PRIME NUMBER CONJECTURE

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ABSTRACT: This paper builds on Goldbach’s weak conjecture, showing that all primes to infinity are composed of 3 smaller primes, suggesting that the modern definition of a prime number may be incomplete and requires revision. The results indicate that the axioms underpinning prime numbers should include one as a prime number and two as a non-prime number, adding a new dimension to the most fundamental of all integers.

KEYWORDS: Prime Number, Goldbach’s weak conjecture

PRIME NUMBER CONJECTURE

In a letter to the great mathematician Leonard Euler, Goldbach posited that all prime numbers \( \mathbb{P}' \) greater than five \( \mathbb{P}' > 5 \) are the sum of 3 smaller primes, known as Goldbach’s weak conjecture (Bruckman (2006); Bruckman (2008); Chang (2013) & Shu-Ping (2013). The conjecture was recently proven true therefore; all primes to infinity are composed of 3 smaller primes. The following theorem builds upon the conjecture to show that the modern definition of a prime number may require revision.

Theorem

Premise #1: Assume that all prime numbers are the sum of 3 smaller primes and not just those \( > 5 \) (as proposed by Goldbach to Euler) with only one exception, the number 1 (1 was assumed prime at the time of Euler and Goldbach).

Premise #2: Assume that the number two is not, prime. This claim is intuitive, not one even prime has been identified for any number up to 17 million digits in length, so why should it be assumed that the even number 2 is prime?

Conversely, the empirical facts suggest that the current definition of a prime number requires revision. Given the two premises, a proof follows that resolves the problem in Goldbach's Weak Conjecture regarding primes of less than 6, en passant proving the primality of 1 and non-primality of 2. Contrary to the work of Goldbach, the numbers 5 and 3 are shown to be composed of 3 smaller prime numbers and the number 2 does not fit the revised definition of a prime number.

Proof

\[ 3 + 1 + 1 = 5 \]
\[ 1 + 1 + 1 = 3 \]
\[ 1 + 1 + ? \neq 2 \]
Quod Erat Demonstrandum (Q.E.D.).

Therefore, the modern definition of a prime number is de facto, incomplete. The number 1 is prime and the number two is not prime. A new prime number definition follows:

**Deduction**

1. A prime number is an odd and natural number,
2. composed of the sum of three smaller prime numbers,
3. with only two factors: 1 and itself.

The only exception is 1, which is presumed to be a special case prime, almost transcendental ($\pi = 3.14, e = 2.71, \Phi = 1.618$), the mother / father of all prime numbers and the only common factor of all prime numbers. A logical proof follows (see Equation 1.1):

$$\{x | x \in \mathbb{P}', 1 \subset x \land 2 \not\subset x\} \Rightarrow \top$$

Hóper édei deixai (OE∆)

Although the results seem trivial on the surface, the findings suggest that the axioms underpinning prime numbers may require adjustment ($2 \neq \mathbb{P}'$), adding a new dimension to the most fundamental of all integers, prime numbers.

**DISCUSSION**

This paper builds on Goldbach’s weak conjecture, showing that all primes to infinity are composed of 3 smaller primes, suggesting that the modern definition of a prime number may be incomplete and requires revision. The results indicate that the axioms underpinning prime numbers should include one as a prime number and two as a non-prime number and adding a new dimension to the most fundamental of all integers, prime numbers.

**REFERENCES**


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