International Journal of Mathematics and Statistics Studies

Vol.1, No.3, pp.38-54, September 2013

Published by European Centre for Research Training and Development UK

ON THE ACCURACY OF BINOMIAL MODEL AND MONTE CARLO METHOD FOR PRICING EUROPEAN OPTIONS

[1*] **Fadugba S. E,** [2] **Nwozo C. R.,** [3] **Okunlola J. T.,** [4] **Adeyemo O.A and** [5] **Ajayi A. O.**

[1*,5] Department of Mathematical Sciences, Ekiti State University, Ado Ekiti, Nigeria

[2] Department of Mathematics, University of Ibadan, Nigeria

[3,4] Department of Mathematical and Physical Sciences, Afe Babalola University, Ado Ekiti, Nigeria

ABSTRACT: *We consider the accuracy of two numerical methods for determining the price of European options namely Binomial model and Monte Carlo method. Then we compare the convergence and accuracy of the methods to the analytic Black-Scholes price of European option. Binomial model is very simple but powerful technique that can be used to solve many complex options pricing problem. Monte Carlo method is very flexible in handling high dimensional financial problems and easy to code. Moreover Binomial model is more accurate and converges faster than Monte Carlo method when pricing European options*.

KEYWORDS: Accuracy, Binomial Model, European option, Monte Carlo Method

Mathematics Subject Classification**:** 65C05, 65C30, 91G60, 60H30, 65N06, 78M31

INTRODUCTION

Option pricing is a major accomplishment of modern finance. It spurred the development and widespread use of familiar financial options, such as puts and calls in common assets, as well as exotic options. A financial derivative is an asset whose value depends on the price of some other asset, the underlying. Derivatives permit investors to customize their exposure to the market: they can speculate, hoping to win a large amount with a small initial investment, or hedge, investing in the new asset in order to offset risks they already have, or arbitrageur (riskless profit).

One of the major contributors to the world of finance was Black and Scholes [1]. They published their seminar work on option pricing. In this paper, the famous Black-Scholes described a mathematical framework for calculating the fair price of a European option in which they used a no-arbitrage to derive a partial differential equation which governs the evolution of the option price with respect to the time to expiry and the price of the underlying asset. Later [6] proposed a jump-diffusion model.

Many option valuation methods have been developed through the years after Black and Scholes published their work in 1973. Most of these methods being devoted to overcome the limitations of the Black and Scholes model, mainly that it can be used to value European options only and that this model demands the underlying asset to follow a lognormal distribution. Also, its solution is exact for a non-dividend paying stock or a stock which pays a continuous dividend proportional to the stock price only. Numerical techniques are needed for pricing options in cases where analytic solutions are either unavailable or easily computable [5]. Now, we present an overview of two popular numerical methods in the context of [1, 6] for pricing European options which are Binomial model for pricing vanilla option under various alternatives, including the absolute diffusion, pre-jump and square root constant elasticity of variance methods derived by [3] based on risk-neutral valuation and Monte Carlo method introduced by [2] which assumes that the asset returns are distributed according to stochastic differential equation and is used for pricing European option and path dependent options. These procedures provide much of the infrastructure in which many contributions to the field over the past three decades have been centered.

In this paper we shall consider the accuracy and convergence of binomial model and Monte Carlo method for pricing European options.

BINOMIAL MODEL AND MONTE CARLO METHOD FOR PRICING EUROPEAN OPTIONS

This section presents the two numerical methods under considerations namely Binomial model and Monte Carlo method.

European Options

An European option contract is an agreement between two parties that gives one party the right to buy or sell the underlying asset at some specified date in the future for a fixed price. This right has a value and the party that provides the option will demand compensation at the inception of the option. Further, once initiated, options can be traded on an exchange much like the underlying asset. European options divided into two classes: calls and puts.

European Call Option

A European call option on a stock gives the buyer the right but not the obligation to buy a number of shares of stock for a specified price K at a specified date T in the future. If the price of the stock S is below the strike price, the call option is said to be “out of the money” whereas a call option with a strike price below the stock price is classified as “in the money” and the strike price and the stock price are the same, the call option is said to be “at the money”. The value of the call option is thus a function of S and time, and its payoff, which describes the option's value at T, is given by

$$C(S,T) = (S - K,0)^{+} \tag{1}$$

The graph below illustrates the payoff for a European call option with strike price $K = 25$

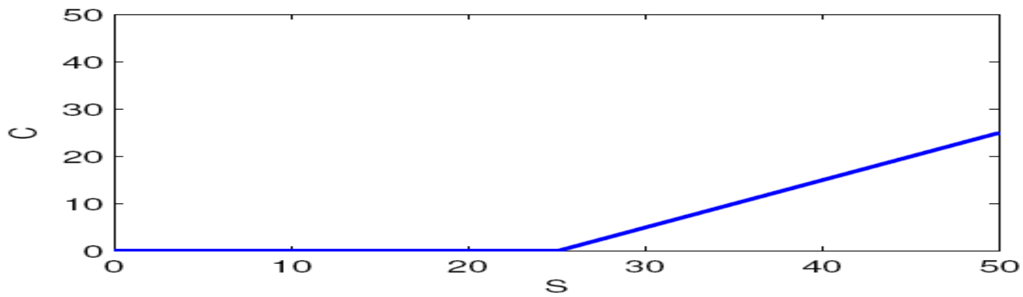

Figure 1: Payoff for a European call Option

European Put Option

A European put option on a stock gives the buyer the right but not the obligation to sell a number of shares of stock for a specified price K at a specified date T in the future. If the price of the stock S is above the strike price, the put option is said to be "out of the money" whereas a put option with a strike price above the stock price is classified as "in the money" and the strike price and the stock price are the same, the put option is said to be "at the money". The value of the put option is thus a function of S and time, and its payoff, which describes the option's value at T, is given by

$$P(S,T) = (K - S,0)^{+} \tag{2}$$

The graph below illustrates the payoff for a European call option with strike price $K = 25$

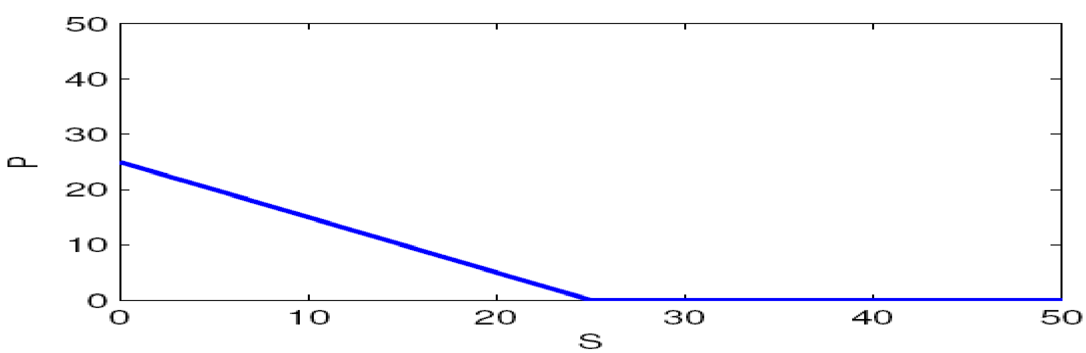

Figure 2: Payoff for a European Put Option

European options are some of the simplest financial derivatives. They are interesting, because their valuation proved to be difficult until 1973. Before that, there was no generally accepted model that could give an option's trader the value of an option before expiry. This age long problem was solved by Black Scholes model.

Binomial Model

This is defined as an iterative solution that models the price evolution over the whole option validity period. For some vanilla options such as American option, iterative model is the only choice since there is no known closed form solution that predicts its price over a period of time.

The binomial model is widely favoured amongst market professionals because of its ease of implementation and its versatility in applying it to all manner of options, simple and complex. The Cox-Ross-Rubinstein "Binomial" model [3] contains the Black-Scholes analytic formula as the limiting case as the number of steps tends to infinity. Next we shall present the derivation and the implementation of the binomial model below.

The Cox-Ross-Rubinstein model [3, 7]

We know that after a period of time, the stock price can move up to Su with probability p or down to Sd with probability ($1-p$), where $u>1$ and $0<d<1$. Therefore the corresponding value of the call option at the first time movement δt is given by

$$\left.\begin{aligned} f_u &= (Su-K,0)^+ \\ f_d &= (S_d-K,0)^+ \end{aligned}\right\} \tag{3}$$

Where f_u and f_d are the values of the call option after upward and downward movements respectively.

We need to derive a formula to calculate the fair price of vanilla options. The risk neutral call option price at the present time is given by

$$f = e^{-r\delta t}[pf_u + (1-p)f_d] \tag{4}$$

Where the risk neutral probability is given by

$$p = \frac{e^{r\delta t}-d}{u-d} \tag{5}$$

Now, we extend the binomial model to two periods. Let f_{uu} denote the call value at time $2\delta t$ for two consecutive upward stock movements, f_{ud} for one downward and one upward movement and f_{dd} for two consecutive downward movements of the stock price [7]. Then we have

$$f_{uu} = (Suu - K, 0)^{+} \tag{6}$$

$$f_{ud} = (Sud - K, 0)^{+} \tag{7}$$

$$f_{dd} = (Sdd - K, 0)^{+} \tag{8}$$

The values of the call options at time δt are

$$\left.\begin{aligned} f_u &= e^{-r\delta t}[pf_{uu} + (1-p) f_{ud}] \\ f_d &= e^{-r\delta t}[pf_{ud} + (1-p) f_{dd}] \end{aligned}\right\} \tag{9}$$

Substituting (9) into (4), we have

$$f = e^{-r\delta t}[pe^{-r\delta t} f_{uu} + (1-p) f_{ud} + (1-p)e^{-r\delta t}(pf_{ud} + (1-p) f_{dd})]$$

$$f = e^{-2r\delta t}[p^2 f_{uu} + 2p(1-p) f_{ud} + (1-p)^2 f_{dd})] \tag{10}$$

Equation (10) is called the current call value, where the numbers p^2, $2p(1-p)$ and $(1-p)^2$ are the risk neutral probabilities for the underlying asset prices Suu, Sud and Sdd respectively.

We generalize the result in (10) to value an option at $T = N\delta t$ as

follows $f = e^{-Nr\delta t} \sum_{j=0}^{N} {}^{N}C_j p^j (1-p)^{N-j} f_{u^j d^{N-j}}$

$$f = e^{-Nr\delta t} \sum_{j=0}^{N} {}^{N}C_j p^j (1-p)^{N-j} (Su^j d^{N-j} - K, 0)^{+} \tag{11}$$

Where $f_{u^j d^{N-j}} = (Su^j d^{N-j} - K, 0)^{+}$ and ${}^{N}C_j = \dfrac{N!}{(N-j)!\, j!}$ is the binomial coefficient. We assume that m is the smallest integer for which the option's intrinsic value in (11) is greater than zero. This implies that $Su^m d^{N-m} \geq K$. Then (11) can be written as

$$f = Se^{-Nr\delta t} \sum_{j=0}^{N} {}^{N}C_j p^j (1-p)^{N-j} u^j d^{N-j} \quad - Ke^{-Nr\delta t} \sum_{j=0}^{N} {}^{N}C_j p^j (1-p)^{N-j} \tag{12}$$

Equation (12) gives us the present value of the call option.

The term $e^{-Nr\delta t}$ is the discounting factor that reduces f to its present value. We can see from the first term of (12) that ${}^{N}C_j p^j (1-p)^{N-j}$ is the binomial probability of j upward movements to occur after the first N trading periods and $Su^j d^{N-j}$ is the corresponding value of the asset after j upward movements of the stock price. The second term of (12) is the present value of the option's strike price. Let $Q = e^{-r\delta t}$, we substitute Q in the first term of (12) to yield

$$f = SQ^{-N} \sum_{j=0}^{N} {}^{N}C_j p^j (1-p)^{N-j} u^j d^{N-j} - Ke^{-Nr\delta t} \sum_{j=0}^{N} {}^{N}C_j p^j (1-p)^{N-j}$$

$$f = S \sum_{j=0}^{N} {}^{N}C_j [Q^{-1} pu]^j [Q^{-1}(1-p)d]^{N-j} - Ke^{-Nr\delta t} \sum_{j=0}^{N} {}^{N}C_j p^j (1-p)^{N-j} \qquad (13)$$

Now, let $\Phi(m; N, p)$ be the binomial distribution function given by $\Phi(m; N, p) = \sum_{j=0}^{N} {}^{N}C_j p^j (1-p)^{N-j}$ (14)

Equation (14) is the probability of at least m success in N independent trials, each resulting in a success with probability p and in a failure with probability $(1-p)$. Then let $p' = Q^{-1} pu$ and $(1-p') = Q^{-1}(1-p)d$. Consequently, it follows that

$$f = S\Phi(m; N, p') - Ke^{-rT} \Phi(m; N, p) \qquad (15)$$

The model in (15) was developed by Cox-Ross-Rubinstein [6], where $\delta t = \frac{T}{N}$ and we will refer to it as CRR model. The corresponding put value of the European option can be obtained using call put relationship of the form $C_E + Ke^{-rt} = P_E + S$ as

$$f = Ke^{-rT} \Phi(m; N, p) - S\Phi(m; N, p') \qquad (16)$$

Where the risk free interest rate is denoted by r, C_E is the European call, P_E is the European put and S is the initial stock price. European option can only be exercised at expiration, while for an American option, we check at each node to see whether early exercise is advisable to holding the option for a further time period δt. When early exercise is taken into consideration, the fair price must be compared with the option's intrinsic value [7].

We shall state a lemma and theorem below as follows:

Lemma 1

The probability of at least m success in N independent trials, each resulting in a success with probability p and in a failure with probability q is given by $\Phi(m; N, p) = \sum_{j=0}^{N} {}^{N}C_j p^j q^{N-j}$.

Let $p' = Q^{-1}pu$ and $q' = Q^{-1}qd$, then it follows that $f = Ke^{-rT}\Phi(m; N, p) - S\Phi(m; N, p')$

Theorem 1

Under the Binomial tree model for stock pricing, the price of a European style option with expiration date $t = T$ is given by

$$f_0 = \frac{E^*(f_T)}{(1+r)^T} \tag{17}$$

Corollary 1

Under the Binomial tree model for stock pricing, the price of a European call style option with expiration date $t = T$ is given by

$$\begin{aligned} f_0 &= \frac{E^*(f_T)}{(1+r)^T} \\ &= \frac{E^*(S_T - K)^+}{(1+r)^T} \\ &= \frac{1}{(1+r)^T} \sum_{j=0}^{T} {}^{T}C_j (p^*)^j (1-p^*)^{T-j} (S_0 u^j d^{T-j} - K, 0)^+ \end{aligned} \tag{18}$$

Where E^* denotes expected value under the risk neutral probability p^* for stock price.

The above Theorem can be written in words as "the price of the option is equal to the present value of the expected payoff of the option under the risk neutral measure"

We shall prove Theorem 1 for $T = 2$, since $T > 2$ case is analogous. To this end we must show that

$$\begin{aligned} f_0 &= \frac{E^*(f_2)}{(1+r)^2} \\ &= \frac{1}{(1+r)^2}[(p^*)^2 f_{2,uu} + 2(p^*)(1-(p^*)) f_{2,ud} + (1-(p^*))^2 f_{2,dd})] \end{aligned} \tag{19}$$

Although we cannot exercise the option at the earlier time $t = 1$, we can sell it, so it does have a "price" at that time which we can view as a potential "payoff". At time $t = 1$, we would know what the new price of the stock is, S_1, and we thus could sell the option which then would have an expiration date of $T = 1$. For example, if $S_1 = uS_0$, then we use $T = 1$ price formula in (4) with outcomes

$$f_u = f_{2,uu},\ f_d = f_{2,dd} \tag{20}$$

The above equation (20) yields the price denoted by $f_{1,u}$, for the upward movement when $t = 1$.

$$f_{1,u} = \frac{1}{(1+r)}[p^* f_{2,uu} + (1-p^*) f_{2,ud}] \tag{21}$$

Similarly, if $S_1 = dS_0$, then

$$f_{1,d} = \frac{1}{(1+r)}[p^* f_{2,du} + (1-p^*) f_{2,dd}] \tag{22}$$

But now we can go one more time step back to $t = 0$: we have these "payoff" values at time $t = 1$ of $f_{1,u}$ and $f_{1,d}$, which we just computed and thus we can now use them in the $T = 1$ formula (4) again to obtain

$$\begin{aligned} f_0 &= \frac{[pf_{1,u} + (1-p) f_{1,d}]}{(1+r)} \\ &= \frac{1}{(1+r)^2}[(p^*)^2 f_{2,uu} + p^*(1-(p^*)) f_{2,ud} + p^*(1-(p^*)) f_{2,du} + (1-(p^*))^2 f_{2,dd})] \end{aligned} \tag{23}$$

In general, the proof proceeds by starting at time *T and* moving back in time step by step to each node on the lattice until finally reaching time $t = 0$. This procedure yields not only f_0 but the entire intermediary price as well.

Monte Carlo Method [2, 8]

Boyle [2] was the first researcher to introduce Monte Carlo method into finance. Monte Carlo method is a numerical method that is useful in many situations when no closed form solution is available. This method is good for pricing both vanilla and path dependent options, quite easy to implement, uses the risk valuation result and can be used without too much difficulty to value a large range of European style exotics [4, 9].

The expected payoff in a risk neutral world is calculated using a sampling procedure. The main procedures are followed when using this method:

Vol.1, No.3, pp.38-54, September 2013
Published by European Centre for Research Training and Development UK

- Simulate a path of the underlying asset under the risk neutral condition within the desired time horizon.
- Discount the payoff corresponding to the path at the risk free interest rate.
- Repeat the procedure for a high number of simulated sample paths.
- Average the discounted cash flows over sample paths to obtain option's value.

A Monte Carlo method followed the geometric Brownian motion for stock price

$$dS = \mu S dt + \sigma S dW(t) \tag{24}$$

Where $dW(t)$ a Brownian motion or Wiener process and S is the stock price. If δS is the increase in the stock price in the next small interval of time δt then,

$$\frac{\delta S}{S} = \mu S dt + \sigma z\sqrt{\delta t} \tag{25}$$

Where z is normally distributed with mean zero and variance one, σ is the volatility of the stock price and μ is the expected return in a risk neutral world, (25) is expressed as

$$S(t+\delta t) - S(t) = \mu S(t)\delta t + \sigma S(t) z\sqrt{\delta t} \tag{26}$$

It is more accurate to estimate $\ln S$ than S, we transform the asset price process using Ito's lemma, and we have

$$d(\ln S) = (\mu - \frac{\sigma^2}{2})dt + \sigma dW(t)$$

So that,

$$\ln S(t+\delta t) - \ln S(t) = \left(\mu - \frac{\sigma^2}{2}\right)\delta t + \sigma z\sqrt{\delta t}$$

Therefore,

$$S(t+\delta t) = S(t)\exp\left(\left(\mu - \sigma^2/2\right) + \sigma z\sqrt{\delta t}\right) \tag{27}$$

This method is particularly relevant when the financial derivatives payoff depends on the path followed by the underlying asset during the life of the option [10, 11]

The fair price for pricing option at maturity date is given by

$$S_T^j = S\exp\left(\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma z\sqrt{T}\right) \tag{28}$$

Where $j = 1,2,\ldots,M$ and M denotes the number of trials. The estimated European call option value is

$$C = \frac{1}{M}\sum_{J=1}^{M}\exp(-rT)(S_T^j - S_t, 0)^+ \tag{29}$$

Similarly, for a European put option, we have

$$C = \frac{1}{M}\sum_{J=1}^{M}\exp(-rT)(S_t - S_T^j, 0)^+ \tag{30}$$

Where S_t is the strike price which can be determined by either arithmetic or geometric mean.

Monte Carlo method can be successfully used to value options since it provides a good approximation to the correct value of the option once adequate variance reduction techniques are applied.

Principles of Theory for Monte Carlo Method

- If a derivative security can be perfectly replicated through trading in other assets, then the price of the derivative security is the cost of the replicating trading strategy.
- Discounted asset prices are martingales under a probability measure associated with the choice of discount factor. Prices are expectation of discounted payoffs under such martingale measure.
- In a complete market, any payoff can be realized through a trading strategy and the martingale measure associated with the discount rate is unique.

Black Scholes Model

Speaking of the continuous-time model to price stock options, few can ignore the fundamental contribution that Fisher Black and Myron Scholes made in the early 1970s. They developed their European option pricing model under the assumption of the lognormal dynamics of derivatives. They also made a major breakthrough in pricing stock options by developing the well-known Black-Scholes model for valuing options especially European call and put options. They further derived the famous Black-Scholes partial differential equation that must be satisfied by the price of any derivative dependent on a non-dividend paying stock.

Black-Scholes model of the market follows these assumptions:

- The stock price follows the geometric Brownian motion with μ and σ constants.
- The short selling of securities with full use of proceeds is permitted.
- There are no transaction costs or taxes; all securities are perfectly divisible.
- There are no dividends during the life of the derivative.
- There is no risk-less arbitrage opportunities.
- The underlying asset trading is continuous and the change of its price is continuous.
- The risk-free rate of interest r is constant and the same for all maturities.
- Fractional shares of the underlying asset may be traded.
- The asset price follows a lognormal random walk.

The continuous Black Scholes Formula

We recall that in a N-period model the price of a European call option is given by

$$f_c(S,t) = SP(X_{P'} \geq m) - Ke^{-rT}P(X_P \geq m) \tag{31}$$

Where $m = \min\left\{ 0 \leq j \leq N \middle| Su^j d^{N-j} - K \geq 0 \right\}$, $X_{P'}$ and X_P are $B(N, p')$ and $B(N, p)$ distributed random variables with

$$\left.\begin{aligned} p &= \frac{e^{r\delta t} - d}{u-d}, \\ p' &= pue^{-r\delta t} \end{aligned}\right\} \tag{32}$$

We shall state below a theorem on continuous black Scholes formula without proof

Theorem 2: (Continuous Black Scholes Formula)

Assume that $T = N\delta t, \quad u > 1, \quad d = \dfrac{1}{u}$ and define $\sigma > 0$ such that $u = e^{\sigma\sqrt{\delta t}}$ and $u = e^{-\sigma\sqrt{\delta t}}$. Then, there holds

$$\left.\begin{aligned} &\lim_{\delta t \to 0} f_c(S,t) = S\phi(d_1) - Ke^{-rT}\phi(d_2), \\ &\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-y^2}{2}}\, dy, \\ &d_2 = d_1 - \sigma\sqrt{T} \end{aligned}\right\} \tag{33}$$

Where

$$\left.\begin{aligned} d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \\ d_2 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \end{aligned}\right\} \tag{34}$$

Equations (33) and (34) are called Black Scholes model for pricing non dividend European call style option. The price of a corresponding put option is $f_p(S,t) = Ke^{-rT}\phi(-d_2) - S\phi(-d_1)$ using put-call parity given by $f_p(S,t) = Ke^{-rT} - S + f_c(S,t)$, where $\phi(.)$ is the cumulative distribution function of the standard normal distribution.

Boundary conditions for Black Scholes Model

The boundary conditions for Black Scholes model for pricing a European call option are given below:

$$\left.\begin{aligned} & f_c(0,t) = 0, \ \forall\, t, \\ & \lim_{S\to\infty} f_c(S,t) = S, \\ & f_c(S,t) = (S - K, 0)^+ \end{aligned}\right\} \tag{35}$$

For example, the price of a European option with strike price of $K = \$10$, T days to maturity and underlying asset price S is given in the Figure 3 below:

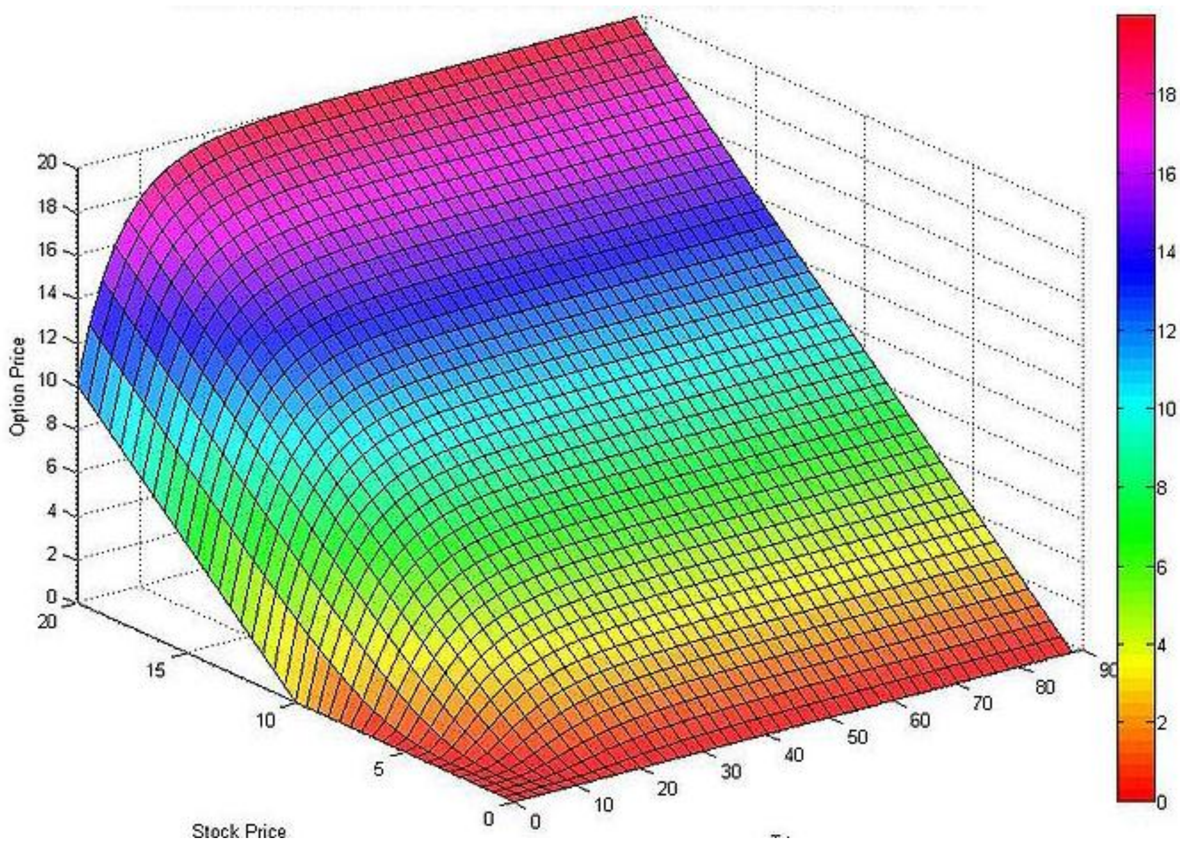

Figure 3: Graph of Black Scholes Model for Pricing European Option.

NUMERICAL EXAMPLES AND RESULTS

This section presents numerical experiment and the results generated from Binomial model and Monte Carlo Method for pricing European options

Numerical Experiment

We consider the accuracy and the convergence of Binomial Model and Monte Carlo method with relation to the Black-Scholes value of the option. We price a European option on non-dividend paying stock with the following parameters:

$$K = 60, r = 0.05, \sigma = 0.25, T = 3$$

The results generated and the errors incurred from the two methods are presented in the Tables 1 and 2 below using MATLAB.

Vol.1, No.3, pp.38-54, September 2013
Published by European Centre for Research Training and Development UK

Table of Results

Table 1: The Performance of the Methods against the 'True' Black Scholes Price for European Option

	European Call			**European Put**		
S	**Black-Scholes Model**	**Binomial Model**	**Monte Carlo Method**	**Black-Scholes Model**	**Binomial Model**	**Monte Carlo Method**
10	0.0002	0.0000	0.0000	41.6426	41.6430	41.8963
20	0.0685	0.0550	0.0000	31.7110	31.6970	31.9589
30	0.8408	0.8270	0.0979	22.4833	22.4700	22.0365
40	3.3320	3.3510	1.7832	14.9744	14.9930	13.8648
50	7.9138	7.8950	6.5806	9.5562	9.5380	8.5794
60	14.3052	14.2220	13.6269	5.9477	5.8640	5.3492
70	22.0135	21.9550	21.2880	3.6560	3.5980	4.1271
80	30.5955	30.6040	30.3936	2.2380	2.2460	2.9875
90	39.7292	39.7630	39.3517	1.3716	1.4050	2.2409
100	49.2020	49.2010	49.3805	0.8445	0.8430	1.6429
110	58.8810	58.8770	59.0032	0.5234	0.5190	1.4314
120	68.6845	68.6870	69.5731	0.3270	0.3300	1.1851
130	78.4416	78.5540	78.7761	0.2061	0.1960	1.0252
140	88.4886	66.4860	87.4763	0.1310	0.1290	0.9822
150	98.4416	98.4380	98.2003	0.0841	0.0800	0.8640
160	108.4120	108.4070	107.9254	0.0545	0.0490	0.8039
170	118.3931	118.3920	116.9221	0.0356	0.0340	0.7200
180	128.3810	128.3760	127.2234	0.0235	0.0190	0.7354
190	138.3731	138.3710	136.3797	0.0156	0.0130	0.6369
200	148.3680	148.3670	147.7091	0.0105	0.0090	0.6271

Table 2: The Comparative Error Analysis of Binomial Model and Monte Carlo Method for Pricing European Options

	European Call		European Put	
S	**Binomial Model**	**Monte Carlo Method**	**Binomial Model**	**Monte Carlo Method**
10	0.0002	0.0002	0.0004	0.2537
20	0.0135	0.0685	0.0140	0.2479
30	0.0135	0.0685	0.0140	0.2479
40	0.0138	0.7429	0.0133	0.4468
50	0.0190	1.5488	0.0186	1.1096
60	0.0188	1.3332	0.0240	0.9826
70	0.0832	0.6783	0.0837	0.5985
80	0.0585	0.7255	0.0580	0.4711
90	0.0085	0.2019	0.0080	0.7495
100	0.0338	0.3775	0.0334	0.8693
110	0.0010	0.1785	0.0015	0.7984
120	0.0040	0.1222	0.0044	0.9080
130	0.0025	0.8886	0.0030	0.8581
140	0.1124	0.3345	0.0101	0.8191
150	0.0036	1.0123	0.0020	0.8512
160	0.0050	0.2413	0.0041	0.7799
170	0.0011	0.4866	0.0055	0.7494
180	0.0005	1.1576	0.0045	0.7119
190	0.0021	1.9934	0.0026	0.6213
200	0.0001	0.6589	0.0015	0.6166

DISCUSSION OF RESULTS

Table 1 above shows the variation of the option price with the underlying price. The results demonstrate that the two methods perform well, are consistent and agree with the Black-Scholes value. The error incurred in binomial model is smaller than that of its counterpart as shown in

Table 2 above. Monte Carlo has appealing characteristics as an option valuation method, such as flexibility to function with different distributions, even empirical distributions of the underlying variables. Additionally, it can incorporate discontinuities such as those that arise from jump processes. However, binomial Model is more accurate than Monte Carlo method when pricing European option.

CONCLUSION

Pricing financial instruments is a recurring problem for many financial institutions. Many different kind of financial instruments exist on the financial market, and new kinds are introduced continually. Furthermore, sophisticated financial instruments involving multiple underlying assets have increased. In general, pricing these financial instruments does not admit explicit analytical solutions. Therefore numerical methods such as binomial model and Monte Carlo method plays an increasingly important role.

The performance, in terms of accuracy, of the binomial model and Monte Carlo method in the context of Black Scholes pricing formula is satisfactory.

Binomial model is good for pricing European options and options with early exercise opportunities and they are relatively easy to implement but can be quite hard to adapt to more complex situations. Monte Carlo is a flexible method that has proved suitable to deal with multiple random factors. The precision of the estimates provided by Monte Carlo can be improved by implementing variance reduction techniques. However, the use of these techniques implies a greater computational effort; thus, there is a trade-off between a lower variance estimator and computational requirements. Monte Carlo method works very well for pricing path dependent options and approximates every arbitrary exotic option, it is flexible in handling varying and even high dimensional financial problems but early exercise remains problematic.

Finally, Binomial model is more accurate and converges faster than Monte Carlo Method when pricing European options as we can see from the Tables above.

.

References

.

[1] F. Black and M. Scholes, The Pricing of Options and Corporate Liabilities, Journal of Political Economy, 81(3), (1973), 637-654.

[2] P. Boyle, M. Broadie and P. Glasserman, Monte Carlo Methods for Security Pricing, Journal of Economic Dynamics and Control, 21(8-9), (1997), 1267-1321.

[3] J. Cox, S. Ross and M. Rubinstein, Option Pricing: A Simplified Approach, Journal of Financial Economics, 7, (1979), 229-263.

[4] S. Fadugba, C. Nwozo and T. Babalola, The Comparative Study of Finite Difference Method and Monte Carlo Method for Pricing European Option, Mathematical Theory and Modeling, USA, 2(4), (2012), 60-66.

[5] J. Hull, Options, Futures and other Derivatives, Pearson Education Inc. Fifth Edition, Prentice Hall, New Jersey, 2003.

[6] R.C. Merton, Theory of Rational Option Pricing, Bell Journal of Economics and Management Science, 4(1), (1973), 141-183.

[7] C. R. Nwozo and S. E. Fadugba, Some Numerical Methods for Options Valuation, Communication in Mathematical Finance, Scienpress Ltd, UK, 1(1), (2012), 57-74.

[8] C. R. Nwozo and S. E. Fadugba, Monte Carlo method for pricing some path dependent options, International Journal of Applied Mathematics, 25(2012), 763-778

[9] S.M. Ross, An Introduction to the Mathematical Finance: Options and Other Topics, Cambridge University Press, 1999.

[10] R.Y. Rubinstein, Simulation and the Monte Carlo Method, John Wiley, New York, 1981.

[11] P. Wilmott, S. Howison, and J. Dewynne, The Mathematics of Financial Derivatives, Cambridge University Press, 2008.