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OPTIMAL WATER TREATMENT COST MODEL FOR GHANA

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ABSTRACT: *In this paper, an optimal water treatment cost model has been developed for Ghana. Linear Programming (LP) was used to formulate the model for Ghana Water Company Limited. It was then tested with real data collected from Weija Water Headworks in Accra using Interior-Point Method. Finally, optimal water treatment cost for Weija Water Headworks in 2014 was found. It is strongly recommended that all Water Headworks under Ghana Water Company Limited (GWCL) should adopt the proposed optimal water treatment cost model so as to save cost. They should also employ at least one Operations Researcher to assist them in their activities.*

KEYWORDS: Linear Programming, Optimal Water Treatment Cost Model, Optimal Water Treatment Cost, Interior-Point Method, Ghana Water Company Limited

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INTRODUCTION

Water is one of the most important natural resources, and can provide support to human survival, sustainable socio-economic development, and ecosystem preservation. Water is essential for all life and human activities and access to freshwater in sufficient amounts and of suitable quality is a precondition to achieving sustainable development. Standard water treatment includes coagulation/flocculation, sedimentation, filtration, and disinfection (Downie, 2005). Ghana Water Company Limited (GWCL) which was formed out of Ghana Water and Sewerage Corporation (GWSC) in 1999 is responsible for providing potable water for urban consumption. The supply of safe potable drinking water in Ghana is characterized by seasonal and persistent shortages. Such shortages are widespread, often as a result of poor management of water resources, irregular rainfall patterns, prolonged drought and inefficient use of available technology (Kumasi et al., 2007). Therefore, effective planning for water quality management is desired. In water quality management, the treatment cost may be as important as the achievement of water quality goals. Water treatment at most GWCL Headworks has been associated with very high cost over the years. The study therefore seeks to develop an optimal water treatment cost model for the company to meet water quality standards while saving cost in order to perhaps reduce tariffs and also make potable water accessible to a larger section of the population.

LITERATURE

A lot of researchers around the world have developed a lot of water optimisation models among which are the following.

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Dorfman (1962) demonstrated the application of LP to a reservoir problem, in which the objective was to maximize the economic benefits of water use. Rossman and Liebman (1974) developed a dynamic programming model with water quality constraints. The model was designed for essentially linear systems and was applied to estuary of Delaware River, USA. Frizzone et al (1997) used a linear programming model to optimize the water resource use in irrigation projects. Sawyer and Lin (1998) presented 'Mixed-integer chance-constrained models for ground-water remediation'. Li et al (2004) developed a hybrid ant colony genetic algorithm model for groundwater long term monitoring to maximize sampling cost-effectiveness. Cho et al (2004) proposed 'A river water quality management model for optimising regional wastewater treatment using a genetic algorithm'. Mousavi et al (2004) applied an interior-point algorithm for optimization of a large-scale reservoir system. They presented a long-term planning model for optimizing the operation of Iranian Karoon-Dez reservoir system using interior-point algorithm. The developed model was applied to Youngsan River in Korea where water quality had decreased due to heavy pollutant loads from Kwangju City and surrounding areas. Karmakar and Mujumdar (2006) presented 'Grey fuzzy optimization model for water quality management of a river system'. Application of the model was illustrated with a case study of the Tunga-Bhadra river system in India. Kumar and Reddy (2006) proposed an Ant Colony Optimisation Algorithm for a multipurpose reservoir system. Qin et al (2007) presented 'An interval-parameter fuzzy nonlinear optimization model for stream water quality management under uncertainty'. A case study for water quality management planning in the Changsha section of the Xiangjiang River was then conducted for demonstrating applicability of the developed model. Lima and Oliveira (2007) presented 'Interior point methods specialized to optimal pump operation costs of water distribution networks'. They solved the water distribution problem using interior point methods and exploited the particular structure of the problem and the specific matrix sparse pattern of the resulting linear systems. Higgins et al (2008) presented 'A stochastic non-linear programming model for a multi-period water resource allocation with multiple objectives'. The model was applied to a case study in South East Queensland in Australia, a region which was facing a severe water shortage. The developed model was applied to Senator Nilo Coelho Project in Brazil. Cheng et al (2009) presented 'An application of a linear programming model to the conjunctive use of surface water and groundwater for optimal water allocation in Taiwan'. Xie et al (2011), presented an inexact chance - constrained programming model for water quality management in Binhai New Area of Tianjin, China. Han et al (2011) presented 'A Multi-objective Linear Programming Model with Interval Parameters for Water Resources Allocation in Dalian City'. Godoy et al (2012) presented 'Multi-Objective Optimisation Method for Water Resources Management: A Multi-Reservoir System Case Study'. The optimisation-simulation model was applied to Wimmera-Mallee Water Supply System which is a complex multi-reservoir system located in Western Victoria (Australia). Kamil and Willis (2013) developed a general optimal control model for the control of saltwater intrusion in coastal aquifer systems. Liu et al (2013) developed an interval-parameter fuzzy robust nonlinear programming model for water quality management. Kurek and Ostfeld (2013) presented 'Multi-objective optimization of water quality, pumps operation and storage sizing of water distribution systems'. A multi-objective methodology utilizing the Strength Pareto Evolutionary Algorithm linked to EPANET for trading-off pumping costs, water quality, and tanks sizing of water distribution systems was developed and demonstrated. The multi-objective model integrated variable speed pumps for modelling the pumps operation, two water quality objectives (one based on chlorine disinfectant concentrations and one on water age), and tanks sizing cost which were assumed to vary with location and diameter. Sweetapple et al (2014) worked on 'Multi-objective optimisation of wastewater treatment plant control to reduce greenhouse gas

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emissions'. They minimised greenhouse gas emissions, operational costs and effluent pollutant concentrations, subject to legislative compliance. It was found that multi-objective optimisation could facilitate a significant reduction in greenhouse gas emissions without the need for plant redesign or modification of the control strategy layout.

METHODOLOGY

Study Area

The study area of the study was Weija Water Headworks in Accra, Ghana. Table 1 gives the technical specifications of Weija Water Headworks.

Table 1: Technical Specifications of Weija Water Headworks

PARAMETER	DESCRIPTION
Region	Greater Accra
Water Source	River Densu
Operator	GWCL
Treatment Plants	Adam Clarke Plant, Bamag Plant and Candy Plant.
Service Areas	Accra Metropolitan Area, Tema Metropolitan Area
Design Capacity	Adam Clark Plant (34.5 million gallons per day), Bamag Plant (5.5 million gallons per day) and Candy Plant (8.75 million gallons per day)
Unit Processes and Systems	Coagulation, Mixing Chamber, Clarifiers, Sand Filtration, Disinfection, pH Adjustment

Source: Weija Water Headworks, 2015

Model Formulation

Linear Programming (LP) was used to formulate the model for Ghana Water Company Limited. It was then tested with real data collected from Weija Water Headworks in Accra using Interior-Point Method.

Linear programming (LP) also called linear optimization is a technique for the optimization of a linear objective function, subject to linear equality or inequality constraints. The objective function may either be maximized or minimized. There are four main assumptions inherent in a LP model that must be taken into account in any application. They are proportionality, additivity, divisibility, and certainty (Hillier and Lieberman, 2000).

Generally Interior-Point Method searches for an optimal solution of a problem by traversing the interior or inside of the feasible region instead of the boundaries as in Simplex Method. The interested reader is referred to Hillier and Lieberman, 2010 pp. 287-298 for a detailed discussion of Interior-Point Method

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The parameters of the water treatment cost model are as follows:

C_T = Total Treatment Cost
C_C = Chemical Cost
E_C = Electricity Cost
F_C = Fuel Cost
P_C = Personnel Cost
M_C = Maintenance Cost
C_{CA} = Average Chemical Cost
M = Average Maintenance Cost
E_{CA} = Average Electricity Cost
F_{CA} = Average Fuel Cost
N = Average Personnel Cost
X_1 = Average Quantity of Chemicals
X_2 = Average Quantity of Electricity
X_3 = Average Quantity of Fuel
X_4 = Number of Personnel
X_5 = Number of Maintenances in a month
α = Unit Chemical Cost
β = Unit Electricity Cost
λ = Unit Personnel Cost
δ = Unit Maintenance Cost
γ = Unit Fuel Cost
μ = Unit Fuel Cost in the Chemical House
ρ = Unit Fuel Cost in the Pumping House
ŋ = Unit Fuel Cost for Transportation
c = Average Fuel Cost in the Chemical House
p = Average Fuel Cost in the Pumping House
t = Average Fuel Cost for Transportation
Ψ = Average Cost in the Chemical House
ϕ = Average Cost in the Pumping House
τ = Average Transportation Cost.

The meanings of the following very important parameters of the model must be well noted:

- $\textit{Unit Chemical Cost, } \alpha = \frac{Average\ Chemical\ Cost}{Average\ Quantity\ of\ Chemicals}$
- $\textit{Unit Electricity Cost, } \beta = \frac{Average\ Electricity\ Cost}{Average\ Quantity\ of\ Electricity}$
- $\textit{Unit Fuel Cost, } \gamma = \frac{Average\ Fuel\ Cost}{Average\ Quantity\ of\ Fuel}$
- $\textit{Unit Personnel Cost, } \lambda = \frac{Average\ Personnel\ Cost}{Number\ of\ Personnel}$
- $\textit{Unit Maintenance Cost, } \delta = \frac{Average\ Maintenance\ Cost}{Number\ of\ Maintenances\ in\ a\ month}$
- $\textit{Unit Fuel Cost in the Chemical House, } \mu = \frac{Average\ Fuel\ Cost\ in\ the\ Chemical\ House}{Average\ Quantity\ of\ Fuel}$
- $\textit{Unit Fuel Cost in the Pumping House, } \rho = \frac{Average\ Fuel\ Cost\ in\ the\ Pumping\ House}{Average\ Quantity\ of\ Fuel}$

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- *Unit Fuel Cost for Transportation*, $\eta = \frac{Average\ Fuel\ Cost\ for\ Transportation}{Average\ Quantity\ of\ Fuel}$

Decision Variables of the Model

The Decision Variables of the model are Average Quantity of Chemicals (X_1), Average Quantity of Electricity (X_2), Average Quantity of Fuel (X_3), Number of Personnel (X_4) and Number of Maintenances in a month (X_5).

Objective Function of the Model

The Chemical Cost (C_C), Electricity Cost (E_C), Fuel Cost (F_C), Personnel Cost (P_C) and Maintenance Cost (M_C) which mostly influence water treatment cost are used to formulate the objective function. Thus,

Total Treatment Cost, $C_T = f\,(C_C, E_C, F_C, P_C, M_C)$

Since we want to minimize water treatment cost, the objective function is then given as:

Minimize $C_T = \alpha\, X_1 + \beta X_2 + \gamma\, X_3 + \lambda\, X_4 + \delta\, X_5$

Constraints of the Model

- Chemical House

Product of the Unit Chemical Cost and the Average Quantity of Chemicals used + Product of the Unit Fuel Cost in the Chemical House and the Average Quantity of Fuel used ≥ Average Cost in the Chemical House.

- Pumping House

Product of the Unit Electricity Cost and the Average Quantity of Electricity used +

Product of the Unit Fuel Cost in the Pumping House and the Average Quantity of Fuel used ≥ Average Cost in the Pumping House.

- Transportation

Product of the Unit Fuel Cost and the Average Quantity of Fuel used ≥ Average Transportation Cost.

- Personnel

Product of the Unit Personnel Cost and the Number of Personnel ≥ Average Personnel Cost.

- Maintenance

Product of the Unit Maintenance Cost and the Number of Maintenances in a month ≥ Average Maintenance Cost.

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RESULTS

The Developed Model

The developed water treatment cost model is therefore given as:

$$Minimize\ C_T = \alpha X_1 + \beta X_2 + \gamma X_3 + \lambda X_4 + \delta X_5$$

$$Subject\ to$$

$$\begin{gathered} \alpha X_1 + \mu X_3 \geq \Psi \\ \beta X_2 + \rho X_3 \geq \phi \\ \gamma X_3 \geq \tau \\ \lambda X_4 \geq N \\ \delta X_5 \geq M \\ X_1, X_2, X_3, X_4, X_5 \geq 0 \end{gathered} \qquad [1]$$

Practical Application of the Model

The developed water treatment cost model has been applied to Weija Water Headworks in Accra.

Secondary data (water treatment/production data for 2014) collected from Weija Water Headworks are given in the following tables:

Table 2: Quantities Of Chemicals, Electricity And Fuel For Water Production At Weija Water Headworks In 2014.

MONTH	CHEMICALS (Kg)	ELECTRICITY (KWh)	FUEL (Litres)
January	676390	2830838	1952.03
February	598725	2176999	2041.08
March	721710	2933564	2122.00
April	595975	2662421	2197.60
May	776845	2878228	2124.00
June	678925	2572475	2416.68
July	787275	2988606	2376.12
August	731695	2820508	2365.12
September	725530	2670512	2099.43
October	647515	2749017	2090.44
November	690345	2678049	2017.46
December	669420	2669726	2080.39
TOTAL	**8300350**	**32630943**	**25882.35**
AVERAGE	**691695.83**	**2719245.25**	**2156.86**

Source: Weija Water Headworks, 2015.

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Table 3: costs of chemicals, electricity, fuel, personnel and maintenance for water production at weija water headworks in 2014.

MONTH	CHEMICAL COST (GH₵)	ELECTRICITY COST (GH₵)	FUEL COST (GH₵)	PERSONNEL COST (GH₵)	MAINTENANCE COST (GH₵)	TOTAL COST (GH₵)
January	550154.30	1437812.58	4555.65	142815.66	13297.90	2148636.09
February	482400.45	1159542.47	4906.92	138948.17	9563.00	1795361.01
March	585473.90	1515848.00	5582.35	162890.08	17637.79	2287432.12
April	592573.00	1340012.36	6077.63	162861.16	19836.34	2121360.49
May	770491.33	1487451.40	5938.20	164424.94	7227.00	2435532.87
June	679833.43	1342699.26	8333.35	164795.29	37689.61	2233350.94
July	779536.98	1726131.58	6544.90	167046.38	25490.97	2704750.81
August	736698.30	1637373.26	7785.79	157902.79	30386.30	2570146.44
September	933843.90	1561136.88	7011.61	170776.23	13909.52	2686678.14
October	836497.95	1706424.45	6835.74	171417.74	13414.77	2734590.65
November	922190.85	1670468.16	6657.61	172909.84	14213.87	2786440.33
December	887812.35	1664566.20	7433.64	169732.44	7008.00	2736552.63
TOTAL	**8757506.74**	**18249466.60**	**77663.39**	**1946520.72**	**209675.07**	**29240832.52**
Average	**729792.23**	**1520788.88**	**6471.95**	**162210.06**	**17472.92**	**2436736.04**

Source: Weija Water Headworks, 2015.

Table 4: Average Cost Allocation Of Chemicals, Electricity And Fuel At Weija Water Headworks In 2014.

	Chemical House	**Pumping House**	**Transportation**	**TOTAL COST**
Chemical Cost	729792.23	0	0	729792.23
Electricity Cost	0	1520788.88	0	1520788.88
Fuel Cost	32.64	1399.15	5040.16	6471.95
TOTAL COST	729824.87	1522188.03	5040.16	2257053.06

Source: Weija Water Headworks, 2015.

The following important information obtained from Weija Water Headworks must also be well noted:

- There were sixty (60) Personnel working in the Headworks in 2014.
- Four (4) maintenance works were done every month in 2014.

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Now, from Tables 2, 3 and 4, the following calculations were done.

- Unit Chemical Cost, $\alpha = \frac{\text{Average Chemical Cost}}{\text{Average Quantity of Chemicals}} = \frac{729792.23}{691695.83} = 1.06$
- Unit Electricity Cost, $\beta = \frac{\text{Average Electricity Cost}}{\text{Average Quantity of Electricity}} = \frac{1520788.88}{2719245.25} = 0.56$
- Unit Fuel Cost, $\gamma = \frac{\text{Average Fuel Cost}}{\text{Average Quantity of Fuel}} = \frac{6471.95}{2156.86} = 3.00$
- Unit Personnel Cost, $\lambda = \frac{\text{Average Personnel Cost}}{\text{Number of Personnel}} = \frac{162210.06}{60} = 2703.5$
- Unit Maintenance Cost, $\delta = \frac{\text{Average Maintenance Cost}}{\text{Number of Maintenances in a month}}$

  $$= \frac{17472.92}{4} = 4368.23$$
- Unit Fuel Cost in the Chemical House, $\mu = \frac{\text{Average Fuel Cost in the Chemical House}}{\text{Average Quantity of Fuel}}$

  $$= \frac{32.64}{2156.86} = 0.015$$
- Unit Fuel Cost in the Pumping House, $\rho = \frac{\text{Average Fuel Cost in the Pumping House}}{\text{Average Quantity of Fuel}}$

  $$= \frac{1399.15}{2156.86} = 0.649$$
- Unit Fuel Cost for Transportation, $\eta = \frac{\text{Average Fuel Cost for Transportation}}{\text{Average Quantity of Fuel}}$

  $$= \frac{5040.16}{2156.86} = 2.34$$
- Ψ = Average Cost in the Chemical House = 729824.87
- ϕ = Average Cost in the Pumping House = 1522188.03
- τ = Average Transportation Cost = 5040.16
- N = Average Personnel Cost = 162210.06
- M = Average Maintenance Cost = 17472.92

Substituting these values into the developed water treatment cost model [1], Water Treatment Cost Model for Weija Water Headworks based on the collected data on water treatment/ production for 2014 is given as follows:

$$\textit{Minimize } C_T = 1.06\, X_1 + 0.56\, X_2 + 3\, X_3 + 2703.5\, X_4 + 4368.23\, X_5$$

Subject to

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$$1.06\, X_1 + 0.015\, X_3 \geq 729824.87$$

$$0.56\, X_2 + 0.649\, X_3 \geq 1522188.03$$

$$3\, X_3 \geq 5040.16 \qquad [2]$$

$$2703.5\, X_4 \geq 162210.06$$

$$4368.23\, X_5 \geq 17472.92$$

$$X_1, X_2, X_3, X_4, X_5 \geq 0$$

Table 5 below gives a detailed and optimal (asterisked) solution of this model [2] using Interior-Point Method.

Table 5: Optimal Solution For Water Production At Weija Water Headworks In 2014 (Using Interior-Point Method).

ITERATION	X_1	X_2	X_3	X_4	X_5	C_T
0	688495.000	2716250.000	1685.000	60.000	4.000	2435642.680
1	688493.969	2716250.495	1682.527	60.000	4.000	2435634.444
2	688492.693	2716249.187	1681.290	60.000	4.000	2435628.649
3	688491.590	2716247.689	1680.672	60.000	4.000	2435624.787
4	688490.926	2716246.772	1680.362	60.000	4.000	2435622.641
5	688490.588	2716246.306	1680.208	60.000	4.000	2435621.558
6	688490.418	2716246.074	1680.131	60.000	4.000	2435621.017
7	688490.334	2716245.958	1680.092	60.000	4.000	2435620.746
8	688490.291	2716245.900	1680.073	60.000	4.000	2435620.610
9	688490.269	2716245.874	1680.063	60.000	4.000	2435620.544
10	688490.258	2716245.861	1680.058	60.000	4.000	2435620.510
11	688490.251	2716245.864	1680.056	60.000	4.000	2435620.497
12	688490.241	2716245.866	1680.055	60.000	4.000	2435620.484
13	688490.241	2716245.867	1680.054	60.000	4.000	2435620.483
14	688490.241	2716245.542	1680.054	60.000	4.000	2435620.300
15*	688490.201*	2716245.543*	1680.053*	60.000*	4.000*	2435620.258*

DISCUSSION

Table 6 gives a detailed comparison of the optimal and original (old) values of the decision variables in relation to the developed model for Weija Water Headworks.

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Table 6: Comparison Of The Optimal And Original (Old) Values Of The Decision Variables Of The Model For Weija Water Headworks.

DECISION VARIABLE	OPTIMAL VALUE	ORIGINAL VALUE	DIFFERENCE
X_1	688490.201	691695.830	3205.629 Kg
X_2	2716245.543	2719245.250	2999.707 KWh
X_3	1680.053	2156.860	476.807 Litres
X_4	60.000	60.000	0.000
X_5	4.000	4.000	0.000
$\mathbf{C_T}$	**2435620.258**	**2436736.040**	**GH₵ 1115.782**

It is very conspicuous from Table 6 that, Weija Water Headworks would have saved an amount of GH₵ 1115.782 monthly and a total amount of GH₵ 13389.384 in 2014 if this optimal water treatment cost model was available.

CONCLUSION

An optimal water treatment cost model has been developed for Ghana. Linear Programming (LP) was used to formulate the model for Ghana Water Company Limited. It was then tested with real data collected from Weija Water Headworks in Accra using Interior-Point Method. Finally, an optimal water treatment cost for Weija Water Headworks in 2014 was found to be GH₵ 2435620.258. Weija Water Headworks would have saved an amount of GH₵ 1115.782 monthly and a total amount of GH₵ 13389.384 in 2014 if this optimal water treatment cost model was available. It is strongly recommended that all Water Headworks under Ghana Water Company Limited (GWCL) should adopt the proposed optimal water treatment cost model so as to save cost in order to perhaps reduce tariffs and also help make potable water accessible to a larger section of the population. They should also employ at least one Operations Researcher to assist them in their activities.

REFERENCES

Cheng, Y., Lee, C. H., Tan, Y. C. and Yeh, H. F. (2009) An optimal water allocation for an irrigation district in Pingtung County, Taiwan. Irrigation and Drainage, 58 (3), 287-306.

Cho, J., Sung, K. S. and Ha, S. R. (2004) A river water quality management model for optimising regional wastewater treatment using a genetic algorithm, Journal of Environmental Management, 73, 229-242.

Dorfman, R. (1962) "Mathematical models: The multi-structure approach, in design of water resources systems." Maass, A. (Ed.), Harward University Press, Cambridge, Mass.

Downie, A. J. (2005) Drinking Water Source Quality Monitoring, USEPA. Drinking Water and Disease. Washington DC. www.cleanwateraction.org.

Frizzone, J. A., Coelho, R. D., Dourado-Neto, D. and Soliani, R. (1997) Linear programming model to optimize the water resource use in irrigation projects: an application to the Senator Nilo Coelho Project, Scientia Agricola, 54.

ISSN 2053-2229 (Print), ISSN 2053-2210 (Online)

Godoy, W., Barton, A. and Perera, B. (2012) Multi-objective Optimisation Method for Water Resources Management: A Multi-Reservoir System Case Study, Hydrology and Water Resources Symposium.

Han, Y., Huang, Y. F., Wang, G. Q. and Maqsood, I. (2011) A multi-objective linear programming model with interval parameters for water resources allocation in Dalian City, Water Resources Management, 25 (2), 449-463.

Higgins, A., Archer, A. and Hajkowicz, S. (2008) A stochastic non-linear programming model for a multi-period water resource allocation with multiple objectives, Water Resources Management, 22 (10), 1445-1460.

Hillier, F. S. and Lieberman, G. J. (2000) Introduction to Operations Research, Seventh Edition, 61-69.

Hillier, F. S. and Lieberman, G. J. (2010) Introduction to Operations Research, Ninth Edition, 287-298.

Karmakar, S. and Mujumdar, P. P. (2006) Grey fuzzy optimization model for water quality management of a river system, Advances in Water, 29 (7), 1088 – 1105.

Kamil, I. M. and Willis, R. (2013) Optimal model for the control of saltwater intrusion, J. Math. Fund. Sci., 45 (2), 124 – 143.

Kumar D. N. and Reddy M. J. (2006) Ant colony optimization for multi-purpose reservoir operation, Water Resources Management, 20, 879–898.

Kumasi, T. C., Obiri-Danso, K. and Ephraim, J. H. (2007) Impacts of land-use change on the water quality of the main source of pipe borne water for Kumasi, Ghana (A case study of the Barekese reservoir catchment), 243–248.

Kurek, W. and Ostfeld, A. (2013) Multi-objective optimization of water quality, pumps operation, and storage sizing of water distribution systems, Journal of Environmental Management, 115, 189 – 197.

Li Y., Chan Hilton A. B., and Tong L. (2004), Development of ant colony optimization for long-term groundwater monitoring, Proceedings of the 2004 World Water and Environmental Resources Congress, Salt Lake City, UT, 1-10.

Liu, M., Nie, G., Hu, M., Liao, R. and Shen, Y. (2013). An Interval-parameter Fuzzy Robust Nonlinear Programming Model for Water Quality Management. *Journal of Water Resource and Protection*, 5, pp 12 – 16.

Lima, A and Oliveira A. (2007) Interior Point Pethods specialized to optimal pump operation costs of water distribution networks, PAMM. Proc. Appl. Math. Mech. 7, 2060021-2060022.

Mousavi, S. J., Moghaddam, K. S. and Seifi, A. (2004) Application of an Interior-Point Algorithm for optimization of a large-scale reservoir system, Water Resources Management, 18 (6), 519-540.

Qin, X. S., Huang, G. H., Zeng, G. M., Chakma, A. and Huang, Y. F. (2007) An interval-parameter fuzzy nonlinear optimization model for stream water quality management under uncertainty, European Journal of Operational Research, 180, 1331 – 1357.

Rossman, L. A. and Liebman, J. C. (1974) Optimal Regionalization of Wastewater Treatment for Water Quality Management. Research Report 89, Water Resources Center, University of Illinois, Urbana, Illinois.

Sawyer, C. and Lin, Y. (1998) Mixed-integer chance-constrained models for ground-water remediation, Journal of Water Resources Planning and Management, 124, 285-294.

Sweetapple, C., Fu, G. and Butler, D (2014) Multi-objective optimisation of wastewater treatment plant control to reduce greenhouse gas emissions, Water Research, 55, 52 - 62.

Xie, Y. L., Li, Y. P., Huang, G. H., Li, Y. F. and Chen, L. R. (2011), An inexact chance - constrained programming model for water quality management in Binhai New Area of Tianjin, China. Science of the Total Environment, 409, 1757 – 1773.