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RELIABILITY AND AVAILABILITY ANALYSIS OF TWO–DISSIMILAR UNITS BY USING LAPLACE TRANSFORMS

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ABSTRACT: *The paper studies the reliability and availability of two dissimilar units. In order to calculate reliability, a state dependent system can be converted into the system of first order ordinary differential equations based on Laplace Transform technique. The system of ordinary differential equations is solved using Inverse Laplace depend on Complex Conjugate roots. Let failure rate and repair rate of each unit are taken as an exponential distribution. Availability, reliability and the mean time to failure are derived. We analysis graphically to observe the effect of various systems Parameters on the availability system and mean time to failure.*

KEYWORDS: Reliability, Availability, State Dependent System, Mttf, Laplace Transform

INTRODUCTION

Studying the reliability of machine repair problem is very important in our life because it is widely used in the industrial system and manufacturing system .any system becomes unreliable due to many reasons. The units of our system have three states up and one down. However, in many cases, the units of the system can have a finite number of states. Most reliability systems assume that the up and down times of the components are exponential distribution. This assumption leads to a Monrovian model with constant transition rates. The analysis in our cases is relatively simple and the numerical results can obtain easily. In [1] evaluate the reliability and sensitivity analysis of a repairable -system with imperfect coverage under service pressure condition.in [2] have studied reliability based measures for a retrial system with mixed standby components. In [3] Reliability analysis of a warm standby repairable system with priority in use.in [4] Comparison of reliability and the availability between four systems with warm-standby components and standby switching failures.in [5] reliability and availability of a warm standby with common cause failure and human- error.in [6] reliability and availability analysis of n-unit outdoor power system subject to an adjustable- repair facility. In [7] Reliability and availability analysis of a standby repairable system with degradation facility, in [8] studies on reliability and availability of a repairable system with multiple degradations. In [9] Reliability and sensitivity analysis of the K-out-of-N: G warm standby parallel repairable system with replacement at common-cause failure using Markova model, in [10] Reliability analysis of a two unit system with common cause shock failures.

The main object is to study system with two dissimilar units where to develop the explicit expressions, for availability function, reliability function and mean time to failure by Laplace transform techniques then we show numerical results to analyze the effects of the various system parameters on the system reliability and system availability.

The objective of this paper is summarized as follows: In Section 1 we show the mathematical preliminaries and notation. Section 2 shows the cubic equations roots and their cases, availability, reliability and mean time to failure for every case. In section 3 presents system behavior through graphs. Finally, in section 4 we outline the main conclusions.

System Description and Assumptions

The system is analyzed under following practical assumptions:

- The system consists of single unit having two dissimilar components, say A and B
- Initially, both the units are working.
- The system fails completely if during the repair of the failed unit, the another unit is also fails
- The failure of component changes the lifetime parameter of the other
- After repair, the unit becomes as good as new.

Table 1. Transition sates

	So	S_1	S_2	S_3
So		λ_1	λ_2	
S_1	μ_1			λ_2
S_2	μ_2			λ_1
S_3		μ_2	μ_1	

Considering these symbols, the system may be in one of the following states

Up state: So = (A_N, B_N) , $S_1 = (A_F, B_N)$, $S_2 = (A_N, B_F)$,

Down state: $S_3 = (A_F, B_F)$

A unit can be in one of the following states:

A_N First unit operative and in normal mode

B_N Second unit operative and in normal mode

A_F First unit failed and under repair

B_F Second unit failed and under repair

Notations and States of the System

λ_1 Failure rate from A_N to A_F

λ_2 Transition rate from B_N to B_F

μ_1 Repair rate from A_F to A_N

μ_2 Repair rate from B_F to B_N

$p_i(t)$ Probability for i=0, 1, 2, 3

$p_i^*(s)$ Laplace transform of $p_i(t)$

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A (t): availability functions of the system.

R (t): reliability functions of the system.

MTTF: mean time to system failure.

Laplace transform of p_i(t) is defined as:

p_i^* (s) = $\int_0^\infty e^{-St}\, p_i(t)$ It, i=0, 1, 2, 3

Mathematical formulation of the model

According to system configuration diagram in table.1, the difference – differential equations for this stochastic process which is continuous in time and discrete in space are given as follows.

$$\frac{dP_0(t)}{dt} = -[\lambda_1 + \lambda_2]\, P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t) \qquad (1)$$

$$\frac{dP_1(t)}{dt} = -[\mu_1 + \lambda_2]\, P_1(t) + \lambda_1 P_0(t) + \mu_2 P_3(t) \qquad (2)$$

$$\frac{dP_2(t)}{dt} = -[\mu_2 + \lambda_1]\, P_2(t) + \lambda_2 P_0(t) + \mu_1 P_3(t) \qquad (3)$$

$$\frac{dP_3(t)}{dt} = -[\mu_1 + \mu_2]\, P_3(t) + \lambda_1 P_2(t) + \lambda_2 P_1(t) \qquad (4)$$

Initial conditions:

$$P_i\,(0) = \begin{cases} 1 & where\ i = 0 \\ 0 & otherwise \end{cases}$$

Taking Laplace transform of equations (1) – (4), we get

$$[\lambda_1 + \lambda_2 + s]\, P_0^*(s) - \mu_1 P_1^*(s) - \mu_2 P_2^*(s) = P_0\,(0) \qquad (5)$$

$$[\mu_1 + \lambda_2 + s]\, P_1^*(s) - \lambda_1 P_0^*(s) - \mu_2 P_3^*(s) = P_1\,(0) \qquad (6)$$

$$[\lambda_1 + \mu_2 + s]\, P_2^*(s) - \lambda_2 P_0^*(s) - \mu_1 P_3^*(s) = P_2\,(0) \qquad (7)$$

$$[\mu_1 + \mu_2 + s]\, P_3^*(s) - \lambda_1 P_2^*(s) - \lambda_2 P_1^*(s) = P_3\,(0) \qquad (8)$$

Solving equations (5-8) by crammer rule, we obtain:

$$P_0^*(s) = \frac{s^3 + As^2 + BS + m}{s[s^3 + a_1 s^2 + a_2 S + a_3]}$$

Cubic equations roots have are 3 cases

First case (D > 0) [1 root is real and 2 complex]

$$P_0^*(s) = \frac{s^3 + As^2 + BS + m}{s(s + A_1 - W)(S + A_1 + w_1 - i\sqrt{3}v_1)(S + A_1 + w_1 + i\sqrt{3}\,v_1)}$$

Where

$$q = \frac{3a_2 - {a_1}^2}{9} \quad , \quad r = \frac{9a_1 a_2 - 2{a_1}^3 - 27a_3}{54}$$

$D = q^3 + r^2$

$u = \sqrt[3]{r + \sqrt{q^3 + r^2}}$, $t = \sqrt[3]{r - \sqrt{q^3 + r^2}}$

$w_1 = \frac{(u+t)}{2}$, $w = (u + t)$, $v_1 = \frac{(u-t)}{2}$, $A_1 = \frac{a_1}{3}$

By taking inverse Laplace transform of equations, we get

$$P_0(t) = \frac{m}{(A_1 - w)(A_1{}^2 + A_1 w + w_1{}^2 + 3v_1{}^2)}$$

$$+ \frac{(-A_1 + w)^3 + A(-A_1 + w)^2 + B(-A_1 + w) + m}{(-A_1 + w)(9w_1{}^2 + 3v_1{}^2)} e^{(-A_1 + W)t}$$

$$+ \left\{ \frac{2[PX + 3HT][\cos\sqrt{3}(v_1)t] - 2\sqrt{3}\,[(HX) - (TP)](\sin\sqrt{3}(v_1)t)}{X^2 + 3T^2} \right\} e^{(-A_1 - w_1)t}$$

Where

$P = (- A_1{}^3 - 3A_1{}^2 w_1 + 9 A_1 v_1{}^2 - w_1{}^3 + 9 w_1 v_1{}^2 - A_1 w_1{}^2)$

$+ A (A_1{}^2 + A_1 w - 3v_1{}^2 + w_1{}^2) + B(-A_1 - w_1) + m$

$H = (6A_1 v_1 w_1 - 3v_1{}^3 + 3w_1{}^2 v_1 + 3A_1{}^2 v_1 - AA_1 v - A v_1 w + Bv_1)$

$X = 3v^2 w + 6v_1{}^2 A_1$

$T = 3wv_1 A_1 + 3w_1{}^2 v - 6v_1{}^3$

$a_3 = \mu_1^2(\mu_2 + \lambda_2) + \lambda_1^2(\mu_2 + \lambda_2) + 2\mu_1\mu_2(\lambda_1 + \lambda_2) + \lambda_1\lambda_2(2\mu_1 + 2\mu_2) + \lambda_2^2(\mu_1 + \lambda_1)$

$+ \mu_2^2(\mu_1 + \lambda_1)$

$a_2 = \mu_1(\mu_1 + 3\mu_2 + 3\lambda_2 + 2\lambda_1) + \mu_2(\mu_2 + 3\lambda_1 + 2\lambda_2) + \lambda_1^2 + 3\lambda_1\lambda_2 + \lambda_2^2$

$a_1 = 2\lambda_1 + 2\lambda_2 + 2\mu_1 + 2\mu_2$

$m = \mu_1\mu_2^2 + \mu_1^2\mu_2 + \mu_1\mu_2\lambda_1 + \mu_1\mu_2\lambda_2$

$B = 3\mu_1\mu_2 + \mu_1\lambda_1 + \mu_1\lambda_2 + \mu_2\lambda_1 + \mu_2\lambda_2 + \lambda_1\lambda_2 + \mu_1^2 + \mu_2^2$

$A = 2\mu_1 + 2\mu_2 + \lambda_1 + \lambda_2$

System availability and reliability

Availability analysis of the System

We find that

$$\boldsymbol{A}(t) = \frac{m}{(A_1 - w)(A_1{}^2 + A_1 w + w_1{}^2 + 3v_1{}^2)}$$

$$+\frac{(-A_1+w)^3+A(-A_1+w)^2+B(-A_1+w)+m}{(-A_1+w)(9w_1^2+3v_1^2)}e^{(-A_1+W)t}$$

$$+\left\{\frac{2[PX+3HT][\cos\sqrt{3}(v_1)t]-2\sqrt{3}\,[(HX)-(TP)](\sin\sqrt{3}(v_1)t)}{X^2+3T^2}\right\}e^{(-A_1-w_1)t}$$

The steady – state availability can be obtained from the following relation

$$\mathbf{A}=\lim_{t\to\infty}A(t)$$

$$\mathbf{A}=\frac{m}{(A_1-w)(A_1^2+A_1w+w_1^2+3v_1^2)}$$

Reliability analysis of the System

To obtain the reliability function for this model, we assume that at least one of failed states is absorbing state and the transition rate from this state equal to zero

$$R(t)=\frac{m}{(A_1-w)(A_1^2+A_1w+w_1^2+3v_1^2)}$$

$$+\frac{(-A_1+w)^3+A(-A_1+w)^2+B(-A_1+w)+m}{(-A_1+w)(9w_1^2+3v_1^2)}e^{(-A_1+W)t}$$

$$+\left\{\frac{2[PX+3HT][\cos\sqrt{3}(v_1)t]-2\sqrt{3}\,[(HX)-(TP)](\sin\sqrt{3}(v_1)t)}{X^2+3T^2}\right\}e^{(-A_1-w_1)t}$$

As we know, we have two failed states this lead to three cases of reliability function.

1- Failed state [components A] is absorbing when. $\mu_1=0$

2- Failed state [components B] is absorbing when. $\mu_2=0$

3- Failed states [components A& B] are absorbing when. $\mu_1=\mu_2=0$

The mean time to failure

The mean time to system failure MTTF can be obtained from the following relation

$$\mathbf{MTTF}=\int_0^\infty R(t)dt=\lim_{t\to\infty}\int_0^t R(t)dt$$

$$\mathbf{MTTF}=\lim_{t\to\infty}\int_0^t R(t)dt=\lim_{s\to 0}SL\left\{\int_0^t R(t)dt\right\}=\lim_{s\to 0}S\frac{R^*(S)}{S}$$

$$\mathbf{MTTF}=\lim_{s\to 0}R^*(S)\qquad,\qquad R^*(S)=\mathrm{L}\,(R(t))$$

As mention above, reliability function has three cases so we find MTTF has following cases:

<u>**When** μ_1=0 we **find**</u>

$$\mathbf{MTTF}=-\left(\frac{(-A_1+w)^2+A(-A_1+w)+B}{(-A_1+w)(9w_1^2+3v_1^2)}\right)-\frac{(2[PX+3HT])(-A_1-w_1)+6v_1\,[HX-TP]}{(X^2+3T^2)[(-A_1-w_1)^2+3v_1^2]}$$

Where

$\mathbf{a_3}=\boldsymbol{\lambda_1^2}(\mu_2+\lambda_2)+\mathbf{2}\lambda_1\lambda_2\mu_2+\boldsymbol{\lambda_2^2}\lambda_1+\mu_2^2\,\lambda_1$

$\mathbf{a_2}=\mu_2\,(\mu_2+\mathbf{3}\lambda_1+\mathbf{2}\lambda_2)+\boldsymbol{\lambda_1^2}+\mathbf{3}\lambda_1\lambda_2+\boldsymbol{\lambda_2^2}$

$\mathbf{a_1}=\mathbf{2}\lambda_1+\mathbf{2}\lambda_2+\mathbf{2}\mu_2$

$\mathbf{m=0}$, $\mathbf{B}=\mu_2\lambda_1+\mu_2\lambda_2+\lambda_1\lambda_2+\mu_2^2$, $\mathbf{A}=\mathbf{2}\mu_2+\lambda_1+\lambda_2$

When $\mu_2=0$ we **find**

$$\mathbf{MTTF}=-\left(\frac{(-A_1+W)^2+A(-A_1+W)+B}{(-A_1+w)(9w_1{}^2+3v_1{}^2)}\right)-\frac{(2[PX+3HT])(-A_1-w_1)+6v_1\,[H\,X-TP\,]}{(X^2+3T^2)[(-A_1-w_1)^2+3v_1{}^2]}$$

Where

$\mathbf{a_3}=\mu_1^2\lambda_2+\boldsymbol{\lambda_1^2}\lambda_2+\mathbf{2}\,\lambda_1\lambda_2\mu_1+\boldsymbol{\lambda_2^2}(\mu_1+\lambda_1)$

$\mathbf{a_2}=\mu_1\,(\mu_1+\mathbf{3}\lambda_2+\mathbf{2}\lambda_1)+\boldsymbol{\lambda_1^2}+\mathbf{3}\lambda_1\lambda_2+\boldsymbol{\lambda_2^2}$

$\mathbf{a_1}=\mathbf{2}\lambda_1+\mathbf{2}\lambda_2+\mathbf{2}\mu_1$

$\mathbf{m=0}$, $\mathbf{B}=\mu_1\lambda_1+\mu_1\lambda_2+\lambda_1\lambda_2+\mu_1^2$, $\mathbf{A=2}\,\mu_1+\lambda_1+\lambda_2$

When μ1= μ2 =0 we **find**

$$\mathbf{MTTF}=-\left(\frac{(-A_1+W)^2+A(-A_1+W)+B}{(-A_1+w)(9w_1{}^2+3v_1{}^2)}\right)-\frac{(2[PX+3HT])(-A_1-w_1)+6v_1\,[H\,X-TP\,]}{(X^2+3T^2)[(-A_1-w_1)^2+3v_1{}^2]}$$

Where

$\mathbf{a_3}=\boldsymbol{\lambda_1^2}\lambda_2+\boldsymbol{\lambda_2^2}\,\lambda_1$, $\mathbf{a_2}=\boldsymbol{\lambda_1^2}+\mathbf{3}\,\lambda_1\lambda_2+\boldsymbol{\lambda_2^2}$, $\mathbf{a_1=2}\,\lambda_1+\mathbf{2}\lambda_2$

$\mathbf{m=0}$, $\mathbf{B}=\lambda_1\,\lambda_2$, $\mathbf{A}=\lambda_1+\lambda_2$

Second case D < 0 [All roots are real and unequal]

$$P_0^*(s)=\frac{s^3+As^2+BS+m}{s(s+A_1-w_0)(S+A_1-w_2)(S+A_1-v_2)}$$

Where

$s_1=2\sqrt{-q}\,\cos(\frac{\theta}{3})-\frac{a_1}{3}$, $s_2=2\sqrt{-q}\,\cos(\frac{\theta}{3}+120)-\frac{a_1}{3}$

$s_3=2\sqrt{-q}\,\cos(\frac{\theta}{3}+240)-\frac{a_1}{3}$, $\theta=\cos^{-1}\frac{r}{\sqrt{-q^3}}$

$w_0=2\sqrt{-q}\,\cos(\frac{\theta}{3})$, $w_2=2\sqrt{-q}\,\cos(\frac{\theta}{3}+120)$

$v_2=2\sqrt{-q}\,\cos(\frac{\theta}{3}+240)$, $A_1=\frac{a_1}{3}$

By taking inverse Laplace transform

$$P_0(t) = \frac{m}{(A_1 - w_0)(A_1{}^2 - A_1 w_2 - A_1 v_2 + w_2 v_2)}$$
$$+ \frac{(-A_1 + w_0)^3 + A(-A_1 + w_0)^2 + B(-A_1 + w_0) + m}{(-A_1 + w_0)(w_0{}^2 - w_0 w_2 - w_0 v_2 + w_2 v_2)} e^{(-A_1 + w_0)t}$$
$$+ \frac{(-A_1 + w_2)^3 + A(-A_1 + w_2)^2 + B(-A_1 + w_2) + m}{(-A_1 + w_2)(w_2{}^2 - w_0 w_2 - w_2 v_2 + w_0 v_2)} e^{(-A_1 + w_2)t}$$
$$+ \frac{(-A_1 + v_2)^3 + A(-A_1 + v_2)^2 + B(-A_1 + v_2) + m}{(-A_1 + v_2)(v_2{}^2 - w_0 v_2 - w_2 v_2 + w_0 w_2)} e^{(-A_1 + v_2)t}$$

System availability and reliability

Availability analysis of the System

We find that

$$A(t) = \frac{m}{(A_1 - w_0)(A_1{}^2 - A_1 w_2 - A_1 v_2 + w_2 v_2)}$$
$$+ \frac{(-A_1 + w_0)^3 + A(-A_1 + w_0)^2 + B(-A_1 + w_0) + m}{(-A_1 + w_0)(w_0{}^2 - w_0 w_2 - w_0 v_2 + w_2 v_2)} e^{(-A_1 + w_0)t}$$
$$+ \frac{(-A_1 + w_2)^3 + A(-A_1 + w_2)^2 + B(-A_1 + w_2) + m}{(-A_1 + w_2)(w_2{}^2 - w_0 w_2 - w_2 v_2 + w_0 v_2)} e^{(-A_1 + w_2)t}$$
$$+ \frac{(-A_1 + v_2)^3 + A(-A_1 + v_2)^2 + B(-A_1 + v_2) + m}{(-A_1 + v_2)(v_2{}^2 - w_0 v_2 - w_2 v_2 + w_0 w_2)} e^{(-A_1 + v_2)t}$$

The steady – state availability can be obtained from the following relation

$$A = \lim_{t \to \infty} A(t)$$

$$A = \frac{m}{(A_1 - w_0)(A_1{}^2 - A_1 w_2 - A_1 v_2 + w_2 v_2)}$$

2.2.3. The mean time to failure

When μ_1=0 we find

$$\mathbf{MTTF} = -\left(\frac{(-A_1 + w_0)^2 + A(-A_1 + w_0) + B}{(-A_1 + w_0)(w_0{}^2 - w_0 w_2 - w_0 v_2 + w_2 v_2)}\right) - \left(\frac{(-A_1 + w_2)^2 + A(-A_1 + w_2) + B}{(-A_1 + w_2)(w_2{}^2 - w_0 w_2 - w_2 v_2 + w_0 v_2)}\right)$$
$$- \left(\frac{(-A_1 + v_2)^2 + A(-A_1 + v_2) + B}{(-A_1 + v_2)(v_2{}^2 - w_0 v_2 - w_2 v_2 + w_0 w_2)}\right)$$

Where

$\mathbf{a_3} = \boldsymbol{\lambda_1^2}(\mu_2 + \lambda_2) + 2\lambda_1\lambda_2\mu_2 + \boldsymbol{\lambda_2^2}\lambda_1 + \mu_2^2\,\lambda_1$

$\mathbf{a_2} = \mu_2\,(\mu_2 + 3\lambda_1 + 2\lambda_2) + \boldsymbol{\lambda_1^2} + 3\lambda_1\lambda_2 + \boldsymbol{\lambda_2^2}$

$\mathbf{a_1} = 2\lambda_1 + 2\lambda_2 + 2\mu_2$

m= 0 , **A=2** $\mu_2 + \lambda_1 + \lambda_2$, **B**=$\mu_2\lambda_1 + \mu_2\lambda_2 + \lambda_1\lambda_2 + \mu_2^2$

When μ_2=0 we find

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$$\mathbf{MTTF} = -\left(\frac{(-A_1+w_0)^2+A(-A_1+w_0)+B}{(-A_1+w_0)(w_0{}^2-w_0w_2-w_0v_2+w_2v_2)}\right)-\left(\frac{(-A_1+w_2)^2+A(-A_1+w_2)+B}{(-A_1+w_2)(w_2{}^2-w_0w_2-w_2v_2+w_0v_2)}\right)$$

$$-\left(\frac{(-A_1+v_2)^2+A(-A_1+v_2)+B}{(-A_1+v_2)(v_2{}^2-w_0v_2-w_2v_2+w_0w_2)}\right)$$

Where

$a_3=\mu_1^2\lambda_2+\lambda_1^2\lambda_2+2\,\lambda_1\lambda_2\mu_1+\lambda_2^2(\mu_1+\lambda_1)$

$a_2=\mu_1\,(\mu_1+3\lambda_2+2\lambda_1)+\lambda_1^2+3\lambda_1\lambda_2+\lambda_2^2$

$a_1=2\lambda_1+2\lambda_2+2\mu_1$

m=0 , $B=\mu_1\lambda_1+\mu_1\lambda_2+\lambda_1\lambda_2+\mu_1^2$, $A=2\,\mu_1+\lambda_1+\lambda_2$

When μ1= μ2 =0 we **find**

$$\mathbf{MTTF} = -\left(\frac{(-A_1+w_0)^2+A(-A_1+w_0)+B}{(-A_1+w_0)(w_0{}^2-w_0w_2-w_0v_2+w_2v_2)}\right)-\left(\frac{(-A_1+w_2)^2+A(-A_1+w_2)+B}{(-A_1+w_2)(w_2{}^2-w_0w_2-w_2v_2+w_0v_2)}\right)$$

$$-\left(\frac{(-A_1+v_2)^2+A(-A_1+v_2)+B}{(-A_1+v_2)(v_2{}^2-w_0v_2-w_2v_2+w_0w_2)}\right)$$

Where

$a_3=\lambda_1^2\lambda_2+\lambda_2^2\,\lambda_1$, $a_2=\lambda_1^2+3\,\lambda_1\lambda_2+\lambda_2^2$, $a_1=2\,\lambda_1+2\lambda_2$

m= 0 , $B=\lambda_1\,\lambda_2$, $A=\lambda_1+\lambda_2$

The system behavior through graphs

For more the concrete study of mean time to system failure and availability. we plot the steady -state availability and MTTF for the models, against λ1 keeping other parameters

$\lambda_2=0.3$, $\mu_2=0.7$, $\mu_1=0.5$ $\lambda_1=0.1, 0.2, 0.3, 0.4, 0.5$

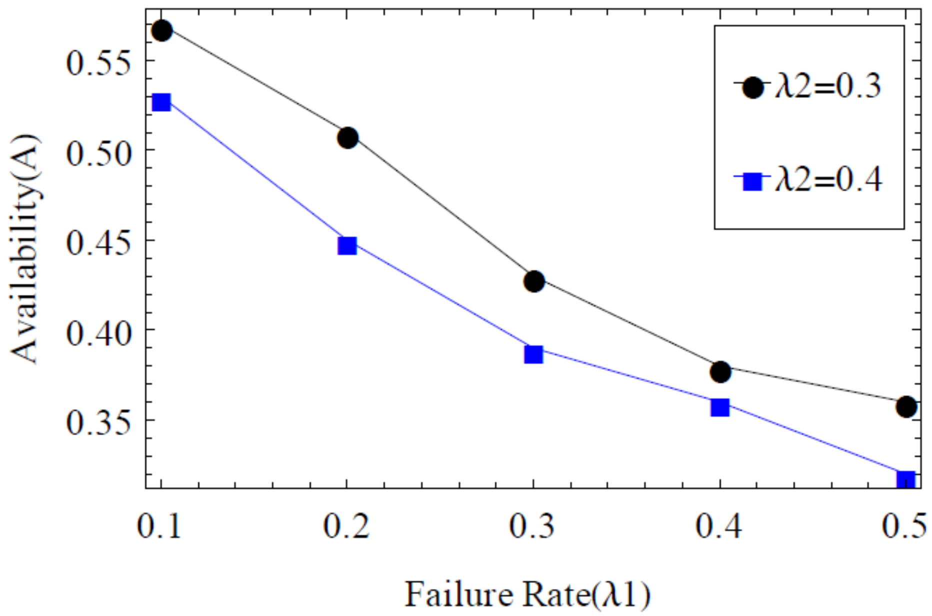

Fig 1

The Steady state Availability w.r.t. Failure Rate λ1

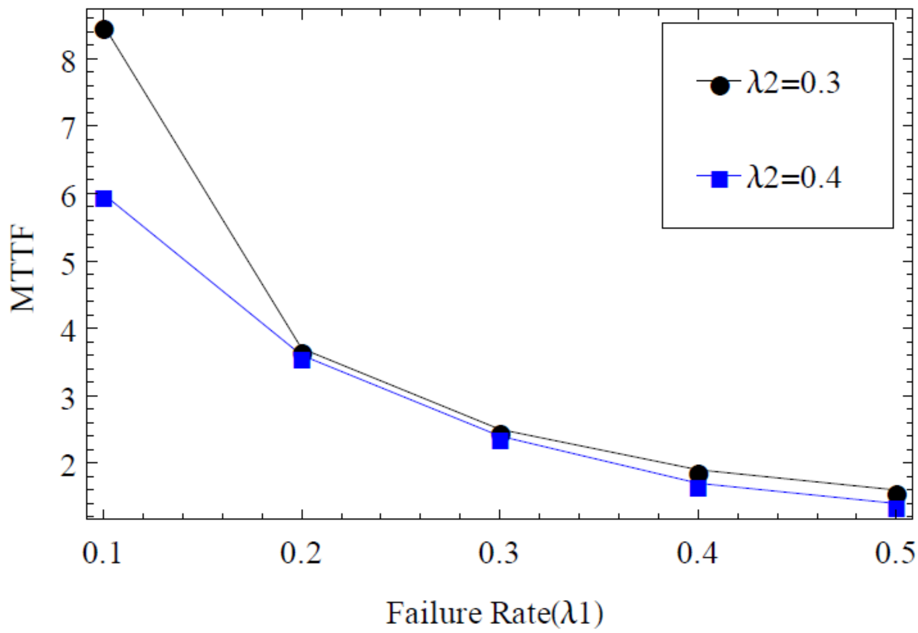

Fig 2

The mean time failure w.r.t. Failure Rate λ1

CONCLUSIONS

We use computer software, to plot system availability and MTTF in fig. 1 and 2 respectively. It is noted that A decrease as λ increases and also MTTF decrease as λ increases.

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