International Journal of Mathematics and Statistics Studies

Vol.2, No.3, pp.14-22, July 2014

Published by European Centre for Research Training and Development UK (www.eajournals.org)

PARAMETERS ESTIMATION FOR THE RAYLEIGH DISTRIBUTION BASED ON A SIMPLE STEP-STRESSES MODEL WITH TYPE-II HYBRID CENSORED DATA

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ABSTRACT: *In this paper, we consider a simple step-Stress model under the Rayleigh distribution when the available data are type-II hybrid censored. The maximum likelihood and Bayes estimators as well as, approximate confidence intervals for the parameters are constructed. Bayes estimators are obtained using the symmetric squared error loss functions and asymmetric LINEX and General Entropy (GE) loss functions using non informative priors, a numerical illustration for these new results are also given.*
MSC: Primary 62N05; 62N01: secondary 62F40; 62F30.

KEYWORDS: Accelerated life testing, cumulative exposure model, Rayleigh distribution, maximum likelihood estimation(MLE) , hybrid censored samples , Symmetric and Asymmetries loss function, Monte-Carlo Simulation, Non-informative prior, step stress model, type.

INTRODUCTION

The accelerated life testing (ALT) experiments are important technical structures in reliability and survival analysis. Such experiments allow the experimenter to obtain adequate life data for the product under accelerated stress conditions, which cause the product to fail more quickly than under the normal operating condition. Some key references in the area of ALT include Nelson and Meeker [16], Nelson [15], Meeker and Escobar [13] and Bagdonavicius and Nikulin [1]. A special class is called the Step-Stress testing. Recently Nelson [14]and Balakrishnan and Xie [2] , suggested a new failure model called a simple step stress model with type-II hybrid censored schemes; with two stress levels based on the exponential distribution when the available data are type–II Hybrid Censored Schemes.

In this article, we considered a simple step-stress model with two stress levels based on the Rayleigh distribution when the available data are type–II hybrid censored schemes. The model is discussed in detail in section (2). We discuss the MLE of the parameters in section (3). In section (4), the Bayes estimators are obtained using both the symmetric and asymmetric loss functions. Section (5) provides results of a practical example consisting of some numerical results using simulation study.

Model Description

Suppose that the data come from a cumulative exposure model, and we consider a simple step stress model based on type-II hybrid censored schemes with only two stress level V_1 and V_2. The life time distribution at V_1 and V_2 were assumed to be Rayleigh distribution with failure rate θ_1 and θ_2, respectively. The probability density function (pdf) and cumulative distribution function (cdf) are given by:

$$f_j(t,\theta_j)=\frac{t}{\theta_j^2}e^{-\frac{1}{2}\left(\frac{t}{\theta_j}\right)^2} \quad , \; t>0 \; , \; \theta>0 \; , \; j=1,2 \qquad , \tag{1}$$

and

$$F_j(t,\theta_j)=1-e^{-\frac{1}{2}\left(\frac{t}{\theta_j}\right)^2} \quad , \; t>0 \; , \; \theta>0 \qquad , \tag{2}$$

where the scale parameter is affected by the stress V_j , $j=1,2$, through the inverse power law model defined as :

$$\theta_j = CS_j^P \qquad , \tag{3}$$

where, $S_j=\dfrac{V^*}{V_j}$ and $V^*=\underset{j=1}{\overset{2}{\pi}} V_j^{b_j}$, $b_j=\dfrac{n_j}{\sum_j n_j}$, $j=1,2$.

Then, we have the cumulative exposure distribution, $G(t)$ as:

$$G(t)=\begin{cases} G_1(t)=F_1(t_i)=1-e^{-\frac{1}{2}\left(\frac{t_i}{C\,S_1^p}\right)^2} & , \; 0<t_i<\tau_1 \\ G_1(t)=F_1\left[t_i-\left(1-\left(\dfrac{S_2}{S_1}\right)^p\right)\tau_1\right]=1-e^{-\frac{1}{2}\left\{\left(\frac{t_i-\tau_1}{C\,S_2^p}\right)^2+\left(\frac{\tau_1}{C\,S_1^p}\right)^2\right\}} & , \; \tau_1\le t_i<\infty \end{cases} \; , \tag{4}$$

and the corresponding pdf is:

$$g(t)=\begin{cases} g_1(t)=\dfrac{t_i}{\left(C\,S_1^p\right)^2}\,e^{-\frac{1}{2}\left(\frac{t_i}{C\,S_1^p}\right)^2} & , \; 0<t_i<\tau_1 \\ g_2(t)=\left(\dfrac{t_i-\tau_1}{\left(C\,S_1^p\right)^2}\right)\left(e^{-\frac{1}{2}\left\{\left(\frac{t_i-\tau_1}{C\,S_2^p}\right)^2+\left(\frac{\tau_1}{C\,S_1^p}\right)^2\right\}}\right) & , \; \tau_1\le t_i<\infty \end{cases} \; , \tag{5}$$

Based on the type – II hybrid censored schemes, we have *n* identical units under an initial stress level V_1. The stress level is changed to V_2 at time τ_1 , and the life–testing experiment is terminated at a random time, here $\tau_2^*=\max(t_r,\tau_2)$ [See, Balakrishnan and Xie (2)].

Maximum Likelihood Estimation (MLE):
For the cumulative exposure function in (4) and the corresponding pdf in (5), we obtain the likelihood function of C and P based on the Type-II hybrid censored schemes as follows:

$$\ell(C, p/t) = \frac{n!}{(n-r^*)!}\left[\underset{i=1}{\overset{N_1}{\pi}} g_1(t)\right]\left[\underset{i=N_1+1}{\overset{r^*}{\pi}} g_2(t)\right]\left\{1-G_2(\tau_2^*)\right\}^{n-r^*} . \quad \textbf{(6)}$$

$$\ell = \frac{n!}{(n-r^*)!}\left[\frac{\sum_{i=1}^{N_1} t_i}{\left(C S_1^p\right)^{2N_1}} \cdot \frac{\sum_{i=N_1+1}^{r^*}(t_i-\tau_1)}{\left(C S_2^p\right)^{2r^*}}\right]\left(e^{-\frac{1}{2}\left\{\frac{D_1}{\left(C S_2^p\right)^2}+\frac{D_2^*}{\left(C S_2^p\right)^2}\right\}}\right) . \quad \textbf{(7)}$$

The natural logarithm of likelihood function is given by:

$$\ell og\,(\ell) = \frac{n!}{(n-r^*)!} - 2(N_1+r^*)\,\ell n\,(C) - 2\,p\left(N_1\,\ell n\,(S_1) + r^*\,\ell n\,(S_2)\right) +$$

$$\sum_{i=1}^{N_1}\ell n\,(t_i) + \sum_{i=N_1+1}^{r^*}\ell n\,(t_i-\tau_1) - \frac{1}{2}\left\{\frac{D_1}{\left(C S_2^p\right)^2} + \frac{D_2^*}{\left(C S_2^p\right)^2}\right\} , \quad \textbf{(8)}$$

where,

$$r^* = N_1 + N_2^*$$

$$D_1 = \sum_{i=1}^{N_1} t_i + (n-N_1)\,\tau_1$$

$$D_2^* = \sum_{i=N_1+1}^{r^*}(t_i-\tau_1) + (n-r^*)(\tau_2^*-\tau_1) .$$

Upon differentiating (8) with respect to C , p and equating each result to zero, two equation must be simultaneously satisfied to obtain MLE $\hat{C}$ and $\hat{p}$. These equations are given by:

$$-\frac{2(N_1+r^*)}{C} + \frac{1}{C^3}\left[\frac{D_1}{S_1^{2p}} + \frac{D_2^*}{S_2^{2p}}\right] = 0 . \quad \textbf{(9)}$$

$$-2\left(N_1\,\ell n(S_1) + r^*\,\ell n(S_2)\right) + \frac{1}{C^2}\left[\frac{\ell n(S_1)}{S_1^{2p}}\,D_1 + \frac{\ell n(S_2)}{S_2^{2p}}\,D_2^*\right] = 0 . \quad \textbf{(10)}$$

The MLE of C is expressed by the fallowing equation,

$$\hat{C}^2 = \frac{\frac{D_1}{S_1^{2p}} + \frac{D_2^*}{S_2^{2p}}}{2(N_1+r^*)} . \quad \textbf{(11)}$$

More ever, it is seen that equation (10) can not be put in explicit form, thus it will be solved simultaneously to obtain $\hat{P}$. Substituting in (9), the MLE of C, $\hat{C}$ is obtained.

Bayes Estimation

Most of the Bayesian inference procedures have been developed under the usual squared error loss function, which is symmetrical, and associates equal importance to the losses due to over estimation and under estimation or equal magnitude. However, such a restriction may be impractical. For example, in the estimation of reliability and failure rate function, an over estimate is usually much more serious than an under estimate; in this case the use of a symmetrical loss function might be in appropriate, as has been recognized by Cacciari and Montanari [4], Basu and Ebrahimi [3] and Soliman [10]. An example of an asymmetrical loss function stated by Faynman [9] that in the disaster of a space shuttle, the management over estimated the average life or reliability of the solid fuel rocket booster. A useful asymmetrical loss known linear exponential (LINEX) loss function was introduced in Zellner [17] and was widely used in several papers as Elshahat [8]. Another useful asymmetric loss function is the General Entropy (GE) loss. This loss function was used in several papers, as an example see Dey et. al. [5], Dey and Liu [6] , Soliman [10, 11, 12] and Elshahat [8]. In the following sub sections, the Bayes estimators are obtained using squared error loss function (SEL), LINEX loss function and GE loss function.

Consider independent non–informative type of priors for the parameters C and p, then the joint prior functions will be

$$\pi(C,p)=\frac{1}{C} \quad , \; C>0 \; , \; 0<p<b \quad and \; b>0 \quad . \tag{12}$$

Combining (12) with equation (7) via Bayes theorem, the joint bivariate posterior distribution is derived as follows:

$$\omega(C,p/t)=\frac{\left[\dfrac{\sum\limits_{i=1}^{N_1} t_i}{\left(S_1^p\right)^{2N_1}} \cdot \dfrac{\sum\limits_{i=N_1+1}^{r^*}(t_i-\tau_1)}{\left(S_2^p\right)^{2r^*}}\right]\left(\dfrac{1}{C}\right)^{2(N_1+r^*)+1} \cdot e^{-\frac{1}{2C^2}.\varphi(p)}}{J_1} \quad , \tag{13}$$

where, $J_1=\int\limits_0^b\int\limits_0^\infty \sum\limits_{i=1}^{N_1} t_i \Big/ \left(S_1^p\right)^{2N_1} \; . \sum\limits_{i=N_1+1}^{r^*}(t_i-\tau_1) \Big/ \left(S_2^p\right)^{2r^*} \left(\dfrac{1}{C}\right)^{2(N_1+r^*)+1} \; . \; e^{-\frac{1}{2C^2}.\varphi(p)} \, dC\, dp$

and,

$$\varphi(p)=\left[\frac{D_1}{S_1^{2p}}+\frac{D_2^*}{S_2^{2p}}\right] \quad .$$

Now, marginal posterior of any parameter is obtained by integrating the joint posterior distribution with respect to other parameters. The posterior pdf of C can be written, after simplification as:

$$\omega(C/t)=\frac{J_2}{J_1} \quad , \tag{14}$$

where, $J_2=\left(\dfrac{1}{C}\right)^{2(N_1+r^*)+1}\int\limits_0^b \sum\limits_{i=1}^{N_1} t_i \Big/ \left(S_1^p\right)^{2N_1} \; . \sum\limits_{i=N_1+1}^{r^*}(t_i-\tau_1) \Big/ \left(S_2^p\right)^{2r^*} \; . \; e^{-\frac{1}{2C^2}.\varphi(p)} \, dp \, .$

Similarly integrating $\omega(C,p/t)$ with respect to C , the marginal posterior of p can be obtained as:

$$\omega(p/t) = \frac{J_3}{J_1} \quad , \tag{15}$$

where, $J_3 = \sum_{i=1}^{N_1} t_i \Big/ \left(S_1^p\right)^{2N_1} . \sum_{i=N_1+1}^{r^*} (t_i - \tau_1) \Big/ \left(S_2^p\right)^{2r^*} \int_0^\infty \left(\frac{1}{C}\right)^{2(N_1+r^*)+1} e^{-\frac{1}{2C^2}.\varphi(p)} \, dC$.

Bayes Estimator under Square Error Loss (SEL) Function:

Under square error loss (Symmetric), the usual estimator of a parameter is posterior mean. Thus, Bayes estimators of the parameters are obtained by using the posterior densities (14) and (15).

The Bayes estimators $\tilde{C}$ and $\tilde{p}$ of parameter C and p are:

$$\tilde{C} = J_4/J_1 \quad , \; \tilde{p} = J_5/J_1 \quad , \tag{16}$$

where, $J_4 = \int_0^b \int_0^\infty \sum_{i=1}^{N_1} t_i \Big/ \left(S_1^p\right)^{2N_1} . \sum_{i=N_1+1}^{r^*} (t_i - \tau_1) \Big/ \left(S_2^p\right)^{2r^*} \left(\frac{1}{C}\right)^{2(N_1+r^*)} . \, e^{-\frac{1}{2C^2}.\varphi(p)} \, dC \, dp$,

and,

$$J_5 = \int_0^b \int_0^\infty p \sum_{i=1}^{N_1} t_i \Big/ \left(S_1^p\right)^{2N_1} . \sum_{i=N_1+1}^{r^*} (t_i - \tau_1) \Big/ \left(S_2^p\right)^{2r^*} \left(\frac{1}{C}\right)^{2(N_1+r^*)+1} . \, e^{-\frac{1}{2C^2}.\varphi(p)} \, dC \, dp \; .$$

Bayes Estimator under LINEX Loss Function:

Under assumption that the minimal loss occurs at $\tilde{\tilde{u}} = u$, the LINEX loss function for $u = u(C, p)$ can be expressed as:

$$L(\Delta) = \wp e^{\wp\Delta} - \wp\Delta - 1 \quad ; \; \wp \neq 0 \quad , \tag{17}$$

where, sign and magnitude of $\wp$ represent the direction and degree of symmetry, respectively, ($\wp > 0$ means over estimation is more serious than under estimation, and $\wp < 0$ means the opposite). For $\wp$ closed to Zero, the LINEX loss function is approximately the squared error loss, and there fore almost symmetric. Following zellner [15], the Bayes estimator $\tilde{\tilde{u}}$ of u under LINEX loss function is:

$$\tilde{\tilde{u}} = -\frac{1}{\wp} \ell og \left\{ E_u \left[\exp(-\wp u) \right] \right. \quad , \tag{18}$$

where, E_u is equivalent to the posterior S-expectation with respect to the posterior pdf (u), provided that $E_u\left[\exp(-\wp u)\right]$ exists, and is finite.

Now, if in (18) $u = c$, then the Bayes estimates $\tilde{\tilde{C}}, \tilde{\tilde{p}}$ of parameter C, p relative to the LINEX loss function in (17) are :

$$\tilde{\tilde{C}} = -\frac{1}{\wp} \ell og \left(\frac{J_6}{J_1}\right) , \quad \tilde{\tilde{p}} = -\frac{1}{\wp} \ell og \left(\frac{J_7}{J_1}\right) , \tag{19}$$

where,

$$J_6 = \int_0^b \int_0^\infty \sum_{i=1}^{N_1} t_i \Big/ \left(S_1^p \right)^{2N_1} \cdot \sum_{i=N_1+1}^{r^*} (t_i - \tau_1) \Big/ \left(S_2^p \right)^{2r^*} \left(\frac{1}{C} \right)^{2(N_1+r^*)+1} \cdot e^{-\frac{1}{2C^2}.\varphi(p) - \wp C} \, dC\, dp \,,$$

$$J_7 = \int_0^b \int_0^\infty \sum_{i=1}^{N_1} t_i \Big/ \left(S_1^p \right)^{2N_1} \cdot \sum_{i=N_1+1}^{r^*} (t_i - \tau_1) \Big/ \left(S_2^p \right)^{2r^*} \left(\frac{1}{C} \right)^{2(N_1+r^*)+1} \cdot e^{-\frac{1}{2C^2}.\varphi(p) - \wp p} \, dC\, dp \,.$$

Bayes Estimator under GE Loss Function:

Another useful asymmetric loss function is GE loss

$$L(\hat{\hat{u}}, u) = \frac{\hat{\hat{u}}}{u} - v\, \ell og\left(\frac{\hat{\hat{u}}}{u} \right) - 1 \quad . \tag{20}$$

Whose minimum occurs at $\hat{\hat{u}} = u$. This loss function is generalization of the Entropy–loss used in several papers where $v = 1$. When $v > 0$, a positive error $(\hat{\hat{u}} > u)$ causes more serious consequences than a negative error. The Bayes estimate $\hat{\hat{u}}$ under GE loss (21) is,

$$\hat{\hat{u}} = E_u \left[u^{-v} \right]^{\frac{1}{v}} \quad . \tag{21}$$

Provided that $\hat{\hat{u}} = E_u(u^{-v})$ exists, and is finite. Set $u = C, p$ in (21), then the Bayes estimates $\hat{\hat{C}}$ and $\hat{\hat{P}}$ of parameters, respectively, C and P relative to GE loss function (20) are,

$$\hat{\hat{C}} = \left(J_8 / J_1 \right)^{\frac{1}{v}} \,, \;\; \hat{\hat{p}} = \left(J_9 / J_1 \right)^{\frac{1}{v}} \quad , \tag{22}$$

where,

$$J_8 = \int_0^b \int_0^\infty \sum_{i=1}^{N_1} t_i \Big/ \left(S_1^p \right)^{2N_1} \cdot \sum_{i=N_1+1}^{r^*} (t_i - \tau_1) \Big/ \left(S_2^p \right)^{2r^*} \left(\frac{1}{C} \right)^{2(N_1+r^*)+v+1} \cdot e^{-\frac{1}{2C^2}.\varphi(p)} \, dC\, dp$$

$$J_9 = \int_0^b \int_0^\infty p^{-v} \sum_{i=1}^{N_1} t_i \Big/ \left(S_1^p \right)^{2N_1} \cdot \sum_{i=N_1+1}^{r^*} (t_i - \tau_1) \Big/ \left(S_2^p \right)^{2r^*} \left(\frac{1}{C} \right)^{2(N_1+r^*)+1} \cdot e^{-\frac{1}{2C^2}.\varphi(p)} \, dC\, dp \,.$$

Simulation Study and Illustrative Example

Simulation Study:

In this sub–section, we present the results of a Mote Carlo simulation study carried out in order to compare the performance of all the methods of estimations described in sections (3) and (4). We choose the values of the parameter is C and p to be (C = 1 and p = 1); for (n, r), we chose (25; (19, 23)) and (35; (25; 30)). For different choices of τ_1 and τ_2 in this case, we determined the values of the estimate and their mean square error (MSE) by all the methods presented in section (3). In addition we also present the corresponding 95% approximate confidence intervals (CIs) for the parameters using the asymptotic S–normality of the MLE's. These results, based on 1000 Monte Carlo simulation, are presented in table (1).

Table (1): MLE's and 95% CI.S for C and p with τ_1=1& τ_2=1.5 and Different n and r under type–II hybrid censored schemes

N	R	Parameter	MLE	MSE	95 % CIs	Average widths of CIs
25	19	$\hat{C}$	1.04	0.043	(0.786, 1.3)	0.514
		$\hat{p}$	0.895	0.39	(0.61, 1.11)	0.50
	23	$\hat{C}$	1.04	0.043	(0.837, 1.24)	0.403
		$\hat{p}$	0.91	0.252	(0.707, 1.11)	0.403
35	25	$\hat{C}$	1.03	0.02	(0.869, 1.187)	0.318
		$\hat{p}$	0.872	0.25	(0.713, 1.031)	0.318
	30	$\hat{C}$	1.03	0.02	(0.892, 1.17)	0.278
		$\hat{p}$	0.92	0.18	(0.77, 1.05)	0.28

The Bayes estimators (SEL, LINEX, GE) for the parameters C and P are computed using the equations (13) to (22) and are given in table (2). All of the results obtained in this article specialized to Type –II hybrid censored schemes. From the results, we observe the following:

i. All of the results obtained in table (1) specialized to type–II hybrid censored schemes. The MSE for the MLE of P and C and the average widths of there CIs are decreasing with increasing sample size n.

ii. Table (2) shows that Bayes estimators relative to asymmetric loss function (LINEX and GE) are sensitive to value of parameters $\wp$ and v. These parameters give one the opportunity to estimate the unknown parameters with more flexibility. But the problem of choosing values of parameters $\wp$ and v on the selected loss function.

Table (2): Bayes estimates of the parameters C and p under type–II hybrid censored schemes

N	R	Parameter	SEL	LINEX			GE		
				$\wp$			V		
				0.5	1	2	0.5	1	2
25	19	C	1.12	1.11	1.10	1.092	0.93	0.93	0.94
		P	0.57	0.56	0.556	0.51	3.3	7.7	218.9
	23	C	1.14	1.13	1.12	1.08	0.91	0.91	0.94
		P	0.62	0.6	0.59	0.52	2.7	5.9	209.9
35	25	C	1.1	1.1	1.1	1.09	0.93	0.93	0.933
		P	0.60	0.59	0.57	5.42	2.8	6.4	183.2
	30	C	1.12	1.1	1.1	1.1	0.913	0.92	0.923
		P	0.64	0.63	0.612	0.58	2.9	4.9	142.97

iii. All Bayes estimators (SEL, LINEX, GE) for the parameters C and P are nearly equal for all values of $\wp$ and v were used.

iv. All of the estimators obtained in tables (1) and (2) are the best good statistical on $r = 0.75n$.

- **v.** The estimated Values of Bayes estimators are not very far from the estimated values of MLE,s .
- **vi.** As anticipated all Bayes estimates relative to both LINEX loss and GE loss (for $\wp$ closed to zero and $v=-1$) are the same as the symmetric Bayes squared error loss.

Illustrative Example:
In this sub–section, we consider one example, use small sample in order to illustrate the method of maximum likelihood estimation which was described in section four.

Example:
We now consider the following data when $n=20$ with $\tau_1=1$, $C=1$ and $p=1$.

Stress level	**Time–to–failure**													
$S_1=1$	**0.03**	**0.4**	**0.54**	**0.69**	**0.65**	**0.8**	**0.98**							
$S_2=2$	**1.1**	**1.17**	**1.44**	**1.46**	**1.463**	**1.468**	**1.63**	**2.11**	**2.2**	**2.3**	**2.6**	**3.14**	**3.3**	

In this case, had we fixed $\tau_2=1.5$ and $r=15,18$, we would be obtained the MLE's of $\hat{\theta}_1$ and $\hat{\theta}_2$ by using equation (3) to be,

r	$\hat{C}$	$\hat{p}$	$\hat{\theta}_1$	$\hat{\theta}_2$
15	0.76	1.3	0.76	1.52
18	0.76	1.2	0.76	1.51

Note that, where r = 15, we have $\tau_2^*=\max(t_{15},\tau_2)=\max(2.1,1.5)=2.1$; similarly, where $r=18$, we find $\tau_2^*=2.6$. The CIs for the parameters obtained. Note that approximate CI and student–t interval are both unsatisfactory.

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