoretical numerical values for all estimation methods which dervivate and find in previous section.

The performance of numerical values to estimate all parameters in (EW).

The simulation procedures are described as follow:

1-Generated random numbers with uniform distributed with [0,1] which denoted by u.

2- Generated random sample which distributed (EW) distribution by using inverse transformation method.

$$u = \left[1 - e^{-(\lambda t_i)^{\beta}}\right]^{\alpha}$$
$$1 - u^{1/\alpha} = e^{-(\lambda t_i)^{\beta}}$$
$$\left[1 - u^{1/\alpha}\right]^{1/\beta} = e^{-\lambda t_i}$$
$$\ln\left[1 - u^{1/\alpha}\right]^{1/\beta} = -\lambda t_i$$
$$t_i = -\frac{\ln\left[1 - u^{1/\alpha}\right]^{1/\beta}}{\lambda}$$

3- Choosing the true values of the parameters, where ($\alpha, \beta = 0.5,1,2, \lambda = 1,2$).

4-Choosing random sample of sizes (n=10,30,50) which generated from (EW) distribution and repeated the sample (1000) replicate.

5-The Newton-Raphson method was used to solving the three nonlinear equations in MLE method given in equations(10,11,12) also the three nonlinear equations in RSSE method given in equations (31,32,33) and the three nonlinear equation in OLS method given in equations (52,53,54).

6-For each samples the Mean squares error measunes (MSE) for all parameters in (EW) distribution were estimated.

$$MSE_j = \sum_{i=1}^{n} \frac{\left(\hat{\theta}_j - \theta\right)^2}{n} \quad where\ \theta = (\alpha, \beta, \lambda)$$

then calculated the average mean squanes error (AMSE) for allgenerated samples

$$AMSE = \sum_{j=1}^{L} \frac{MSE_j}{L}$$

7-The A MSE of the estimators for the three parameters for all sample sizes were tabulated in the following table.

Table:the MSE of the parameters $\theta = (\alpha, \beta, \lambda)$ for all methods

n	A	β	λ	MLE	RSSE	OLS	Min value eq.
10	0.5	0.5	1	0.3792	0.7431	1.8256	MLE
			2	0.5142	0.9211	1.9934	MLE
		1	1	0.3914	0.7992	2.0761	MLE
			2	0.6785	0.9415	2.6321	MLE
		2	1	0.4221	0.8321	2.1311	MLE
			2	0.6998	0.9884	2.441	MLE
	1	0.5	1	0.3893	0.7744	1.8572	MLE
			2	0.5144	0.9525	2.1212	MLE
		1	1	0.3965	0.8133	2.4378	MLE
			2	0.6818	0.9676	2.6935	MLE
		2	1	0.4271	0.8851	2.4217	MLE
			2	0.7021	1.013	2.6138	MLE
	2	0.5	1	0.3895	0.8214	1.8813	MLE
			2	0.5164	0.9699	2.2007	MLE
		1	1	0.3982	0.8334	2.5019	MLE
			2	0.6855	1.1022	2.7131	MLE
		2	1	0.4343	0.9245	2.6078	MLE
			2	0.7108	1.482	2.6991	MLE
30	0.5	0.5	1	0.6721	0.6935	0.9219	MLE
			2	0.8621	0.8824	1.2002	MLE
		1	1	0.7825	0.7892	1.3214	MLE
			2	0.8907	0.9393	1.3532	MLE
		2	1	0.8216	0.8603	1.5617	MLE
			2	1.0056	1.1924	1.7417	MLE
	1	0.5	1	0.6851	0.7136	1.2514	MLE
			2	0.9524	0.9581	1.4130	MLE
		1	1	0.7902	0.8185	1.9213	MLE
			2	0.9732	1.7321	1.921	MLE
		2	1	0.8421	0.9030	2.2017	MLE
			2	1.3434	2.6633	2.8754	MLE
	2	0.5	1	0.6291	0.9133	1.9341	MLE
			2	1.0799	1.7724	2.5417	MLE
		1	1	0.8010	0.8418	2.5420	MLE
			2	1.4215	1.8541	2.5921	MLE
		2	1	0.8542	0.9521	2.8412	MLE
			2	1.6566	1.8921	2.6363	MLE
50	0.5	0.5	1	0.7773	0.7882	0.9329	MLE
			2	0.9105	1.1147	1.6632	MLE
		1	1	0.8245	0.8909	1.2073	MLE
			2	0.9333	1.2995	1.7570	MLE
		2	1	0.8932	0.9311	1.4029	MLE
			2	0.1311	1.7336	1.9321	MLE
	1	0.5	1	0.7954	0.8250	1.1661	MLE
			2	1.2156	1.7549	1.9907	MLE
		1	1	0.8702	0.9030	1.1787	MLE
			2	1.4433	1.8721	2.0141	MLE

		2	1 2	1.0995 1.8821	1.1441 2.9079	1.35341 2.092	MLE MLE
	2	0.5	1 2	0.8340 1.2715	1.2321 1.9907	1.5007 2.7013	MLE MLE
		1	1 2	0.9214 1.5507	1.4027 2.5656	1.6921 2.7456	MLE MLE
		2	1 2	1.1752 1.8732	1.5958 2.8792	1.9025 2.8773	MLE MLE

CONCLUSIONS

1-For all methods also for all values of sample size and for all true values of the parameters we found that MLE method have the smallest MSE.
2-For all methods espesically MLE method noting that the values of MSEare increasing.
3-For all methods espesically MLE method showing that the values of MSE are increasing when the true values of all parameters are increasing.

REFERENCE

1.Ashour S.and Afify W.(2007),"Statistical analysis of exponentiated weibull family under type I progressive interval censoring with random removals", Journal of applied science research,3(12):1851-1863.
2.Dhifaf F.(2013),"Parameters estimation of the modified weibull distribution based on complete data with application",MS.c thesis ,Baghdad University, college of science for women.
3.Eisa M.and Mitra S. (2012),"Exponentiated weibull-Geometric distribution and its applications",
4.Hirose H.(2007), "Maximum likelihood estimation in the 3-parameter weibull distribution:a look through the generalized extreme-value distribution", IEEE transactions on dielectrics and electrical insulation Vol.14 No.1.
5.Iden H. and Hind J.(2014), "Srvival estimation for singly type one censored sample based on generalized Rayleigh distribution", Journal of Baghdad for science Vol.11(2)
6.Ling J. and Pan J.(1998),"Anew method for selection of population distribution and parameter estimation", Reliability Engineering and system safety,Vol.60, pp.247-255.
7.Mudholkar G. Srivastava D.(1993),"Exponentiated weibull family for analyzing bathtub failure-rate data", IEEE transactions on reliability Vol. 42,No.2.
8.Mudholkar G. Srivastava D. (1995),"The exponentiated weibull family: a reanalysis of the bus-motor-failure data", Technometrics 37(436-445).
9.Mudholkar G. Huston A. (1996),"The exponentiated weibull family: some properties and a flood data application", Communications in statistics-theory and methods 25(3059-3083).
10.Mohammad Z. and Mohamed T.(2009),"Bayesian analysis for the exponentiated Rayleigh distribution", International Journal of Statistics Vol.LXVII, n. 3,pp. 269-288.
11.Mclntyre, G. (1952),"A method of unbiased selective sampling using rank sets", Australian J. Agricultural Research, Vol.3,pp.385-390.
12.Nassar M. and Eissa F.(2003),"On the Exponentiated weibull distribution", communications in statistics-theory and methods, Vol.32,pp1317-1336.
13.Pai M. and Ali M.(2006),"Exponentiated weibull distribution", statistca anno LXVI,n.2.

14.Salem A. and Abo-kasem O. (2011),"Estimation for the parameters of the exponentiated weibull distribution based on progressive hybrid censored samples",Int.J. Contemp.Math.Sciences,Vol.6,no.35,1713-1724.

15.Debaushee D. and Datta D.(2013),"Comparison of weibull distribution and Exponentiated weibull distribution based estimation of Mean and Variance of Wind data",Interrational Journal of Energy .Intormation and communications Vol.4. Issue 4.