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# PARAMETER SENSITIVITY AND ELASTICITY ANALYSIS OF A MATHEMATICAL MODEL FOR NON–HOMOGENOUS POPULATION DENSITY OF A WEED SPECIES

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**ABSTRACT:** *In this work, a stage-structured model for non- homogenous population density of an annual weed is analysed for parameter sensitivity and elasticity. The steady state solution of the model is obtained. In order to determine the contribution of identified parameters to the model steady state, the sensitivity and elasticity analyses are performed using matrix calculus approach. The result of the sensitivity analysis shows that the steady state is very responsive to change in established seedling survival rate (e). While, elasticity analysis indicates that, both established and matured weeds steady-state densities are equally affected by small additive changes in maturity rate (m) and establishment rate (e). Besides, seed bank seed density is most sensitive to small additive change in seed production (b) as compared to weed maturity rate (m). Hence, we conclude that increase in the survival and maturity rates possibly may lead to an increase in weed population density.*

**KEYBOARDS:** Steady-state, Parameter, Sensitivity, Elasticity, Matrix calculus, Partial derivative.

## INTRODUCTION

Weeds are unwanted plants that always grow naturally. They exist only in natural environments that have been disturbed by humans' activities such as agricultural lands, recreational parks, and irrigation dams [1]. In fact, weed is a term applied to any plant that grows in a place it is not wanted.

Generally, models for the dynamics of weed in arable requires component to describe changes in the number of vegetative plants stages. This paper considered a deterministic discrete-time model for stage-structure population densities of single species for a hypothetical annual weed. Sensitivity and elasticity analysis of the model parameters was performed in order to determine the absolute contribution of each identified parameter to the steady state population density. We developed the model equation in section 2, parameters sensitivity and elasticity analyses are carried out in section 3 while the conclusion is given in section 4.

## MODEL EQUATIONS

In this paper, the life-cycle of an annual weed is divided into three stages; seed (S) in the seed bank, established seedling (E) and mature weeds (M). The life-cycle events of weeds are assumed to occur in one time step, because they are usually understood to be synchronous. So, this requires system of difference equations that relate the numbers of seeds, seedling and mature weeds at time $t$ to the numbers at time ($t$+1).



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The individuals that are classed as dormant seeds may stay as viable dormant seeds in the soil after surviving dry season or they may germinate at some rate to become established seedling. The established seedling may survive and grow at some rate to become reproductive mature weeds. Those individuals classed as mature weeds have two possible routes after flowering and seeds production: Newly produced seeds may either survived predation and germinate late in the season to become establish seedling or survive and become part of the soil dormant seed.

The stage structured population model for the abundant densities of seeds ($n_{1,t}$), established seedlings ($n_{2,t}$) and mature-weeds ($n_{3,t}$) is given by the following system of difference equations:

$$n_{1,t+1} = d(1-g_1(n_{2,t}))n_{1,t} + bd(1-g_1(n_{2,t}))n_{3,t} \quad (1)$$

$$n_{2,t+1} = edg_1(n_{2,t})n_{1,t} + bedg_2(n_{2,t})n_{3,t} \quad (2)$$

$$n_{3,t+1} = m(n_{2,t})n_{2,t} \quad (3)$$

Biologically, it has been observed that any of the parameters $g_i, e, m$ and $b$ may experience density-dependence due to resource limitation (such as space, nutrient, water and light). The Beaverton-Holt density-dependence function type in [2] is adopted for the functions $g_i(n_{2,t})$ and $m(n_{2,t})$ due to the assumption that seedling recruitment and the established seedling growth to mature weed are density dependent and there is competition among the weed for the available micro site. Thus;

$$g_i(n_{2,t}) = \frac{g_i}{1+\alpha n_{2,t}} \quad (4)$$

$$m(n_{2,t}) = \frac{m}{1+\alpha n_{2,t}} \quad (5)$$

So, substituting (4) and (5) into (1) – (3) and expresses in matrix form becomes;

$$\begin{pmatrix} n_{1,t+1} \\ n_{2,t+1} \\ n_{3,t+1} \end{pmatrix} = \begin{pmatrix} d\left(1-\frac{g_1}{1+\alpha n_{2,t}}\right) & 0 & bd\left(1-\frac{g_2}{1+\alpha n_{2,t}}\right) \\ de\frac{g_1}{1+\alpha n_{2,t}} & 0 & bde\frac{g_2}{1+\alpha n_{2,t}} \\ 0 & \frac{m}{1+\alpha n_{2,t}} & 0 \end{pmatrix} \begin{pmatrix} n_{1,t} \\ n_{2,t} \\ n_{3,t} \end{pmatrix}. \quad (6)$$

a density–dependence stage-structured model for non-homogeneous population density of an annual weed.

Where

$g_i$ The maximum value of $g_i(n_{2,t})$ at a low density of established seedling $(n_{2,t})$.

$d$ Fraction of dormant seeds surviving in the seed bank

$g_2$ Fraction of viable new (fresh) seeds germination within the growing season

$g_1$ Fraction of seeds older than one year germination out of the seed bank.

$e$ Fraction of germinated seeds that become established seedlings

$m$ Fraction of the established seedlings that survive to mature weeds

$b$ Average number of seeds produced by the mature weed per unit area.

Note: mature weed density is not included in the density–dependence term because after seed production they will die been monocarpic annual.

## SENSITIVITY ANALYSIS

The matrix calculus approach in [4] is adopted to perform the sensitivity analysis of model parameters in order to determine the absolute contribution of each identified parameter to the steady state population density. To do this, the steady state solution (population density) is obtained next.

### Positive Steady-state Solution

The Steady–state solution of the equations (1) – (3) is found by setting $\mathbf{n}_{t+1} = \mathbf{n}_t = \mathbf{n}$ in (6) [6] where $\mathbf{n} = (n_1 \quad n_2 \quad n_3)^T$, using substitution method and assuming that $d \approx 1$. The analytical expression for the non-zero solution, $\mathbf{n} = (n_1, \quad n_2, \quad n_3) \in \operatorname{int} \Re_+^3$ is obtained thus;

$$E_1(n_1, \quad n_2, \quad n_3) = \left( \frac{(bme-1)(bme-g_2)}{eg_1\alpha}, \frac{bme-1}{\alpha}, \frac{bme-1}{be\alpha} \right) \tag{7}$$

They all exist and are positive.

### Parameters Sensitivity Analyses of the Steady State.

The sensitivity analysis is to identify which life-cycle stage of weed that affects the steady-state density. To carry out the analysis, equation (6) is expressed in a general matrix form as;

$$\mathbf{n_{t+1}} = \mathbf{P}(\theta, n_t)\mathbf{n_t}, \tag{8}$$

At the steady state (equilibrium) (8) satisfies

$$\bar{\mathbf{n}} = \mathbf{P}(\theta, \bar{n})\bar{\mathbf{n}} \tag{9}$$

Finding the rate of change of the elements of $\bar{\mathbf{n}}$ with respect to each and every one of the parameters in $\theta$ through taking the differential of both sides of (9) gives


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$$d\bar{\mathbf{n}} = (d\mathbf{P})\bar{n} + \mathbf{P}(d\bar{\mathbf{n}}) \tag{10}$$

Rewrite this as

$$d\bar{\mathbf{n}} = \mathbf{I}_3(d\mathbf{P})\bar{n} + \mathbf{P}(d\bar{\mathbf{n}}) \tag{11}$$

where $\mathbf{I}_3$ is an identity matrix of dimension 3. Then the vec operator is applied to both sides of (11) [3]. Remembering that vec $\bar{\mathbf{n}} = \bar{\mathbf{n}}$ since $\bar{\mathbf{n}}$ is a column vector. Then applying Roth's theorem [4], to obtain

$$d\bar{\mathbf{n}} = (\bar{\mathbf{n}}^T \otimes \mathbf{I}_3) dvec\mathbf{P} + \mathbf{P}d\bar{\mathbf{n}} \tag{12}$$

However, $\mathbf{P}$ is a function of $\bar{\mathbf{n}}$ and $\theta$ so,

$$dvec\mathbf{P} = \frac{\partial vec\mathbf{P}}{\partial \theta^T} d\theta^T + \frac{\partial vec\mathbf{P}}{\partial \bar{\mathbf{n}}^T} d\bar{\mathbf{n}} \tag{13}$$

Substituting (12) into (13) and applying the chain rule gives

$$\frac{d\bar{\mathbf{n}}}{d\theta^T} = (\bar{\mathbf{n}}^T \otimes \mathbf{I}_3)\left(\frac{\partial vec\mathbf{P}}{\partial \theta^T} + \frac{\partial vec\mathbf{P}}{\partial \mathbf{n}^T}\frac{d\bar{\mathbf{n}}}{d\theta^T}\right) + \mathbf{P}\frac{d\bar{\mathbf{n}}}{d\theta^T} \tag{14}$$

Expand right side of (14) and solve for $\frac{d\bar{\mathbf{n}}}{d\theta^T}$ to obtain

$$\frac{d\bar{\mathbf{n}}}{d\theta^T} = \left(\mathbf{I}_3 - \mathbf{P} - (\bar{\mathbf{n}}^T \otimes \mathbf{I}_3)\frac{\partial vec\mathbf{P}}{\partial \mathbf{n}^T}\right)^{-1} (\bar{\mathbf{n}}^T \otimes \mathbf{I}_3)\frac{\partial vec\mathbf{P}}{\partial \theta^T} \tag{15}$$

where $\mathbf{P}$, $\frac{\partial vec\mathbf{P}}{\partial \theta^T}$ and $\frac{\partial vec\mathbf{P}}{\partial \mathbf{n}^T}$ are evaluated at $\bar{\mathbf{n}}$.

Then, apply (15) to determine the sensitivity of non-zero equilibrium state of our density-dependent weed population model. The parameters vector is defined as; $\theta = (g_1, g_2, b, e, m)$ and the steady state solution; $\mathbf{n} = (n_1, \quad n_2, \quad n_3)$ . So

$$\mathbf{P}(\theta, \bar{\mathbf{n}})\Big| = \begin{pmatrix} \frac{bme - g_1}{bme} & 0 & \frac{b(bme - g_2)}{bme} \\ \frac{g_1}{bm} & 0 & \frac{g_2}{m} \\ 0 & \frac{1}{be} & 0 \end{pmatrix} \tag{16}$$

Then, the partial derivatives of $P(\theta, \mathbf{n})$ with respect to $\theta$ and $\boldsymbol{n}$ are obtained thus

$$\frac{\partial vecP(\theta, \mathbf{n})}{\partial g_1} = vec\begin{pmatrix} -(1+\alpha n_2)^{-1} & 0 & 0 \\ e(1+\alpha n_2)^{-1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{17}$$


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$$\frac{\partial vecP(\theta,\mathbf{n})}{\partial g_2} = vec\begin{pmatrix} 0 & 0 & -b(1+\alpha n_2)^{-1} \\ 0 & 0 & be(1+\alpha n_2)^{-1} \\ 0 & 0 & 0 \end{pmatrix} \quad (18)$$

$$\frac{\partial vecP(\theta,\mathbf{n})}{\partial b} = vec\begin{pmatrix} 0 & 0 & 1-g_2(1+\alpha n_2)^{-1} \\ 0 & 0 & eg_2(1+\alpha n_2)^{-1} \\ 0 & 0 & 0 \end{pmatrix} \quad (19)$$

$$\frac{\partial vecP(\theta,\mathbf{n})}{\partial e} = vec\begin{pmatrix} 0 & 0 & 0 \\ g_1(1+\alpha n_2)^{-1} & 0 & bg_2(1+\alpha n_2)^{-1} \\ 0 & 0 & 0 \end{pmatrix} \quad (20)$$

$$\frac{\partial vecP(\theta,\mathbf{n})}{\partial m} = vec\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & (1+\alpha n_2)^{-1} & 0 \end{pmatrix} \quad (21)$$

The partial derivative of $P$ with respect to $\theta$ is a $(9\times 5)$ matrix

$$\frac{\partial vecP(\theta,\mathbf{n})}{\partial \theta} = \begin{pmatrix} -(1+\alpha n_2)^{-1} & 0 & 0 & 0 & 0 \\ e(1+\alpha n_2)^{-1} & 0 & 0 & g_1(1+\alpha n_2)^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1+\alpha n_2)^{-1} \\ 0 & -b(1+\alpha n_2)^{-1} & 1-g_2(1+\alpha n_2)^{-1} & 0 & 0 \\ 0 & be(1+\alpha n_2)^{-1} & eg_2(1+\alpha n_2)^{-1} & bg_2(1+\alpha n_2)^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (22)$$

Evaluating (22) at $\mathbf{n} = (n_1,\ \ n_2,\ \ n_3)$ gives

$$\left.\frac{\partial vecP(\theta,\mathbf{n})}{\partial \theta}\right|_{n_2} = \begin{pmatrix} -(bme)^{-1} & 0 & 0 & 0 & 0 \\ (bm)^{-1} & 0 & 0 & g_1(bme)^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (bme)^{-1} \\ 0 & -(me)^{-1} & 1-g_2(bme)^{-1} & 0 & 0 \\ 0 & (m)^{-1} & g_2(bm)^{-1} & g_2(me)^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (23)$$

The derivative of $P(\theta,\mathbf{n})$ with respect to $n_2$ is


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$$\frac{\partial vecP(\theta,\mathbf{n})}{\partial \mathbf{n}^T}=\begin{pmatrix} 0 & g_1\alpha(1+\alpha n_2)^{-2} & 0 \\ 0 & -eg_1\alpha(1+\alpha n_2)^{-2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -m\alpha(1+\alpha n_2)^{-2} & 0 \\ 0 & bg_2\alpha(1+\alpha n_2)^{-2} & 0 \\ 0 & -beg_2\alpha(1+\alpha n_2)^{-2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (24)$$

Evaluating (24) at $\mathbf{n}=(n_1, \quad n_2, \quad n_3)$ gives

$$\left.\frac{\partial vecP(\theta,\mathbf{n})}{\partial \mathbf{n}^T}\right|_{n_2}=\begin{pmatrix} 0 & g_1\alpha(bme)^{-2} & 0 \\ 0 & -eg_1\alpha(bme)^{-2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -m\alpha(bme)^{-2} & 0 \\ 0 & bg_2\alpha(bme)^{-2} & 0 \\ 0 & -beg_2\alpha(bme)^{-2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (25)$$

$$\mathbf{n}^T \otimes I_3=\begin{pmatrix} \frac{(\text{bme-1})(\text{bme-g}_2)}{\text{e}\alpha\text{g}_1} & \frac{\text{bme-1}}{\alpha} & \frac{\text{bme-1}}{\text{be}\alpha} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (26)$$

$$\mathbf{n}^T \otimes I_3=\begin{pmatrix} \frac{(\text{bme-1})(\text{bme-g}_2)}{\text{e}\alpha\text{g}_1} & 0 & 0 & \frac{\text{bme-1}}{\alpha} & 0 & 0 & \frac{\text{bme-1}}{\text{be}\alpha} & 0 & 0 \\ 0 & \frac{(\text{bme-1})(\text{bme-g}_2)}{\text{e}\alpha\text{g}_1} & 0 & 0 & \frac{\text{bme-1}}{\alpha} & 0 & 0 & \frac{\text{bme-1}}{\text{be}\alpha} & 0 \\ 0 & 0 & \frac{(\text{bme-1})(\text{bme-g}_2)}{\text{e}\alpha\text{g}_1} & 0 & 0 & \frac{\text{bme-1}}{\alpha} & 0 & 0 & \frac{\text{bme-1}}{\text{be}\alpha} \end{pmatrix} \quad (27)$$

$$(\mathbf{n}^T \otimes I_3)\frac{\partial vecP(\theta,\mathbf{n})}{\partial \mathbf{n}^T}=\begin{pmatrix} n_1 & 0 & 0 & n_2 & 0 & 0 & n_3 & 0 & 0 \\ 0 & n_1 & 0 & 0 & n_2 & 0 & 0 & n_3 & 0 \\ 0 & 0 & n_1 & 0 & 0 & n_2 & 0 & 0 & n_3 \end{pmatrix}\begin{pmatrix} 0 & g_1\alpha(bme)^{-2} & 0 \\ 0 & -eg_1\alpha(bme)^{-2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -m\alpha(bme)^{-2} & 0 \\ 0 & bg_2\alpha(bme)^{-2} & 0 \\ 0 & -beg_2\alpha(bme)^{-2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (28)$$


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$$(\mathbf{n}^T \otimes I_3)\frac{\partial vecP(\theta,\mathbf{n})}{\partial \mathbf{n}^T} = \begin{pmatrix} 0 & \frac{(\text{bme}-1)(\text{bme}-\text{g}_2)}{\text{e}(bme)^2} + \frac{(\text{bme}-1)g_2}{\text{e}(bme)^2} & 0 \\ 0 & -\frac{(\text{bme}-1)(\text{bme}-\text{g}_2)}{(bme)^2} - \frac{(\text{bme}-1)g_2}{(bme)^2} & 0 \\ 0 & -\frac{\text{m(bme}-1)}{(bme)^2} & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{(\text{bme}-1)}{\text{e}(bme)} & 0 \\ 0 & -\frac{(\text{bme}-1)}{(bme)} & 0 \\ 0 & -\frac{\text{m(bme}-1)}{(bme)^2} & 0 \end{pmatrix} \quad (29)$$

Using $\mathbf{I}_3$, (16) and (29) to obtain (30) after simplification

$$\mathbf{I}_3 - \mathbf{P} - (\mathbf{n}^T \otimes I_3)\frac{\partial vecP(\theta,\mathbf{n})}{\partial \mathbf{n}^T} = \begin{pmatrix} \frac{g_1}{bme} & -\frac{(\text{bme-1})}{\text{e}(bme)} & -\frac{(bme-g_2)}{me} \\ -\frac{g_1}{bm} & \frac{2(\text{bme})\text{-}1}{bme} & -\frac{g_2}{m} \\ 0 & \frac{1}{(bme)^2} & 1 \end{pmatrix} \quad (30)$$

So

$$\left(\mathbf{I}_3 - \mathbf{P} - (\mathbf{n}^T \otimes I_3)\frac{\partial vecP(\theta,\mathbf{n})}{\partial \mathbf{n}^T}\right)^{-1} =$$

$$\begin{pmatrix} \frac{\text{bm}^2\text{e}[2(\text{bme})-1+\text{g}_2]}{g_1(bm^2e+1)} & -\frac{bm^2(b^2me^2-bme-beg_2+1}{g_1(bm^2e+1)} & \frac{\text{bme}(2\text{bme}-1)-\text{g}_2(bme+2)}{eg_1(bm^2e+1)} \\ \frac{bm^2e^2}{(bm^2e+1)} & \frac{bm^2e}{(bm^2e+1)} & \frac{(\text{bme})^2}{(bm^2e+1)} \\ \frac{-1}{b(bm^2e+1)} & -\frac{1}{be(bm^2e+1)} & \frac{bm^2e}{(bm^2e+1)} \end{pmatrix} \quad (31)$$

Also, use (23) and (30) to obtain (32)

$$(\mathbf{n}^T \otimes I_3)\frac{\partial vecP(\theta,\mathbf{n})}{\partial \theta^T} = \begin{pmatrix} \frac{(\text{bme-1})(\text{bme-g}_2)}{\alpha\text{g}_1(bme^2)} & -\frac{\text{bme-1}}{\alpha bme^2} & \frac{(\text{bme-1})(\text{bme-g}_2)}{\alpha(b^2me^2)} & 0 & 0 \\ \frac{(\text{bme-1})(\text{bme-g}_2)}{\alpha\text{g}_1 bme} & \frac{\text{bme-1}}{\alpha\text{bme}} & \frac{\text{g}_2(\text{bme-1})}{\alpha\text{b}^2\text{me}} & \frac{(\text{bme-1})}{\alpha e} & 0 \\ 0 & 0 & 0 & 0 & \frac{\text{bme-1}}{\alpha bme} \end{pmatrix} \quad (32)$$

Putting (31) and (32) into (15) after simplification gives


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$$\times\begin{pmatrix} \frac{(\mathrm{bme}-1)(\mathrm{bme}-\mathrm{g}_2)}{\alpha g_1(bme^2)} & -\frac{\mathrm{bme}-1}{\alpha bme^2} & \frac{(\mathrm{bme}-1)(\mathrm{bme}-\mathrm{g}_2)}{\alpha(b^2me^2)} & 0 & 0 \\ \frac{(\mathrm{bme}-1)(\mathrm{bme}-\mathrm{g}_2)}{\alpha g_1 bme} & \frac{\mathrm{bme}-1}{\alpha\mathrm{bme}} & \frac{\mathrm{g}_2(\mathrm{bme}-1)}{\alpha\mathrm{b}^2\mathrm{me}} & \frac{(\mathrm{bme}-1)}{\alpha e} & 0 \\ 0 & 0 & 0 & 0 & \frac{\mathrm{bme}-1}{\alpha bme} \end{pmatrix}$$

$$\frac{d\bar{\mathbf{n}}}{d\theta^T}=\begin{pmatrix} \frac{\mathrm{bm}^2\mathrm{e}[2(\mathrm{bme})-1+\mathrm{g}_2]}{g_1(bm^2e+1)} & -\frac{bm^2(b^2me^2-bme-beg_2+1)}{g_1(bm^2e+1)} & \frac{\mathrm{bme}(2\mathrm{bme}-1)-\mathrm{g}_2(bme+2)}{eg_1(bm^2e+1)} \\ \frac{bm^2e^2}{(bm^2e+1)} & \frac{bm^2e}{(bm^2e+1)} & \frac{(\mathrm{bme})^2}{(bm^2e+1)} \\ \frac{-1}{b(bm^2e+1)} & -\frac{1}{be(bm^2e+1)} & \frac{bm^2e}{(bm^2e+1)} \end{pmatrix}$$

$$\frac{d\bar{\mathbf{n}}}{d\theta^T}=\begin{pmatrix} \frac{EGm(A+E-beG)}{\alpha g_1^2eB} & \frac{-Em[A+beG-E]}{\alpha eg_1B} & \frac{E[G-mg_2(beG-E)]}{\alpha beg_1B} & \frac{-mE(beG-E)}{\alpha eg_1B} & \frac{(bmeA-2g_2)E}{\alpha bme^2g_1B} \\ \frac{2mEG}{\alpha g_1B} & 0 & \frac{mE(G+g_2)}{\alpha bB} & \frac{bm^2E}{\alpha B} & \frac{bmE}{\alpha B} \\ \frac{-2EG}{\alpha g_1b^2me^2} & 0 & \frac{-E(G+g_2)}{\alpha b^3me^2B} & \frac{-E}{\alpha be^2B} & \frac{mE}{\alpha B} \end{pmatrix} \quad (33)$$

where,

$A=(2\mathrm{bme}+\mathrm{g}_2-1)$ , $B=(bm^2e+1)$, $\mathrm{E}=(\mathrm{bme}-1)$ and $\mathrm{G}=(\mathrm{bme}-\mathrm{g}_2)$ and

A>B>G>E provided $\mathrm{g}_2<1$. (33) gives the steady-state (population density) sensitivities to changes in the identified parameters. Compactly written, it is

$$\frac{d\bar{\mathbf{n}}}{d\theta}=\begin{pmatrix} \frac{d\bar{\mathbf{n}}}{dg_1} & \frac{d\bar{\mathbf{n}}}{dg_2} & \frac{d\bar{\mathbf{n}}}{db} & \frac{d\bar{\mathbf{n}}}{de} & \frac{d\bar{\mathbf{n}}}{dm} \end{pmatrix}. \quad (34)$$

Each column of (34) is changing rate of the vector $\bar{\mathbf{n}}$ to one of the parameters $\theta$. Thus, the steady-state density is very susceptible to alteration in established seedling survival (*e*) than maturation rate to mature weeds (*m*) because $\frac{bm^2E}{\alpha B}>\frac{bmE}{\alpha B}$. Increase in establishing rate will increase established weed density than maturation rate to adult due to competitive pressure.

It has been observed that one major challenge in interpreting sensitivities analysis is that demographic parameters (variables) are measured in different units. The sensitivity of changes in seedling survival (e) and maturation rate of weeds (m) (a probability with value between 0 and 1) may be difficult to compare to the sensitivity of seed production rates (*b*) (which has no restriction of value). This is where elasticity comes into play in order to overcome this problem.



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**Elasticity Analyses of the Steady State Solution**

The elasticity analysis estimates the effect of a proportional change in the vital parameter values on population growth. When population is density-dependent, elasticities of the equilibrium (steady-state) population size is examine [5]. This is to determine the effects of each parameter on the steady-state population density. In essence, elasticities are proportional sensitivities, scaled so that they are dimensionless.

The elasticity of the steady-state densities are calculated from the sensitivity by the expression given [4] as

$$S_{n\theta} = diag[\bar{\mathbf{n}}]^{-1} \frac{d\bar{\mathbf{n}}}{d\theta^T} diag[\theta] \tag{35}$$

So, using $\bar{\mathbf{n}}$ , $\theta$ and (35)after simplification we have

$$S_{\bar{n}\theta} = \begin{pmatrix} \frac{EGm(A+E-beG)}{g_1 B(\text{bme}-1)(\text{bme}-g_2)} & \frac{-Em[A+(beG-E)]}{B(\text{bme}-1)(\text{bme}-g_2)} & \frac{mE[AG-g_2(beG-E)]}{bB(\text{bme}-1)(\text{bme}-g_2)} & \frac{-mE(beG-E)}{B(\text{bme}-1)(\text{bme}-g_2)} & \frac{E(bmeA-2g_2)}{bmeB(\text{bme}-1)(\text{bme}-g_2)} \\ \frac{2m\text{EG}}{g_1 B(\text{bme}-1)} & 0 & \frac{mE(G+g_2)}{bB(\text{bme}-1)} & \frac{bm^2 E}{B(\text{bme}-1)} & \frac{bmeE}{B(\text{bme}-1)} \\ \frac{-2\text{EG}}{g_1 bme(\text{bme}-1)} & 0 & \frac{-\text{E}(\text{G}+g_2)}{b^2 \text{me} B(\text{bme}-1)} & \frac{-E}{eB(\text{bme}-1)} & \frac{\text{be}mE}{B(\text{bme}-1)} \end{pmatrix} \tag{36}$$

(Note; A, B, E and G retained their previous representations.)

The values in the matrix (35) give the results of small additive change of the parameters, which is the proportional result of small proportional deviation. Since they are proportions values measured without units, it is very easy to compare all the parameters. Here, it indicates that, both establish and mature weeds steady-state densities are equally affected by small additive changes in maturity rate (*m*). Seed bank seed density is most sensitive to small additive change in seed production rate (*b*) as compared to weed maturity rate (*m*). While above ground steady-state density mostly affected by additive changes in establishment rate (*e*).

## CONCLUDING REMARKS

In this work, non-zero positive steady–state density of a developed stage-structured model for a species of weed was obtained. The sensitivity and elasticity analyses of model parameters were performed using matrix calculus approach to determine the absolute contribution of each identified parameter to the steady state population density. The results of the sensitivity analysis indicated that an increase in establishing rate will increase weed population density than maturation rate to adult. This might be due-to weeds' competition for micro site pressure. Besides, elasticity analysis made it possible to compare more parameters' contributions to the steady state density. It revealed that the above ground steady-state weed density is mostly affected by the establishment rate (*e*) rather than the maturity rate (*m*).



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