OPTIMAL PORTFOLIO MIX FOR MULTIGROW INSURANCE COMPANY IN GHANA USING LINEAR PROGRAMMING

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ABSTRACT: In this paper, the concept of Linear Programming (LP) was applied to Multigrow Insurance Company in Ghana which had a portfolio problem. The company had obtained GH₵ 200000 cash but had a difficulty in determining how much to invest in each of five investment areas in order to maximize return. Based on the data collected, the problem was formulated as a Linear Programming Problem and solved using Management Scientist Version 5 Software. Optimal portfolio mix was obtained for the Insurance Company. Finally, the total optimal return on the investments of the company was found to be GH₵ 15980. It is strongly recommended that the Company should adhere to the proposed optimal portfolio mix and also employ at least one operations researcher to assist the Company in its activities.

KEYWORDS: Portfolio, Investment, Linear Programming, Optimal Portfolio Mix, Optimal Return.

INTRODUCTION

In finance, a portfolio is a collection of investments held by an investment company, hedge fund, financial institution or individual (Investopedia, 2011). These investments often include stocks, which are investments in individual businesses; bonds, which are investments in debt that are designed to earn interest; and mutual funds, which are essentially pools of money from many investors that are invested by professionals or according to indices. It is a generally accepted principle that a portfolio is designed according to the investor's risk tolerance, time frame and investment objectives. The monetary value of each asset may influence the risk/reward ratio of the portfolio and is referred to as the asset allocation of the portfolio (Investopedia, 2011). When determining a proper asset allocation one aims at maximizing the expected return.

Multigrow Insurance Company which was established in 2007 and whose main office is in Adum-Kumasi (Ghana) had a portfolio problem. The company had obtained GH₵ 200000 cash but had a difficulty in determining how much to invest in each of five investment areas in order to maximize return. The objective of the study was to find an optimal portfolio mix for the company so as to maximize return.

METHODOLOGY

The concept of Linear Programming was applied to Multigrow Insurance Company in Ghana which had a difficulty in determining how much to invest in each of five investment areas in order to maximize return on GH₵ 200000.00 cash. Linear programming (LP), also called linear optimization, is a method used to achieve the best outcome for an objective (such as maximum profit or minimum cost) in a mathematical model whose requirements are represented by linear relationships. More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality or inequality constraints. The general form of the LP model is stated as:

Optimize \( f(\mathbf{x}) = \sum_{j=1}^{n} c_j x_j \)

Subject to:
\[
\begin{align*}
\sum_{j=1}^{n} a_{ij} x_j & \leq b_i & 1 \leq i \leq p \\
\sum_{j=1}^{n} a_{ij} x_j & = b_i & p + 1 \leq i \leq k \\
\sum_{j=1}^{n} a_{ij} x_j & \geq b_i & k + 1 \leq i \leq m
\end{align*}
\]
\[ x_j \geq 0 \quad 1 \leq j \leq n \]  

where \( f(x) \) is the objective function, \( x_j \) is the \( j^{th} \) decision variable, \( c_j \) is the \( j^{th} \) cost coefficient, \( a_{ij} \) is the \( j^{th} \) technological coefficient in the \( i^{th} \) constraint, \( b_i \) is the \( i^{th} \) right-hand-side parameter (resource availability) and \( p, k, m, \) and \( n \) are integers. The general form LP [1] can be transformed into the standard form as:

Optimize \( f(x) = \sum_{j=1}^{n} c_j x_j \)

Subject to \( \sum_{j=1}^{n} a_{ij} x_j = b_i \) \quad 1 \leq i \leq m

\[ x_j \geq 0 \quad 1 \leq j \leq n \]  

The standard form LP [2] is obtained by adding to or subtracting from each inequality constraint slack or surplus variables. A slack variable is a non-negative variable which when added to the left-hand-side (LHS) of a less-than-or-equal-to constraint transforms it into an equality constraint. A surplus variable on the other hand transforms a greater-than-or-equal-to constraint into an equality constraint. The standard form LP is necessary for the application of solution algorithms, since the algorithms work only with equality conditions (Williams, 2013). The objective function may either be maximized or minimized. There are four main assumptions inherent in a LP model that must be taken into account in any application. They are proportionality, additivity, divisibility, and certainty (Hillier and Lieberman, 2000).

Secondary data (The Company’s projected annual rates of return) was collected from the Manager of the Company as shown in Table 1

### Table 1:

<table>
<thead>
<tr>
<th>INVESTMENT</th>
<th>RATE OF RETURN (in percentage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile Oil</td>
<td>10.5</td>
</tr>
<tr>
<td>Shell Oil</td>
<td>7.2</td>
</tr>
<tr>
<td>Anglo Gold</td>
<td>7.4</td>
</tr>
<tr>
<td>Tarkwa Gold</td>
<td>6.4</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>4.4</td>
</tr>
</tbody>
</table>


The company had imposed the following investment guidelines:

1. None of the industries should receive more than 50% of the total investment
2. Government bonds should be at least 25% of the mining industry’s investments
3. The investment in mobile oil though has the highest return, is a high risk one and therefore should not be more than 60% of the total oil industry investment.
RESULTS

Problem Formulation

Based on the data collected, the problem was formulated as a Linear Programming Problem as follows:

Let

\[ x_1 = \text{amount of cedis to be invested in Mobile Oil} \]
\[ x_2 = \text{amount of cedis to be invested in Shell Oil} \]
\[ x_3 = \text{amount of cedis to be invested in Anglo Gold} \]
\[ x_4 = \text{amount of cedis to be invested in Tarkwa Gold} \]
\[ x_5 = \text{amount of cedis to be invested in Government Bonds} \]

Objective function: \[ P = 0.105x_1 + 0.072x_2 + 0.074x_3 + 0.064x_4 + 0.044x_5 \]

Constraint functions:

Available funds: \[ x_1 + x_2 + x_3 + x_4 + x_5 = 200,000 \]

Oil industry investment: \[ x_1 + x_2 \leq 100,000 \]

Mining industry investment: \[ x_3 + x_4 \leq 100,000 \]

Government bond investment:

\[ x_5 \geq 0.25(x_3 + x_4) \text{ OR} \]
\[ -0.25x_3 - 0.25x_4 + x_5 \geq 0 \]

Investment in mobile oil:

\[ x_1 \leq 0.6(x_1 + x_2) \text{ OR} \]
\[ 0.4x_1 - 0.6x_2 \leq 0 \]

The Linear Programming (LP) model is then given as:

Maximize \[ P = 0.105x_1 + 0.072x_2 + 0.074x_3 + 0.064x_4 + 0.044x_5 \]

Subject to:

\[ x_1 + x_2 + x_3 + x_4 + x_5 = 200,000 \]
\[ x_1 + x_2 \leq 100,000 \]
\[ x_3 + x_4 \leq 100,000 \]
\[ -0.25x_3 - 0.25x_4 + x_5 \geq 0 \]
\[ 0.4x_1 - 0.6x_2 \leq 0 \]
\[ x_1, x_2, x_3, x_4, x_5 \geq 0. \]
Optimal Solution

Objective Function Value = 15980.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>60000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$x_2$</td>
<td>40000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$x_3$</td>
<td>80000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$x_4$</td>
<td>0.000</td>
<td>0.010</td>
</tr>
<tr>
<td>$x_5$</td>
<td>20000.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

DISCUSSION

It follows that, the optimal portfolio mix for the insurance company is as follows. Multigrow Insurance Company should invest an amount of GH₵ 60000.00, GH₵ 40000.00, GH₵ 80000.00 and GH₵ 20000.00 in Mobile Oil, Shell Oil, Anglo Gold and Government Bonds respectively. Also, the company should not invest in Tarkwa Gold. Finally, the optimal return on the investments will be GH₵ 15980.00 if the insurance company goes by the proposed optimal portfolio mix.

CONCLUSION

The concept of Linear Programming (LP) was applied to Multigrow Insurance Company in Ghana which had a portfolio problem. The company had obtained GH₵ 200000.00 cash but had a difficulty in determining how much to invest in each of five investment areas in order to maximize return. Based on the data collected, the problem was formulated as a Linear Programming Problem and solved using Management Scientist Version 5 Software. Optimal portfolio mix was obtained for the Insurance Company. The Multigrow Insurance Company should invest an amount of GH₵ 60000.00, GH₵ 40000.00, GH₵ 80000.00 and GH₵ 20000.00 in Mobile Oil, Shell Oil, Anglo Gold and Government Bonds respectively. Also, the company should not invest in Tarkwa Gold. Finally, the total optimal return on the investments of the company was found to be GH₵ 15980. It is strongly recommended that Multigrow Insurance Company should adhere to the proposed optimal portfolio mix and also employ at least one operations researcher to assist the Company in its activities.
REFERENCES