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STATISTICAL INFERENCE FOR PARAMETERS OF GOMPERTZ DISTRIBUTION BASED ON GENERAL PROGRESSIVELY TYPE-II RIGHT CENSORED ORDER STATISTICS

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ABSTRACT: *In this article, we concluded the Bayesian estimations for the parameters of the Gompertz distribution (GD) based on general progressively type-II right censored order statistics (GPTIIRCO). The squared error loss function (SEL) and linear-exponential loss function (LINEX) and the generalization of the entropy loss function (GE) and Al-Bayyati loss function (ABL) are considered for Bayesian estimation. Finally, a numerical example is established to clear the theoretical procedures.*

KEYWORDS: Gompertz Distribution; General Progressively type-II Right Censored Order Statistics.

INTRODUCTION

Kim [11] derived on estimating Burr Type XII parameter based on GPTIICO. Mohie El-Din et al. [12] derived Estimation for parameters of Feller-Pareto distribution from progressive Type-II censoring and some characterizations. Mohie El-Din et al. [13] and [14] derived characterization for Gompertz and linear failure rate distributions using recurrence relations of single and product moments based on GPTIICO.

In failure data analysis, it is common that some individuals cannot be observed for the full failure times. GPTIICOS is a useful and more general scheme in which a specific fraction of individuals at risk may be removed from the study at each of several ordered failure times. Progressively censored samples have been considered, among others, as solved by Davis and Feldstein [7], Balakrishnan et al. [4], and Guilbaud [10]. This scheme of censoring was generalized by Balakrishnan and Sandhu [3] as follows: at time $X_0 \equiv 0$, $n$units are placed on test; the first r failure times, $X_1, \ldots, X_r$, are not observed; at time $X_i + 0$, where X_i is the i^{th} ordered failure time $(i = r+1, \ldots, m-1)$, R_i units are removed from the test randomly, so prior to the $(i+1)^{th}$ failure there are $n_i = n - i - \sum_{j=r+1}^{i} R_j$ units on test; finally, at the time of the m^{th} failure, X_m, the experiment is terminated, i.e., the remaining R_m units are removed from the test. The R_i 's, m and r are prespecified integers which must satisfy the conditions: $0 \le r < m \le n,\ 0 \le R_i \le n_{i-1}\ for\ i = r+1, \ldots, m-1$ with $n_r = n - r$ and $R_m = n_{m-1} - 1$.

If the failure times are based on an absolutely continuous distribution function (cdf) F with probability density function (pdf) f, the joint probability density function based on GPTIICOS failure times $X_{r+1:m:n}, X_{r+2:m:n}, \ldots, X_{m:m:n}$, is given by

$$f_{X_{r+1:m:n},X_{r+2:m:n},\ldots,X_{m:m:n}}(x_{r+1}, x_{r+2}, \ldots, x_m) = A_{(n,m-1)}[F(x_{r+1},\theta)]^r \times \prod_{i=r+1}^{m} f(x_i,\theta)[1 - F(x_i,\theta)]^{R_i}, \qquad x_{r+1} < x_{r+2} < \cdots < x_m, \tag{1.1}$$

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where,

$$A_{(n,m-1)} = \frac{n!}{r!\,(n-r)!}\left(\prod_{j=r}^{m-1} n_j\right), \quad n_i = n - i - \sum_{j=r+1}^{i} R_j\,, \quad i = r+1, \dots, m-1.$$

Through ut this paper, we concluded the Bayesian estimations for the parameters of the GD based on GPTIIRCO. The SEL, ABL, LINEX and GE loss functions are considered for Bayesian estimation. Finally, a numerical example is established to clear the theoretical proceduresare obtained. For more details of the SEL, ABL, LINEX and GE loss functions see Al-Aboud [1], Al-Bayyati [2], Bernardo and Smith [5], Calabria and Pulcini [6], and Figueiredo [8].

Let $X_{r+1:m:n}^{(R_{r+1},R_{r+2},\dots,R_m)} < X_{r+2:m:n}^{(R_{r+1},R_{r+2},\dots,R_m)} < \cdots < X_{m:m:n}^{(R_{r+1},R_{r+2},\dots,R_m)}$ be the m ordered observed failure times in a sample of size $(n-r)$ under GPTIICOS scheme from the GD with probability density function (pdf) given by

$$f(x,\alpha,\beta) = \alpha e^{\beta x - \frac{\alpha}{\beta}(e^{\beta x}-1)}, \quad \alpha > 0,\ \beta > 0 \quad x \ge 0 \tag{1.2}$$

and the corresponding cumulative distribution function (cdf) is given by

$$F(x,\alpha,\beta) = 1 - e^{-\frac{\alpha}{\beta}(e^{\beta x}-1)}, \quad \alpha > 0,\ \beta > 0 \quad x \ge 0. \tag{1.3}$$

The hazard function at time *t* is given, by

$$H(t) = \frac{f(t)}{F(t)} = \alpha e^{\beta t}. \tag{1.4}$$

The GD was introduced by Gompertz [9].

Bayes estimation

In this section, we have three types of loss functions, which are SEL, ABL, LINEX and GE loss functions. We apply these types in two cases, the first is known shape parameter β and the second is

unknown shape β and scale parameter.

The likelihood function is

$$\mathrm{L}(\alpha,\beta|\mathbf{x}) = A_{(n,m-1)}\left[1 - e^{-\frac{\alpha}{\beta}(e^{\beta x_{r+1}}-1)}\right]^{r} \prod_{i=r+1}^{m} \alpha e^{\beta x_i - \frac{\alpha}{\beta}(e^{\beta x_i}-1)}\left[e^{-\frac{\alpha}{\beta}(e^{\beta x_i}-1)}\right]^{R_i}. \tag{2.1}$$

The case of known shape parameter β

For Bayesian estimation, we use here the natural conjugate prior density function for α and the known shape parameter β is given by

$$\pi(\alpha|\mathbf{x},\beta) = \frac{b^a}{\Gamma(a)}\alpha^{a-1}e^{-b\alpha}, \quad a,b > 0, \tag{2.2}$$

Combining the likelihood function given by Eq.(2.1) and prior function Eq.(2.2), the posterior density of α given $\mathbf{x}$ is

$$\pi^*(\alpha|\mathbf{x},\beta) = \frac{L(\alpha|\mathbf{x},\beta)\pi(\alpha|\mathbf{x},\beta)}{\int_0^\infty L(\alpha|\mathbf{x},\beta)\pi(\alpha|\mathbf{x},\beta)d\alpha}$$

$$= \frac{L(\alpha|\mathbf{x},\beta)\pi(\alpha|\mathbf{x},\beta)}{I(a,b)} . \tag{2.3}$$

Bayes estimator based on squared error loss function (SEL) The Bayes estimators $\hat{\alpha}_{BS}$ of α based on SEL loss function given by

$$\hat{\alpha}_{BS} = E(\alpha|\mathbf{x},\beta)$$

$$= \int_0^\infty \alpha\pi^*(\alpha|\mathbf{x},\beta)d\alpha$$

$$= \frac{a\, I(a+1,b)}{b\, I(a,b)}. \tag{2.4}$$

The hazard function H(t) at time under the SEL loss function given by

$$\hat{H}_{BS}(t) = E(H(t)|\mathbf{x},\beta)$$

$$= e^{t\beta}\hat{\alpha}_{BS} . \tag{2.5}$$

Bayes estimator based on Al-Bayyati loss function (ABL) The Bayes estimators $\hat{\alpha}_{BA}$ of α based on ABL loss function given by

$$\hat{\alpha}_{BA} = \frac{E(\alpha^{q+1}|\mathbf{x},\beta)}{E(\alpha^q|\mathbf{x},\beta)}$$

$$= \frac{\int_0^\infty \alpha^{q+1}\pi^*(\alpha|\mathbf{x},\beta)d\alpha}{\int_0^\infty \alpha^q\pi^*(\alpha|\mathbf{x},\beta)d\alpha}$$

$$= \frac{(a+q)\, I(a+q+1,b)}{b\, I(a+q,b)}. \tag{2.6}$$

Similarly, hazard function H(t) at time t under the ABL loss function given by

$$\hat{H}_{BA}(t) = \frac{E(H^{q+1}(t)|\mathbf{x},\beta)}{E(H^q(t)|\mathbf{x},\beta)}$$

$$= e^{t\beta}\,\hat{\alpha}_{BA}. \tag{2.7}$$

Bayes estimator based on linear-exponential loss function (LINEX)

The Bayes estimators $\hat{\alpha}_{BL}$ of α based on **LINEX** loss function given by

$$\hat{\alpha}_{BL} = \frac{-1}{p}\log\left(E(e^{-p\alpha}|\mathbf{x},\beta)\right)$$

$$= \frac{-1}{p}\log\left(\int_0^\infty e^{-p\alpha}\pi^*(\alpha|\mathbf{x},\beta)d\alpha\right)$$

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$$= \frac{-1}{p}\log\left(\frac{b^{a}\ \ I(a, b+p)}{(b+p)^{a}\ \ I(a, b)}\right). \tag{2.8}$$

Similarly, hazard function H(t) at time t under the LINEX loss function given by

$$\hat{H}_{BA}(t) = \frac{-1}{p}\log\left(E\left(e^{-pH(t)}|\mathbf{x},\beta\right)\right)$$

$$= \frac{-1}{p}\log\left(\frac{b^{a}\ \ I\left(a, b+p\,e^{t\beta}\right)}{(b+p\,e^{t\beta})^{a}\ \ I(a, b)}\right). \tag{2.9}$$

Bayes estimator based ongeneralization of the entropy loss function (GE)

The Bayes estimators $\hat{\alpha}_{BG}$ of α based on GE loss function given by

$$\hat{\alpha}_{BG} = \left(E(\alpha^{-\upsilon}|\mathbf{x},\beta)\right)^{\frac{-1}{\upsilon}}$$

$$= \left(\int_0^{\infty} \alpha^{-\upsilon}\pi^{*}(\alpha|\mathbf{x},\beta)d\alpha\right)^{\frac{-1}{\upsilon}}$$

$$= \left(\frac{b^{\upsilon}\ \Gamma(a-\upsilon)I(a-\upsilon, b)}{\Gamma(a)\qquad I(a, b)}\right)^{\frac{-1}{\upsilon}}. \tag{2.10}$$

The hazard function H(t) at time t under the GE loss function given by

$$\hat{H}_{BG}(t) = \left(E(H^{-\upsilon}(t)|\mathbf{x},\beta)\right)^{\frac{-1}{\upsilon}}$$

$$= e^{t\beta}\ \hat{\alpha}_{BS}\ . \tag{2.11}$$

The case of unknown shape and scale parameters α and β

For Bayesian estimation, we use here the natural conjugate prior density function for (α,β) given by

$$\pi(\alpha,\beta|\mathbf{x}) = \pi_1(\beta)\,\pi_2(\alpha|\beta)$$

$$= \frac{{b_1}^{a_1}{b_2}^{a_2}}{\Gamma(a_1)\Gamma(a_2)}\alpha^{a_2-1}\beta^{a_1-a_2-1}e^{-\frac{b_2\alpha}{\beta}-b_1\beta},\quad a_1,a_2,b_1,b_2>0, \tag{2.12}$$

where

$$\pi_1(\beta) = \frac{{b_1}^{a_1}}{\Gamma(a_1)}\beta^{a_1-1}e^{-b_1\beta}\ and\ \pi_2(\alpha|\beta) = \frac{\left(\frac{b_2}{\beta}\right)^{a_2}}{\Gamma(a_2)}\alpha^{a_2-1}e^{-\frac{b_2\alpha}{\beta}}.$$

Combining the likelihood function given by Eq. (2.1) and prior function Eq. (2.12), the posterior

density of α and β given $\mathbf{x}$ is

$$\pi^{*}(\alpha,\beta|\mathbf{x}) = \frac{L(\alpha,\beta|\mathbf{x})\pi(\alpha,\beta|\mathbf{x})}{\int_0^{\infty}\int_0^{\infty} L(\alpha,\beta|\mathbf{x})\pi(\alpha,\beta|\mathbf{x})d\alpha\, d\beta}$$

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$$= \frac{L(\alpha, \beta|\mathbf{x})\pi(\alpha, \beta|\mathbf{x})}{I(a_1, b_1, a_2, b_2)} . \tag{2.13}$$

Bayes estimator based on squared error loss function (SEL)

The Bayes estimators α *and* β based on SEL loss function are given respectively, by

$$\hat{\alpha}_{BS} = E(\alpha|\mathbf{x})$$

$$= \int_0^\infty \int_0^\infty \alpha\pi^*(\alpha, \beta|\mathbf{x}) d\alpha \; d\beta$$

$$= \frac{a_1 \, a_2 \, I(a_1 + 1, b_1, a_2 + 1, b_2)}{b_1 \, b_2 I(a_1, b_1, a_2, b_2)}, \tag{2.14}$$

and

$$\hat{\beta}_{BS} = E(\beta|\mathbf{x})$$

$$= \frac{a_1 \;\; I(a_1 + 1, b_1, a_2, b_2)}{b_1 \;\; I(a_1, b_1, a_2, b_2)}. \tag{2.15}$$

The hazard function H(t) at time under the SEL loss function given by

$$\hat{H}_{BS}(t) = E(H(t)|\mathbf{x})$$

$$= \frac{a_1 \, a_2 \, b_1{}^{a_1} \qquad I(a_1 + 1, b_1 - t, a_2 + 1, b_2)}{b_1 b_2 (b_1 - t)^{a_1} \; I(a_1, b_1, a_2, b_2)} . \tag{2.16}$$

Bayes estimator based on Al-Bayyati loss function (ABL) The Bayes estimators α *and* β based on ABL loss function given by

$$\hat{\alpha}_{BA} = \frac{E(\alpha^{q+1}|\mathbf{x})}{E(\alpha^q|\mathbf{x})}$$

$$= \frac{\int_0^\infty \int_0^\infty \alpha^{q+1}\pi^*(\alpha|\mathbf{x}) d\alpha \, d\beta}{\int_0^\infty \int_0^\infty \alpha^q \pi^*(\alpha|\mathbf{x}) d\alpha \, d\beta}$$

$$= \frac{(a + q) \, I(a + q + 1, b)}{b \, I(a + q, b)}, \tag{2.17}$$

and

$$\hat{\beta}_{BS} = \frac{a_1 \;\; I(a_1 + 1, b_1, a_2, b_2)}{b_1 \;\; I(a_1, b_1, a_2, b_2)}. \tag{2.18}$$

Similarly, hazard function H(t) at time t under the ABL loss function given by

$$\hat{H}_{BA}(t) = \frac{E(H^{q+1}(t)|\mathbf{x})}{E(H^q(t)|\mathbf{x})}$$

$$= e^{t\beta} \, \hat{\alpha}_{BA}. \tag{2.19}$$

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Bayes estimator based on linear-exponential loss function (LINEX)

The Bayes estimators α *and* β based on **LINEX** loss function given by

$$\hat{\alpha}_{BL} = \frac{-1}{p}\log\left(E\left(e^{-p\alpha}|\mathbf{x}\right)\right)$$

$$= \frac{-1}{p}\log\left(\int_0^\infty \int_0^\infty e^{-p\alpha}\pi^*(\alpha|\mathbf{x})d\alpha\ d\beta\right)$$

$$= \frac{-1}{p}\log\left(\frac{b_2^{\,a_2}\ \mathrm{I}(a_1, b_1, a_2, b_2 + p)}{(b_2 + \mathrm{p})^{a_2}\ \mathrm{I}(a_1, b_1, a_2, b_2)}\right), \tag{2.20}$$

and

$$\hat{\beta}_{BL} = \frac{-1}{p}\log\left(E\left(e^{-p\beta}|\mathbf{x}\right)\right)$$

$$= \frac{-1}{p}\log\left(\frac{b_1^{\,a_1}\ \mathrm{I}(a_1, b_1 + p, a_2, b_2)}{(b_1 + \mathrm{p})^{a_1}\ \mathrm{I}(a_1, b_1, a_2, b_2)}\right). \tag{2.21}$$

Similarly, hazard function H(t) at time t under the LINEX loss function given by

$$\hat{H}_{BA}(t) = \frac{-1}{p}\log\left(E\left(e^{-pH(t)}|\boldsymbol{x}\right)\right)$$

$$= \frac{-1}{p}\log\left(\sum_{l=0}^{\infty}\frac{(-p)^l\, b_1^{\,a_1}\,\Gamma(a_1 + l)\,\Gamma(a_2 + l)\,\mathrm{I}(a_1 + l, b_1 - tl, a_2 + l, b_2)}{\Gamma(a_1)\,\Gamma(a_2)\,\Gamma(l + 1)\,(b - tl)^{a_1}\,\mathrm{I}(a_1, b_1, a_2, b_2)}\right). \tag{2.22}$$

Bayes estimator based ongeneralization of the entropy loss function (GE)

The Bayes estimators α *and* β based on GE loss function given by

$$\hat{\alpha}_{BG} = \left(E\left(\alpha^{-v}|\mathbf{x}\right)\right)^{\frac{-1}{v}}$$

$$= \left(\int_0^\infty \int_0^\infty \alpha^{-v}\pi^*(\alpha|\mathbf{x})d\alpha\ d\beta\right)^{\frac{-1}{v}}$$

$$= \left(\frac{(b_1\ b_2)^v\ \Gamma(\mathrm{a}_1 - v)\ \Gamma(\mathrm{a}_2 - v)\ \mathrm{I}(a_1 - v, b_1, a_2 - v, b_2)}{\Gamma(\mathrm{a}_1)\ \Gamma(\mathrm{a}_2)\qquad\qquad \mathrm{I}(a_1, b_1, a_2, b_2)}\right)^{\frac{-1}{v}}, \tag{2.23}$$

and

$$\hat{\beta}_{BG} = \left(E\left(\beta^{-v}|\mathbf{x}\right)\right)^{\frac{-1}{v}}$$

$$= \left(\frac{(b_1)^v\ \Gamma(\mathrm{a}_1 - v)\ \ \mathrm{I}(a_1 - v, b_1, a_2, b_2)}{\Gamma(\mathrm{a}_1)\qquad\qquad \mathrm{I}(a_1, b_1, a_2, b_2)}\right)^{\frac{-1}{v}}. \tag{2.24}$$

The hazard function H(t) at time t under the GE loss function given by

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$$\hat{H}_{BG}(t) = \left(E(\mathrm{H}^{-\nu}(t)|\mathbf{x})\right)^{\frac{-1}{\nu}}$$

$$= \left(\frac{(b_1)^{a_1+\nu}\,(b_2)^{\nu}\,\Gamma(\mathrm{a}_1-\nu)\;\Gamma(\mathrm{a}_2-\nu)}{\Gamma(\mathrm{a}_1)\,\Gamma(\mathrm{a}_1)\,(b_1+t\nu)^{a_1}}\frac{\mathrm{I}(a_1-\nu,b_1+t\nu,a_2-\nu,b_2)}{\mathrm{I}(a_1,b_1,a_2,b_2)}\right)^{\frac{-1}{\nu}}. \tag{2.25}$$

Numerical results

In this section, we obtained some numerical results to compare the Bayes estimation under SEL, ABL

(with $q = 0.5$ and $q = -0.5$), LINEX (with $p = 0.5$) and GE (with $\nu = -0.5$) loss functions according to the following steps:

Algorithm

1- Specify the values $of\;\; n, m, r, t, q, p\;and\;\nu.$

2- Specify the values of the parameters $\alpha\;and\;\beta.$

3- For given values of the prior parameters.

4- Generate a simple random sample of size m from $U(0,1)$ distribution, $(U_{r+1}, \dots, U_m)$.

5- Determine the values of the censoring scheme $R_i,\;\; i = r+1, \dots, m.$

6- Set $E_i = U_i^{\frac{1}{\sum_{j=m-i+1}^{m} R_j}}, \quad i = r+1, \dots, m.$

7- Obtain the General Progressively type-II censored sample $(U^*_{r+1:m:n}, \dots, U^*_{m:m:n})$, where $U^*_{i:m:n} = 1 - \prod_{j=m-i+1}^{m} E_i\,, \;, \quad i = r+1, \dots, m.$

8- From steps (4)-(7), the order observations $y_{r+1:m:n}, y_{r+2:m:n}, \dots, y_{n_1:m:n}, y_{n_1+1:m:n}, \dots, y_{m:m:n}$ are calculated as follows

$$y_{i:m:n} = \frac{1}{\beta}\log\left[1 - \frac{\beta}{\alpha}\log(1 - U^*_{i:m:n})\right], \;\; i = r+1, \dots, m.$$

9- We perform Monte Carlo simulation to compare th performaces of the different estimators for different sampling schemes. Monte Carlo simulations were performed utilizing 1000 general progressively type-II censored samples for each simulations. The mean squared error (MSE) is used to compear the estimators, where $MSE = \frac{1}{1000}\sum_{i=1}^{1000}\left(\theta - \hat{\theta}\right)^2$ and $\hat{\theta}$ is the estimator of θ.

10- Compute the BEs of the model parameters relative to SEL, ABL, LINEXL and GE loss functions.

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The following is the generated sample from Gompertz distribution pdf in (1).

Scheme	n	m	r	R
1	30	23	2	-,-, 0,0,0,0,0,1,0,0,0,0,0,1,0,1,0,0,0,1,1,1,1
2	30	23	5	-,-, -,-,-,0,0,1,0,0,0,0,0,1,0,1,0,0,0,1,1,1,1
3	30	17	2	-,-, 0,0,0,0, 1,1,1,1,1,1,1,2,2,1,1
4	30	17	5	-,-, -,-,-,0, 1,1,1,1,1,1,1,2,2,1,1
5	10	07	1	-,0,0,0,1,1,1

The case of known parameter β

Table 1: The values of BIAS and MSE of the Bayesian estimates of α based on SEL, ABL, LINEX and GE loss functions when $\beta = 0.3$ *and* 0.5.

n	m	r	α	β	$\hat{\alpha}_{BS}$		$\hat{\alpha}_{BA}$ $q=-0.5$		$\hat{\alpha}_{BA}$ $q=0.5$		$\hat{\alpha}_{BL}$		$\hat{\alpha}_{BE}$	
					BIAS	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*
30	23	2	0.3	0.3	0.01511	0.00517	0.00829	0.00480	0.02193	0.00564	0.01399	0.00506	0.01172	0.00497
				0.5	0.01947	0.00522	0.01256	0.00479	0.02638	0.00575	0.01832	0.00510	0.01603	0.00500
			0.5	0.3	0.02669	0.01381	0.01530	0.01277	0.03809	0.01512	0.02358	0.01333	0.02103	0.01326
		5	0.3	0.3	0.02091	0.00590	0.01396	0.00543	0.02786	0.00648	0.01974	0.00577	0.01745	0.00565
				0.5	0.01025	0.00431	0.00354	0.00404	0.01697	0.00468	0.00917	0.00423	0.00691	0.00416
			0.5	0.3	0.02358	0.01330	0.01224	0.01234	0.03492	0.01452	0.02050	0.01285	0.01794	0.01279
	17	2	0.3	0.3	0.01873	0.00701	0.00941	0.00637	0.02804	0.00784	0.01715	0.00682	0.01410	0.00667
				0.5	0.01827	0.00703	0.00897	0.00639	0.02758	0.00785	0.01671	0.00684	0.01366	0.00669
			0.5	0.3	0.03043	0.01808	0.01493	0.01639	0.04593	0.02028	0.02612	0.01727	0.02274	0.01717
		5	0.3	0.3	0.02085	0.00671	0.01147	0.00604	0.03024	0.00756	0.01927	0.00652	0.01620	0.00635
				0.5	0.02093	0.00714	0.01154	0.00645	0.03032	0.00802	0.01934	0.00694	0.01627	0.00677
			0.5	0.3	0.03357	0.01981	0.01796	0.01793	0.04918	0.02221	0.02918	0.01889	0.02582	0.01881
10	7	1	0.3	0.3	0.05179	0.02691	0.02703	0.02167	0.07654	0.03362	0.04670	0.02483	0.03963	0.02415
				0.5	0.05211	0.02489	0.02734	0.01991	0.07689	0.03132	0.04709	0.02300	0.03995	0.02226
			0.5	0.3	0.08072	0.06527	0.03986	0.05236	0.12159	0.08209	0.06729	0.05746	0.06066	0.05844

Table 2: The values of BIAS and MSE of the Bayesian estimates of $H(t)$ when $t = 3$ based on SEL, ABL, LINEX and GE loss functions when $\beta = 0.3$ *and* 0.5.

n	m	r	α	β	$\hat{H}_{BS}$		$\hat{H}_{BA}$ $q=-0.5$		$\hat{H}_{BA}$ $q=0.5$		$\hat{H}_{BL}$		$\hat{H}_{BE}$	
					BIAS	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*
30	23	2	0.3	0.3	0.03717	0.03127	0.02040	0.02902	0.05394	0.03411	0.03043	0.02970	0.02883	0.03008
				0.5	0.08725	0.10487	0.05627	0.09626	0.11823	0.11549	0.06453	0.09497	0.07184	0.10034
			0.5	0.3	0.06566	0.08354	0.03763	0.07726	0.09369	0.09148	0.04703	0.07680	0.05172	0.08021
		5	0.3	0.3	0.05143	0.03571	0.03433	0.03283	0.06852	0.03921	0.04441	0.03381	0.04293	0.03420
				0.5	0.04596	0.08660	0.01584	0.08112	0.07607	0.09398	0.02457	0.07982	0.03098	0.08364
			0.5	0.3	0.05800	0.08043	0.03011	0.07467	0.08589	0.08782	0.03958	0.07419	0.04413	0.07737
	17	2	0.3	0.3	0.04606	0.04243	0.02315	0.03852	0.06897	0.04746	0.03665	0.03968	0.03469	0.04035
				0.5	0.08190	0.14121	0.04021	0.12837	0.12359	0.15775	0.05117	0.12543	0.06121	0.13437

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			0.5	0.3	0.07485	0.10936	0.03672	0.09913	0.11298	0.12267	0.04919	0.09805	0.05593	0.10389
		5	0.3	0.3	0.05130	0.04057	0.02821	0.03655	0.07439	0.04573	0.04179	0.03784	0.03984	0.03843
				0.5	0.09382	0.14343	0.05173	0.12954	0.13590	0.16109	0.06257	0.12655	0.07293	0.13606
			0.5	0.3	0.08257	0.11983	0.04417	0.10845	0.12097	0.13436	0.05647	0.10701	0.06351	0.11379
10	7	1	0.3	0.3	0.12737	0.16279	0.06649	0.13109	0.18827	0.20337	0.09760	0.13435	0.09748	0.14612
				0.5	0.23355	0.49993	0.12251	0.39992	0.34461	0.62902	0.13973	0.3592	0.17904	0.44720
			0.5	0.3	0.19855	0.39484	0.09804	0.31677	0.29907	0.49664	0.12127	0.29300	0.14920	0.35355

The case of unknown parameters α *and* β

Table 3: The values of BIAS and MSE of the Bayesian estimates of α based on SEL, ABL, LINEX and GE loss functions

n	m	r	α	β	$\hat{\alpha}_{BS}$		$\hat{\alpha}_{BA}$ $q=-0.5$		$\hat{\alpha}_{BA}$ $q=0.5$		$\hat{\alpha}_{BL}$		$\hat{\alpha}_{BE}$	
					BIAS	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*
30	23	2	0.3	0.3	0.11413	0.02057	0.10177	0.01748	0.12651	0.02398	0.11149	0.01977	0.10799	0.01899
				0.5	0.19652	0.04644	0.18184	0.04045	0.21121	0.05288	0.19280	0.04476	0.18923	0.04341
			0.5	0.3	0.12476	0.03210	0.10639	0.02691	0.14314	0.03799	0.11886	0.03002	0.11564	0.02944
		5	0.3	0.3	0.11381	0.02017	0.10145	0.01711	0.12617	0.02356	0.11117	0.01939	0.10767	0.01861
				0.5	0.19663	0.04612	0.18195	0.04014	0.21132	0.05254	0.19292	0.04444	0.18934	0.04309
			0.5	0.3	0.11667	0.03107	0.09853	0.02617	0.13482	0.03665	0.11090	0.02908	0.10767	0.02855
	17	2	0.3	0.3	0.08188	0.01364	0.07062	0.01153	0.09314	0.01602	0.07964	0.01311	0.07629	0.01256
				0.5	0.13706	0.02658	0.12422	0.02278	0.14990	0.03073	0.13417	0.02558	0.13069	0.02466
			0.5	0.3	0.08994	0.02588	0.07267	0.02205	0.10721	0.03034	0.08465	0.02430	0.08137	0.0239
		5	0.3	0.3	0.07655	0.01163	0.06544	0.00972	0.08766	0.01379	0.07439	0.01117	0.07104	0.01065
				0.5	0.13512	0.02560	0.12234	0.02189	0.14791	0.02965	0.13226	0.02464	0.12878	0.02372
			0.5	0.3	0.08037	0.02223	0.06337	0.01888	0.09736	0.02618	0.07527	0.02087	0.07193	0.02049
10	7	1	0.3	0.3	0.10079	0.02819	0.07254	0.02084	0.12906	0.03731	0.09463	0.02576	0.08692	0.02436
				0.5	0.16657	0.05056	0.13370	0.03756	0.19945	0.06596	0.15832	0.04586	0.15043	0.04388
			0.5	0.3	0.10816	0.05879	0.06526	0.04483	0.15108	0.07694	0.09394	0.05083	0.08709	0.05142

Table 4: The values of BIAS and MSE of the Bayesian estimates of β based on SEL, ABL, LINEX and GE loss functions

n	m	r	α	β	$\hat{\beta}_{BS}$		$\hat{\beta}_{BA}$ $q=-0.5$		$\hat{\beta}_{BA}$ $q=0.5$		$\hat{\beta}_{BL}$		$\hat{\beta}_{BE}$	
					BIAS	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*
30	23	2	0.3	0.3	0.19074	0.03639	0.19566	0.03830	0.18584	0.03455	0.19101	0.03649	0.19315	0.03732
				0.5	0.38918	0.15147	0.3941	0.15532	0.38424	0.14765	0.38945	0.15168	0.39162	0.15337
			0.5	0.3	0.19062	0.03634	0.19536	0.03817	0.18587	0.03455	0.19088	0.03644	0.19297	0.03724
		5	0.3	0.3	0.19071	0.03638	0.19560	0.03827	0.18582	0.03454	0.19098	0.03649	0.19313	0.03731
				0.5	038924	0.15151	0.39415	0.15536	0.38431	0.14770	0.38951	0.15172	0.39167	0.15341

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n	m	r	α	β	BIAS	MSE	BIAS	MSE	BIAS	MSE	BIAS	MSE	BIAS	MSE
			0.5	0.3	0.19051	0.03630	0.19526	0.03813	0.18574	0.03451	0.19077	0.03640	0.19286	0.03720
	17	2	0.3	0.3	0.19345	0.03743	0.19830	0.03933	0.18858	0.03557	0.19371	0.03753	0.19585	0.03836
				0.5	0.39228	0.15389	0.39715	0.15773	0.38739	0.15008	0.39254	0.15409	0.39469	0.15578
			0.5	0.3	0.19255	0.03708	0.19727	0.03892	0.18781	0.03528	0.19281	0.03718	0.19489	0.03799
		5	0.3	0.3	0.19352	0.03745	0.19837	0.03935	0.18865	0.03559	0.19377	0.03755	0.19592	0.03839
				0.5	0.39238	0.15396	0.39724	0.15780	0.38749	0.15015	0.39264	0.15417	0.39478	0.15586
			0.5	0.3	0.19262	0.03710	0.19734	0.03895	0.18787	0.03530	0.19287	0.03720	0.19496	0.03801
10	7	1	0.3	0.3	0.19436	0.03778	0.19919	0.03968	0.18952	0.03592	0.19462	0.03788	0.19675	0.03871
				0.5	0.39353	0.15487	0.39834	0.15868	0.38870	0.15109	0.39378	0.15507	0.39591	0.15675
			0.5	0.3	0.19290	0.03721	0.19762	0.03906	0.18815	0.03540	0.19315	0.03731	0.19524	0.03812

Table 5: The values of BIAS and MSE of the Bayesian estimates of $H(t)$ when $t = 3$ based on SEL, ABL, LINEX and GE loss functions

n	m	r	α	β	$\hat{H}_{BS}$		$\hat{H}_{BA}$ $q=-0.5$		$\hat{H}_{BA}$ $q=0.5$		$\hat{H}_{BL}$		$\hat{H}_{BE}$	
					BIAS	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*	*BIAS*	*MSE*
30	23	2	0.3	0.3	0.16250	0.04092	0.18046	0.04611	0.14446	0.03639	0.16788	0.04208	0.17143	0.04341
				0.5	0.65076	0.43850	0.67290	0.46679	0.62847	0.41106	0.65863	0.44810	0.66177	0.45245
			0.5	0.3	0.35963	0.16203	0.38776	0.18080	0.33128	0.14481	0.37229	0.16926	0.37363	0.17116
		5	0.3	0.3	0.16297	0.04038	0.18091	0.04563	0.14495	0.03579	0.16833	0.04159	0.17189	0.04290
				0.5	0.65067	0.43775	0.67281	0.46608	0.62838	0.41026	0.65854	0.44736	0.66168	0.45171
			0.5	0.3	0.37056	0.17199	0.39834	0.19095	0.34257	0.15455	0.38296	0.17917	0.38439	0.18122
	17	2	0.3	0.3	0.21084	0.05778	0.22769	0.06427	0.19389	0.05186	0.21550	0.05925	0.21922	0.06093
				0.5	0.73882	0.56094	0.75846	0.58932	0.71901	0.53315	0.74450	0.56945	0.74860	0.57497
			0.5	0.3	0.41184	0.20502	0.43885	0.22552	0.38458	0.18591	0.42341	0.21253	0.42529	0.21503
		5	0.3	0.3	0.21830	0.05874	0.23488	0.06551	0.20161	0.05252	0.22278	0.06031	0.22655	0.06204
				0.5	0.74176	0.56431	0.76129	0.59271	0.72207	0.53650	0.74786	0.57279	0.75148	0.57835
			0.5	0.3	0.42540	0.21215	0.45192	0.23325	0.39864	0.19237	0.43653	0.21993	0.43861	0.22247
10	7	1	0.3	0.3	0.18518	0.06978	0.22602	0.08121	0.14400	0.06213	0.19756	0.07088	0.20529	0.07493
				0.5	0.69890	0.53419	0.74707	0.59679	0.65026	0.47639	0.71578	0.55187	0.72263	0.56437
			0.5	0.3	0.38481	0.24465	0.44885	0.28282	0.32000	0.2159	0.41452	0.25238	0.41639	0.26225

CONCLUSION

The Bayes estimators under both symmetric and asymmetric loss functions are obtained. When we compare the loss functions, we find that the ABL loss function is better than SEL, LINEX and GE loss functions in tables 1,3,4, and 5, while find that the ABL and LINEX loss functions are better than SEL and GE loss functions in table 2.

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