THE GEOMETRIC THINKING LEVELS OF SENIOR HIGH SCHOOL STUDENTS IN GHANA

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ABSTRACT: *This study was to measure the Van Hiele's levels of geometric thinking attained by Ghanaian final year (SHS 3) students before leaving School. A quantitative research approach was employed in the study and sample of 200 students randomly selected from the three participated schools. The results showed that 42.5% of the students could not attain any VHG level at all, 33% of the students attained Van Hiele's level 1, 22.5% reached level 2, 1.5% reached level 3 and only 0.5% reached level 4.The findings indicated that most of the Ghanaian SHS form 3 students do not attain any level of VHGT.*

KEYWORDS: Geometry, Geometric Thinking, Van Hiele's Levels, Visualization, Cognitive Development

INTRODUCTION

Mathematics educators have put up maximum efforts aimed at identifying the major problems associated with the teaching and learning of mathematics in the nation's schools. Despite all these maximum efforts, the problem of poor performance in mathematics has continued to rear its head in the nation's public examinations (Adolphus, 2011).In 2003 and 2007, Ghana participated in Trends in International Mathematics and Science Study (TIMSS) in order to find out how the performance of her (JHS2) in science and mathematics compared with those of other countries. The analysis of the Ghanaian students' performance in mathematics indicated that, Algebra,Measurementand Geometry were the Students weak content areas (Anamuah-Mensah, Mereku & Asabere-Ameyaw, 2004). According to TIMSS 2011 report, Ghanaian students performance in mathematics also indicated that, algebra and geometry were the weak content areas (Mullis, Martin, Foy & Aron, 2011).

Geometry provides a more complete appreciation of the world we live in (Atebe, 2008). For example, geometry appears naturally in the structure of the solar system, in geological formation of plants and flowers, and even in animals. It is also a major part of our synthetic world such as art, architecture, cars, machines, and virtually everything humans create. There has been a great deal of concern about the level of students understanding of geometry in Ghanaian schools.

A number of factors have been put forward to explain why the learning of geometry is difficult. Some of these factors are language of geometry, visualization abilities, and ineffective instruction. Poor reasoning skills are also another area of concern among secondary school students. Many students are unable to extract necessary information from given data and many more are unable to interpret answers and make conclusions. Traditional approaches in learning geometry emphasize more on how much the students can remember and less on how well the students can think and reason. Thus, learning becomes forced and seldom brings satisfaction to the students (Baffoe & Mereku, 2010).

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These difficulties have been rectified in some Western counties and few African nations who have used the Van Hiele's Model of Learning in Geometry effectively to improve performance of students in geometry. It has also helped inform curriculum developers and teachers about the importance of the Van Hiele's Model. The Van Hiele's Geometric Test have also improved the students' geometric thinking levels in the countries which have adopted it (Van Hiele, 1999; Clements, 2004; Adolphus, 2011, Abdullah & Zakaria, 2013).

Currently, research works on assessing students' geometric thinking levels in Ghana is scarce and as a result there is little or no comprehensive descriptions of Ghanaian students' geometric thinking level sufficient to inform the interventional plan of teachers. Thus, this gap has made earlier interventions of enhancing students geometric thinking to have little effect on students. Also, since mathematics at the Tertiary level builds on the knowledge and competencies developed at the SHS level, it is important to identify the geometric thinking levels of students leaving SHS 3. This study sought to find out which Van Hiele's level of Geometric Thinking do students in SHS 3 attain. However, the assessment tools used in our classrooms do not provide comprehensive description of our students' geometric thinking levels in order for teachers to plan appropriate interventions.

The Van Hiele's levels of geometric thinking

Van Hiele's levels of geometric thinking was developed by Van Hiele and modified by Usiskin to categorize mental activity by quantity and quality attributes of the activities required by the students in five levels (Usiskin, 1982; Hoffer, 1983; Clement, 2004; Atebe& Schafer, 2010). As shown in table 1 below.

Table 1: The Van Hiele's levels of geometric thinking

Levels	Types
Level 1	Visualization or Recognition
Level 2	Analysis
Level 3	Ordering
Level 4	Deduction
Level 5	Rigor

From Table 1, it is observed that the lowest level of geometric thinking was described by van Hiele as Visualization or Recognition. The student who attains the level 1 is able to identify, name, compare and operates on geometric figures such as triangles, angles, or intersections according to the physical appearance of the figure. The level 2 is the analysis stage because at this level, the student is able to analyze the figures in terms of their components and relationship among components and discovers properties/rules of a class of shapes empirically such as folding, measuring, using grid or diagram. Level 3 is named as the Ordering stage because at this level, the student logically interrelates previously discovered properties and rules by giving or following informal arguments. The Deduction stage is a student who attains level 4. Here the student proves theorems deductively and established interrelationships among networks of theorems. Finally, level 5 is a student at the Rigor stage where the student establishes theorems in different postulation systems and analyzes or compares these systems

Methods

The instruments used for data collection were Van Hiele's Geometry Test (VHGT) items and curriculum analysis guide. The test modified by Usiskin (1982) to assess Van Hiele's

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Geometric Thinking level based on the Van Hiele's descriptions of the five levels of geometric thinking was adapted for this study. The VHGT was administered to compare the students' geometry performance and Van Hiele's levels of Geometric Thinking. The test involved 20 item multiple-choice tests. The first five questions dealt with identification, naming and comparing of geometric shapes such as triangles, squares and rectangles. The next set of five questions dealt with recognizing and naming properties of geometric figures, the third five questions dealt with logical order of the properties of figures previously identified, and begins to perceive the relationships between these properties, whilst the last set dealt with questions that require students to understand the significance of deduction and the role of postulates, axioms, theorems and proof (Pegg, 1995).

The stratified sampling technique was used to select three (3) Municipalities from the Central Region while simple random sampling technique was employed to select one (1) public SHS from each of the Municipalities. The sample comprised a total of 200 final year Senior High School students comprising 88 (44%) males and 112 (56%) females randomlyselected. The ages of these learners ranged from 14 to 20. Out of the 200 students, 41 were drawn from school A, 84 from school B whilst 75 students were drawn from school C.

The researchers also analyzed the content of the mathematics curriculum (Ministry of Education, Science and Sports, 2012. *Teaching syllabus for mathematics, Senior High School*) and some mathematics text books at SHS to assess the level of consideration of Van Hiele's model of Geometric Thinking Level.

RESULTS

The Overall Scores of Students in the VHGT Item Test.

The purpose of this study was to use Van Hiele's levels of Geometric Thinking to assess and classify the geometric thinking levels of the final year students (SHS3) in Ghana. The researchers organized the results of the study by first analyzing the general performance of the students in the Van Hiele's levels of Geometric Thinking. The items in the various levels were also analysed to find the specific levels of students in geometric thinking. There were 20 items that were used to assess the students'levels with each item allotted with one mark. Table 2 shows the general performance of students in the 20 test items.

Table 2: Total Scores Obtained by Students in the VHGT Item Test

Score	Number of Students (n)	Cumulative(n)	%	Cumulative (%)
1	1	1	0.5	0.5
2	4	5	2.0	2.5
3	11	16	5.5	8.0
4	11	27	5.5	13.5
5	17	44	8.5	22.0
6	39	83	19.5	41.5
7	29	112	14.5	56.0
8	31	143	15.5	71.5
9	23	166	11.5	83.0

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10	15	181	7.5	90.5
11	7	188	3.5	94.0
12	6	194	3.0	97.0
13	5	199	2.5	99.5
14	1	200	0.5	100.0

The results in Table 2 shows that 83.0% ($n = 166$) of the students obtained less than half of the total score, 7.5% of the students (n=15) scored half of the total marks allotted to the test while 9.0% ($n = 19$) obtained more than half. In spite of the low performance of the students in the test, no student scored zero with only one of the students obtaining the minimum mark of 1. Interestingly, the highest mark scored in the test was 14 out of the 20 and one student obtained that. Moreover, no students could score marks above 15. This indicates that the general performance of the final year (SHS 3) students in the VHGT Item test was very weak. A descriptive analysis of the general performance was performed and the results presented in table 3.

Table 3: Descriptive Statistics on the Total Score of Students.

	$(\boldsymbol{n})$	Minimum Score	Maximum Score	Mean	Std. Deviation
Total score	200	1	14	7.21	2.51

The results in Table 3 shows that out of a total score of twenty (20) marks, the mean score of students was 7.21 and the standard deviation was 2.51. The descriptive analysis is an indication that most students scored very low marks.

Table 4 presents percentages of the proportion of students reaching the four levels of the Van Hiele's Geometric thinking levels. As can be seen in the Table 4, each level had five items with four multiple choice options. For each item, the number in bold font represents the total number $(\boldsymbol{n})$ and $(\%)$ of students who answered that item correctly. In this section the participants' overall performance on the items in the four subtests are discussed.

Table 4: Overall Participants' Performance on each Item in the VHGT.

Levels 1	Choice Items	1 $(\boldsymbol{n})(\%)$	2 $(\boldsymbol{n})(\%)$	3 $(\boldsymbol{n})(\%)$	4 $(\boldsymbol{n})(\%)$	5 $(\boldsymbol{n})(\%)$
	A	2 (2.0)	**151 (75.0)**	8 (4.0)	24 (12.0)	128 (64.0)
	B	21 (10.5)	3 (1.5)	45 (22.5)	46 (23.0)	5 (2.5)
	C	3 (1.5)	23 (11.5)	**72 (36.0)**	**116 (58.0)**	36 (18.0)
	D	3 (1.5)	2 (1.0)	19 (9.5)	6 (3.0)	2 (1.0)
	E	**170 (85.0)**	17 (8.5)	48 (24.0)	8 (4.0)	**21 (10.5)**
Level 2	Choice Items	6 $(\boldsymbol{n})(\%)$	7 $(\boldsymbol{n})(\%)$	8 $(\boldsymbol{n})(\%)$	9 $(\boldsymbol{n})(\%)$	10 $(\boldsymbol{n})(\%)$
	A	20 (10.0)	16 (8.0)	14 (7.0)	20 (10.0)	**90 (45.0)**

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	B	**100 (50.0)**	19 (9.5)	30 (15.0)	74 (37.0)	32 (16.0)
	C	45 (22.5)	**115 (57.5)**	**60 (30.0)**	20 (10.0)	13 (6.5)
	D	21 (10.5)	27 (13.5)	45 (22.5)	23 (11.5)	19 (9.5)
	E	7 (3.5)	19 (9.5)	36 (18.0)	**55 (27.5)**	38 (19.0)
Level 3	Choice Items	11 $(\boldsymbol{n})$(%)	12 $(\boldsymbol{n})$(%)	13 $(\boldsymbol{n})$(%)	14 $(\boldsymbol{n})$(%)	15 $(\boldsymbol{n})$(%)
	A	46 (23.0)	54 (27.0)	36 (18.0)	56 (28.0)	48 (24.0)
	B	**37 (18.5)**	29 (14.5)	27 (13.5)	37 (18.5)	**39 (19.5)**
	C	46 (23.0)	21 (10.5)	**26 (13.0)**	**48 (24.0)**	42 (21.0)
	D	35 (17.5)	**52 (26.0)**	38 (19.0)	37 (18.5)	35 (17.5)
	E	20 (10.0)	33 (16.5)	66 (33.0)	12 (6.0)	22 (11.0)
Level 4	Choice Items	16 $(\boldsymbol{n})$(%)	17 $(\boldsymbol{n})$(%)	18 $(\boldsymbol{n})$(%)	19 $(\boldsymbol{n})$(%)	20 $(\boldsymbol{n})$(%)
	A	**25 (12.5)**	**22 (11.0)**	36 (18.0)	**1 (0.5)**	37 (18.5)
	B	48 (24.0)	26 (13.0)	66 (33.0)	29 (14.5)	71 (35.5)
	C	39 (19.5)	35 (17.5)	39 (19.5)	38 (19.0)	**17 (8.5)**
	D	35 (17.5)	44 (22.0)	31 (15.5)	58 (29.0)	25 (12.5)
	E	43 (21.5)	64 (32.0)	**22 (11.0)**	31 (15.5)	34 (17.0)

NB: The figures in bold represent the total number $(\boldsymbol{n})$(%) of students who answered that item correctly.

It can be observed from Table 4 that most of the students were able to score most of the items in level 1 (visualization stage). With the exception of item 3 and 5, more than half of the students scored the rest of the items, thus items 1, 2 and 4.

In the level 2 (analysis stage), the students could not score well because it is only item 7 that more than half of the students were able to score and exactly half of them scored item 6. For items 8, 9 and 10, 60 (30%), 55 (27.5%) and 90 (45%) of the students respectively scored them. The students could not handle the items in the 3rd and 4th levels at all. It was only item 12 that 26% out of the 200 students were able to score and the rest not even a quarter of the students could score. The results in the table 4 is a clear indication that as the students move from one level to the other increasing manner of geometric thinking then the performance of the number of students at the levels would be decreasing.

Table 5summarizes the students' performance into the various levels with percentage of students reaching the various levels of the Van Hiele's levels. It was observed from table 4 that most of the students scored majority of the items in level 1. However, this observation does not reflect in the number of students who have attained the level 1. This implies that most of the students could not score 3 out of 5 items in the level which was the bench mark for the attainment of the level.

Table 5: Levels of the Van Hiele Geometric thinking Reached by Students

Levels	No Level Reached	Visualization level 1	Analysis level 2	Ordering level 3	Deduction level 4	**Total**
n	85	66	45	3	1	**200**
%	42.5	33	22.5	1.5	0.5	**100**

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From Table 5, 42.5% ($n = 85$) of the students could not reach any of the levels, 33% ($n = 66$) of the students reached the Visualization (level 1), while 22.5% ($n = 45$) reached the Analysis (level 2) of the Van Hiele Geometric thinking levels. Furthermore, 1.5% ($n = 3$) reached the Ordering (level 3). This leaves the number of students reaching the level 4 of the Van Hiele's Geometric thinking levels as 0.5% ($n = 1$). Students who did not reach any of the levels of Van Hiele Geometric thinking means that the students could not meet the criteria for attaining VHGT level, that is the students could not answer three (3) questions correctly fromthe items 1 to 5.

From the Table 5, it can be seen that 1.5% ($n = 3$), students reached the Orderingstage (level 3) level; this is an indication that out of 200 students only three (3) of the studentscould reach the levels 1, 2 and 3. This means that only three (3) students could perform in level 3, where students can logically order the "litany" of properties of figures previously identified.

Finally, only 0.5% ($n = 1$) of the students reached the Deductionstage (level 4)of the Van Hiele Geometric thinking levels. This indicates only one (1) out of 200 students was able to meet the criteria 3 of 5 correct suggested by Usiskin (1982) in all the levels, that means only 1 of the students could answer 3 items correctly in questions items; 1 to 5, 6 to 10, 11 to 15, 16 to 20. It shows that, at this level a student understands the significance of deduction and the role of postulates, axioms, theorems and proofs. These are in fact fundamental geometric knowledge which students need to study in geometry related courses at the tertiary level.

DISCUSSION

In this study, the researchers aimed at using Van Hiele's levels of Geometric Thinking as a theoretical framework to classify students' geometric thinking level. A test item of four levels of difficulty was used to gather information on students' Geometric thinking level. The focus was on students' performance in the test items. The researchers developed a VHGT Item test in line with earlier VHGT Item test used by (Atebe& Schafer, 2010) to assess Nigerian students and pre-service teachers' Geometric thinking level. Students' barriers in solving the VHGT Item test could be attributed to the following:

- Comprehension of the VHGT Item test itself
- Inadequate Basic Computational skills

The findings from the analysis showed that most students did not perform well in the VHGT Item test and therefore could not solve the Levels 3 and 4 items. Students who attempted the questions used wrong working processes in their attempt to solve the items in the VHGT Item test. This resulted in some students arriving at various answers. Furthermore, students inability to solve the Ordering and Deductive levels questions agree with the findings of Atebe and Schafer (2010); Baffoe and Mereku (2010) who stated that students 'weaknesses had obstructed the progress of mapping the steps appropriately to finding the solution.

Also the analysis on levels reached by students on the Van Hiele's Geometric thinking levels showed that, majority of the students had not reach any level or reached the first and second levels of the Van Hiele's Geometric thinking levels, that is the Visualization and Analysis level. The number of students who reached levels 3 and 4, thus ordering and deductive levels shows that most students were not able to classify and generalize by attributes and develop proofs

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using axioms and definitions. The findings in the study showed that students who reached the Ordering and Deductive levels could classify and generalize by attributes and develop proofs using axioms and definitions. Also, the different working processes used by the students who reached the Deductive level shows that problem solving should not be treated as a topic but should be incorporated in all mathematics topic.It was observed from the curriculum analysis that that the objectives on geometry topics in the syllabus meet only the first three levels of the Van Hiele's levels of intellectual developments. Similarly, it was observed that none of the geometry activities in the SHS textbooks could be described as one at the Van Hiele's Geometric Thinking model for level 4 (Deductive) where students can prove theorems deductively and establish interrelationships among networks of theorems. This implies that less attention is given to the Van Hiele's model of Geometric Thinking level. This finding is in line with the study by Usiskin (1982); Hoffer (1988); Atebe and Schafer (2010); Baffoe and Mereku (2010) who found out that most African students at the high school were not able to solve a variety of geometric problem and that most of the students encountered difficulty in reaching the ordering and deductive levels.

CONCLUSION

From the results of the content analysis, it emerged that the highest possible Van Hiele's level attainable by a student leaving Senior High School in Ghana is level 3 (Ordering). This situation leaves the students in Ghana not being compatible with other students in the rest of the world where geometry is required to be taught and learnt up to Van Hiele's level 4 (Deduction). Mathematics at the Higherinstitutions is more abstract and Conceptual, and heavily founded on the basis of deductions. For this reason, any student who enters these institutions of higher learning is expected to think at Van Hiele's level 4.

RECOMMENDATIONS

Based on the findings of the study, the following recommendations are made for the improvement:

- Since only 0.5% (n = 1) out of the 200 SHS 3 students reached the level 4 of the Van Hiele levels, the researchers recommend that SHS teachers combine routine and non-routine, theorems and proofs problems in their teaching and learning activity. This will help students see that Geometry relies almost exclusively on written symbolic forms as the tool to make representation, generalization and interpretation to the applied problem.
- Mathematics teachers must adopt the VHGT Item test to assess thinking levels of students in every topic taught. This will enable them plan an appropriate intervention for each student. It will also enable the teachers make an informed decision on how to help students improve their geometric knowledge.
- Curriculum developers should consider revisiting the Senior High School mathematics curriculum, with specific reference to Van Hiele's geometric thinking level 4. These aspects include students using the properties that they already know to formulate definitions of simple geometric shapes, and class inclusions are understood and are able

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to draw inferences from various shapes leading to students developing proofs using axioms and definitions. Since the Van Hiele's theory forms the foundation of mathematics curricula, I recommend that the Ghanaian mathematics curriculum should also be aligned to the said theory.

- Mathematics Heads of Departments (HODs) in consultation with the school heads should organize workshops and seminars for mathematics teachers on the Van Hiele's Model

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