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QUASI- BAYESIAN ESTIMATION FOR INVERSE WEIBULL DISTRIBUTION

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ABSTRACT: *This article is concerned with quasi- likelihood estimation of the unknown parameters of a new form for inverse Weibull distribution, we have performed a simulation study in order to compare the proposed quasi- likelihood estimators with the maximum likelihood estimators. Also, we used a real life data set to illustrate the result derived.*

KEYWORDS: Quasi- Bayesian Estimation, Inverse Weibull Distribution

INTRODUCTION

The Inverse Weibull distribution (IW) is an important life time model in reliability and survival analysis which can be used to model a variety of failure characteristics such as infant mortality, useful life, wear out period, relays, ball bearings, electron tubes, capacitors, germanium transistors, photo-condue Live cells, motors, automotive radiators, regulators, generators, turbine blades, fatigue in textiles, corrosion resistance, leakage of dry batteries, return of products after shipment, marketing life, expectancy of drugs, the number of downtimes per shift and solids subjected to fatigue stresses. It can also be used to determine the coast effectiveness and maintenance periods of reliability centered maintenance activities.

Recently, Ismail (2013). Suggested a new form for inverse Weibull distribution. EL-Shahat and Ismail (under publication). obtained quasi- likelihood estimators for the new form. The probability density function (pdf) and cumulative distribution function (cdf) of the new form of the inverse Weibull distribution $(NIWD)$ are given by

$$f_{NIWD}(x) = m\theta^c x^{-(m+1)} e^{-\theta^c x^{-m}}, \quad x \geq 0, m, \theta > 0, -\infty < c < \infty . \quad (1)$$

$$F_{NIWD}(x) = e^{-\theta^c x^{-m}} \quad (2)$$

Where m is the shape parameter and c, θ are the scale parameters

If c = 1 , we have the form given by Sultan (2008),

c = -1 , we have forms given by Aleem (2005) and Khan et al. (2008),

c = m , we have forms given by Pawalas & Szynal (2000), Mahmoud et al. (2003 a) and De Gusmao et al. (2011),

c = -m , we have forms given by Mahmoud et al. (2003 b) and AL-Hidairah and AL- Adayian (2008).

In recent years, the IW distribution has gained widely attention in various fields. Sultan (2008) used the lower record values to drive and discuss Bayesian estimation method. Kundo and Howlader (2010) described the Bayesian inference and predication for type II censored data. Yao et al. (2011) discussed the Expected Bayes estimation and Emprical Bayes Estimation.

From (1) and (2) , it is easy to write the reliability and hazard functions R(t) and H(t), respectively, as

$$R_{NIWD}(t) = 1 - e^{-\theta^c t^{-m}} \quad . \qquad (3)$$

$$H_{NIWD}(t) = \frac{m\theta^c t^{-(m+1)} e^{-\theta^c t^{-m}}}{1-e^{-\theta^c t^{-m}}} \quad . \qquad (4)$$

The rth moments of the NIWD is denoted by $\mu_r^`$ and it is given by

$$\mu_r^` = \theta^{\frac{cr}{m}} \Gamma\left(1 - \frac{r}{m}\right), \qquad r = 1,2, \dots\dots \qquad (5)$$

The mean and variance of IWD are

$$\mu = \mu_1^` = \theta^{\frac{c}{m}} \Gamma\left(1 - \frac{1}{m}\right) \quad , \qquad (6)$$

And the variance

$$var(x) = \mu^2 \left[\frac{\Gamma\left(1-\frac{2}{m}\right)}{\Gamma^2\left(1-\frac{1}{m}\right)} - 1\right] = \emptyset\, \mu^2, \qquad (7)$$

The quasi-likelihood function was introduced by Wedderburn (1974), for estimating the unknown parameters in generalized linear model when only the variance of each observations is specified to be or either equal, or proportional to some function of its expectation. He defined the quasi-likelihood, strictly the quasi-log –likelihood, Q for an observation X with μ and variance V(μ) by the equation:

$$\frac{\partial Q(x;\mu)}{\partial \mu} = \frac{x-\mu}{V(\mu)} \qquad (8)$$

Or equivalent by

$$Q(x,\mu) = \int^{\mu} \frac{x-\mu}{V(\mu)}\, d\mu + function\ of\ X \qquad (9)$$

Where, μ = E(x), V(μ) = var (x).

The variance assumption is generalized to var(X) = ϕ V(μ),where the variance function V(.) is assumed to be known and the parameter ϕ may be unknown. Wedderburn (1974) found that the quasi-likelihood function is the ln-likelihood function of the distribution if x comes from a one-parameter exponential family and the quasi-likelihood function has properties similar to those of the ln-likelihood function.

In this article, the Bayesian and quasi – Bayesian procedures will be used to estimate the unknown parameters of IWD. The quasi- Bayesian procedures can be applied without specifying the likelihood function of the sample observations, if the relationship between the mean and variance is known. To derive the posterior distribution of the unknown parameters based on quasi- likelihood, the likelihood function could be replaced with the natural exponential of the quasi- likelihood Ashour and Elsherpieny [5]. The quasi- Bayesian estimation reduces to the usual Bayesian procedures, if the quasi-likelihood and the log likelihood are identical.

In section 2, the Bayesian estimates of unknown parameters of IW distribution are derived. Section 3 deals with the quasi- Bayesian estimations. A numerical illustration is presented in section 4.

THE BAYESIAN ESTIMATION

In this section, Bayesian method is used to obtain the estimators for the unknown parameters of the NIWD given in (1) using symmetric squared error loss function and asymmetric LINEX loss function.

We design an experiment in which n units are placed on the test. All units are independent and have identical IW distribution. So the likelihood function is

$$L = \prod_{i=1}^{n} f_{Giwd}(x_i; m, \theta) = m^n \theta^{nc} \prod_{i=1}^{n} x_i^{-(m+1)} e^{-\theta^c v^*} . \qquad (8)$$

Where $\upsilon^* = \sum_{i=1}^{n} x_i^{-m}$.

Lemma 1

Bayes estimates of θ , c and m under symmetric square error loss function are

$$\tilde{\vartheta} = E(h_1(\theta\backslash x)) = \int_0^\infty \theta \; h_1(\theta\backslash x)\, d\theta = \int_0^\infty \frac{e^{-b\theta}\, \theta^a \int_{-\infty}^{\infty}\int_0^\infty \theta^{cn} m^{n+\ell-1}\, q\, e^{-(v^*\theta^c + m\,k)} \prod_{i=1}^{n} x_i^{-(m+1)}\, dm\, dc}{\Im_2}\, d\theta \; . \qquad (9)$$

$$\tilde{m} = E(h_2(m\backslash x)) = \int_0^\infty m\; h_2(m\backslash x)\, dm = \int_0^\infty \frac{m^{n+\ell-1}\, e^{-km} \prod_{i=1}^{n} x_i^{-(m+1)} \int_{-\infty}^{\infty}\int_0^\infty \theta^{cn+a-1}\, q\, e^{-(b\theta+v^*\theta^c)}\, d\theta\, dc}{\Im_2}\, dm \; . \qquad (10)$$

$$\tilde{c} = E(h_3(c\backslash x)) = \int_{-\infty}^{\infty} c\; h_3(c\backslash x)\, dc = \int_{-\infty}^{\infty} \frac{c\, q \int_0^\infty\int_0^\infty \theta^{cn+a-1}\, m^{n+\ell-1}\, e^{-(b\theta+v^*\theta^c+km)} \prod_{i=1}^{n} x_i^{-(m+1)}\, dm\, d\theta}{\Im_2}\, dc \; . \qquad (11)$$

It assumed that θ and m each has independent gamma (a, b), and gamma (k,ℓ) priors respectively follow **Yao. et al. (2011),** for $a > 0, b > 0$, $k > 0$,$\ell > 0$, $i.e.$

$$g(\theta) = \frac{b^a}{\Gamma(a)}\ \theta^{a-1}\ e^{-b\theta}\ ,\theta > 0\ ,\ \ a,b > 0\ . \tag{12}$$

and

$$g(m) = \frac{k^\ell}{\Gamma(\ell)}\ m^{\ell-1}e^{km}\ ,\qquad m > 0,\ k,\ell > 0\ . \tag{13}$$

It is assumed that c has a non-informative prior distribution for $-\infty < q < \infty$

$$g(c) = q. \tag{14}$$

Therefore the joint prior for θ, c and m is given by

$$g(\theta,c,m) = \frac{b^q k^\ell}{\Gamma(a)\Gamma(\ell)}\ \theta^{a-1}\ m^{\ell-1}\ q\ \ e^{-(b\theta+km)}\ . \tag{15}$$

From Bayes theorem, by combining likelihood function (8) with joint prior function (15) the posterior density of θ , c and m is given by

$$h(\theta,c,m\backslash x) = \frac{m^{n+\ell-1}\theta^{cn+a-1}\ q\ e^{-(\theta^c v^* + b\theta + km)}\prod_{i=1}^{n} x_i^{-(m+1)}}{\Im_2}\ . \tag{16}$$

where

$$\Im_2 = \int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} q\ m^{n+\ell-1}\ \theta^{cn+a-1}\ e^{-(\theta^c v^* + b\theta+km)}\prod_{i=1}^{n} x_i^{-(m+1)}\ dm\ d\theta\ dc\ . \tag{17}$$

$$v^* = \sum_{i=1}^{n} x_i^{-m}\ .$$

Now, marginal posterior of any parameter is obtained by integrating the joint posterior distribution with respect to other parameters. The posterior pdf of θ can be written, after simplification as

$$h_1\ (\theta\backslash x) = \frac{e^{-b\theta}\ \theta^{a-1}\int_{-\infty}^{\infty}\int_0^{\infty}\theta^{cn}\, m^{n+\ell-1}\ q\ e^{-(v^*\theta^c + m\ k)}\ \prod_{i=1}^{n} x_i^{-(m+1)}\ dm\ dc}{\Im_2}\ . \tag{18}$$

Similarly integrating the joint posterior with respect to θ and c, the marginal posterior of m can be obtained as

$$h_2(m\backslash x) = \frac{m^{n+\ell-1}\ e^{-km}\ \prod_{i=1}^{n} x_i^{-(m+1)}\int_{-\infty}^{\infty}\int_0^{\infty}\theta^{cn+a-1}\ q\ e^{-(b\theta+v^*\theta^c)}\ d\theta\ dc}{\Im_2}\ . \tag{19}$$

$$h_3(c\backslash x) = \frac{q\int_0^{\infty}\int_0^{\infty}\theta^{cn+a-1}\ m^{n+\ell-1}\ e^{-(b\theta+km+v^*\theta^c)}\prod_{i=1}^{n} x_i^{-(m+1)}\ dm\ d\theta}{\Im_2}\ .$$

(20)

Bayes risk for the parameters θ, c and m based on the square error loss function can be obtained as

$$var(\tilde{\vartheta}) = \int_0^\infty \frac{e^{-b\theta}\, \theta^{a} \int_{-\infty}^{\infty}\int_0^\infty \theta^{cn} m^{n+\ell-1}\, q\, e^{-(v^*\theta^c + m\,k)} \prod_{i=1}^n x_i^{-(m+1)}\, dm\, dc}{\Im_2} (\tilde{\vartheta} - \theta)^2 \, d\theta. \quad (21)$$

$$var\,(\tilde{m}) = \int_0^\infty \frac{m^{n+\ell-1}\, e^{-km} \prod_{i=1}^n x_i^{-(m+1)} \int_{-\infty}^{\infty}\int_0^\infty \theta^{cn+a-1}\, q\, e^{-(b\theta+v^*\theta^c)}\, d\theta\, dc}{\Im_2} (\tilde{m} - m)^2\, dm. \quad (22)$$

$$var(\tilde{c}) = \int_{-\infty}^{\infty} \frac{c\, q \int_0^\infty \int_0^\infty \theta^{cn+a-1} m^{n+\ell-1}\, e^{-(b\theta+v^*\theta^c+km)} \prod_{i=1}^n x_i^{-(m+1)}\, dm\, d\theta}{\Im_2} (\tilde{c} - c)^2 dc. \quad (23)$$

Lemma 2

Under LINEX loss function the Bayes estimators for parameters θ, c and m of NIWD can be expressed as

$$\bar{\vartheta} = -\frac{1}{\Omega} \ln E\left(e^{-\Omega\theta}\right) \quad , \Omega \neq 0 \;,$$

$$= -\frac{1}{\Omega} \ln\left[\int_0^\infty e^{-\Omega\theta}\, h_1\,(\theta\backslash x)\; d\theta\right],$$

$$= -\frac{1}{\Omega} \ln\left[\int_0^\infty e^{-\Omega\,\theta} \frac{e^{-b\theta}\, \theta^{a-1} \int_{-\infty}^{\infty}\int_0^\infty \theta^{cn} m^{n+\ell-1}\, q\, e^{-(v^*\theta^c + m\,k)} \prod_{i=1}^n x_i^{-(m+1)}\, dm\, dc}{\Im_2} d\theta\right]. \quad (24)$$

$$\bar{m} = -\frac{1}{\Omega} \ln E\left(e^{-\Omega m}\right) \quad , \Omega \neq 0 \,,$$

$$= -\frac{1}{\Omega} \ln\left[\int_0^\infty e^{-\Omega m}\; h_2(m\backslash x)\, dm\right],$$

$$\bar{m} = -\frac{1}{\Omega} \ln\left[\int_0^\infty e^{-\Omega\, m} \frac{m^{n+\ell-1}\, e^{-km} \prod_{i=1}^n x_i^{-(m+1)} \int_{-\infty}^{\infty}\int_0^\infty \theta^{cn+a-1}\, q\, e^{-(b\theta+v^*\theta^c)}\, d\theta\, dc}{\Im_2} dm\right]. \quad (25)$$

$$\bar{c} = -\frac{1}{\Omega} \ln E\left(e^{-\Omega m}\right) \quad , \Omega \neq 0 \;,$$

$$= -\frac{1}{\Omega} \ln\left[\int_0^\infty e^{-\Omega c}\; h_3(c\backslash x)\, dc\right],$$

$$= -\frac{1}{\Omega} \ln\left[\int_0^\infty e^{-\Omega\, c} \frac{q \int_0^\infty \int_0^\infty \theta^{cn+a-1} m^{n+\ell-1}\, e^{-(b\theta+km+v^*\theta^c)} \prod_{i=1}^n x_i^{-(m+1)}\, dm\, d\theta}{\Im_2} dc\right]. \quad (26)$$

Bayes risk for the parameters θ , c and m based on LINEX error loss function can be obtained as:

$$var(\bar{\vartheta}) = -\frac{1}{\Omega} \ln[\ \int_0^\infty e^{-\Omega\,\theta} \frac{e^{-b\theta}\,\theta^{a-1} \int_{-\infty}^{\infty}\int_0^\infty \theta^{cn} m^{n+\ell-1}\, q\, e^{-(v^*\theta^c + mk)} \prod_{i=1}^n x_i^{-(m+1)}\, dm\, dc}{\Im_2} (\bar{\vartheta} - \theta)^2 d\theta\,]. \tag{27}$$

$$var(\bar{m}) = -\frac{1}{\Omega}\ \ln\ [\int_0^\infty e^{-\Omega\, m}\ \frac{m^{n+\ell-1}\, e^{-km} \prod_{i=1}^n x_i^{-(m+1)} \int_{-\infty}^{\infty}\int_0^\infty \theta^{cn+a-1}\, q\, e^{-(b\theta+v^*\theta^c)}\ d\theta\, dc}{\Im_2} (\bar{m} - m)^{\,2} dm\,] \tag{28}$$

$$var(\bar{c}) = -\frac{1}{\Omega}\ \ln\ [\int_0^\infty e^{-\Omega\, c}\ \frac{q \int_0^\infty\int_0^\infty \theta^{cn+a-1}\, m^{n+\ell-1}\, e^{-(b\theta+km+v^*\theta^c)} \prod_{i=1}^n x_i^{-(m+1)}\ dm\, d\theta}{\Im_2}\ (\bar{c} - c)^2\ dc\,]. \tag{29}$$

3. The Quasi-Bayesian Estimation:

For a random sample of size n, the quasi- likelihood function of the IWD is given by

$$Q(x;\mu) = -\frac{\sum_{i=1}^n x_i}{\mu} - n \ln \mu \tag{30}$$

Substituting for μ from (6), the quasi-likelihood function (30) as a function of θ, c *and* m becomes

$$Q(x;\theta,m) = -\frac{\sum_{i=1}^n x_i}{\theta^{\frac{c}{m}}\Gamma(1-\frac{1}{m})} - n[\frac{c}{m}\ln\theta + \ln\Gamma(1-\frac{1}{m})]\ . \tag{31}$$

In quasi-Bayesian we replace the likelihood function by the natural exponential of quasi-likelihood function. From equation (31), the natural exponential of the quasi-likelihood function for a sample of size n from the IWD is given by

$$\exp[Q(x;\theta,m)] = \frac{e^{-\varphi\theta^{-\frac{c}{m}}\ \Gamma^{-1}\left(1-\frac{1}{m}\right)}}{[\ \theta^{\frac{c}{m}}\ \Gamma\left(1-\frac{1}{m}\right)]^n}\ . \tag{32}$$

Where

$\varphi = \sum_{i=1}^n x_i$.

Lemma 3

Quasi-Bayes estimators for the three unknown parameters θ, c and m under square error loss function and its variances may be obtained by solving numerically, the following equations.

$$\Theta^* = E(\theta\backslash x) = \int_0^\infty \int_{-\infty}^\infty \int_0^\infty \theta \ \ \pi^*(x;\theta,m)\ dm\ \ dc\ \ d\theta \ \ ,$$

$$= \int_0^\infty \int_{-\infty}^\infty \int_0^\infty \frac{\theta^{a-\frac{cn}{m}}\, m^{\ell-1}\, q\ \Gamma^{-n}\left(1-\frac{1}{m}\right)\ e^{-\varphi_1}}{\Im_s}\ dm\ \ dc\ \ d\theta \ \ . \tag{33}$$

$$C^* = E(c\backslash x) = \int_{-\infty}^\infty \int_0^\infty \int_0^\infty c \ \ \pi^*(x;\theta,c,m)\ dm\ \ d\theta\ \ dc\ ,$$

$$= \int_{-\infty}^\infty \int_0^\infty \int_0^\infty c\ \ \frac{\theta^{a-\frac{cn}{m}-1}\, m^{\ell-1}\, q\ \Gamma^{-n}\left(1-\frac{1}{m}\right)\ e^{-\varphi_1}}{\Im_s}\ dm\ \ d\theta\ \ dc\ . \tag{34}$$

$$\mathcal{M}^* = E(m\backslash x) = \int_0^\infty \int_{-\infty}^\infty \int_0^\infty m \ \ \pi^*(x;\theta,c,m)\ d\theta\ \ dc\ \ dm\ ,$$

$$= \int_0^\infty \int_{-\infty}^\infty \int_0^\infty \frac{\theta^{a-\frac{cn}{m}-1}\, m^{\ell}\, q\ \Gamma^{-n}\left(1-\frac{1}{m}\right)\ e^{-\varphi_1}}{\Im_s}\ d\theta\ dc\ \ dm \qquad . \tag{35}$$

$$var\ (\Theta^*) = \int_0^\infty \int_{-\infty}^\infty \int_0^\infty [\theta - \Theta^*]^2\ \ \pi^*(x;\theta,m)\ dm\ \ dc\ \ d\theta\ ,$$

$$= \int_0^\infty \int_{-\infty}^\infty \int_0^\infty [\theta - \Theta^*]^2\ \frac{\theta^{a-\frac{cn}{m}-1}\, m^{\ell-1}\, q\ \Gamma^{-n}\left(1-\frac{1}{m}\right)\ e^{-\varphi_1}}{\Im_s}\ dm\, dc\, d\theta \qquad .$$

(36)

$$var\ (C^*) = \int_{-\infty}^\infty \int_0^\infty \int_0^\infty [c - C^*]^2\ \ \pi^*(x;\theta,c,m)\ dm\ \ d\theta\ \ dc,$$

$$= \int_{-\infty}^\infty \int_0^\infty \int_0^\infty [c - C^*]^2\ \ \frac{\theta^{a-\frac{cn}{m}-1}\, m^{\ell-1}\, q\ \Gamma^{-n}\left(1-\frac{1}{m}\right)\ e^{-\varphi_1}}{\Im_s}\ dm\ \ d\theta\ \ dc.$$

(37)

$$var(\mathcal{M}^*) = \int_0^\infty \int_{-\infty}^\infty \int_0^\infty [m - \mathcal{M}^*]^2\ \pi^*(x;\theta,c,m)\ d\theta\ \ dc\ \ dm\ ,$$

$$= \int_0^\infty \int_{-\infty}^\infty \int_0^\infty [m - \mathcal{M}^*]^2\ \frac{\theta^{a-\frac{cn}{m}-1}\, m^{\ell-1}\, q\ \Gamma^{-n}\left(1-\frac{1}{m}\right)\ e^{-\varphi_1}}{\Im_s}\ d\theta\, dc\, dm\ .$$

(38)

Numerical evaluations, using computer facilities are needed to evaluate equations (33), (34), (35), (36), (37) and (38).

Using the same priors distributions for $\theta, m \; and \; c$ which were defined in equations (12) ,(13) and (14). and the joint prior distribution for $\theta, m \; and \; c$ defined in (15).

From equation (15) and (32), the resulting posterior pdf of θ, c and m after simplification is

$$\pi^*(x;\theta,c,m) = \frac{\exp[Q(x;\theta,c,m)]\; g(\theta,c,m)}{\Im_3} ,$$

$$= \frac{\theta^{a-\frac{cn}{m}-1}\, m^{\ell-1}\, q\, \Gamma^{-n}\left(1-\frac{1}{m}\right)\, e^{-\varphi_1}}{\Im_3}. \qquad (39)$$

Where

$\varphi_1 = b\theta + km + \varphi^*$,

$\varphi^* = \varphi\, \theta^{-\frac{c}{m}}\; \Gamma^{-1}\left(1-\frac{1}{m}\right)$,

$\Im_3 = \int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} \theta^{a-\frac{cn}{m}-1}\, m^{\ell-1}\, q\; \Gamma^{-n}\left(1-\frac{1}{m}\right)\; e^{-\varphi_1}\, d\theta\, dm\, dc$.

NUMERICAL ILLUSTRATION:

numerical example:

In this section we consider a real life data set and illustrate the methods proposed in the previous sections. The data set is fromKundu&Howlader (2010), the data set represents the survival times (in days) of guinea pigs injected with different doses of tubercle bacilli. It is known that guinea pigs have high susceptibility of human tuberculosis and that is why they were used in this particular study. The regimen number is the common logarithm of the number of bacillary units per 0.5 ml. (log (4.0 $\times 10^6$) $=$ 6.6). Corresponding to regimen 6.6, there were 72 observations listed below:

x	12	15	22	24	24	32	32	33	34	38	38	43	44	48
x	52	53	54	54	55	56	57	58	58	59	60	60	60	60
x	61	62	63	65	65	67	68	70	70	72	73	75	76	76
x	81	83	84	85	87	91	95	96	98	99	109	110	121	127
x	129	131	143	146	146	175	175	211	233	258	258	263	297	341

x	341	376

Kundu&Howlader(2010) calculated The mean, standard deviation and the coefficient of skewness as 99.82,80.55 and 1.80, respectively. The measure of skewness indicates that the data are positively skewed. For computational ease, each data point has been divided by 1000. The empirical hazard function of the observed data examined by the scaled Total Time on Test (TTT) plot, see Aarset(1987). This provides a very good idea about the shape of the hazard function of a distribution.

Table (1): the average values of posterior risk and MSE for Bayesian estimates and quasi-Bayesian estimates.

parameters	SEL		LINEX						Quasi-Bayesian	
	Risk	MSE	$\Omega = 0.5$		$\Omega = 5$		$\Omega = -0.5$		Risk	MSE
			Risk	MSE	Risk	MSE	Risk	MSE		
θ	.023	.039	.02087	.02247	.028	.0416	.023	.038	.027	.0286
c	.019	.942	.0026	.0346	.018	.946	.314	5.264	.0021	.0040
m	.03	.4144	.011	.0202	.0298	.4019	.00992	.0131	.0098	.0128

Simulation study:

in order to evaluate the performance of all the different methods of posterior estimator`s and posterior risks discussed in the preceding sections, a Monte Carlo simulation study was conducted and the results are presented in this section. We investigate the performance of the proposed estimators using quasi-Bayesian estimation method through a simulation study. The simulation study is carried out using (18), (19) and (20) for small, medium and large sample sizes, in particular we take sample sizes n= 10, 25 and 50.

For comparison purpose we compute Bayesian estimation method in order to illustrate the symmetric (SEL) and asymmetric (LINEX) loss function. Using (5.24), (5.25) and (5.26) for parameters values ($\theta = 0.24, c = 0.88, m = 0.97$). we report average estimates obtained by all the methods along with mean squared error [MSE $= \frac{\sum Bayes\ risk}{100} + (parameter\ bais)\ ^2$] in parentheses

Table (2): the average values of posterior risk and MSE for Bayesian estimates.

parameters	n	SEL		LINEX					
		Risk	MSE	$\Omega = 0.5$		$\Omega = 5$		$\Omega = -0.5$	
				Risk	MSE	Risk	MSE	Risk	MSE
θ	10	.334	.889	.513	2.143	.259	.429	.418	1.1085
c		.213	.248	.189	.245	.431	.7215	1.491	4.652
m		.291	.397	.311	.4212	.287	.3868	.289	.392
θ	25	.308	.657	.421	1.1165	.246	.4044	.2617	.5127
c		.184	.1936	.186	.1978	.387	.6088	1.352	4.474
m		.284	.3788	.2871	.3863	.2816	.3734	.2857	.3824
θ	50	.293	.601	.225	.302	.218	.28	.26	.432
c		.165	.1694	.173	.1807	.315	.4616	1.278	3.799
m		.239	.3254	.2813	.3719	.243	.3306	.2436	.3318

Table (3): the average values of posterior risk and MSE for Quasi-Bayesian estimates.

Parameters	n	Risk	MSE
θ	10	.313	.389
c		.184	.2256
m		.218	.2835
θ	25	.1853	.233
c		.1759	.1838
m		.176	.2038
θ	50	.154	.186
c		.1538	.1543
m		.149	.1492

CONCLUDING REMARKS

From the results in table 1 it indicates that the performance of Quasi-Bayesian estimator for θ is close to the performance of Bayesian estimators. And the performance of quasi-Bayesian estimator for c is close to the Bayesian (LINEX $\Omega = 0.5$) estimator, and it performs better than the performance of other Bayesian estimators. Here also it is observed that the performance for m estimator is very close to the performance of Bayesian (LINEX $\Omega = -0.5$) and close to the performance of Bayesian (LINEX $\Omega = 0.5$) estimator and perform better than other Bayesian estimators.

From table (2) and(3) we conclude that:

- The quasi-Bayesian estimator of θ perform better than the Bayesian (SEL) and Bayesian (LINEX) estimators at all sample sizes , as sample size increases MSE of the estimated parameter decrease .
- And the quasi- Bayesian estimator of c perform better than the Bayesian (SEL) and Bayesian (LINEX) estimators at all sample sizes.
- From the results in the above tables it is observed that the performance of Bayesian (SEL) estimator for m is very close to the performance of Bayesian (LINEX) estimators at all sample size , and the quasi-Bayesian estimator for m perform better than the Bayesian (SEL) and Bayesian (LINEX) estimator at all sample sizes.
- It is also noticed that the Ω value affect the Bayesian (LINEX) estimators results. The Bayesian (LINEX) estimator of θ at $\Omega = 5$ is perform better than Bayesian (LINEX) estimator at different values of Ω at all sample sizes and it is also perform better than the Bayesian (SEL) at sample size 10 and 25 but its performance is very close to Bayesian (SEL) at sample size 50.
- The performance of Bayesian (LINEX) estimator of c at $\Omega = 0.5$ is better than Bayesian (LINEX) estimator at different values of Ω at all sample sizes and its performance is very close to the performance of Bayesian (SEL) estimator at all sample sizes.
- As the sample size increase MSE of the estimated parameters $(\theta, c \text{ and } m)$ decreases.

- It is observed that the quasi-Bayesian estimate is the closest method to the real parameters values.

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