ON THE METRIC DIMENSION OF INNER AND OUTER CYCLES OF GRAPH PN,1,2

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ABSTRACT: *In this paper we have found the metric dimension of Dodecahedral Other Embedding (also called Pn,1,2) for inner cycle and outer cycle graphs. We have proved that metric dimension of Pn,1,2 is bounded And only three vertices chosen appropriately suffice to resolve all the vertices of these graphs for $n = 0 \pmod 4$, $n \geq 16$, $n = 2 \pmod 4$, $n \geq 18$ and $n = 3 \pmod 4$, $n \geq 11$ for inner cycle and outer cycle graphs respectively and only four vertices chosen appropriately suffice to resolve all the vertices of these graphs for $n = 1 \pmod 4$, $n \geq 17$.*

KEYWORDS: Metric Dimension, Basis, Resolving Set, Dodecahedral Other Embedding Called $P_{n,1,2}$

OPEN PROBLEM

Further it can be proved that the metric dimensions of inner cycle, outer cycle graphs of $P_{n,1,2}$ may be constant.

Notations and preliminary results

Let G (V, E) be a connected graph where V and E represents the vertex and edge Set of G respectively. If $x_1, x_2 \in V(G)$ are the two vertices of connected graph G, if there is an edge between x_1 and x_2 then distance of these two vertices i.e $x_1, x_2 \in V(G)$ described as $d(x_1, x_2)$ and it would be the shortest length or smallest x_1-x_2 path in the connected graph G. Let w = $\{w_1, w_2, w_3, \ldots, w_m\}$ be the set of vertices of G which must be an ordered set i.e while $x \in V(G)$. Then $r(x / W)$ will be the representation of x with respect to w and it is called m-tuple and is denoted by $(d(x / w_1), d(x / w_2), \ldots, d(x / w_m))$.

[12- 27] Then " W " is a "Resolving Set " for G, if vertices of G which are distinct have distinct representation with respect to W. [8] A " Basis " for G is actually a set of minimum cardinality and when we take Cardinality of the basis of G then it would be the metric dimension of G written as Dim (G).

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For an order set of vertices $w = \{w_1, w_2, w_3, \ldots, w_m\}$ of a graph G, The i-th component of $r(x/W)$ is 0 if and only if $x = w_i$, Thus to show that W is resolving set it suffices to verify that $r(x_1 / W) \neq r(x_2 / W)$ for each pair of distinct points $x_1, x_2 \in V(G)$

A useful property in finding dim (G) is the following:

Lemma 1: [27]

Let W be the resolving set for a connected graph G and $x_1, x_2 \in V(G)$ if $r(x_1 / W) = r(x_2 / W)$ for all vertices $w \in V(G) \setminus \{x_1, x_2\}$, Then $\{x_1, x_2\} \cap W \neq \varnothing$

[24 - 25] Slater (1975) was the first mathematician who introduces the idea orConcept of metric dimension of graphs and after this the number of researchers in Graph Theory have been projected their work on the problem of metric dimension of Different types of graphs.

By denoting G + H the join of G and H a wheel Wn is defined as $W_n = K_1 + C_n$ for $n \geq 3$, a Fan is $F_n = K_1 + P_n$ for $n \geq 1$ and Jahangir graph J_{2n}, $(n \geq 2)$ which is obtained from wheel graph W_{2n} by alternating deleting n-spokes . [8] Buczkowski *et al*. determine the dimension of wheel W_n, [10-11] Caceres *et al*. The dimension of Fan graph F_n and [17] Tomesko and Javed the dimension of Jahangir graph J_{2n}, $(n \geq 2)$.

[14] In P(m, n) when we take m = 1, P(n, 1) is called prism and n-prism graph
Has 2n-nodes and 3n-edges it is denoted by D_n and Caceres *et al.* (2005) while
Working with metric dimension of some families of graphs it is shown that

$$\text{Dim}(P_m \times C_n) = \begin{cases} 3 & \text{if } n = \text{even} \\ 2 & \text{if } n = \text{odd} \end{cases}$$

Since Prism is infact the cross product $P_m \times C_n$ and this suggest that

$$\text{Dim}(D_n) = \begin{cases} 3 & \text{if } n = \text{even} \\ 2 & \text{if } n = \text{odd} \end{cases}$$

Thus it is obvious that prism contains a class of 3-regular graphs with bonded metric dimension. The generalized Peterson graph P (n, 2) becomes a useful example for many problems in the field of graph theory. We consider the metric dimension of Generalized Peterson graph P(n, m), for m = 2, $\{x_1, x_2, . . . , x_n\}$ induces a cycle in P(n, 2) with $x_i x_{i+1}$ $(1 \leq i \leq n)$ as edges. When n is odd then $\{y_1, y_2, . . . , y_m\}$ induce a cycle of length n with $y_i y_{i+2}$ $(1 \leq i \leq n)$ as edges, with indices taken modulo n, and when n is even i.e n = 2k, ($k \geq 3$) , { y1, y2, . . . , yn } generate two cycle of length k with $y_i y_{i+2}$ $(1 \leq i \leq n)$ as edges. [17, 20] Javaid *et al.* (2008).proved that some regular Graphs namely Generalized Peterson graph P (n, 2), Antiprism A n and Harary graph $H_{4;n}$ are families of graph with constant metric dimension and it is shown that Dim (P (n, 2)) = 3, for every $n \geq 5$. When we take for m = 3, $\{x_1, x_2, . . . , x_n\}$ generate a cycle in P(n, 3) with with $x_i x_{i+1}$ $(1 \leq i \leq n)$ as edges, If n = 3k ($k \geq 3$) then $\{y_1, y_2, . . . , y_m\}$ generate three cycles of length k or else generate cycle of length " n " with $y_i y_{i+3}$ $(1 \leq i \leq n)$as edges. Generalized Peterson graph produce an important class of 3-regular graph with 2n –vertices And 3n - edges so it is necessary to determine their metric dimension. [14] M. Imran found that generalized Peterson graph consists a family of 3-regular graph having bonded metric dimension, and for n = 0, 3, 4, 5 (mod 6) resolving sets consisting of only four vertices are chosen that resolves all the vertices of Generalized Peterson graph P(n, 3), except n = 2 (mod 6). For n = 1 (mod 6) resolving set consisting of three vertices is taken. In graph P(n, 3) all the indices " i " which do not satisfy the inequality $1 \leq i \leq n$ will be taken modulo n. [14] Upper bounds for metric dimension of Generalized Peterson graphs P(n, 3) as proved by M. mran
are given below,
For Generalized Peterson graph P (n, 3) we have
a) dim (P (n, 3)) ≤ 4 , for n = 0, 3, 4, 5 (mod 6) for $n \geq 17$
b) dim (P (n, 3)) ≤ 3 , for n = 1 (mod 6) for $n \geq 13$
c) dim (P (n, 3)) ≤ 5, for n = 2 (mod 6) for $n \geq 8$

THE GRAPH OF $P_{N,1,2}$
In this paper we have found and studied the metric dimension of the graph $P_{n,1,2}$. This graph has the following set of vertices and the set of edges denoted by $V(P_{n,1,2})$ and $E(P_{n,1,2})$ for the inner cycle and the outer cycle are as under:

$V(P_{n,1,2}) = \{u_1, u_2, . . . , u_n, v_1, v_2, . . . , v_n\}$

And

$E(P_{n,1,2}) = \{ u_i \ u_{i+2}, u_i v_i \} \cup \{ v_i v_{i+1} \}$
for $1 \le i \le n$, where the indices n + 1 and n + 2 must be replaced by 1 and 2 respectively.

For our convenience, we represent the cycle induced by $\{u_1, u_2, \ldots, u_n\}$ the inner cycle, the cycle induced by $\{ v_1, v_2, \ldots, v_n\}$ the outer cycle. Again the vertices choice chosen is crucial for the basis. Note that throughout our discussion remember that $P_{n,1,2}$ stands for the Inner and Outer Cycle graphs respectively.

Case 1: n = 0 (mod 4), n ≥ 16

In this case it can be written as $n = 4k$, $k \ge 4$, and $k \in Z^+$, The resolving set in general form is $W = \{u_1, u_7, v_{2k+3}\} \in V(P_{n,1,2})$, $k \ge 4$.

Theorem 1:

Prove that Metric Dimension of $P_{n,1,2}$ denoted by $\dim(P_{n,1,2}) \le 3$ for inner and outer cycles for $n \ge 16$.

Proof:

In this case it can be written as $n = 4k$, $k \ge 4$, and $k \in Z^+$, the resolving set in general form is $W = \{u_1, u_7, v_{2k+3}\} \in V(P_{n,1,2})$, $k \ge 4$.

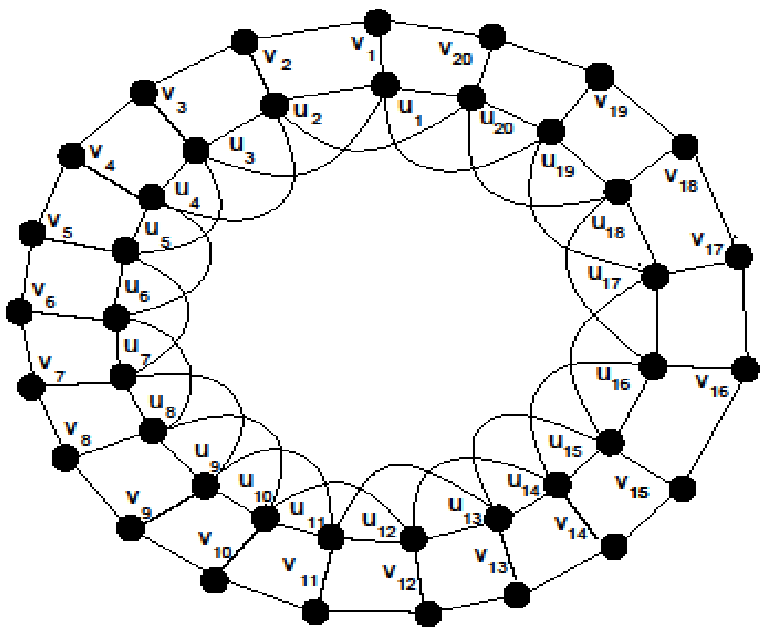

Figure 1: The graph of $P_{20,1,2}$

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Representations of vertices w.r.t w in general form are:

Representation of even vertices of inner cycle:

$$
r(u_{2i} / w) = \begin{cases} (1, 3, k+1) & \text{if} \quad i = 1 \\ (2, 2, k+1) & \text{if} \quad i = 2 \\ (2, 2, k+1) & \text{if} \quad i = 2 \\ (3, 1, k) & \text{if } i=3 \\ (I, i-3, k-i+3) & \text{if} \quad 4 \le i \le k \\ (k, k-2, 2) & \text{if} \quad i = k+1 \\ (k-1, k-1, 2) & \text{if} \quad i = k+2 \\ (k-2, k, 3) & \text{if} \quad i = k+3 \\ (2k-i+1, 2k-i+4, i-k) & \text{if} \quad k+4 \le i \le 2k \end{cases}
$$

And

Representation of odd vertices of inner cycle:

$$
r(u_{2i-1} / w) = \begin{cases} (1, 2, k+1) & \text{if } i = 2 \\ (2, 1, k) & \text{if } i = 3 \\ (i, i-3, k-i+2) & \text{if } 4 \le i \le k \\ (k-1, k-2, 1) & \text{if } i = k+1 \\ (k-2, k-1, 2) & \text{if } i = k+2 \\ (2k-i, 2k-i+3, i-k) & \text{if } k+3 \le i \le 2k+1 \end{cases}
$$

Representation of the vertices of Outer cycle

Representation of even vertices of outer cycle:

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$$r(v_{2i}/w) = \begin{cases} (3, 3, k+2) & \text{if } i = 2 \\ (4, 2, k+1) & \text{if } i = 3 \\ (i+1, i-2, k-i+4) & \text{if } 4 \le i \le k-1 \\ (k+1, k-2, 3) & \text{if } i = k \\ (k+1, k-1, 1) & \text{if } i = k+1 \\ (k, k, 2) & \text{if } i = k+2 \\ (k-1, k+1, 4) & \text{if } i = k+3 \\ (2k-i+2, 2k-i+5, i-k+1) & \text{if } k+4 \le i \le 2k-1 \\ (2, 2, 5) & \text{if } i = 2k \end{cases}$$

Representation of odd vertices of outer cycle:

$$r(v_{2i-1}/w) = \begin{cases} (1, 4, k+1) & \text{if } i = 1 \\ (2, 3, k+2) & \text{if } i = 2 \\ (3, 2, k+1) & \text{if } i = 3 \\ (I, i-3, k-i+4) & \text{if } 4 \le i \le k+1 \\ (k, k, 1) & \text{if } i = k+2 \\ (k-1, k, 2) & \text{if } i = k+3 \\ (2k-i+2, 2k-i+5, i-k) & \text{if } k+5 \le i \le 2k \end{cases}$$

Case 2: n = 1 (mod 4), n ≥ 17

In general form it can be written as $n = 4k+1$, $k \geq 4$, and $k \in Z^+$ and the resolving set , In general form is $W = \{u_1, u_2, u_7, v_{2k+2}\} \in V(Pn,1,2)$, $k \geq 4$, and $k \in Z^+$.

The Graph $P_{n,1,2}$ for particular value of n for inner and outer cycle is shown in figure for n = 21.

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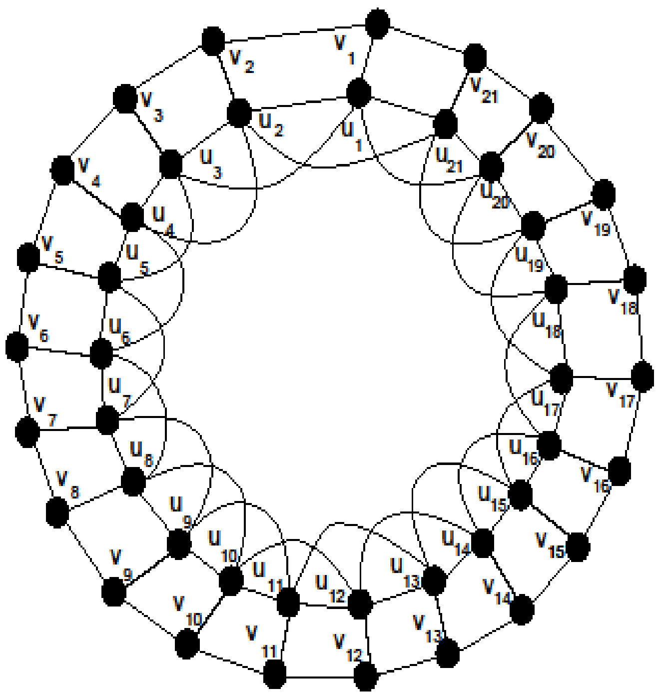

Figure 2: The Graph Of $P_{21,1,2}$

Theorem 2:
Prove that Metric Dimension of $P_{n,1,2}$ denoted by dim $(P_{n,1,2}) \leq 4$ for Inner and outer cycles for $n \geq 17$

Proof:
In this case it can be written as $n = 4k + 1$, $k \geq 4$, and $k \in Z^{+}$, TheResolving set in general form is $W = \{u_1, u_2, u_7, v_{2k+2}\} \in V(P_{n,1,2})$, $k \geq 4$.This graph has the following set of vertices and the set of edges denoted by $V(P_{n,1,2})$ and $E(P_{n,1,2})$ for the inner cycle and the outer cycle are as under:$V(P_{n,1,2}) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$

And

$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\}$

For $1 \leq i \leq n$, where the indices $n + 1$ and $n + 2$ must be replaced by 1 and 2 Respectively.

Representation of the vertices of inner cycle:

Representation of even vertices of inner cycle:

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$$r(u_{2i}/w) = \begin{cases} (2, 1, 2, k) & \text{if } i = 1 \\ (3, 2, 1, k-1) & \text{if } i = 2 \\ (i, i-1, k-i+2) & \text{if } 3 \le i \le k \\ (k, k, k-2, 1) & \text{if } i = k+1 \\ (k-1, k, k-1, 2) & \text{if } i = k+2 \\ (k-2, k-1, k, 3) & \text{if } i = k+3 \\ (2k-i+1, 2k-i+2, 2k-i+4, i-k) & \text{if } k+4 \le i \le 2k \end{cases}$$

Representation of odd vertices of inner cycle:

$$r(u_{2i+1}/w) = \begin{cases} (1, 1, 2, k+1) & \text{if } i = 1 \\ (2, 2, 1, k) & \text{if } i = 2 \\ (i, i, i-3, k-i+2) & \text{if } 4 \le i \le k \\ (k, k, k-2, 2) & \text{if } i = k+1 \\ (k-1, k-1, k-1, 3) & \text{if } i = k+2 \\ (k-2, k-2, k, 4) & \text{if } i = k+3 \\ (2k-i+1, 2k-i+1, 2k-i+4, i-k+1) & \text{if } k+4 \le i \le 2k \end{cases}$$

Representation of the vertices of Outer cycle:

Representation of even vertices of outer cycle:

$$r(v2i / w) = \begin{cases} (2, 1, 4, k+2) & \text{if } i = 1 \\ (3, 2, 3, k+1) & \text{if } i = 2 \\ (4, 3, 2, k+1) & \text{if } i = 3 \\ (i+1, I, i-2, k-i+3) & \text{if } 4 \le i \le k-1 \\ (k, k, k-2, 2) & \text{if } i = k \\ (k, k+1, k, 2) & \text{if } i = k+2 \\ (k-1, k, k+1, 4) & \text{if } i = k+3 \\ (2k-i+2, 2k-i+3, i-k+1) & \text{if } k+4 \le i \le 2k \end{cases}$$

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Representation of odd vertices of outer cycle:

$$r(v2i-1 / w) = \begin{cases} (1, 2, 4, k+2) & \text{if} \quad i = 1 \\ (2, 2, 3, k+2) & \text{if} \quad i = 2 \\ (3, 3, 2, k+1) & \text{if} \quad i = 3 \\ (i, i, i-3, k-i+4) & \text{if} \quad 4 \le i \le k-1 \\ (k, k, k-3, 3) & \text{if} \quad i = k \\ (k+1, k+1, k-2, 3) & \text{if} \quad i = k+1 \\ (k+1, k+1, k-1, 1) & \text{if} \quad i = k+2 \\ (k, k, k, 3) & \text{if} \quad i = k+3 \\ (k-1, k-1, k+1, 5) & \text{if} \quad i = k+4 \\ (2k-i+3, 2k-i+3, 2k-i+6, i-k) & \text{if} \quad k+5 \le i \le 2k+1 \end{cases}$$

Case 3: n = 2 (mod 4), n ≥ 18 :

In this case it can be written as $n = 4k + 2$, $k \ge 4$ and $k \in Z^+$, The resolving set in general form is $W = \{u_1, u_4, v_{2k+4}\} \in V(P_{n,1,2})$, $k \ge 4$.

Theorem 3:

Prove that Metric Dimension of $P_{n,1,2}$ denoted by dim $(P_{n,1,2}) \le 3$ for Inner and outer cycles for $n \ge 18$

Proof:

In this case it can be written as $n = 4k + 2$, $k \ge 4$, and $k \in Z^+$, The resolving set in general form is $W = \{u_1, u_4, v_{2k+4}\} \in V(P_{n,1,2})$, $k \ge 4$.

The Graph $P_{n,1,2}$ has the following set of vertices and the set of edges denoted by $V(P_{n,1,2})$ and $E(P_{n,1,2})$ for the inner cycle and the outer cycle are as under:

$V(P_{n,1,2}) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$

And the edge set

$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\}$

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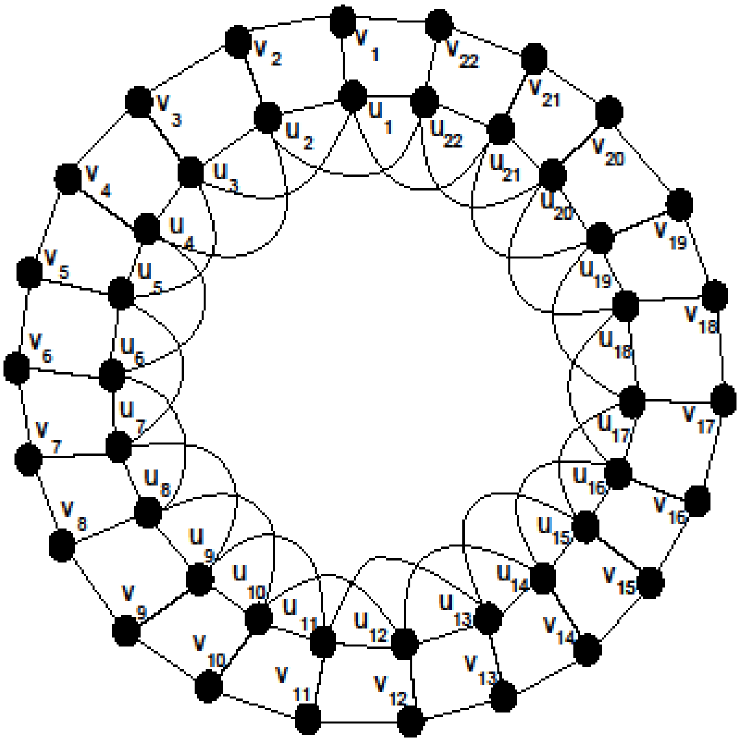

Figure 3. The graph $P_{22,1,2}$

Representation of the vertices of inner cycle:

Representation of even vertices of inner cycle:

$$r(u_{2i} / w) = \begin{cases} (1, 1, k + 1) & \text{if} \quad i = 1 \\ (i, i - 2, k - i + 3) & \text{if} \quad 3 \leq i \leq k + 1 \\ (k, k, 1) & \text{if} \quad i = k + 2 \\ (2k - i + 2, 2k - i + 3, i - k - 1) & \text{if} \quad k + 3 \leq i \leq 2k + 1 \end{cases}$$

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Representation of odd vertices of inner cycle:

$$r(u_{2i+1} / w) = \begin{cases} (1, 1, k + 2) & \text{if} \quad i = 1 \\ (i, i - 1, k - i + 3) & \text{if} \quad 3 \leq i \leq k + 1 \\ (k, k, 2) & \text{if} \quad i = k + 2 \\ (2k - i + 1, 2k - i + 3, i - k) & \text{if} \quad k + 3 \leq i \leq 2k + 1 \end{cases}$$

Representation of the vertices of Outer Cycle.

Representation of even vertices of outer cycle:

$$r(v2i / w) = \begin{cases} (2, 2, k + 2) & \text{if} \quad i = 1 \\ (3, 1, k + 2) & \text{if} \quad i = 2 \\ (i + 1, i - 1, k - i + 4) & \text{if} \quad 3 \leq i \leq k \\ (k + 2, k, 2) & \text{if} \quad i = k + 1 \\ (k, k + 1, 2) & \text{if} \quad i = k + 3 \\ (2k - i + 3, 2k - i + 4, i - k) & \text{if} \quad k + 4 \leq i \leq 2k + 1 \end{cases}$$

Representation of odd vertices of outer cycle:

$$r(v2i+1 / w) = \begin{cases} (1, 3, k + 2) & \text{if} \quad i = 1 \\ (2, 2, k + 3) & \text{if} \quad i = 2 \\ (i, i - 1, k - i + 5) & \text{if} \quad 3 \leq i \leq k \\ (k, k, 3) & \text{if} \quad i = k + 1 \\ (k + 1, k + 1, 1) & \text{if} \quad i = k + 2 \\ (k, k + 2, 1) & \text{if} \quad i = k + 3 \\ (k - 1, k + 1, 3) & \text{if} \quad i = k + 4 \\ (2k - i + 3, 2k - i + 5, i - k) & \text{if} \quad k + 5 \leq i \leq 2k + 1 \end{cases}$$

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Case 4: n = 3 (mod 4), $n \geq 11$

In general form it can be written as $n = 4k+3$, $k \geq 2$ and $k \in Z^+$, The resolving set in general form is $W = \{u_1, u_4, v_{2k+3}\} \in V(P_{n,1,2})$, $k \geq 2$.
and $k \in Z^+$.

Theorem 4: Prove that Metric Dimension of $P_{n,1,2}$ denoted by $\dim(P_{n,1,2}) \leq 3$ for Inner and outer cycles for $n \geq 11$

Proof:

In this case it can be written as $n = 4k+3$, $k \geq 2$ and $k \in Z^+$, The resolving set in general form is $W = \{u_1, u_4, v_{2k+3}\} \in V(P_{n,1,2})$, $k \geq 2$. This graph has the following set of vertices and the set of edges denoted by $V(P_{n,1,2})$ and $E(P_{n,1,2})$ as under:

$V(P_{n,1,2}) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ And

$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\}$

For $1 \leq i \leq n$, where the indices $n + 1$ and $n + 2$ must be replaced by 1 and 2 respectively.

The Graph $P_{n,1,2}$ for particular value of n for inner and outer cycle is shown in figure for n = 19.

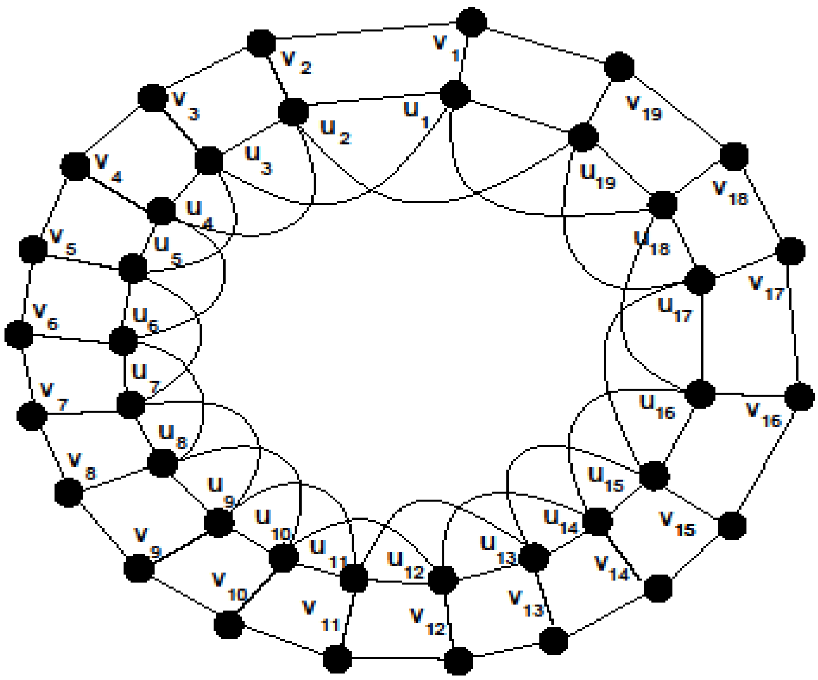

Figure 4. The graph $P_{19,1,2}$

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Representation of the vertices of inner cycle:

Representation of even vertices of inner cycle:

$$r(u_{2i} / W) = \begin{cases} (1, 1, k + 2) & \text{if } i = 1 \\ (i, i - 2, k - i + 3) & \text{if } 3 \leq i \leq k + 1 \\ (k, k, 2) & \text{if } i = k + 2 \\ (2k - i + 2, 2k - i + 4, i - k) & \text{if } k + 3 \leq i \leq 2k + 1 \end{cases}$$

Representation of odd vertices of inner cycle:

$$r(u_{2i+1} / W) = \begin{cases} (1, 1, k + 1) & \text{if } i = 1 \\ (i, i - 1, k - i + 2) & \text{if } 2 \leq i \leq k + 1 \\ (2k - i + 2, 2k - i + 3, i - k) & \text{if } k + 2 \leq i \leq 2k + 1 \end{cases}$$

Representation of the vertices of Outer cycle:

Representation of even vertices of outer cycle:

$$r(v_{2i} / W) = \begin{cases} (2, 2, k + 3) & \text{if } i = 1 \\ (i + 1, i - 1, k - i + 4) & \text{if } 2 \leq i \leq k - 1 \\ (k + 1, k - 1, 3) & \text{if } i = k \\ (k + 2, k, 1) & \text{if } i = k + 1 \\ (k + 1, k + 1, 1) & \text{if } i = k + 2 \\ (k, k + 2, 3) & \text{if } i = k + 3 \\ (2k - i + 3, 2k - i + 5, i - k + 1) & \text{if } k + 4 \leq i \leq 2k + 1 \end{cases}$$

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Representation of odd vertices of outer cycle:

$$r(v_{2i+1} / w) = \begin{cases} (1, 3, k+3) & \text{if} \quad i = 1 \\ (2, 2, k+2) & \text{if} \quad i = 2 \\ (i, i-1, k-i+4) & \text{if} \quad 3 \le i \le k \\ (k+1, k, 2) & \text{if} \quad i = k+1 \\ (k+1, k+2, 2) & \text{if} \quad i = k+3 \\ (2k-i+4, 2k-i+5, i-k) & \text{if} \quad k+4 \le i \le 2k+2 \end{cases}$$

CONCLUSION

The purpose of this paper was to find the metric dimension of $P_{n,1,2}$ by using the technique of shortest distance, showing that the distinct vertices has distinct representation with respect to the resolving set W. The resolving set W is a set of vertices which is subset of the set of vertices of the graph G denoted by V (G). Keeping in view the very fact that no two vertices of G has same representation with respect to the resolving set W. We have found the metric dimension for inner and outer cycle of the graph $P_{n,1,2}$. We have also observed that the metric dimension of all these graphs is Bounded and n = 1 (mod 4) for n $\geq$ 14 for inner and outer cycle graph which is bounded above by 4. Note that only four vertices are appropriately chosen suffice to resolve all the vertices of these graphs except for n = 0 (mod 4) for n $\geq$ 16 for for inner and outer cycle graph, also for n = 2 (mod 4) for n $\geq$ 18 and n = 3 (mod 4) for n $\geq$ 11 for for inner and outer cycle graphs for which only three vertices are appropriately chosen suffices to resolve all the vertices of these graphs for which the metric dimension of these cases is bounded by 3.

We have proved that the metric dimension of $P_{n,1,2}$ is bounded for inner and outer cycles as given below:

a) dim $(P_{n,1,2}) \leq 3$ for n = 0 (mod 4) and n $\geq$ 16
b) dim $(P_{n,1,2}) \leq 4$ for n = 1 (mod 4) and n $\geq$ 17
c) dim $(P_{n,1,2}) \leq 3$ for n = 2 (mod 4) and n $\geq$ 18
d) dim $(P_{n,1,2}) \leq 3$ for n = 3 (mod 4) and n $\geq$ 11

According to results of this paper the metric dimension is bounded for all cases of $P_{n,1,2}$ and there is an open problem for constant metric dimension of these cases.

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