European Journal of Statistics and Probability

Vol.6, No.2, pp.1-11, May 2018

Published by European Centre for Research Training and Development UK (www.eajournals.org)

# ON PRICING OF UNEMPLOYMENT INSURANCE ASSUMING NON-ZERO MORTALITY FOR EMPLOYEES WITH APPLICATION TO THE USA ECONOMY

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**ABSTRACT:** *Unemployment insurance may be availed to those employees still employed but may become temporary unemployed for some short period of time before returning to employment. In a recent study an approach for valuing the benefits and thus pricing the policy was derived, it results in a lower premium than the one proposed for the states by the federal government of United States of America (USA). The model, however assumes zero mortality for the lives involved. In this study the zero mortality assumption is not made and further, the probability mass function of the duration of unemployment is incorporated directly into the modeling. The new approach is formulated basing on the actuarial expected present value ' benefit-event' benefits valuation method and the payment of premiums conform to a life annuity payment. The new premium is 4.07 %, less than the 5.1% reported in the recent study, and much less than the 6% recommended by the USA federal government.*

**KEYWORDS:** Unemployment Insurance, Survivorship Probability, Unemployment Duration Distribution, Expected Present Value, Equation of Value.

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## INTRODUCTION

The study relies on the estimates of parameters , interest rates and probabilities. In particular we consider the distribution of unemployment duration for some particular population of those in employment. In the previous study it is noted that the three parameter Barr XII distribution is recommended for the USA data, and the maximum likelihood estimates of the parameters are thus obtained. To estimate the interest rate, the capital asset pricing model is applied in the previous study.

There are two sets of probabilities required in this model, namely the discrete probability mass function to be derived from the continuous distribution for the unemployment duration , determined in the previous study, and the survivorship probabilities of the employees.

The new approach is developed as an improvement to the previous one by appropriately incorporating these probabilities in the valuation and in the pricing equations. The previous equations appear in the next Section , under literature review while the proposed equations for the improved approach are discussed in the Section on methodology . Results on the application of the new approach to the USA data, and also the results under the various assumptions are obtained and compared.



ISSN: 2055-0154(Print), ISSN 2055-0162(Online)

## LITERATURE REVIEW

In this section, we review the relevant literature on unemployment insurance, following the approach in Simwa et al(2016), the main reference for this study. The literature on the topic is discussed in general while in the next section followed by the discussion which is specifically with respect to United States of America (USA) economy, since the application in this paper is on the USA economic data. A discussion on the main relevant results from the previous study in Simwa et al(2016).

### General literature on unemployment Insurance

Some of the problems highlighted in Malinvaud E. (1985) were moral hazard, dis-utility and adverse effects while classifying the risk groups.

In Beenstock M. ( 1985) a model is developed to address the above problems by diversifying the unemployment risk and assuming that the unemployment benefits are deterministic. According to the model, the unemployment insurance contract would automatically be enacted when a person starts working and the insured was required to pay premiums right from the onset of their employment. They would then receive unemployment benefits in the event that they become involuntarily unemployed until they secure another job if this occurs before the contract expires. To be able to determine the amount of premiums payable for the cover, Beenstock (1985) assumed that the insurer has identified various risk groups, just as is the case in car insurance, and considered each risk group as a stationary fund. Since the benefits are deterministic, then equating the discounted value of the benefits gives the amount of premiums payable.

Capital asset pricing model is used in Bronars G. S.( 1985) to determine the fair premiums in a theoretical model of a hypothetical regulated private market for unemployment insurance. This is an improvement of the existing work in Beenstock M. ( 1985) where unemployment risk is undiversified and an appropriate risk-adjusted interest rate is specified for the unemployment insurance.

In Hwei-LinC.andYuM.T. (2010) , the authors extend the results in Bronars G. S.( 1985) by incorporating survival analysis models to estimate the unemployment duration and to calculate the fair premium rate for the unemployment insurance program. In their study they used data from the unemployment insurance program in Taiwan. In the development of the model, the Weibull distribution was used to estimate the average unemployment duration while the capital asset pricing model was used to determine the interest rate used to discount the benefits.

The author in Bowers N. (1980) probed issues surrounding unemployment duration ranging from methodological, measurement and results interpretation of existing statistics on unemployment duration and observes that most unemployment spells are of short durations of less than 10 weeks, although with some fluctuations especially during recessions. The author uses transition probabilities among the three states namely employed, unemployed and not in the labour force to estimate the duration of unemployment. According to their study, the short unemployment durations do not imply an active labour market so that in the event of a job loss, one is able to find his usual type of a job in a relatively short period. This is because a large portion of job changes occurs without any intervening spells of unemployment. More so, the ambiguity in labour force classification, particularly in differentiating between the unemployed and not in the labour force states, is problematic. This is because some of those who withdraw


ISSN: 2055-0154(Print), ISSN 2055-0162(Online)

from the labour force experience a brief spell outside and soon enter the labour force as unemployed again.

The sorting model considered in Salant S.W. (1977) assumed a constant individual hazard rate which was allowed to vary among different individuals. The constant hazard rate was accounted for by the exponential distribution while the variation among the individuals was accounted for by the gamma distribution. The resulting mixture model, that is Pareto, yielded a decreasing hazard rate for the whole cohort of unemployed individuals.

In McDonald and Butler (1987) several mixture distributions of generalized beta distributions were reviewed. Statistical tests on Salant's model in Salant S.W. (1977) and Bur XII together with the need to conform to job search theory revealed that the Burr XII distribution, a mixture of Weibull and the inverse generalized gamma distributions, is better than Pareto in estimating the spells of unemployment since then there is allowance for heterogeneity in unemployment data.

In Cummins (1991), the problem is posed in research on insurance pricing due to parallelism in research on the three major paradigms on insurance. These are statistical modeling, financial modeling and economics. Although few attempts have been made to integrate research in the three areas, the technicality and high specialization exhibited in each have posed a great challenge in the exercise. Hwei andYum (2010) made an attempt of integrating the three by considering both statistical and financial models and how they are applied in insurance together with some of the errors made in application. Some of the statistical models, concepts and laws looked at include individual and collective risk models, central limit theorem, law of large numbers and the concept of homogeneity of risks. The financial models explored include the application of capital asset pricing model in determining underwriting rate of return, discrete time discounted cash flow models, option pricing models and sensitivity analysis of the assets and liabilities of the insurance firm. Although economic models were not considered in his integration, Cummins notes that financial models consider insurance variables in an economic setup which in a way incorporates economic models.

This study seeks to improve the model in Simwa et al (2016) by incorporating the necessary probabilities, basing on the benefit provision regulations and the unemployment duration distribution discussed in Simwa et al (2016).

## Unemployment Insurance in the United States of America

The Unemployment insurance scheme in the USA is a federal-state partnership based upon federal law. The arrangement is anchored on a strong use of incentives to enhance efficiency. The Federal government ensures conformity and compliance of state programs through Federal Unemployment Tax Act (FUTA) and Social Security Act (SSA). States have enacted their own laws to regulate their individual schemes. The arrangement can be explained by considering separately, the method of financing the scheme, eligibility, unemployment benefits and the waiting period of the scheme.

### 1.) Financing the scheme

The program is entirely funded by employer taxes, both federal and state, although the states of Alaska, Pennsylvania and New Jersey levy unemployment taxes on employees to supplement employer contributions. Unemployment taxes to the federal government have been at a rate of 6% per annum of the first $7, 000 wage base per employee following the decline


ISSN: 2055-0154(Print), ISSN 2055-0162(Online)

from 6.2% per annum in July 2011. Tax credit is available up to a maximum of 5.4% of FUTA taxable wages. However, the maximum tax discount is offered to employers who pay their respective state unemployment taxes in full, on time and on all forms of income subject to FUTA tax.

The Unemployment taxes to the federal government are used to pay for administrative costs incurred in the running the Unemployment Insurance programs in all the states together with other associated programs, federal share of extended benefits and to pay for other third tier programs like loans to states with deficits in payment of benefits.

All states finance their Unemployment Insurance programs through taxes from subject employers on the wages of their covered employees. The taxes are deposited into the state's Unemployment Tax Fund (UTF) and are withdrawn by the state to pay the benefits or tax overpayment refunds. Contrary to the federal tax rate, most states use experience rate system to set the tax rate for each employer. However, new employers are given a standard rate before their experience rate is determined. States sets their own tax base with some preferring to use the federal government's tax base.

**2.) Eligibility to the scheme**

An application brought to the State unemployment agency is reviewed to determine if the applicant qualifies to receive the benefits. To qualify, one must have worked for the base period or have earned the required wages as provided for in the State's labour laws and the cause of the unemployment must be out of control of the insured.

Upon commencement of the benefits, one must file weekly or biweekly claims and reports regarding any incomes from work or job offers refused as well as respond to any questions from the state labour office. Additionally, one must report to the Unemployment Insurance Claims office when required to do so.

**3.) Waiting period**

Workers are required to file a claim with the Unemployment Insurance Agency of the state they worked for immediately before they become unemployed. During claim, workers furnish the agency with the details of their immediate former employer to aid in authentication of the claim. According to the United States Department of Labour, it takes an average of two to three weeks after a claim is filed for one to receive the first benefits. However, some states take only one week to process a claim.

**4.) Unemployment Benefits**

States pay a benefit of between 40%–50% of average monthly earnings in the past one year before unemployment, up to a state's maximum amount. Benefits are advanced on weekly basis up to a maximum of 26 weeks unless in the case of extended benefits during periods of high unemployment.

**Main relevant results from previous study**

Several results appear in the main reference by Simwa et al(2016), and since this study is an extension of the same , we highlight these results and use them to derive more practical results in the methodology Section . In next section we highlight that the Burr XII distribution is the best choice for the distribution of unemployment for the USA



ISSN: 2055-0154(Print), ISSN 2055-0162(Online)

unemployment data. Pricing of the unemployment premiums using the approach by Simwa et al(2016), is also noted.

## Distribution of Unemployment Duration

The distribution of unemployment duration , the length of the period of unemployment, is required in the proposed approach for determining premiums for the unemployment insurance. In the following section, the Burr XII is noted to be the best choice , among other distributions, as the distribution for the unemployment duration with respect to the USA data. The estimates of the parameters of the Burr XII distribution are highlighted thereafter.

### Goodness of fit of the distributions on USA unemployment duration data (Simwa et al (2016))

In order to determine the appropriate unemployment duration, we need to establish which distribution best fits the data.

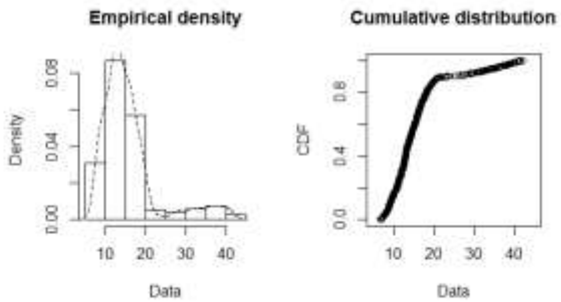


**Fig . 1:** ***Empirical pdf and cdf.***

An empirical plot of both the density and distribution functions of the raw unemployment duration as shown in Fig-1 indicates that the data follows one of the tailed distributions.

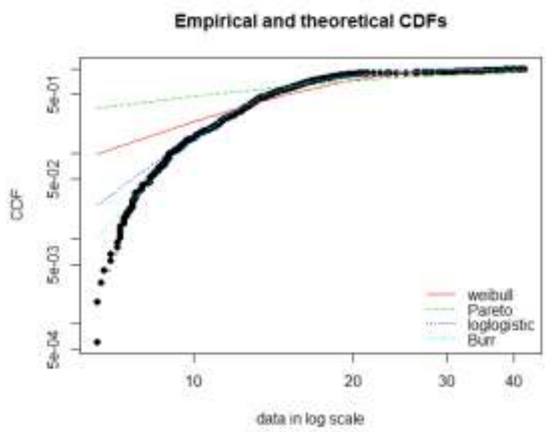


**Fig. 2:** ***Empirical and theoretical cdfs***



ISSN: 2055-0154(Print), ISSN 2055-0162(Online)

A plot of both empirical and theoretical cumulative distribution functions of Weibull, Pareto, log-logistic and Burr XII of distributions shown in Fig-2 indicates that both Burr XII distribution and log-logistic provide better fits than the Weibull and Pareto.

**Table 1**: ***Goodness of fit criteria***

| | Weibull | loglogistic | Pareto | Burr |
|---|---|---|---|---|
| Akaike's Information Criterion | 5436.162 | 5099.712 | 6158.474 | 5057.250 |
| Bayesian Information Criterion | 5445.581 | 5109.130 | 6167.893 | 5071.378 |

From the goodness of fit indices outlined in Table -1 the Burr XII corresponds to the lowest indices under both AIC and BIC and is therefore the preferred model in estimating the duration of unemployment.

**Parameter estimation ( Simwa et al(2016) )**

The parameters of the Burr distribution were then estimated using Maximum likelihood estimation in R programming. The results are as outlined in Table-2. The R codes used appear in Appendix 1.

**Table 2:** ***Estimation of parameters***

| Parameter | Estimate | Standard Error |
|---|---|---|
| Shape 1 (a) | 0.4955088 | 0.050208998 |
| Shape 2 (b) | 6.6921700 | 0.390146800 |
| Rate (1/s) | 0.0853068 | 0.002159988 |

**Equation of value for the Premium Rate (Simwa et al(2016))**

According to the equivalence principle for premium calculation in insurance, a fair premium is one that equates the expected present value of the benefits to that of the premium income. For a state premium rate W of the taxable wage base S, the mean present value of the premium income with zero mortality assumption, $MPV(P)$ is given by

$$MPV(P) = W * 12S * \sum_{k=0}^{47} (1 + r_f)^{-k} \qquad (1)$$

where $r_f$ is the risk-free rate.

We use the benefit-event valuation approach (see AEC-ST4(2014) and Okasha M.K. a (2015) ) to discount the contingent claims. The mean present value of a monthly benefit, $MPV(B)$, of 45% of the taxable wage base per month, payable weekly during spells of unemployment is given by

$$MPV(B) = \sum_{k=0}^{2444} (1 + r_b)^{-\left(\frac{k+m}{52}\right)} \times q_k^u \times \left\{\frac{(0.45 - W)}{4} \left( \sum_{t=0}^{d} (1 + r_b)^{-t/52} \right)\right\} (2)$$



ISSN: 2055-0154(Print), ISSN 2055-0162(Online)

where k is the number of weeks since becoming involuntarily unemployed, m is the waiting period after applying for the unemployment benefits, $q_k^u$ is the probability of a claim in week k, 52 is the number of weeks in a year and $r_b$is the expected risk-adjusted rate of return.

From the Capital Asset Pricing Model (CAPM), $r_b$ is given by

$$r_b = E(r'_b) = r_f + (r'_m - r_f)\beta_u \qquad (3)$$

where $r'_b$ is the corresponding random rate of return and $r'_m$ is the expected market rate of return. $\beta_u$, denotes the correlation between unemployment rate and the market rate of return,and is given by:

$$\beta_u = \frac{cov(unemployment, market)}{var(market)} = \frac{cov(r_u, r_m)}{var(r_m)} \qquad (4)$$

In this case $r_u$ is the rate of unemployment. W is determined by equating the right hand side of Equations (2) and (3).

## METHODOLOGY

In this section, we discuss the discrete equivalent of the unemployment duration distribution, the survivorship probabilities and the equation of value for determining the premium rate.

### Unemployment Duration Distribution

Let F(t) denote the cumulative distribution function of the unemployment duration. This is the Burr XII distribution noted in the literature. To apply the distribution in the new approach discussed in this paper, let $f_t$ denote the probability mass function defined on F(t) by

.$f_t = F(t) - F(t-1)$, t = 1, 2, 3,…..

Then $f_t$ is a probability mass function, since

$$\sum_{1}^{\infty} f_t = F(\infty) = 1$$

### Survivorship Probabilities

To account for non -zero mortality, we assume life table function, $l_x$ , and the corresponding survivorship probability, ${}_tp_x$ , see Bowers et al(1997). It follows that ${}_tp_x = l_{x+t} / l_x$ , where x is the age of a life and t is any positive real number, x=0,1,2….and $l_x$, the life table function, for the number of lives aged exactly x years ( see Bowers et al(1997)) .

### Equation of value for the Premiums : New Approach Case 1.

The previous results, as noted in Equation (2) the unemployment duration distribution is incorporated through the use of the estimate for the expected duration of unemployment, d, such that the relevant summation in the model is upto the value *d*. In this approach we begin by incorporating the distribution by using its discrete equivalent mass function, $f_t$ . . Also the 26 weeks assumption on USA unemployment data, a regulation by the USA federal



ISSN: 2055-0154(Print), ISSN 2055-0162(Online)

government, listed as assumption number four under the literature review section, is now adhered to. Thus the corresponding equations are as follows.

The Expected present value of the premium income, $EPV(P)$,

$$EPV(P) = W * 12S * \sum_{k=0}^{47}(1+r_f)^{-k} \quad (5)$$

where $r_f$ is the risk-free rate.

The expected present value of a monthly benefit, , EPV(B) , of 45% of the taxable wage base per month, payable weekly during spells of unemployment is given by

$$EPV(B) = \sum_{k=0}^{2444}(1+r_b)^{-\left(\frac{k+m}{52}\right)} \times q_k^u \times \left\{\frac{(0.45-W)}{4}\left(\sum_{t=0}^{26}(1+r_b)^{-t/52} ft\right)\right\} \quad (6)$$

where ft is the probability mass function corresponding to the Burr XII unemployment distribution , k is the number of weeks since becoming involuntarily unemployed, m is the waiting period after applying for the unemployment benefits, $q_k^u$ is the probability of a claim in week k, 52 is the number of weeks in a year and $r_b$ is the expected risk-adjusted rate of return.

**Equation of value for the Premiums: New Approach Case 2**

The results in the previous section are now improved further by introducing non-zero mortality assumption, such that we incorporate the survivorship probabilities, both in the premium payment by employees' sponsor and the payment of benefits to the unemployed. We assume that the sponsor pays premiums only for the employees who are alive and also the sponsor pays the benefits only to the unemployed, who are alive.

The corresponding equations are as follows.

The expected present value for the premiums contributed, EPV(P), is given by

$$EPV(P) = W * 12S * \sum_{k=0}^{47}(1+r_f)^{-k} \, {}_kp_{18} \quad (7)$$

while the expected present value for the benefits, EPV(B), is given by

$$EPV(B) = \sum_{k=0}^{2444}(1+r_b)^{-\left(\frac{k+m}{52}\right)} \times r(k) \times \left\{\frac{(0.45-W)}{4}\left(\sum_{t=0}^{26}(1+r_b)^{-t/52} g(t)\right)\right\} \quad (8)$$

where,

$$r(k) = {}_kp_{18} \times q_k^u \quad (9)$$

$$g(t) = {}_yp_{18+k} \times f_t \ , \quad (10)$$

for $y = t/52$

Equation (7) has the summation portion that is just a life annuity that is payable annually in advance , limited to 47 years (see Bowers et al(1997), while Equation (8) , on the right hand side, is such that the summand conforms to the benefit-event present value method of valuing benefits (see AEC-ST4(2014)).


ISSN: 2055-0154(Print), ISSN 2055-0162(Online)

## RESULTS

This section is on fitting USA economic data into the two cases proposed by the new approach in section 3. Note that the results in Simwa et al(2016) are assumed, thus we use results in the literature section. In addition, from Simwa et al(2016),

$$r_b = 4.11\%.$$

In the new approach case 2, a service table for the mortality of employees in the USA is required. We have used, for the service table, the Life Table for USA, see Bowers et al(1997). The life table is a good estimate of the possible service table for the employees, since the former is general while the later is the employee specific mortality experience.

To compute the the premium rate, w , we equate the EPV(P) to the corresponding EPV(B) in each case and solve for w. Thus for the new approach case 1, we equate right hand side of Equation (5) to the right hand side of Equation (6), while for the new approach case 2, we equate the right hand side of Equation (7) to the right hand side of Equation (8).

### Results on the premium rating

The solution for the premium rate, w, for each case found by equating the relevant expressions, is as given in Table 3, which also highlights results from the previous study, for comparison.

**Table 3:** Premium Rate for each approach

| **Approach** | **Premium Rate, w** |
|---|---|
| Recommended by USA government | 6 % |
| Simwa et al(2016) approach | 5.1% |
| New Approach Case 1 | 4.6 % |
| New Approach Case 2 | 4.07% |

## DISCUSSION

The results given in Table 3 show that the New approach case 2, has the lowest premium rating of 4.07 % followed by the New approach case 1 of this study. The premium rating recommended by the USA federal government is the largest in size. The choice of which approach to use in practice depends on many factors, however an approach that accounts for the benefit provision regulations and relies more on the relevant data is likely to be preferred to one that ignores these model inputs. In the methodology we have used the expected present value (EPV) to signify the inclusion of the survival and unemployment probabilities as opposed to the mean present value (MPV) discussed in the literature, where these probabilities are not included.



ISSN: 2055-0154(Print), ISSN 2055-0162(Online)

## CONCLUSION

Thus holding all other factors constant, the New approach case 2 is the best, among these four approaches, since it leads to the lowest premium rating hence reduced budgetary allocation for the provision of unemployment insurance benefits, by both the federal government and the governments of affected states in the USA.

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ISSN: 2055-0154(Print), ISSN 2055-0162(Online)