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ORTHOGONAL SYMMETRIC BI DERIVATIONS IN SEMIPRIME RINGS

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ABSTRACT: *In this paper we introduce the notation of orthogonal symmetric bi derivations in semiprime ring and we consider R be a 2-torsion free semiprime ring, B and D are two symmetric bi derivations on R,then B and D are orthogonal if and only if one of the following equivalent conditions holds for all $x, y, z \in R$: (1) $BD = 0$. (2) $B(x, y)D(y, z) = 0$ or $D(x, y)B(y, z) = 0$. (3) BD is a bi derivation. (4) $B(x, y)D(y, z) + D(y, z)B(x, y) = 0$,for every $x, y, z \in R$.*

KEYWORDS: Semi Prime Ring, Derivation, Bi Derivation, Orthogonal Derivation, Orthogonal Symmetric Bi Derivation, Jordan Bi Derivation.

INTRODUCTION

In [3] Bresar.M and Vukman.J have introduced the notation of orthogonality for a pair of derivations (d, g) of a semiprime ring, and they gaves several necessary and sufficient conditions for d and g are to be orthogonal and they gave the related results to a classical result of E.Posner [6]. Ashraf.M and Jamal M.R in [2] have studied orthogonality for a pair of derivations (d, g) of a semiprime gamma ring and gave the several necessary and sufficient conditions for d and g are to be orthogonal. In [1] Argac.N, Nakajima.A and Albas.E have studied some results orthogonal generalized derivations of semiprime rings. Daif.M.N,Tammam EI-Sayiad.M.S and Haetinger.C [4] have studied some results orthogonal derivations and bi derivations in semiprime ring. Ozturk.M.A and Sapanci.M [5] have studied some results orthogonal symmetric bi derivation on semiprime Gamma rings. Vukman.J [7, 8] investigated symmetric bi derivations on prime and semi prime rings in connection with centralizing mappings. In this paper we proved some results on orthogonal symmetric bi derivations in semiprime rings.

Preliminaries

Throughout this work R will be an associative ring. A ring R is said to be 2-torsion free if $2x = 0, x \in R$ implies $x = 0$. Recall that R is a prime if $xRy = 0$ implies $x = 0$ or $y = 0$, and R is a semiprime if $xRx = 0$ implies $x = 0$. Let us write $[x, y] = xy - yx$, for all $x, y \in R$ and the identities $[xy, z] = [x, z]y + x[y, z]$, $[x, yz] = [x, y]z + y[x, z]$. An additive map $d: R \to R$ is called derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. An additive map $d: R \to R$ is called Jordan derivation if $d(x^2) = d(x)x + xd(x)$, for all $x \in R$.A mapping $B(.,.): R \times R \to R$ is a symmetric mapping if $B(x, y) = B(y, x)$, for all $x, y \in R$. A symmetric bi additive mapping $D(.,.): R \times R \to R$ is called a symmetric bi derivation if $D(xy, z) = D(x, z)y + xD(y, z)$, for all $x, y, z \in R$.Obviously, in this case also the relation $D(x, yz) = D(x, y)z + yD(x, z)$, for all $x, y, z \in R$. A symmetric bi additive mapping $D(.,.): R \times R \to R$ is called a symmetric Jordan bi derivation if $D(x^2, y) = D(x, y)x + xD(x, y)$, for all $x, y \in R$. Obviously, in this case also the relation $D(x, y^2) = D(x, y)y + yD(x, y)$, for all $x, y \in R$. Let R be a semiprime ring, the two derivations d and g are called orthogonal if $d(x)Rg(y) = 0 =$

$g(y)Rd(x)$, for all $x, y \in R$ [3]. Let R be a semiprime ring, the two symmetric bi derivations B and D are called orthogonal if $B(x, y)RD(y, z) = (0) = D(y, z)RB(x, y)$, for all $x, y, z \in R$.

Lemma 1

[3] Let R be a 2-torsion free semiprime ring and $a, b \in R$, then the following conditions are equivalent.

(i) $axb = 0$, for all $x \in R$.

(ii) $bxa = 0$, for all $x \in R$.

(iii) $axb + bxa = 0$, for all $x \in R$.

If one of the above conditions is fulfilled then $ab = ba = 0$, too.

Lemma 2

Let R be a semiprime ring. Suppose that two bi additive mappings $B: R \times R \rightarrow R$ and $D: R \times R \rightarrow R$ satisfies $B(x, y)RD(x, y) = 0$, for all $x, y \in R$. Then $B(x, y)RD(y, z) = 0$, for all $x, y, z \in R$.

Proof:

We have $B(x, y)RD(x, y) = 0$, for all $x, y \in R$. (1)

We replace y by $y + z$ in equation (1), we get

$B(x, y + z)RD(x, y + z) = 0$.

$(B(x, y) + B(x, z))R(D(x, y) + D(x, z)) = 0$.

$B(x, y)RD(x, y) + B(x, y)RD(x, z) + B(x, z)RD(x, y) + B(x, z)RD(x, z) = 0$.

By using equation (1), we get

$B(x, y)RD(x, z) + B(x, z)RD(x, y) = 0$.

$B(x, y)rD(x, z) = -B(x, z)rD(x, y)$, for all $x, y, z, r \in R$. (2)

$B(x, y)rD(x, z)RB(x, y)rD(x, z) = -B(x, z)rD(x, y)RB(x, y)rD(x, z)$

By using (1) in the above equation, we get

$B(x, y)rD(x, z)RB(x, y)rD(x, z) = 0$, for all $x, y, z, r \in R$.

Since R is semiprime, we have

$B(x, y)rD(x, z) = 0$.

Therefore $B(x, y)RD(x, z) = 0$, for all $x, y, z, r \in R$.

Lemma 3

Let R be a 2-torsion free semiprime ring. Two bi derivations B and D are orthogonal if and only if $B(x,y)D(y,z) + D(x,y)B(y,z) = 0$, for all $x,y,z \in R$.

Proof

Suppose $B(x,y)D(y,z) + D(x,y)B(y,z) = 0$, for all $x,y,z \in R$. (3)

We replace z by zx in equation (3), we get

$B(x,y)D(y,zx) + D(x,y)B(y,zx) = 0$

$B(x,y)zD(y,x) + B(x,y)D(y,z)x + D(x,y)zB(y,x) + D(x,y)B(y,z)x = 0$

$B(x,y)zD(y,x) + D(x,y)zB(y,x) + \big(B(x,y)D(y,z) + D(x,y)B(y,z)\big)x = 0$

By using equation (3) in the above equation, we get

$B(x,y)zD(y,x) + D(x,y)zB(y,x) = 0.$

By using lemma (1), we get

$B(x,y)RD(y,x) = 0$, for all $x,y \in R$.

Again by using lemma (2), we get

$B(x,y)RD(y,z) = 0$, for all $x,y \in R$.

Therefore B and D are orthogonal.

Conversely, suppose B and D are orthogonal

$B(x,y)RD(y,z) = 0$ also $D(x,y)RB(y,z) = 0$, for all $x,y,z \in R$.

Since $1 \in R$, $B(x,y)D(y,z) = 0$ also $D(x,y)B(y,z) = 0$, for all $x,y,z \in R$.

$B(x,y)D(y,z) + D(x,y)B(y,z) = 0$, for all $x,y,z \in R$.

Lemma 4

Let R be a 2-torsion free semi-prime ring and let $B: R \times R \to R$ be a Jordan bi derivation. Then B is a bi derivation.

Proposition 1

Let B and D are two bi derivations of a ring R. The following identity holds, for all $x,y,z,m \in R$.

$BD(xy,z) = B(D(xy,z),m)$

$= B(xD(y,z) + D(x,z)y, m)$

$= B(xD(y,z),m) + B(D(x,z)y,m)$

$= xB(D(y,z),m) + B(x,m)D(y,z) + D(x,z)B(y,m) + B(D(x,z),m)y$

$= xBD(y,z) + B(x,m)D(y,z) + D(x,z)B(y,m) + BD(x,z)y.$

Theorem 1

Let R be a 2-torsion free semiprime ring. Two bi derivations B and D are orthogonal if and only if $BD = 0$.

Proof

Let B and D are bi derivations such that $BD = 0$.

According to proposition 1, we have

$B(x,m)D(y,z) + D(x,z)B(y,m) = 0.$

In particular $m = z$ in the above equation, we get

$B(x,z)D(y,z) + D(x,z)B(y,z) = 0$

$B(x,z)D(z,y) + D(x,z)B(z,y) = 0$

By using lemma 3, we get

B and D are orthogonal.

If B and D are orthogonal then $B(x,y)rD(y,z) = 0$, for all$x,y,z,r \in R$.

Hence $B(B(x,y)rD(y,z)) = 0$

$B(B(x,y)rD(y,z),m) = 0$

$B(x,y)rBD(y,z) + B(B(x,y)r,m)D(y,z) = 0$

$B(x,y)rBD(y,z) + B(x,y)B(r,m)D(y,z) + B(B(x,y),m)rD(y,z) = 0$

$B(x,y)rBD(y,z) + B(x,y)B(r)D(y,z) + B(x,y)B(x,y)rD(y,z) = 0$

Therefore $B(x,y)rBD(y,z) = 0$.

In particular $x = D(y,z)$

$B(D(y,z),y)rBD(y,z) = 0$

$BD(y,z)rBD(y,z) = 0$

Since R is semiprime ring we have $BD = 0$.

Theorem 2

Let R be a 2-torsion free semiprime ring. Two bi derivations B and D are orthogonal if and only if $B(x,y)D(y,z) = 0$ or $D(x,y)B(y,z) = 0$.

Proof

We have $\mathrm{B(x,y)D(y,z)} = 0$, for all $x, y, z \in R$. (4)

We replace z by zx in (4), we get

$\mathrm{B(x,y)D(y,zx)} = 0$

$\mathrm{B(x,y)zD(y,x)} + \mathrm{B(x,y)D(y,z)x} = 0.$

By using (4) in the above equation, we get

$\mathrm{B(x,y)zD(y,x)} = 0$

$\mathrm{B(x,y)RD(x,y)} = 0$

By using lemma (2)

$\mathrm{B(x,y)RD(y,z)} = 0$, for all $x, y, z \in R$.

That is B and D are orthogonal.

Conversely suppose Band D are orthogonal

$\mathrm{B(x,y)RD(y,z)} = 0 = \mathrm{D(y,z)RB(x,y)}$

Since $1 \in R$, which implies $\mathrm{B(x,y)D(y,z)} = 0$, for all $x, y, z \in R$.

Similarly D and B are orthogonal if and only if $D(x,y)B(y,z) = 0$.

Theorem 3

Let R be a 2-torsion free semiprime ring. Two bi derivations B and D are orthogonal if and only if BD is a bi derivation.

Proof

Let B and D are two bi derivations and BD is a Bi derivation then

$\mathrm{BD(xy,z)} = \mathrm{xBD(y,z)} + \mathrm{BD(x,z)y}$, for all $x, y, z \in R$ (5)

From proposition (1), we get

$\mathrm{B(x,m)D(y,z)} + \mathrm{D(x,z)B(y,m)} = 0.$

In particular m = z in the above equation, we get

$\mathrm{B(x,z)D(y,z)} + \mathrm{D(x,z)B(y,z)} = 0$

$\mathrm{B(x,z)D(z,y)} + \mathrm{D(x,z)B(z,y)} = 0.$

That is from lemma (3), we get

B and D are orthogonal.

Conversely, let B and D are orthogonal.

Then from lemma (3), implies

$\mathrm{B(x,z)D(z,y) + D(x,z)B(z,y) = 0}$

Suppose $\mathrm{B(x,m)D(y,z) + D(x,z)B(y,m) \neq 0}$

In particular $m = z$, in the above equation, we get

$\mathrm{B(x,z)D(y,z) + D(x,z)B(y,z) \neq 0}$

Since B and D are orthogonal, our supposition is wrong

Hence $\mathrm{B(x,m)D(y,z) + D(x,z)B(y,m) = 0}$.

BD is a bi derivation.

Theorem 4

Let R be a 2-torsion free semiprime ring. Two bi derivations B and D are orthogonal if and only if $B(x,y)D(y,z) + D(y,z)B(x,y) = 0$, for all $x, y \in R$.

Proof

Let B and D satisfy

$B(x,y)D(y,z) + D(y,z)B(x,y) = 0$, for all $x, y \in R$. (6)

In proposition [1], take $y = x$ then we get

$BD(x^2, z) = \mathrm{xBD(x,z) + B(x,m)D(x,z) + D(x,z)B(x,m) + BD(x,z)x}$, for all $x, y, z, m \in R$. (7)

We replace x by m and y by x in (6), we get

$\mathrm{B(m,x)D(x,z) + D(x,z)B(m,x) = 0}$

$\mathrm{B(x,m)D(x,z) + D(x,z)B(x,m) = 0}$, for all $x, y, z, m \in R$. (8)

By using (8) in (7), we get

Therefore, $BD(x^2, z) == \mathrm{xBD(x,z) + BD(x,z)x}$

BD is a Jordan bi derivation

Since every Jordan bi derivation is a bi derivation on 2-torsion free ring.

Then from theorem 3, we have B and D are orthogonal.

Conversely, B and D are orthogonal

$B(x,y)D(y,z) = 0 = D(y,z)B(x,y)$, for all $x, y, z \in R$.

Therefore $B(x,y)D(y,z) + D(y,z)B(x,y) = 0$, for all $x, y, z \in R$.

Theorem 5

Let R be a 2-torsion free semiprime ring. Two bi derivations B and D are orthogonal if there exists a fixed element $a \in R$ such that $(BD)(x,y) = xay + yax$, for all $x, y \in R$.

Proof

Let a be any fixed element of R and B, D are two bi derivations such that

$$BD(x,y) = xay + yax, for\ all\ x, y \in R \quad (9)$$

By proposition 1, we have

$BD(xy,z) = \mathrm{xB(D(y,z),m) + B(x,m)D(y,z) + D(x,z)B(y,m) + B(D(x,z),m)y}$, for all $x, y, z, m \in R$. (10)

$= x\mathrm{BD(y,z) + B(x,m)D(y,z) + D(x,z)B(y,m) + BD(x,z)y}$

By using equation (9) in (10)

$$xyaz + zaxy = xyaz + xzay + \mathrm{B(x,m)D(y,z) + D(x,z)B(y,m)} + xazy + zaxy$$

$$(xza + xaz)y + \mathrm{B(x,m)D(y,z) + D(x,z)B(y,m)} = 0, \text{ for all } x, y, z, m \in R. \quad (11)$$

Replace y by yx in (11), we get

$$(xza + xaz)yx + \mathrm{B(x,m)D(yx,z) + D(x,z)B(yx,m)} = 0$$

$$(xza + xaz)yx + \mathrm{B(x,m)yD(x,z)} + \mathrm{B(x,m)D(y,z)x + D(x,z)B(y,m)x + D(x,z)yB(x,m)} = 0$$

$$((xza + xaz)y + \mathrm{B(x,m)D(y,z) + D(x,z)B(y,m))x} + \mathrm{B(x,m)yD(x,z) + D(x,z)yB(x,m)} = 0$$

$\mathrm{B(x,m)yD(x,z) + D(x,z)yB(x,m)} = 0$, for all $x, y, z, m \in R$.

By using lemma (1), we get

$\mathrm{B(x,m)RD(x,z)} = (0)$ or $\mathrm{D(x,z)RB(x,m)} = (0)$

B and D are orthogonal.

Theorem 6

Let R be a 2-torsion free semiprime ring. Two bi derivations B and D are orthogonal if and only if the following conditions are equivalent:

(1) $BD = 0$.

(2) $B(x,y)D(y,z) = 0\ or\ D(x,y)B(y,z) = 0$.

(3) BD is a bi derivation.

(4) $B(x,y)D(y,z)+D(y,z)B(x,y)=0$,for every $x,y,z\in R$.

Proof

It follows easily from theorems (1), (2), (3), (4) these are equivalent.

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