

STUDENTS' ERRORS AND MISCONCEPTIONS IN ALGEBRA: EXPLORING THE IMPACT OF REMEDY USING DIAGNOSTIC CONFLICT AND CONVENTIONAL TEACHING APPROACHES

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ABSTRACT: *This study explored the impact of 'Diagnostic Conflict Teaching' and 'Conventional Teaching' approaches on the remediation of algebraic errors and misconceptions among second year high school students in the Cape Coast Metropolis of Ghana. A validated algebra diagnostic test covering the fundamental algebra concepts was adopted, adapted, piloted and used to collect data from 114 participants. A quasi-experimental design with a non-equivalent (pre-test, post-test) control-group was employed. Descriptive statistics, bar charts, dependent and independent t-tests were used to analyse students' pre-test and post-test scores. Though findings of the study showed significant impact with both teaching methods, the Diagnostic Conflict Teaching approach was more effective in addressing students' algebraic errors and misconceptions than the Conventional approach. Recommendations made underscore the need for curriculum and educational programmes to provide classroom or teaching experiences that have the potential of helping pre-service and in-service mathematics teachers to improve on their knowledge for teaching algebra as well as their pedagogical content knowledge, to better equip them to address students' misconceptions and errors effectively.*

KEYWORDS: Algebra, Errors and Misconceptions, Diagnostic Conflict Teaching, Conventional Teaching.

INTRODUCTION

Apart from arithmetic, one can neither be successful in mathematics without algebra (Makonye & Stepwell, 2016) nor understand well a lot of important mathematical concepts in science, statistics, business, or today's technology (Katz, 2007). This is an indication that algebraic thinking is vital and ought to be within reach of all learners if they are to participate fully in society (Fletcher, 2008). A good understanding of algebra is therefore, a prerequisite (the gate keeper) (Usisin, 2004) for proficiency and success in higher mathematics and life. However, students are beset by conceptual difficulties in algebra (Chow, 2011; Chow & Treagust, 2013; Matzin & Shahrill, 2015) which require consistent attention. For example, misconceptions (the cause of systematic errors) are conceptual difficulties widely reported in literature (e.g., Chow, 2011; Ling, Shahrill & Tan, 2016; Pournara, Sanders, Adler & Hodgen, 2016; Swan, 2001; Olivier, 1989). Misconceptions and its errors in algebra among high school students have been documented. Egodawatte (2011) and Seng (2010) found out that secondary school students possess misconceptions (and make systematic errors) with the fundamental algebra concepts - variables, algebraic expressions, equations and word problems. Errors and misconceptions hinder further understanding and learning (Booth, Barbieri, Eyer & Paré-Blagojev, 2014; Mulungye, O'Connor & Ndethiu, 2016; Ling, Shahrill & Tan, 2016; Booth & Koedinger, 2008; Falle, 2007) as well as problem solving (Knuth, Alibali, McNeil, Weinberg

& Stephens, 2005). These discoveries appear more crucial when related studies reiterate that if students' misconceptions and errors in algebra are not purposefully detected and corrected immediately, it could lead to incorrect understanding of the concepts and consequently affect students' achievement (Radhiah, Noor and Zaid, 2015) not only in mathematics (Olivier, 1989) but other fields that require the use mathematics such as the sciences (Clement, 1982).

It is important to understand that algebra is a very important topic in the school curriculum in Ghana. Every student who is admitted to the senior high school studies algebra; one of the first math topic in core (general) mathematics that builds on their understanding of arithmetic. All students in this category are expected to study this subject throughout their study period in the high school, irrespective of the programme. Algebra is also an integral component of Elective mathematics which is normally a mandatory subject offered to students who specialise in the Science and Technical programmes as well as some programmes in the Arts and Business. The end of the school mathematics is marked by an external examination conducted by the West African Examination Council (WAEC). A study of the WAEC chief examiner's report for mathematics (both core and elective) from 2014 to 2017 revealed that among others topics, students had weakness/difficulties in the following areas: word problems, variations, binary operations, logarithms, calculus, - which are heavily dependent on algebra. "Students have difficulty in translating work/story problems into mathematical statements and solving them" (Chief Examiner's Report in Mathematics, 2017, p. 210), and "demonstrated weakness in using algebra to solve probabilities" (Chief Examiner's Report in Mathematics, 2014, p. 13). Again, it was observed that even though students could communicate the meaning of topics like compound interest, calculus and so on, they made algebra related errors as they solve problems; such as simplifying $15,000(1 - 0.5r)^2$ as $15,000(1^2 - 0.5r^2)$; and $\int \sqrt{x^2 + x} dx$ as $\int \sqrt{x^2} + \sqrt{x} dx$. The question then remains if these arguments on errors and misconceptions confirm the manifestation of the report that a "weak learner in algebra struggles to handle mathematics topics" as pointed out by Makonye and Stepwell (2016, p. 1)? The authors contend that students' misconceptions and errors in algebra have a strong tendency to affect learning mathematics and other related subjects. If so, then a search for teaching approaches that can offer the optimum redress is vital. Some studies (Perso, 1991; Bell, 1993; Swan, 2001) have recommended the cognitive conflict teaching strategy, which focuses on destabilizing students' confidence in their existing conceptions through the presentation of contradictory experiences and then substituting their alternative conceptions with accepted ones as a useful strategy in remedying students' misconceptions and errors in mathematics; but it also behoves on researchers to explore the extent of impact existing teaching approaches effect on students formulation of misconception and errors in the mathematics classrooms.

In this study, we explore the impact of two teaching approaches: the diagnostic conflict teaching and the existing mathematics teaching approach (also known traditional method and hereafter referred to as the conventional teaching approach) and their impact in remedying students' errors and misconceptions in Algebra in the mathematics classroom within the context of this study. While the diagnostic teaching approach is not a common practice in this context, Agyei (2013) pointed out that the most frequently teaching strategy used in mathematics classrooms is the Conventional approach. According to Agyei and Voogt (2011) this type of teaching is heavily dominated by teachers (while students are silent), involves whole class teaching, lots of notes being copied, and hardly any hands-on activities. Agyei (2013) further reiterated that this strategy has been strongly criticized for failing to prepare students to attain high achievement levels in mathematics.

Theoretical Framework

The Conceptual change model, which mainly originated from the Piagetians idea of accommodation and Kuhnian concept of scientific revolution (diSessa, 2006) has served as the theoretical support for many educational researches on students' alternative conceptions (misconceptions). According to Kang, Noh, and Koh (2005), conceptual change is the most significant learning model that evolved from the 'alternative conceptions movement'. Conceptual change posits that learning consists of iterative interactions that take place between students' existing conceptions and their new experiences. To Dhindsa and Anderson (2004), "Conceptual change is interpreted as a context appropriate change in the breadth and composition of conceptual knowledge occasioned by challenging experiences that require learners to rethink their understandings based on evidence from experience" (p. 64). Conceptual change is essential for meaningful learning; with students learning most effectively by constructing their own knowledge through the modification of their conceptual framework (Chow & Treagust, 2013). Posner et al. suggested four conditions—dissatisfaction, intelligibility, plausibility; fruitfulness—under which alternative conceptions held by students can be changed; which has inspired many teaching strategies (Kang, Noh, & Koh, 2005).

Three approaches to dealing with errors and misconceptions are identified from literature: 1) trying to avoid them; 'If I warn learners about the misconceptions as I teach, they are less likely to happen - prevention is better than cure.'; 2) Re-teaching the concept from the beginning and patiently giving detail explanation called didactic approach (Swan, 2001) hereafter called the conventional teaching approach (or the traditional approach); and 3) The use of errors and misconception in a diagnostic (cognitive) conflict teaching approach in a cooperative environment (Swan, 2001; Lucariello, Tine, Ganley, 2014; Olivier, 1989) with conceptual change as a yardstick. Literature has indicated that it is impossible to avoid errors and misconceptions as we teach since students will always make errors and incorrect generalizations and many of these misconceptions will remain hidden unless the teacher makes specific efforts to identify them (Askew & Williams, 1995) since misconceptions and errors are essential stepping stones for effective instruction (Confrey, 1990). In this regard, we employed the latter two approaches of dealing with misconceptions (the diagnostic conflict and the conventional approaches) as teaching interventions in this study. Preceded by identifying year two senior high students' errors and misconceptions in algebra, we explore the impact of remedy using these teaching approaches.

Diagnostic (Cognitive) Conflict Teaching and Conceptual Change

Cognitive conflict is created when one's expectations and predictions, based on one's current reasoning, are not conformed to. It is disequilibrium (Wadsworth, 1996). Lee and Yi (as cited in Potvin, Sauriol and Riopel, 2015) defines it as "a perceptual state in which one notices the discrepancy between one's cognitive structure and environment (external information) or between the components of one's cognitive structure (i.e., one's conceptions, beliefs, substructures, etc., which are part of the cognitive structure)". Citing other literatures, Potvin, Sauriol and Riopel indicated that cognitive conflict helps students discover that their preconceptions are inadequate to explain new experience or realize that, they must "replace or reorganize" their "central concepts" because they are "inadequate to allow them [students] to grasp some new phenomenon successfully. It has also been suggested that conflicts could "arouse attention" or make students "show more curiosity and interest when the given phenomenon or information is not consistent with their expectations". Many researchers (Perso, 1991; Bell, 1993; Baser, 2006; Swan, 2001) have found diagnostic (or cognitive)

conflict effective in fostering conceptual change. Chow and Treagust (2013) indicated that the initiating factor for conceptual change is disequilibrium, dissatisfaction, or cognitive conflict.

Susilawati, Suryadi and Dahlan (2017) revealed in their study that mathematical spatial visualization ability of students who were exposed to cognitive conflict strategy demonstrated higher improvement levels than students who were exposed to expository teaching based on overall and prior mathematical knowledge. Chow and Treagust (2013) researched into using cognitive conflict to foster conceptual change. Their study suggested possible effectiveness of conflict teaching in achieving conceptual gain and positive changes in students' attitudes. They indicate that it helped students to learn by actively identifying and challenging their existing conceptions and the views of their classmates. Dahlan, Rohayati (2014) found that development of subject matter with Piaget or Hasweh cognitive conflict base is descriptively able to enhance students' ability of mathematics understanding and connection. Bell (1993) and Swan (2001) suggested that the benefit to long-term learning is greater when students remedy misconceptions through their own work through cognitive conflict than when teachers choose to draw attention to potential errors and misconceptions. Swan proposed the following principles for effective diagnostic conflict teaching: 1) Assessing pupils' conceptual framework before teaching; 2) Making existing concepts and methods explicit in the classroom; 3) Sharing methods and results and provoke conflict discussion; 4) Resolving conflict through discussion and formulate new concepts and methods; and 5) Consolidating learning by using the new concepts and methods through problem solving (pp.158 – 159). The diagnostic conflict teaching in this study was implemented along these principles.

Research Design

The design used for the research was the quasi-experimental paradigm. In quasi-experiments, the investigator uses control and experimental groups but does not randomly assign participants to groups and involves pre-testing and post-testing of non-randomized control and experimental groups (Creswell, 2013). The non-equivalent (Pre-test and Post-test) Control-Group design was employed. This provided practical option to work with intact classes in both the experimental group and the control group by ensuring that the systematic arrangement and normal running of the participating schools were not interfered with or disrupted. It further ensured that participants with similar characteristics were selected. The essence of the pre-test was to identify the algebraic errors and misconceptions which are necessary for the design of the interventions. It also helped to establish the baseline performance of the groups and possibly differentiate between the groups before intervention approaches (Fletcher and Anderson, 2012). The post-test helped to establish the effectiveness or the impact of the diagnostic conflict and the conventional teaching interventions. Consequently, one intact class each (consisting of year two students) from two senior high schools were chosen to participate as experimental and control groups.

Methods

Participants

The subjects for the study comprised year two students (average age of 16 years) from two purposefully selected senior high schools in the Cape Coast Metropolis of Ghana. The purposive sampling procedure was employed, where the goal was to select samples that are likely to be "information-rich" with respect to the anticipated outcomes of the study (Gall, Gall & Borg, 2007). The problem as indicated was initially identified in the first school (the

Experimental group) but the second school (the Control group) was purposefully chosen to have similar characteristics as the former and was located at the farthest distance of about 12 km. Based on the pre-test conducted, two low performed (Ling, Shahrill & Tan, 2016) and comparable intact classes (one from each school) were selected. The experimental group (class) consisted of 58 students while the control group consisted of 60 students. However, one student from the experimental group and three students from the control group were absent for the post-test, so their data were discarded. Therefore, data analyses in the study were presented on $N = 57$ for experimental group and $N = 57$ for the control group.

Test Instrument

The instruments for this study were Algebra Diagnostic pre-test and post-test. The test items covered basic concepts of algebra such as variables, expressions, equations and word problems. To ensure content validity of the tests, items (weighing 1 mark each) which require two to three steps to be solved, were adopted from Chow's (2011) algebra diagnostic test. The items were consistent with the scope of algebra as prescribed by Ghana's general mathematics curriculum, and also with the fundamental algebra content or concepts areas. Also, the items were adapted to reflect the Ghanaian contexts. For Instance, the US currency in some of the questions was changed into the Ghana Cedis, the Ghanaian currency, and uncommon items and names as well as prices of the items were also changed to reflect market values in Ghana at the time of the study. For example 'plum' was changed to 'pineapple'. In Addition, the test was subjected to peer reviews and suggestions were fully implemented. The test was piloted in another school with similar characteristics as those used in the main study. The papers were marked and items coded 0 and 1 (0 for wrong and 1 for right answers) and analysed. Analyses of the pilot test produced a Cronbach's alpha of 0.78, a discriminatory index of 0.33 and a difficulty index of 64.26%. These values were considered enough to attest the reliability of the test according to benchmarks reported by Nunnally (as cited in Venter, 2006) and Ebel and Frisbie (as cited in Matlock-Hetzel, 1997). The same test was administered twice, first as a pre-test before the interventions and again as a post-test about a month after the teaching experiments. Students were asked to show working beside each question to help avoid guessing in choosing answers and also to provide some insight into the errors and misconceptions made. The test consisted of 20 multiple choice items.

The Interventions

The experimental group was taught with the diagnostic teaching approach by one of the researchers (who is also a teacher) at the school and the control group was taught by another teacher with same qualification and similar experience who is also a teacher at the control school using the conventional approach. Both groups took the algebraic diagnostic pre-test on the same day. Papers were marked, and guided by literature; algebraic errors and misconceptions were identified before the interventions. Both groups were taught basic algebra concepts (variables, expressions, equations, word problem) but with focus on addressing errors and misconceptions identified from the pre-test since the students had already studied algebra in their first year.

The diagnostic conflict teaching was implemented as proposed by Swan (2001) in general. The approach involved preparing and giving tasks/activities (on worksheet) inbuilt with the misconceptions together with the teaching plans and teaching materials on a particular subject matter in a given lesson. The tasks were given to the students to do first individually, then in pairs for discussions. Feedbacks were then given, either by result comparisons, allowing time

for students to think, result checks, working task using two or more methods, practically performing the activity, all with the intention to create cognitive conflicts. The teacher guided and responded to students question by not telling them the answers but by asking further leading and cognitive conflict provoking questions. Conflict resolutions were done through whole class discussion in a non-judgemental atmosphere to ensure reformulation of ideas. Finally, further problems were given for consolidation of new ideas.

Regarding the conventional teaching, teaching plans were developed as well, based on the same errors and misconceptions as the diagnostic conflict teaching. The general approach however was that: students were provided with clear and detailed lectures and careful explanations concerning the errors and misconceptions. The classroom typically consisted of the instructor presenting the “right way” to solve the problems and providing reasons why the students’ concepts, procedures and solutions were incorrect. The teacher directly provided answers to students’ questions and provided more explanations where necessary.

The interventions lasted for two weeks with the implementation of four lessons each (the last been a tutorial lesson with the same problems) in both the control and experimental groups. The post-test was conducted for both groups on the same day about the same time, approximately four weeks after the interventions.

RESULTS

The data used for measuring the impact (level of reduction in errors and misconceptions) for the interventions were the score obtained from both the pre-test and the post-test. To ascertain the impact of the diagnostic teaching approach bar graphs, descriptive statistics and paired sample t-test were used. Figure 1 shows the distribution of scores (out of 20) for both pre and post-tests of the students in the experimental group.

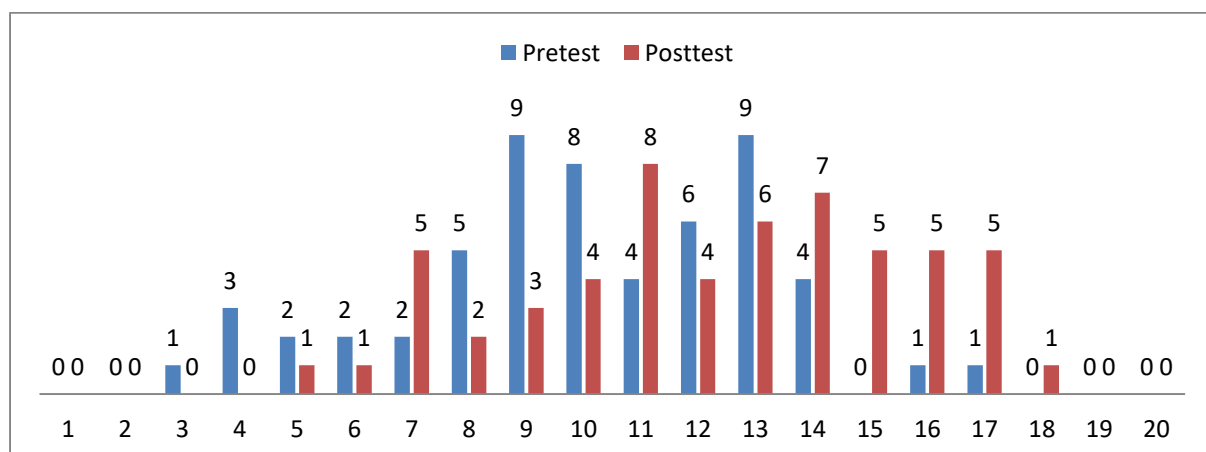


Figure 1: Pre-test and Post-test overall scores for Experimental group

From the figure, it can be seen that generally, more students scored higher marks in the post-test than in the pre-test. Five students scored 15 out of 20 questions in the post-test while no student scored 15 in the pre-test. A total of 23 students scored from 14 to 18 in the post-test while 6 students scored from 14 to 18 in the pre-test. This is an indication that for the experimental group, students performed better (committed smaller number of errors) in the post-test than in the pre-test, and could be attributed to the impact of the diagnostic conflict

teaching approach. The number of students who scored each question correctly is shown in Figure 2

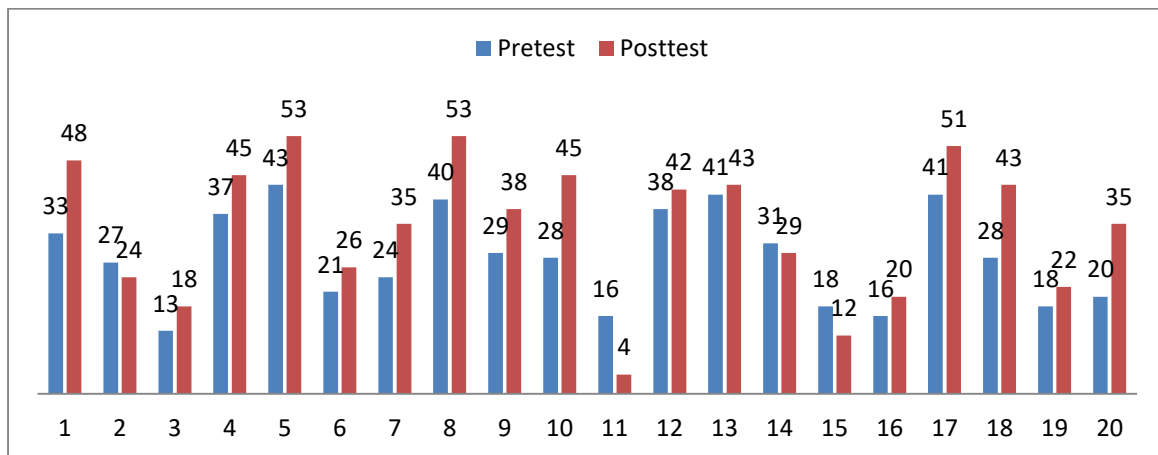


Figure 2: Distribution of students scoring each item (Pre-test – Post-test) correctly for the Experimental group

Figure 2 reveals a general increase in the number of students who scored each item correctly. That is, higher numbers of students scored each item correctly in the post-test than in the pre-test. For example, 33 students had item 1 correct in the pre-test whilst 48 (an increase of 15) students had item 1 correct in the post-test. Apart from questions 2, 11, 14, 15 (where there were observed drops in the number of students), the number of students who scored each item correctly has increased on the post-test. This further suggested that more students gained better understanding after the diagnostic conflict teaching. This is an indication that the diagnostic teaching approach had some positive impact on the students' performance. Table 1 displays and compares the mean and the standard deviations of the pre-test and the post-test for the experimental group.

Table 1: Mean Scores (Pre-test and Post-test) for Experimental Group

Type of test	Mean	N	SD	<i>p</i> -value	Effect size	<i>t</i>
Pre-test	10.07	57	3.12	0.000	0.84	6.35
Post-test	12.26	57	3.30			

Shapiro-Wilk test: p -value = 0.189; ($p > 0.05$)

The pre-test mean score was 10.07 while the post-test mean was 12.26 indicating a gain of 2.19. The standard deviation for the pre-test was 3.12 and 3.30 for the post test. This result showed that there was statistically significant difference between the pre-test and post-test ($t = 6.35$, $p < 0.05$) indicating that students' gain in understanding improved at a statistically significant level. To estimate the degree of the differences in addition to statistical significance, the effect size (Cohen's d) was determined as recommended by Morris and Deshon (2002). From table 1, the effect size for algebra diagnostic test pre-test and post-test was 0.84. According to Cohen's interpretation, an effect size of 0.84 indicated that the mean of the treated experimental group is at the 79th percentile of the untreated experimental group (Becker, 2000; Coe, 2002). Therefore, it could be suggested that the diagnostic conflict teaching method had

a positive and significant educational impact (improved understanding) in reducing or remedying senior high students' algebraic errors and misconceptions.

Similar analyses were done to ascertain the impact of the conventional teaching approach.

Figure 3 displays the distribution of scores for both pre and post-tests of the students in the control group.

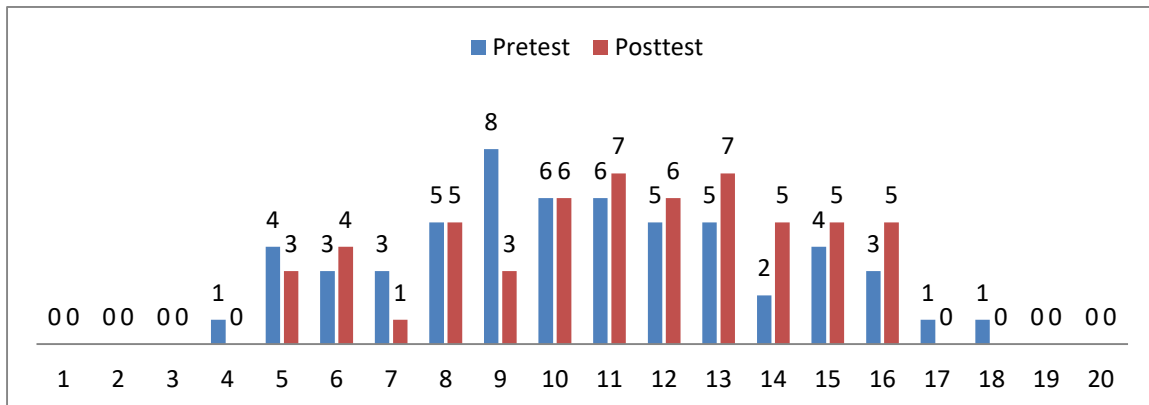


Figure 3: Pre-test and Post-test overall scores for the Control group

From Figure 3, it can be seen that generally, more students scored higher marks in the post-test than in the pre-test. Five students scored 16 out of 20 questions in the post-test while 3 students scored 16 in the pre-test. A total of 15 students scored from 14 to 18 in the post-test while 11 students scored from 14 to 18 in the pre-test. The results suggested that students gained more understanding after the conventional teaching approach. This is an indication that for the control group, students performed better (i.e., committed reduced number of errors) in the post-test than in the pre-test, and could be attributed to the impact of the conventional teaching approach. Figure 4 compared the number of students who scored each item correctly in pre-test and post-test for the control group.

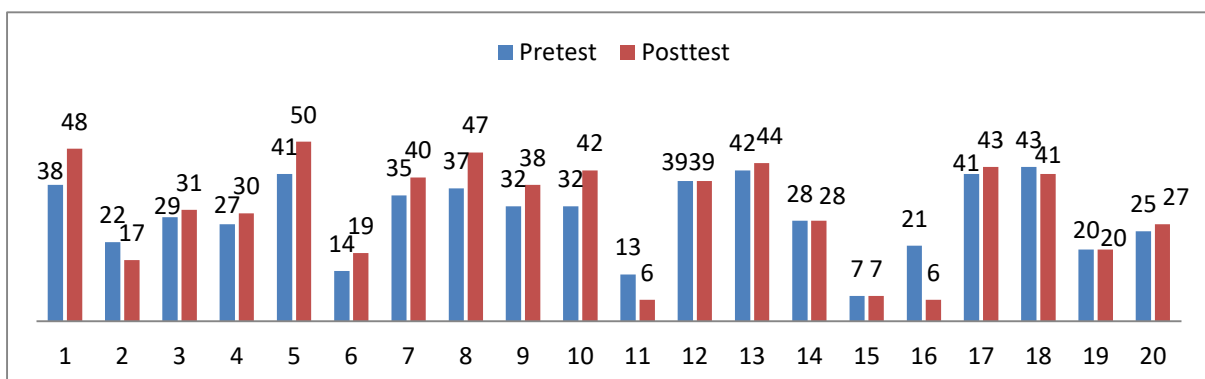


Figure 4: Distribution of students scoring each item (Pre-test – Post-test) correctly for the Control group

Figure 4 shows a general but marginal increase in the number of students who scored each item correctly. That is, higher numbers of students scored each item correctly in the post-test than in the pre-test in general. For example, 38 students had item 1 correct in the pre-test whilst 48

(an increase of 10) students scored item 1 correctly in the post-test. Apart from items 2, 11, 16, 18 (where there were observed drops in the number of students), and 12, 14, 15, 19 (where the number of students remained the same), the number of students who scored each item correctly has increased on the post-test. This could mean that more students gained better understanding of algebra concepts due to the impact of conventional teaching approach. The mean and the standard deviation of the scores from both tests for the control group are displayed in Table 2.

Table 2: Mean Scores (Pre-test and Post- test) for Control Group

Type of test	Mean	N	SD	<i>p</i> -value	Effect size	<i>t</i>
Pre-test	10.47	57	3.41	0.036	0.30	2.14
Post-test	11.19	57	3.19			

Shapiro-Wilk test: p -value = 0.104; ($p > 0.05$)

From Table 2, the pre-test mean score was 10.47 while the post-test mean was 11.19 indicating a gain of 0.72, which could be attributed to the conventional teaching approach. The results showed that there was statistically significant differences between the pre-test and post-test ($t = 2.14$, $p < 0.05$) indicating that students' gain in understanding improved at a statistically significant level. This also means that there is significant reduction in students' errors and misconceptions. To estimate the degree of the differences in addition to statistical significance, the effect size (Cohen's d) was determined. From table 2, the effect size for algebra diagnostic test pre-test and post-test was 0.30. An effect size of 0.30 indicated that the mean of the treated control group is at the 62nd percentile of the untreated control group (Becker, 2000; Coe, 2002). Therefore, it can be concluded that the conventional teaching approach had positive and an educational impact on the reduction of second year senior high students' algebraic errors and misconceptions.

To analyse data concerning the performances of senior high students on the algebra diagnostic test for both the experimental and control groups, descriptive statistics, bar graphs, and independent t -test were used. Figure 5 shows the overall post-test scores for both experimental group and control groups.

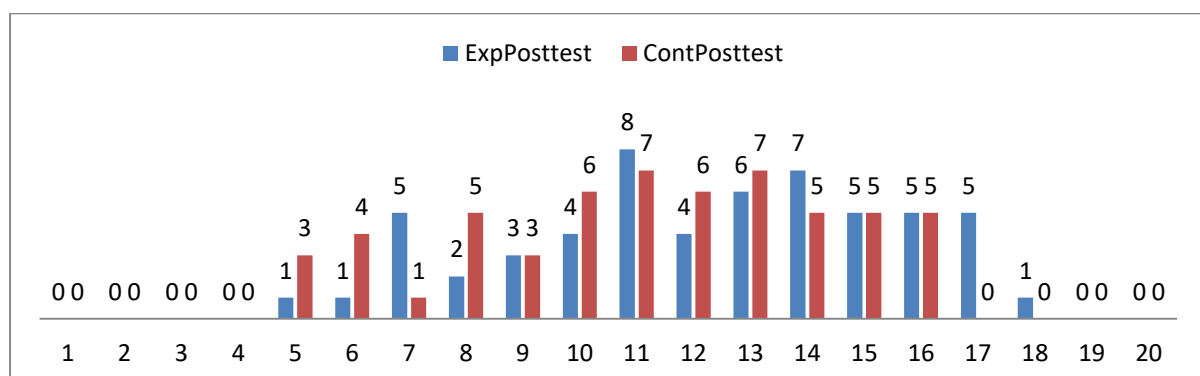


Figure 5: Comparison of overall Post-test scores for both Experimental and Control groups

From Figure 5, it can be observed that more students scored higher marks in the experimental group than in the control group. Five students from the experimental group scored 17 out of 20 questions while no student from the control group scored 17. A total of 23 students from the experimental group scored from 14 to 18 while 15 students from the control group scored from 14 to 18. This was an indication that students of the experimental group performed better in the post-test than students of the control group. The results from Figure 5 suggested that the diagnostic teaching approach made more impact than the conventional teaching approach in remedying or reducing the algebraic errors and misconceptions of second year senior high students. Figure 6 compares the performance in the two groups based on the number of students scoring each item correctly.

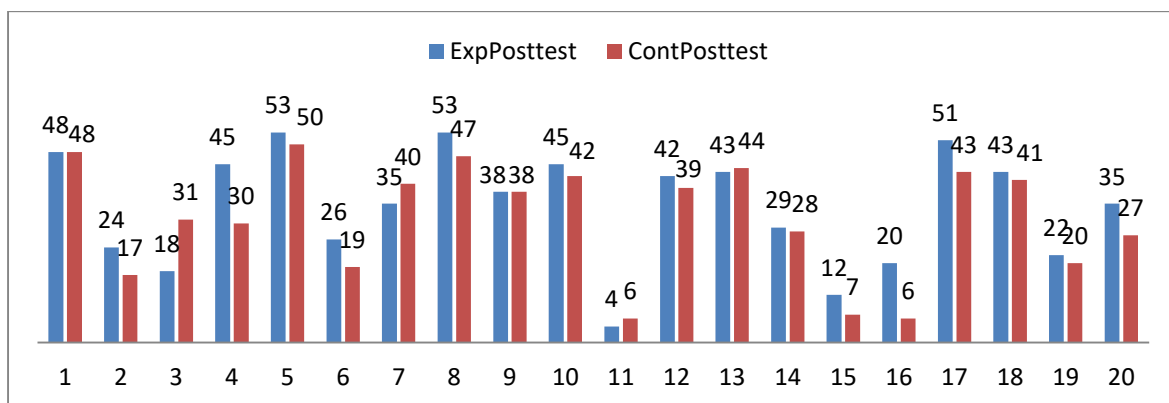


Figure 6: Distribution of students scoring each item correctly for Experimental and Control Post-test

As shown in Figure 6, the numbers of students who scored each item correctly in the post-test for the experimental group are higher as compared to the control group. For example, 45 students from the experimental group had item 4 correct while 30 students from the control group had item 4 correct. Apart from items 1, 3, 11, the numbers of students who scored the other items correctly are higher for the experimental group than the control group, suggesting that the diagnostic teaching approach improved students' understanding better than the conventional teaching approach. The mean and the standard deviations are displaced in Table 3.

Table 3: Mean Score (standard deviation) of the Experimental and Control Groups on the Post-test

	Control Group		Experimental Group	
	Mean	Standard deviation	Mean	standard deviation
Post-test	11.19	3.19	12.26	3.30
Pre-test	10.47	3.41	10.07	3.12

From Table 3, the mean score of the control group on the post-test was 0.72 (i.e., 11.19 – 10.47) higher than their mean score on the pre-test whereas the mean score of the experimental group in the post-test was 2.19 (i.e., 12.26 – 10.07) higher than the mean

score in their pre-test. Judging from the face of it, the gain in performance made by the experimental group is higher than that of the control group. The mean score of the experimental group in the post-test was 12.26 while that of the control group was 11.19. Thus, while the control group led the experimental group by 0.40 marks before the intervention, it now trailed by 1.07 marks after the interventions. The independent t-test (shown in Table 4) also suggested a significant gain in the performance between the experimental and control groups.

Table 4: The independent t-test for Mean Gains of the Experimental and Control Groups in the Post-test

Group	Mean	N	SD	<i>p</i> -value	Effect size	<i>t</i>
Experimental	12.26	57	3.30	0.003	0.54	3.06
Control	11.19	57	3.19			

Levene's test: p -value = 0.481; ($p > 0.05$)

Whilst there was no significant difference between the pre-test means of the experimental group and control group before the interventions as indicated by the Levene's test [$p = 0.481$; ($p > 0.05$)], the results from Table 4 shows that there was a statistically significant ($t = 2.14$, $p < 0.05$) difference in the performance between the experimental and control groups in the post-test. The effect size of 0.54 found also confirmed the significant difference between the post-test means as established by the t-test. This suggested that the diagnostic conflict teaching had more impact (gain in understanding) than the conventional teaching approach in remedying second year high school students' errors and misconceptions in algebra.

DISCUSSION OF RESULTS

The study aimed at exploring the impact of diagnostic conflict and conventional teaching approaches on remedying year two senior high students' errors and misconceptions in algebra. The results showed that students in both the control and experimental group gained understanding of fundamental algebra concepts indicating that both teaching approaches had significant impact on remedying students' errors and misconceptions in algebra. This significant improvement in the students' understanding can be attributed not only to the mere fact that there were interventions, but also to the fact that both the diagnostic conflict and conventional teaching approaches were purposefully designed to remedy identified errors and misconceptions (Confrey, 1990). These findings are in line with other studies (e.g., Zohar & Aharon-Kravetsky, 2005; Swan, 2001; Makonye & Stepwell, 2016; Chow, 2011). Zohar and Aharon-Kravetsky (2005) found that students with low academic achievements benefited from the conventional teaching approaches (direct verbal exposition) while Makonye and Stepwell (2016) and Chow (2011) indicated that the diagnostic conflict teaching significantly reduced students' errors and misconceptions in algebra. Swan (2001) on the other hand, advocates for the diagnostic conflict approach but suggested that learning gains from the conventional teaching approach are normally minimal. Though this study seemed to contradict Swan's position on the efficacy of the conventional teaching approach in addressing students'

misconceptions and errors, it aligned with his advocacy for the diagnostic conflict teaching approach.

The study further showed that though both teaching approaches had significant impact on improving learning outcomes and remedying students' errors and misconceptions in algebra, the diagnostic conflict teaching had more effect than the conventional teaching approach. Apparently, the potential of the approach that allowed for students' active construction (or reconstruction) of their own knowledge, students interaction with each other to share their ideas about anomalous situations and possible solutions, the creation of cognitive conflict situations (Baser, 2006) through feedbacks; either by result comparisons, allowing time for students to think, result checks, working tasks using two or more methods (Swan, 2001) as well as practically performing the lesson activities were possible reasons which led to a superior performance of the diagnostic conflict teaching approach. This finding is consistent with similar studies. For example, Dahlan and Rohayati (2012) found out that using cognitive conflict strategy increased students' critical and creative thinking more than the traditional teaching approaches in mathematics. Chow and Treagust (2013) also noted that the diagnostic teaching has significantly improved students' achievement in mathematics and students' attitudes towards inquiry of mathematics lessons. Another reason that explains the superior performance of the diagnostic conflict approach in improving learning outcomes and reducing errors and misconception could be the improved students' attitude exhibited during the study period. This gives the indication that although both interventions had positive impact on the remediation of students' misconceptions and errors in algebra, the diagnostic conflict teaching holds a greater prospect to achieving optimum results in the context of this study.

CONCLUSION AND RECOMMENDATIONS

Both the diagnostic conflict and the conventional teaching approaches implemented made significant impact in remedying students' misconceptions and errors in algebra; with the diagnostic conflict teaching being the superior. The findings reported here highlight areas that require further attention to enhance teacher's concepts in algebra to reduce students' formulation of misconceptions and errors. The need for teacher training and professional development programmes to expose teachers to students' misconceptions (or preconceptions) and errors to enrich their knowledge for teaching algebra (Wilmot, 2015) and to equip teachers with knowledge of the strategies (e.g. cognitive conflict strategies) as part of pedagogical content knowledge, that will be useful in reorganizing the understanding of learners; since learners are unlikely to appear in class as blank slates (Shulman, 1986). Such programme need not differ in content but in format for both pre-service and in-service teachers.

REFERENCES

- Agyei, D. D. (2013). Effects of using interactive spreadsheet as a demonstrative tool to enhance teaching and learning of mathematics concepts. *International Journal of Educational Planning & Administration*, 3(1), 81-99
- Agyei, D. D., & Voogt, J. (2011). ICT use in the teaching of mathematics: Implications for professional development of pre-service teachers in Ghana. *Education and Information Technologies*, 16(4), 423-439.

- Askew, M. and Wiliam, D. (1995). *Recent Research in Mathematics Education*, (London: HMSO), 5-16.
- Baser, M. (2006). Fostering conceptual change by cognitive conflict based instruction on students' understanding of heat and temperature concepts. *Eurasia Journal of Mathematics, Science and Technology Education*, 2(2), 96-114.
- Bell, A. (1993). Some Experiments in Diagnostic Teaching. *Educational Studies in Mathematics*, 24(1), 115–137. Retrieved from <http://www.jstor.org/stable/3482981>
- Berk, L.E. (1997) *Child Development*, 4th edn. (Boston: Allyn and Bacon)
- Booth, L. R. (1988). Children's difficulties in beginning algebra. *The ideas of algebra*, K-12, 20-32.
- Booth, J. L., Barbieri, C., Eyer, F., & Paré-Blagoev, E. J. (2014). Persistent and pernicious errors in algebraic problem solving. *The Journal of Problem Solving*, 7(1), 3.
- Booth, J. L., & Koedinger, K. R. (2012). Are diagrams always helpful tools? Developmental and individual differences in the effect of presentation format on student problem solving. *British Journal of Educational Psychology*, 82(3), 492-511
- Chow, T. C. F. (2011). Students' difficulties, conceptions and attitudes towards learning algebra: an intervention study to improve teaching and learning (Doctoral dissertation, Curtin University, Science and Mathematics Education Centre).
- Chow, T. C., & Treagust, D. (2013). An Intervention Study Using Cognitive Conflict to Foster Conceptual Change. *Journal of Science and Mathematics*, 36(1), 44-64.
- Confrey, J. (1990). A review of the research on student conceptions in mathematics, science, and programming. *Review of research in education*, 16, 3-56
- Clement, J., Lochhead, J., & Monk, G. S. (1981). Translation difficulties in learning mathematics. *The American Mathematical Monthly*, 88(4), 286-290.
- Coe, R. (2002). It's the effect size, stupid: What effect size is and why it is Important.
- Creswell, J. W. (2013). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage publications.
- Dahlan, J. A., & Rohayati, A. (2012). Implementasi Strategi Pembelajaran Konflik Kognitif Dalam Upaya Meningkatkan High Order Mathematical Thinking Siswa. *Jurnal Pendidikan*, 13(2), 65-76.
- Dahlan J. A., Rohayati A. (2014). The comparison of mathematical understanding and connection through cognitive conflict of piaget and hasweh *Proceeding of International Conference On Research, Implementation And Education Of Mathematics And Sciences 2014, Yogyakarta State University, 18-20 May 2014*
- Dhindsa, H. S., & Anderson, O. R. (2004). Using a conceptual-change approach to help preservice science teachers reorganize their knowledge structures for constructivist teaching. *Journal of Science Teacher Education*, 15(1), 63-85.
- DiSessa, A. A. (2006). *A History of Conceptual Change Research: Threads and Fault Lines*. Cambridge University Press.
- Egodawatte G. (2011) Secondary School Student's Misconceptions in Algebra, Department of Curriculum, Teaching and Learning, University of Toronto. Thesis PhD. Mathematics, Texas A&M University, 1-12.
- Fletcher, J. A. & Anderson, S. (2012). Improving students' performance in mensuration at the senior high school level using Geometer' sketchpad. *Journal of Science and Mathematics Education*, 6(1), 63-79.
- Gall, M. D., Gall, J. P., & Borg, W. R. (2007). *Educational research: An introduction* (8th Ed.). United States of America: Pearson Education, Inc.

- Kang, S., Scharmann, L. C., Noh, T., & Koh, H. (2005). The influence of students' cognitive and motivational variables in respect of cognitive conflict and conceptual change. *International Journal of Science Education*, 27(9), 1037-1058.
- Ling, G. C. L., Shahrill, M., & Tan, A. (2016). Common Misconceptions of Algebraic Problems: Identifying Trends and Proposing Possible Remedial Measures. *Advanced Science Letters*, 22(5-6), 1547-1550.
- Lucariello, J., Tine, M. T., & Ganley, C. M. (2014). A formative assessment of students' algebraic variable misconceptions. *The Journal of Mathematical Behavior*, 33, 30-41.
- Makonye, J. P. (2011). Learner Errors in Introductory Differentiation Tasks: A Study of Learner Misconceptions in the National Senior Certificate Examinations. *University of Johannesburg, Auckland Park, South Africa*.
- Makonye, J. P., & Stepwell N. (2016). Eliciting Learner Errors and Misconceptions in Simplifying Rational Algebraic Expressions to Improve Teaching and Learning. *Kamla-Raj 2016 Int J Edu Sci*, 12(1): 16-28.
- Matlock-Hetzel, S. (1997). Basic Concepts in Item and Test Analysis. <http://files.eric.ed.gov/fulltext/ED406441.pdf>.
- Matzin, E. S., & Shahrill, M. (2015). A preliminary study of year 7 students' performance on algebraic concepts. In *In Pursuit of Quality Mathematics Education for All: Proceedings of the 7th ICMI-East Asia Regional Conference on Mathematics Education* (pp. 233-239).
- Morris, S. B., & DeShon, R. P. (2002). Combining effect size estimates in meta-analysis with repeated measures and independent-groups designs. *Psychological methods*, 7(1), 105.
- Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Math literacy and civil rights*. Beacon Press (MA).
- Mulungye, M. M., O'Connor, M., & Ndethiu, S. (2016). Sources of Student Errors and Misconceptions in Algebra and Effectiveness of Classroom Practice Remediation in Machakos County--Kenya. *Journal of Education and Practice*, 7(10), 31-33.
- Olivier, A. (1989). Handling pupils' misconceptions. In *Pythagoras*.
- Perso, T. F. (1991). *Misconceptions in algebra: Identification, diagnosis and treatment*. Unpublished doctoral thesis, Curtin University of Technology, Perth.
- Potvin, P., Sauriol, É., & Riopel, M. (2015). Experimental evidence of the superiority of the prevalence model of conceptual change over the classical models and repetition. *Journal of Research in Science Teaching*, 52(8), 1082-1108.
- Pournara, C., Sanders, Y., Adler, J., & Hodgen, J. (2016). Learners' errors in secondary algebra: insights from tracking a cohort from Grade 9 to Grade 11 on a diagnostic algebra test. *Pythagoras*, 37(1), 1-10.
- Rahim, R., Noor, N. M., & Zaid, N. M. (2015). Meta-analysis on element of cognitive conflict strategies with a focus on multimedia learning material development. *International Education Studies*, 8(13), 73.
- Seng, L. K. (2010). An error analysis of form 2 (grade 7) students in simplifying algebraic expressions: A descriptive study. *Electronic Journal of Research in Educational Psychology*, 8(1), 139-162.
- Star, J. R., Foegen, A., Larson, M. R., McCallum, W. G., Porath, J., Zbiek, R. M., & Lyskawa, J. (2015). Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students. Educator's Practice Guide. What Works Clearinghouse.™ NCEE 2015-4010. What Works Clearinghouse.

- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, 15(2), 4-14.
- Susilawati, W., Suryadi, D. & Dahlan, J.A. (2017). The Improvement of Mathematical Spatial Visualization Ability of Student through Cognitive Conflict. *IEJMEMathematics Education*, 12(2), 155-166
- Swan, M. (2006). *Collaborative learning in mathematics: a challenge to our beliefs*
- Swan, M. (2001). Dealing with misconceptions in mathematics. In P. Gates (Ed.), *Issues in Mathematics Teaching* (pp. 147-165). London: Routledge Falmer.
- Tsai, C. C., & Chang, C. Y. (2005). Lasting effects of instruction guided by the conflict map: Experimental study of learning about the causes of the seasons. *Journal of research in science teaching*, 42(10), 1089-1111.
- Usiskin, Z. (2004). A significant amount of algebra. *Nieuw Archief Voor Wiskunde*, 5, 147-152.
- Venter, I. (2006). Development of a valid and reliable test for higher-educated young adults measuring dietary fibre food source and health-disease association knowledge. *Journal of Family Ecology and Consumer Sciences/Tydskrif vir Gesinsekologie en Verbruikerswetenskappe*, 34(1).
- WAEC (2014) Mathematic Chief Examiners' report (2014) on WASSCE <https://www.waecgh.org/Exams/ChiefExaminersReport.aspx>
- WAEC (2015) Mathematic Chief Examiners' report (2014) on WASSCE <https://www.waecgh.org/Exams/ChiefExaminersReport.aspx>
- WAEC (2016) Mathematic Chief Examiners' report (2014) on WASSCE <https://www.waecgh.org/Exams/ChiefExaminersReport.aspx>
- WAEC (2017) Mathematic Chief Examiners' report (2017) on WASSCE
- Wadsworth, B. J. (1996). *Piaget's theory of cognitive and affective development: Foundations of constructivism*. Longman Publishing.
- Wilmot, E. M. (2015). HIGH SCHOOL TEACHERS' KNOWLEDGE FOR TEACHING ALGEBRA: A COMPARISON OF GHANA AND THE US. *British Journal of Education*, 3(2), 61-74.
- Zohar, A., & Aharon-Kravetsky, S. (2005). Exploring the effects of cognitive conflict and direct teaching for students of different academic levels. *Journal of Research in Science Teaching*, 42(7), 829-855.