International Journal of Mathematics and Statistics Studies

Vol. 2.No.1, pp.34-44, March 2014

Published by European Centre for Research Training and Development UK (www.ea-journals.org)

THE BRAIDING MATRICES LINK BETWEEN YANG-BAXTER EQUATIONS AND QUANTUM INFORMATION

Rizwana Parveen

Lecturer, Department of Mathematics Aljouf University,

Aljouf, Saudi Arabia.

ABSTRACT: *The current review examines the correlation between Yang–Baxter structure and quantum information using secondary literature. Several recent papers were examined, like the study of the properties of geometric figures based on Temperley–Lieb (TL) algebra, Birman–Wenzl (BW) algebra and other subjects. Currently Yang-Baxter equation is useful in solving statistical prototypes, quantum integrable prototypes and many-body problems. A number of researchers in the past have explored the Yang–Baxter Equation and the braiding operators in relation to the field of quantum computation and quantum information. In the recent past, braiding operators and YBE are initiated with the fields of quantum computation processing and quantum information. Several researchers have indicated a widespread adaptation of the YBE. Sometimes, the unitary solutions related to the comprehensive YBE further facilitates braid group depictions which can then be used for quantum information processing. The paper concludes that generalized Yang–Baxter Equations provide robust technique for unravelling depictions of the braid set. Further, these depictions are useful in several fields; in specific, the resultant braiding quantum circuits are vigorously examined in quantum information science.*

KEYWORDS: Braiding Matrices, Yang-Baxter Equations, Quantum Information

INTRODUCTION

In statistical mechanics, Onsager made available renowned elucidation of the Ising Model known as ‘star triangle transformation’, called Yang-Baxter equation (Perk & Au-Yang, 2006). The Yang–Baxter equation is a vital system in physics and mathematics associated to a number of disciplines like quantum field theory, statistical mechanics, quantum groups, quantum topology, and of late, quantum information science. Currently Yang-Baxter equation has a vital part in statistical prototypes, quantum integrable prototypes and many-body problems (Jimbo, 1990; Yang, 1967; Yang, 1968; Baxter, 1982; Baxter, 1972; Takhtadzhan & Faddeev, 1979; Kulish & Sklyanin, 1982a; Kulish & Sklyanin, 1982b). Several attempts were made to examine the braiding operators and the Yang–Baxter Equation (YBE) (Yang, 1967; Yang, 1968; Baxter, 1982; Baxter, 1972) to the discipline of quantum computation and quantum information (Kauffman & Lomonaco, 2004; Zhang *et al*., 2007; Chen *et al*., 2007; Chen *et al*., 2008; Wang *et al*., 2009). Of late, braiding operators and YBE are initiated in the disciplines of quantum computation processing and quantum information (Kauffman & Lomonaco, 2004). Kauffman and Lomonaco (2004) examined the contribution of unitary Yang-Baxter Ř matrices in quantum computation. It revealed that braid matrices and Yang-Baxter Ř matrices could be recognized to be universal quantum gates (Chen *et al*., 2007). Based on unitary Ř matrices, Chen *e. al*. (2008) developed a group of Hamiltonians, and examined the Berry phase and quantum criticality pertaining to the Yang-Baxter Equation. The review is projected as an opening to YBE for non-specialists. This study endeavoured to incorporate certain latest progress in this field, highlighting the part of quantum sets. The review is categorized in the following manner: Section 2 pertains to fundamental definition, properties and basic exemplars of explanation of YBE; Section 2 various studies that linked YBE with quantum information using different algebra such as Temperley-Leib (TL) algebra and Birman-Wenzel (BW) algebra.

THE YANG BAXTER EQUATION

Formulation

The Yang–Baxter Equation in dimension *d, that is* dimension d^2, is a matrix equation designed for an invertible complex matrix $R = (R_{ij}^{kl})$, *i, j, k, l* = 1, 2, . . . , *d*. The simplest method to note the YBE is by utilizing the style of linear operators amid vector spaces. If *V is the* d-dimensional complex vector space with a selected basis $\{e_i\}$, *i* = 1, 2, . . . , *d*. The matrix *R* delineates an invertible operator
$R: V \otimes V \to V \otimes V$ by $R(e_i \otimes e_j) = \sum_{k,l=1}^{d} R_{ij}^{kl}\, e_k \otimes e_i$, *i, j* =*1*, ..., *d,* where $\{e_a \otimes e_b\}$, a, b = 1, . . . , d, is a basis of $V \otimes V$. By abusing notation, it has been indicated that the operator connected to *R* and by *R*. Letting *IV* be the identity operator on *V*, two operators are created $R \otimes I_V$ and $I_V \otimes R$ from $V \otimes V \otimes V$ to itself. Subsequently the YBE, noted similar to a equation for linear operators, is what follows:

$$\text{(YBE)}(R \otimes I_V)(I_V \otimes R)(R \otimes I_V) = (I_V \otimes R)(R \otimes I_V)(I_V \otimes R)$$

If one utilizes the formula $\{e_a \otimes e_b \otimes e_c\}$, *a, b, c* = 1, . . . , *d*, the foresaid equation pertaining to linear operators turns out to be an equation pertaining to matrix *R*. This matrix equation intended for *R* is generally known as the Yang–Baxter Equation, and a solution is known as *R*-matrix. In both the equations one finds similar association if the bases of the concerned vector spaces are predetermined, therefore we will consider the two as the YBE presuming that certain conditions were selected pertaining to the operator equation. Accurately stating, the YBE in this situation happens to be the *braided* variety of the constant quantum YBE, yet we state it as just the Yang–Baxter Equation for briefness.

The YBE comprises of a group of polynomial equations for the entries $\{R_{ij}^{kl}\}$, of the matrix *R*. Unambiguously, for whichever option of two ordered triples (*x, y, z*) and (*u, v,w*) from {1, 2, . . . , *d*}, we have

$$\text{(YBE)} \sum_{a,b,c=1}^{d} R_{uv}^{ab} R_{bw}^{cz} R_{ac}^{xy} = \sum_{m,n,p=1}^{d} R_{vw}^{np} R_{un}^{xm} R_{mp}^{yz}\,.$$

One function of unitary *R*-matrices is to quantum information science. Unitary R-matrix results in a unitary illustration of the braid set, and the consequential unitary matrices connected to braids could be utilized to develop quantum information (Nayak *et al*., 2008).

Encouraged by this function, Rowell *et al.* (2010) suggested a widespread adaptation of the YBE. Unitary solutions pertaining to the comprehensive YBE at times further allow braid group depictions and consequently could be utilized for quantum information processing. An 8×8 answer to a comprehensive YBE is utilized to develop the Greenberger–Horne–Zeillinger states (Rowell *et al.*, 2010). Answers to the YBE are hard to discover. This could be witnessed by calculating the quantity of variables and the quantity of equations. In case the vector space V is of dimension d, then R has d^4 entries, so there are d^4 unknowns. The YBE comprises of d^6 cubic polynomial equations for the d^4 variables $\{R_{ij}^{kl}\}$. For d = 1, the equation is mechanically fulfilled by whichever nonzero complex number. For d = 2, the YBE comprises of 64 cubic uniform polynomial equations for 16 complex unknowns. This is the single instance where the YBE was resolved totally, although utilizing a computer. The current study examines the various papers pertaining to braiding matrices and answers of Yang–Baxter Equation and connected to quantum information.

Generalized Yang–Baxter Equations

The YBE or Yang–Baxter Equation is listed by a distinct natural number d, the dimension of the vector space V. The generalized Yang–Baxter Equation or YBE suggested in (Chen, 2011) is listed by yet another two natural numbers, namely, m and l. In order for easy reference, we will carry on the utilization of operator language at the same time bear in thought that, by the option of a base of the vector space, the gYBE is a matrix equation.

Definition 1

Let V be a complex vector space of dimension d. The (d, m, l)- gYBE is an equation for an invertible operator $R: V^{\otimes m} \to V^{\otimes m}$ such that

$$\text{(gYBE)}\ (R \otimes I_V^{\otimes l})\ (I_V^{\otimes l} \otimes R)\ (R \otimes I_V^{\otimes l}) = (I_V^{\otimes l} \otimes R)\ (R \otimes I_V^{\otimes l})\ (I_V^{\otimes l} \otimes R)$$

where d, m, and l are natural numbers and $I\otimes lV$ is the identity operator on $V \otimes l$. Any matrix answers to the (d, m, l)-gYBE is known as a (d, m, l)-R-matrix. Taking into consideration that presuming m = 2 and l = 1, the gYBE lessens to the customary Yang–Baxter equation. Usually, nonetheless, the gYBE is more difficult to resolve than the Yang–Baxter equation, barring a few fundamental instances. If d = 1, R is merely a scalar, so whichever nonzero complex number is an answer. If $m = l = 1$, the gYBE turns out to be the equation $R^2 \otimes R = R \otimes R^2$, wherein R is an invertible operator on V. $R^2 \otimes R = R \otimes R^2$ is similar to $R \otimes I = I \otimes R$ since R is invertible. It is understood that $R = \lambda I_v$ for certain nonzero scalar λ, if $m = l =$ 1. For the purpose of quantum information science, we will centre on d = 2 since, if d = 2, V is isomorphic to $\mathbb{C}^2$, the supposed qubit state space. The (2, 2, 1)-gYBE is the Yang–Baxter equation in dimension 2, so the initial nontrivial gYBE for qubits is the (2, 3, 1) - *generalized Yang–Baxter Equation*.

Definition 2

The (2, 3, 1)-gYBE for $R:\ (\mathbb{C}^2)^{\otimes 3} \to (\mathbb{C}^2)^{\otimes 3}$ is $R_1R_2R_1 = R_2R_1R_2$

where $R_1 = R \otimes I_2$ and $R_2 = I_2 \otimes R$ act on$(\mathbb{C}^2)^{\otimes 4}$. One finds numerous varieties of principles pertaining to the tensor product of matrices. Here we will utilize the supposed Kronecker product: for two matrices $A = (a_{ij})_{m\times n}$ and $B = (b_{kl})_{p\times q}$, $A \otimes B$ is the $(mp \times nq)$-matrix attained by substituting every one of the entries a_{ij} of A by the block $a_{ij}B$. We will employ *In* to indicate the $n \times n$ identity matrix. In the event that no uncertainty ensues, we will merely note down *In* as *I*. For both matrices *X* and *Y*, *X* ⊗ *Y* indicates the block diagonal matrix $\begin{pmatrix} X & 0 \\ 0 & Y \end{pmatrix}$

Different Solutions to the Yang-Baxter Equation

Chen (2012) suggested various answers to the YBE; in specific, unitary depictions of the braid set are required since they produce braiding quantum gates. These issues are keenly examined in the present study into topological quantum computing. A *gYBE* was suggested some years previously by Rowell *et al.* (2010). By obtaining answers to the *gYBE*, researchers in this investigation found latest unitary braid group depictions. These depictions have given ascend to braiding quantum gates and therefore contain the probability to assist in the creation of helpful quantum computers. Likewise, in this study, Hu, Xue and Wu (2009) create a latest 8X8 matrix as of the 4X4 *M* matrix, wherein M/*M* is the representation of the braid group depiction. The 8X8 M matrix and the 4X4 M matrix together assure additional exceptional 2-group algebra associations. By Yang–Baxteration technique, we obtain a unitary *R (ø)* matrix as of the M matrix with strictures *ø* and φ. Three-qubit entwined states could be developed by the *R (ø,* φ) matrix. A Hamiltonian for three qubits is developed from the unitary *R (ø,* φ) matrix. Researchers also examined the entanglement and Berry stage of the Yang–Baxter System.

Ge and Xue (2012) examined the connection between Yang–Baxter System and Quantum information founded on the study of the properties of geometric figures based on Temperely-Lieb (TL) algebra and Birman-Wenzel (BW) algebra. Researchers in this investigation further revealed certain of the outcomes which can take place owing to the branding matrices associated with quantum information. The Birman–Wenzl algebra (BWA) associations are created in addition $\check{R}_{i,j+1} = S_{i,j+1}$, there happens to be a latest operator $E_{i,j+1}$. Indicating the eigen values of a braiding matrix S with three eigen values by λ_1, λ_2, and λ_3, wherein S gratifies braid association
$S_{12}S_{23}S_{12} = S_{23}S_{12}S_{23}\ (S_1 \equiv S_{12} = S \otimes I, S_2 = S_{23} = I \otimes S)$ (1)

And with no loss of generality by setting $\lambda_1\lambda_2 = -1, W = \lambda_1 + \lambda_2, \lambda_3 = \sigma$ (2) we include for S with three distinctive eigen values

$$S - S^{-1} = WI + \frac{1}{\sigma}(I + WS - S^2). \quad (3)$$

Defining $E = \frac{1}{\sigma W}(S^2 - WS - I)$ (4)

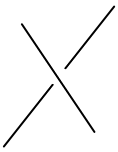
Eq. (3) becomes

$S - S^{-1} = W(I - E)$ (5)

They form BWA. Noting that a loop takes the value

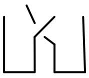
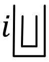
$$d = 1 + \frac{1}{W}(\sigma^{-1} - \sigma) \qquad (6)$$

And extending the topological basis $|e_1\rangle$ and $|e_2\rangle$ for T–L algebra, we shall find the uni-orthogonal basis $|e_1\rangle$, $|e_2\rangle$ and $|e_3\rangle$ such that: S12$|e_\mu\rangle = \lambda_\mu |e_\mu\rangle$ (μ = 1, 2, 3) (7)

With S_{12} = and  E$_i$ = , d= o (i= 1,2) (8)

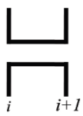
Where the eigen values λ_μ may be complex. The proposed topological basis for BWA is:

$|e_3\rangle = d^{-1} \coprod \coprod$ (8)

$|e_i\rangle = f_i\{$  $+\alpha i$  $+\beta i \coprod\coprod\}$ (i=1, 2) (9)

We shall prove that the (7) together with-

$\langle e3|e_i\rangle = 0$, $\langle e_i|e_j\rangle = \delta_{ij}$ $(i, j = 1, 2)$ (10)

Lead to the constraints to the parameters  α_i and β_i and normalization constant f_i:

$\alpha_i = \lambda_i\,(\lambda_1 \lambda_2 = -1)$, $\alpha_i + \beta_i d =$ $-\sigma^{-1}$ $(i = 1, 2)$ (11)

And

$$f_i = \{d(\lambda_i^2 + 1)[-\lambda_i^{-1} d^{-1}(\sigma^{-1} + \lambda_i) + \lambda_i^{-1}\sigma + d]\}^{-1/2} \qquad (12)$$

for $\lambda_\mu^* = \lambda_\mu (\mu = 1, 2, 3), i.e., S^\dagger = S$ (hermitian), whereas

$$f_i = \{(d - 1)(\lambda_i + \lambda_i^{-1})(\sigma + \lambda_i^d + \lambda_i^{-1})\}^{-1/2} \qquad (13)$$

for $\lambda_\mu^* = \lambda_\mu^{-1}$ $(\mu = 1, 2, 3), i.e., S^\dagger = S^{-1}$ (unitary).

The (11) acquires a similar structure for S being hermitian or unitary. The single variation among hermitian and unitary is in the variation. The study of the properties of geometric figures based on BWA were developed on the basis of the graphic method in the knot theory which is to some degree advanced to expand whichever theory connected with T–L algebra. The speed additivity of the spectrum strictures for the key of Yang–Baxter Equation connected with quantum information complies with Lorentz procedure sooner than Galileo's theories (Ge & Xue, 2012).

QUANTUM PHASE TRANSITION

Likewise, Wang, Xue, Sun, Hu, Zhou and Du (2010) examined quantum stage transition pertaining to the "q-deformed" wherein the computation illustrates that at the time the deformed stricture q moves towards 1, a quantum critical point is present for spectral stricture θ. taking into account YBS, quantum entanglement and the geometric stage could typify quantum stage transition. The matrix recognitions of Temperley-Lieb Algebra (TLA) U-matrix, YBE answer $\check{R}$ -matrix and Braid Group Realization or BGR, S-matrix are 4 × 4 matrices functioning on the tensor creation space $V \times V$, wherein V is a 2d vector space. Since U, S and $\check{R}$ operate on the tensor product$Vi \times V_{i+1}$, we indicate them through U_i , b_i and $\check{R}i$, correspondingly, is illustrated. The study revealed a short assessment of the theory of braid sets, the Yang–Baxter Equation and Yang-Baxterization method (Ge *et al.*, 1991). Presume B_n indicate the braid set on n strands. B_n is created by basic braids $\{I, b_1, b_2, . . . , b_{n-1}\}$ with the braid associations, _

$$\begin{aligned} b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1}, \quad & 1 \leq i < n-2, \\ b_i b_j = b_j b_i \quad & |i-j| \geq 2, \end{aligned} \tag{14}$$

Where in the data $b_i \equiv b_{i,i+1}$is utilized. The bi stands for$1_1 \otimes 1_2 \otimes 1_3 \cdot \cdot \cdot \otimes 1_{i-1} \otimes S \otimes 1_{i+2} \otimes \cdot \cdot \cdot \otimes 1_n$, and 1_j is the unit matrix of the j -th element. After that we identify S the Braid Group Representation. Additionally, the braid group is effortlessly comprehended in as knot illustrations in Ref (Kauffman & Lomonaco, 2002). As recognized, a key of Yang–Baxter Equation could be noted through Yang-Baxterization performing on the solution of the braid association. For instance, presuming S contains two eigen values (λ_1, λ_1), after that the Yang- Baxterization of the unitary braiding operator S is

$$\check{R}(x) = \rho(x)(xS + x^{-1}\lambda_1\lambda_2 s^{-1} \tag{15}$$

Let $\check{R}_i$ denote $\check{R}_{i,i+1}$. The unitary Ř-matrix gratifies the Yang–Baxter Equation that is of the structure,

$$\check{R}_i(x)\check{R}_{i+1}(xy)\check{R}_{i(y)} = \check{R}_{i+1}(y)\check{R}_i(xy)\check{R}_{i+1}(x) \tag{16}$$

Where in multiplicative strictures *x* and *y* are recognized as the spectral strictures. In general, multi-spin interaction Hamiltonians could be developed on the basis of the Yang–Baxter Equation. Since Ř is unitary, it could describe the development of a state $|\Psi(0)$

$$|\Psi(t) = \check{R}_i(t)|\Psi(0) \tag{17}$$

in this place $\check{R}_i(t)$ depends on a time frame that could be recognized by identifying equivalent strictures that depend on time pertaining to $\check{R}_i$. Taking into consideration partial derivative of the state $|\Psi(t)$ pertaining to time t, we arrived at the equation

$$i\hbar\frac{\partial|\Psi(t)}{\partial t} = i\hbar\left[\frac{\partial\,\check{R}_i\,(t)}{\partial t}\,\check{R}_i^{\dagger} i\,(t)\right]\check{R}_i(t)|\Psi(0) = H(t)|\Psi(t) \tag{18}$$

$$\text{where } H(t) = i\hbar\frac{\partial\,\check{R}_i\,(t)}{\partial t}\,\check{R}_i^{\dagger}\,(t)$$

$i\,(t)$ is the Hamiltonian leading the evolution of the state $|\Psi(t)\rangle$.

Therefore, the Hamiltonian $H(t)$ for the YBS is gained with the help of the Yang-Baxterization method.
In this study, the highlight is on the standard spin-1/2 six-vertex BGR (Wadati *et al.*, 1989),

$$S = \begin{pmatrix} q & 0 & 0 & 0 \\ 0 & 0 & -n & 0 \\ 0 & -\eta^{-1} & q-q^{-1} & 0 \\ 0 & 0 & 0 & q \end{pmatrix} = q\,(I - q^{-1}U) \tag{19}$$

Wherein U gratifies Temperley-Lieb associations, $U_iU_{i\pm1}U_i = U_i\,, U_iU_j = U_jU_i$ (for $|i - j| \geq 2$) and $U^2 = dU$. In this instance, $d = q + q^{-1}$. In the study of the properties of geometric figures (topology), the stricture d matches to a distinct loop "O". The matrix structure for U being $U = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & q & n & 0 \\ 0 & -\eta & q^{-1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ (20)

Presuming I_4 and O_4 indicate 4 dimensional identity matrix and zero matrixes, correspondingly.

We can confirm that the BGR contains two distinctive eigen values $\lambda_{12} = q$ and $\lambda_2 = -q^{-1}$ $(i.e. (S - qI_4)(S + q^{-1}I_4) = O_4)$. Replacing both these eigen values into, we attain a Yang-Baxter ˇR matrix as shown forthwith,

$$\check{R}(x) = [q^2 + q^{-2} - (x^2 + x^{-2})]^{-1/2}[(qx - q^{-1}x^{-1})I - (x - x^{-1})U] \tag{21}$$

The inverse matrix of $\check{R}$ *(x)* could be attained as shown forthwith,

$$[\check{R}(x)]^{-1} = [q^2 + q^{-2} - (x^2 + x^{-2})]^{-1/2}[(qx^{-1} - q^{-1}x)I + (x - x^{-1})U] \tag{22}$$

To make things simpler, we have initiated two strictures ϑ and ϕ wherein

$x = e^{i\vartheta}$ and $\eta = e^{i\varphi}$.

The unitary form

$\check{R}(\vartheta,\phi)\left[\check{R}(\vartheta,\phi)\right]^{\dagger} = \left[\check{R}(\vartheta,\phi)\right]^{\dagger}\check{R}(\vartheta,\phi) = I$ provides us ϑ and ϕ, which are authentic.

Subsequently the here said Yang-Baxter Hamiltonian could be re-organized as shown below,

$$H = \hbar\omega sin\vartheta(Q^{2} + sin^{2}\vartheta)^{-1}[sin\vartheta(s_1^3 - s_2^3) + Q(e^{-i(\varphi+\frac{\pi}{2})}S_1^+S_2^- + e^{i(\varphi+\frac{\pi}{2})}S_1^-S_2^+)]$$
(23)

Where $Q = (q - q^{-1})/2$. Let

$$v_1 = \begin{pmatrix} 0 & e^{i\frac{\vartheta}{4}} \\ e^{-i\frac{\vartheta}{4}} & 0 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 0 & e^{i\frac{\vartheta}{4}} \\ e^{-\frac{i\vartheta}{4}} & 0 \end{pmatrix},$$

It is possible to put forth a time-independent local unitary alteration as shown forthwith, $H' = VHV^{-1}$, wherein $V = v_1 \otimes v_2$. It should be noted that this local unitary alteration depends on a time frame, thus the entanglement and geometric stage characteristics do not change under these alterations. If we set $\vartheta = \pi/2 - \theta$ and $\varphi(t) = \phi(t) - \pi/2 = wt$, it is possible to attain the subsequent Yang-Baxter Hamiltonian,

$$H' = \hbar\omega cos\,\theta(Q^{2} + cos^{2}\theta)^{-1}H0, \qquad (24)$$

where $H0 = cos\,\theta(S_1^3 - s_2^3) + Q(e^{i\phi}S_1^+S_2^- - + e^{i\phi}S_1^-S_2^+)$

In its matrix structure,

$$H0 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos\theta & Qe^{-i\phi} & 0 \\ 0 & Qe^{i\phi} & -\cos\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (25)$$

In effect, the here said Hamiltonian is described on the subspace covered by $\{|01\rangle, |10\rangle\}$. This resembles a Hamiltonian of a turn 1/2 within a magnetic field. In effect, it is possible to initiate a group of $SU(2)$ operators$\{S^{(+)}, S^{(-)}, S^{(3)}\}$, where$S^{(+)} = S_1^+\ S_2^-$ and $S^{(3)} = (S_1^3 - S_2^3)/2$.

Pertaining to this group of $SU(2)$ operators the Hamiltonian $H0$ could be re-organized as shown forthwith,

$$H_0 = 2\sqrt{Q^2 + cos^2\theta}\ \mathbf{B.S}$$

where $\boldsymbol{B} = (sin\, \theta' \,cos\, \phi, sin\, \theta' \,sin\, \phi, cos\, \theta')$ with $\theta' = arctan(Q/cos\, \theta)$ and $\boldsymbol{S} = (S^{(1)}, S^{(2)}, S^{(3)})$ with $S^{(1)} = \frac{1}{2}(S^{(+)} + S^{(-)})$ and $S^{(2)} = \frac{1}{2i}(S^{(+)} + S^{(-)})$.

It is possible to effortlessly attain the eigen energies of H_0 as follows,

$$E_{\pm} = \pm\sqrt{Q^2 + cos^2\theta} \qquad (26)$$

Perceptibly, the ground state energy of the said YBS is $E_g = E_- = -\sqrt{Q^2 + cos^2\theta}$. and the equivalent eigen state is

$$|\psi g\rangle = \frac{1}{\sqrt{Q^2+\left(E_g - cos\theta\right)^2}} \left(Qe^{-i\phi}\, |01\rangle + (E_g - cos\,\theta)|10\rangle\right). \qquad (27)$$

Among E_+ and E_-, an energy gap is present $\Delta E = 2\sqrt{Q^2 + cos^2\theta}$ While $Q = 0$ (that is, q = 1 or a solitary loop d = 2), one finds a intersecting point $\theta c = \pi/2$. In case,$Q = 0$, one finds a finite gap $2|Q|$ at the intersecting point. It is evident that in case Q moves towards 0 (that is, d moves towards 2), the energy gap between the ground state and excited state turns out to be lesser. It is possible to attain the initial and the next derivative of the ground state energy E_g pertaining to the stricture θ as shown forthwith,

$$\frac{\partial E_g}{\partial \theta} = -\frac{sin\, 2\theta}{2E_g}, \frac{\partial^2 E_g}{\partial \theta^2} = -\frac{cos\, 2\theta}{E_g} - \frac{\sin^2 2\theta}{4(E_g)^3} \qquad (28)$$

To observe the quantum criticality clearly, we design the primary and the next derivative of the ground state energy E_g pertaining to the stricture θ.
Apparently, while $Q \to 0$, there is quantum critical point $\theta_c = \pi/2$.

CONCLUSION

Finally we would be gratified to comprehend if there are actual physical structures that will recognize our braid group depictions. This type of physical structures will be quantum computers, superior instruments for investigating the quantum spectrum and for helping mankind to develop latest technologies that would be of immense advantage to the society. Generalized Yang–Baxter Equations offer a latest technique for discovering depictions of the braid set. These depictions have relevance in several fields; in specific, the resultant braiding quantum circuits are vigorously examined in quantum information science (Kauffman & Lomonaco, 2004; Rowell *et al.*, 2010; Nayak *et al.*, 2008). In this segment, we record some of the numerous open queries and courses for future investigation. (1) What is the way to discover every unitary (2, 3, 1)-R-matrices $R = X \oplus Y$ so that X, Y are 2 × 2 diagonal and unitary? Be aware that their 2×2 blocks $A, B, C, D, Y_i, i = 1, 2, 3, 4,$ are not essentially unitary. (2) What is the way in which to discover every unitary (2, 3, 1) - or (2, 3, 2)-R-matrices with the zero entries in similar positions as in the key R_X? (3) Which quantum circuits could be recognized by the braiding quantum circuits consequential from our (2, 3, 1)-R-matrices? Scientifically, the query is to discover the representations of the given braid group images. (4) Which entangled states could be created from product states utilizing our braiding quantum circuits? (5) Pertaining to the three groups of answers in Theorem 3.3, do any two (2, 3, 1)-R matrices R and R' belonging to similar groups result in similar braid group depictions? To decide if the depictions from R and R' are alike, we require to discover if there is a solitary matrix P so that $P^{-1}(R \otimes I)\, P \;=\; R' \otimes I$, $P^{-1}\,(I \;\otimes R)P \;=\; I \otimes R'$. We expect to resolve this query sometime in future. It should be noted that the solution does not have any relevance on the application to braiding quantum gates.

REFERENCES

Baxter, R. J. (1972) 'Partition function of the eight-vertex lattice model', *Ann. Phys*, 70(2), 193-238.

Baxter, R. J. (1982) *Exactly Solvable Models in Statistic Mechanics*, New York: Academic.

Chen, J. L., Xue, K. and Ge, M. L. (2007) 'Braiding transformation, entanglement swapping and Berry phase in entanglement space', *Physical Review*, A 76, 042324.

Chen, J. L., Xue, K., Ge, M. L. (2008) 'Berry phase and quantum criticality in Yang-BaxterSystems', *Annals of Physics*, 323, pp. 2614.

Chen, R. S. (2011) *Generalized Yang-Baxter Equations And Braiding Quantum Gates*, Available at: http://arxiv.org/pdf/1108.5215.pdf

Chen, R. S. (2012) 'Generalized Yang–Baxter Equations And Braiding Quantum Gates, *J. Knot Theory Ramifications*, 21(1).

Faddeev, L. D. (1981) 'Quantum completely integrable models in field theory, in Mathematical Physics Reviews', *Soviet Sci. Rev. Sect. C: Math. Phys. Rev.*, 1, pp. 107.

Ge, M. L., Xue, K. and Wu, Y-S. (1991) 'Explicit trigonometric Yang Baxterization', *Int. J.Mod. Phys. A*., 6, pp. 37-35.

Jimbo, M. (eds.) (1990) *Yang–Baxter Eq. in Integrable Systems*, Singapore: World Scientific.

Hu, T., Xue, K. and Wu, C. (2009) 'Berry phase and entanglement of three qubits in a new Yang–Baxter system', *Journal of Mathematical Physics*, 50(8).

Kauffman, L.H. and Lomonaco, S.J. (2004) 'Braiding operators are universal quantum gates', *New J.Phys*., 6(134), pp.1023 – 27.

Kauffman, L. H. and Lomonaco, S. J. (2004) 'Braiding operators are universal quantum gates', *New J. Phys.*, 6, pp. 134-139.

Kulish, P. P. and Sklyanin, E. K. (1982a) 'Quantum Spectral transform method, Recent developments,'*Lecture Notes in Physics*, 151, pp. 61–119.

Kulish, P. P. and Sklyanin, E. K. (1982b) 'Solutions of the Yang-Baxter equation,' J.Soviet Math., 19, pp.1596–1620.

Nayak, C. S., Simon, H., Stern, A., Freedman, M. and Das Sarma, S. (2008) 'Non-abelian anyons and topological quantum computation', *Rev. Mod. Phys*. 80, pp.1083.

Perk, J. H. H. and Au-Yang, H. (2006) 'Yang–Baxter Equation', in: Franoise, J. P., Naber, G. L. and Tsou, S. T. (eds.) *Encyclopedia of Mathematical Physics*, pp. 465–47.

Rowell, E. C., Zhang, Y., Wu, Y. S. and Ge, M. L. (2010) 'Extraspecial Two-Groups, Generalized Yang-Baxter Equations and Braiding Quantum Gates', *Quantum Inf. Comput*, 10 (7-8), pp. 685–702.

Takhtadzhan, L. A. and Faddeev, L. (1979) 'The Quantum Method Of The Inverse Problem And The Heisenberg Xyz Model', *Russ. Math. Surv*. 34(5), pp. 11-68.

Yang, C. N. (1967) 'Exact Solution of a Model of Two-Dimensional Ferroelectrics in an Arbitrary External Electric Field', *Phys. Rev. Lett.*, 19(10).

Yang, C. N. (1968) ‘S matrix for the one dimensional N-body problem with repulsive or attractive delta function interaction’, *Phys. Rev.*, 168, pp. 1920.

Yang, C. N. and Ge, M. L. (eds.) (1990) *Braid Group, Knot Theory and Statistical Mechanics*, Singapore: World Scientific.

Wadati, M., Deguchi, T.and Akutsu, Y. (1989) ‘Exactly solvable models and knot theory’, *Phys. Rep.*, 180, pp. 247-332.

Wang, G., Xue, K., Wu, C., Liang, H., Oh, C. H. (2009) ‘Entanglement and the Berry Phase in a new Yang-Baxter System’, *J. Phys. A Math. Theor*. 42, pp. 125-207.

Wang, G., Xue, K., Sun, C., Hu, T., Zhou, C. and Du, G. (2010) ‘Quantum Phase Transition Like Phenomenon in a Two-Qubit Yang-Baxter System’, *International Journal of Theoretical Physics*, 49(1), pp. 2499-2505.

Zhang, Y., Rowell, E. C., Wu, Y. S., Wang, Z.H. and Ge, M. L. (2007) ‘Extraspecial Two-Groups, Generalized Yang-Baxter Equations and Braiding Quantum Gates’, Quantum Inf. Comput. 10 (7 & 8), pp. 0685-0702.