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ON THE ADOMIAN DECOMPOSITION METHOD FOR THE SOLUTION OF SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT: *This paper presents the Adomian decomposition method for the solution of second order ordinary differential equations. The Adomian decomposition method (ADM) is a creative and effective method for exact solution of functional equations of various kinds. It is important to note that a large amount of research work has been devoted to the application of the Adomian decomposition method to a wide class of linear and non-linear, ordinary or partial differential equations. The decomposition method provides the solution as an infinite series in which each term can be easily determined*.

KEYWORDS: Adomian Decomposition Method, Differential Equation, Order

INTRODUCTION

The Adomian Decomposition Method is a semi-analytical method for solving differential equations. The method was developed from the 1970s to the 1990s by George Adomian, chair of the center for applied Mathematics at the University of Georgia. It is further extensible to stochastic systems by using the Ito integral. The aim of this method is towards a unified theory for the solution of partial differential equations (PDE). The crucial aspect of the method is employment of the Adomian polynomials which allow convergence of solution for the non-linear portion of the equation without linearizing the system.

In this paper, we introduce a non-local approximation of the non-linear terms of the usual Adomian polynomials in the Adomian Decomposition method.

ADOMAIN DECOMPOSITIONMETHOD

The take-off point is to give a review of the Adomian decomposition method as follows;

It is assumed that $y = f(x)$ is sufficiently differentiable and that the solution of

$y' = f(x, y); y(a) = y_0$
(1)

Exists and satisfies the Lipchitz condition.

Let $L = \frac{d}{dx}$ and the inverse operator L^{-1} to be the one fold integral operator defined by

$L^{-1} = \int_0^x (.)dx$
(2)

for the first order differential equation; and

Let $L = \frac{d^2}{dx^2}$ and the inverse operator L^{-1} to be the two fold integral operator defined by

$L^{-1} = \int_0^x \int_0^x (.)dxdx$
(3)

For the second order ordinary differential equation

Thus, the numerical solution to $y'' = f(x, y)$ is given by

$y(x) = y_0 + y_1 x + L^{-1}[f(x,y)]$
(4)

Where $y_1 = y^1(x_0)$

$$\sum_{n=0}^{\infty} y_n(x) = y(x) \quad (5)$$

This is as a result of Adomian's Decomposition Method, which assumes a series solution for y(x) by given it as an infinite sum of the components of (2).

$$y(x) = \sum_{n=0}^{\infty} y_n x \quad (6)$$

The components $y_n(x)$ will be determined recursively. Adomian decomposition method defined the non-linear function $f(x,y)$ by the infinite series of polynomials.

$$f(x,y) = \sum_{n=0}^{\infty} A_n \tag{7}$$

The A_n is called Adomian's polynomials. This can be calculated for various classes of non-linearity according to

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\aleph^n} N\left[\sum_{k=0}^{n} \aleph^k U_k\right]\right]_{\aleph=0} \tag{8}$$

Or

$$A_n = \sum_{k=1}^{n} C(k,n) f^{(k)}(U_0) \qquad n = 1, 2, 3\,, \tag{9}$$

$$f^{(k)}(U_0) = \frac{d^k f(U_0)}{dU_0^k} \tag{10}$$

and C (k, n) means the sum of possible products of K components of U, whose subscripts add to n, divided by the factorial of the number of repetitions.

Substituting (6) and (7) into (4) we have;

$$\sum_{n=0}^{\infty} y_n(x) = y_0 + y_1(x) + L^{-1}\left[\sum_{n=0}^{\infty} A_n\right] \tag{11}$$

Each term of the series (11) is given by the recurrent relation

$y_0(x) = y_0 + y_1(x)$

(12)

and

$y_{n+1}(x) = L^{-1} A_{n,}\ \ n \geq 0.$

(13)

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Numerical Examples and Results of Adomian Decomposition Method

Here, we focus our attention on the applications of Adomian decomposition method to generate numerical results, in each problem, only the first ten term of the decomposition series will be used in computing the results.

Example 1

We consider the second order differential equation of the form;

$$y''(x) = x + y,\ y(0) = 1,\ y'(0) = 1,\ h = 0.01$$

With exact solution of the form

$$y(x) = e^x - x$$

(14)

The solution is then compared with the exact solution. The detail of the results is given in the Table 1, Figures 1A and 1B below.

Table 1

x	Exact Solution	Numerical Solution	Error
0.00	1.00000000	1.00000000	0.00000000
0.10	1.00517094	1.00517094	0.00000000
0.20	1.02140272	1.02140284	0.00000012
0.30	1.04985881	1.04985881	0.00000000
0.40	1.09182477	1.09182477	0.00000000
0.50	1.14872122	1.14872134	0.00000012
0.60	1.22211885	1.22211874	0.00000012
0.70	1.31375265	1.31375277	0.00000012
0.80	1.42554104	1.42554104	0.00000000
0.90	1.55960321	1.55960333	0.00000012
1.00	1.71828210	1.71828198	0.00000012

Figure1

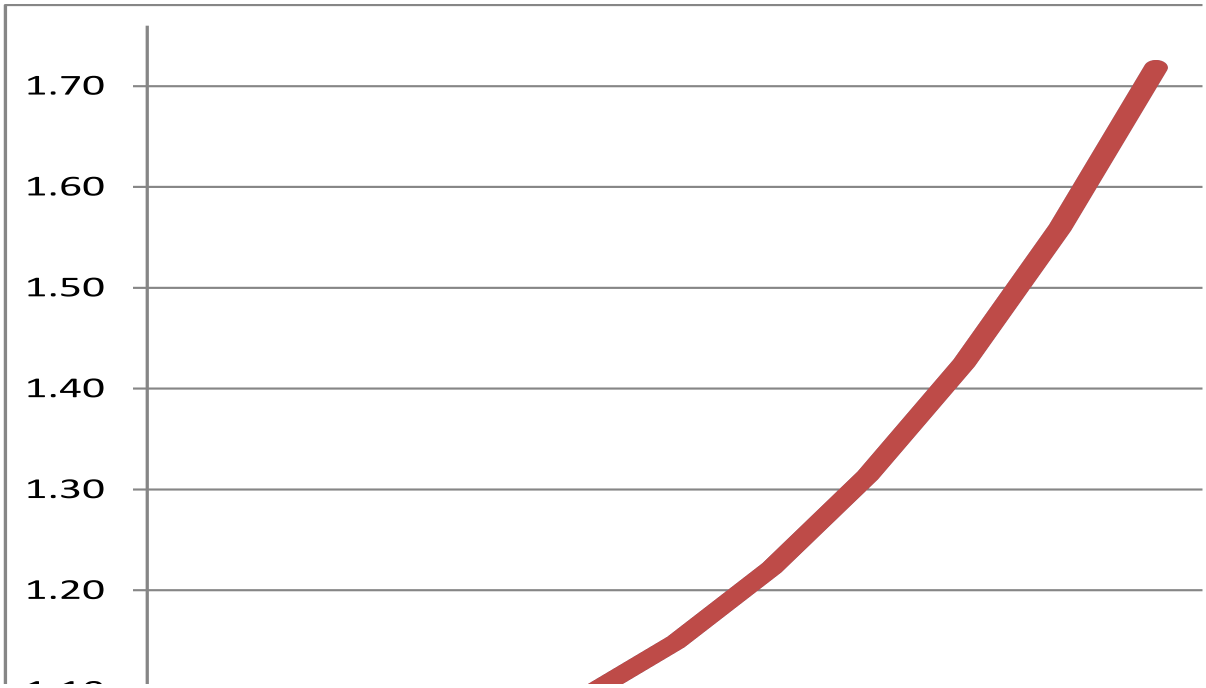
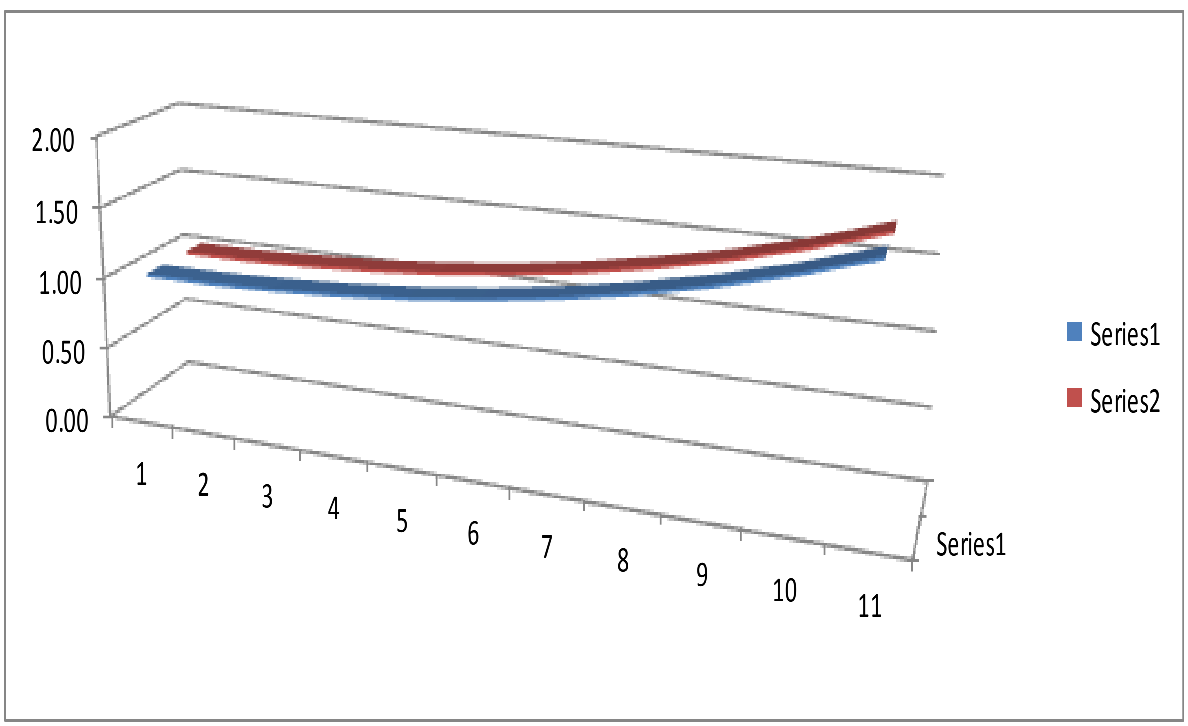


Figure 1B: The 3-D view



Example 2

We consider the second order differential equation of the form

$$y'' = -2y^1 - 5y, \quad y(0) = 2, \quad y'(0) = -1$$

$$y(x) = Ae^{-x} + Be^{-5x}, \quad A = \frac{19}{4}, \quad B = -\frac{3}{4}, \quad h = 0.01$$

The results generated are shown in Table 2, Figures 2A and 2B below.

Table 2

X	Exact Solution	Numerical Solution	Error
0.00	2.00000000	2.00000000	0.00000000
0.01	1.98960352	1.98960185	0.00000167
0.02	1.97842801	1.97841477	0.00001323
0.03	1.96649432	1.96645010	0.00004423
0.04	1.95382380	1.95371914	0.00010467
0.05	1.94043684	1.94023347	0.00020337
0.06	1.92635429	1.92600477	0.00034952
0.07	1.91159725	1.91104472	0.00055254
0.08	1.89618587	1.89536524	0.00082064
0.09	1.88014126	1.87897825	0.00116301
0.10	1.86348355	1.86189568	0.00158787

Figure 2A

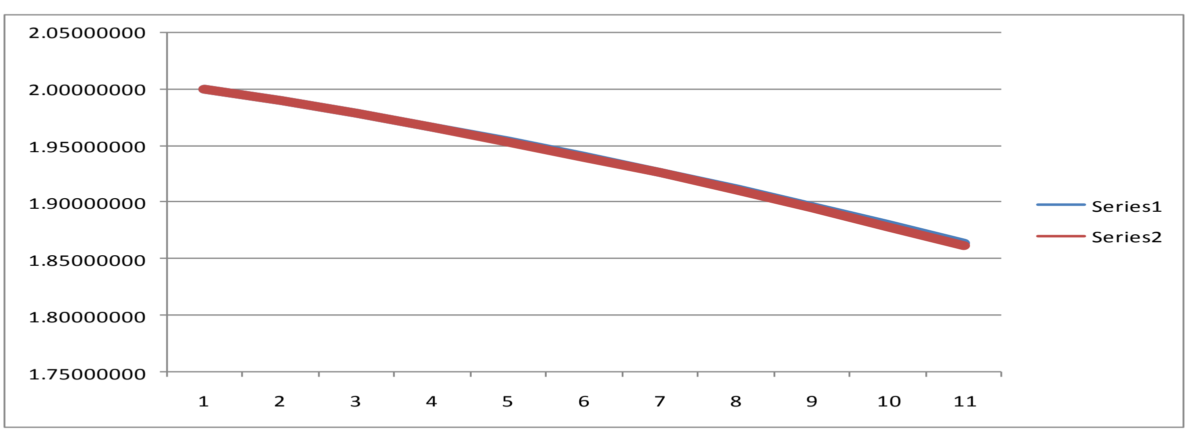
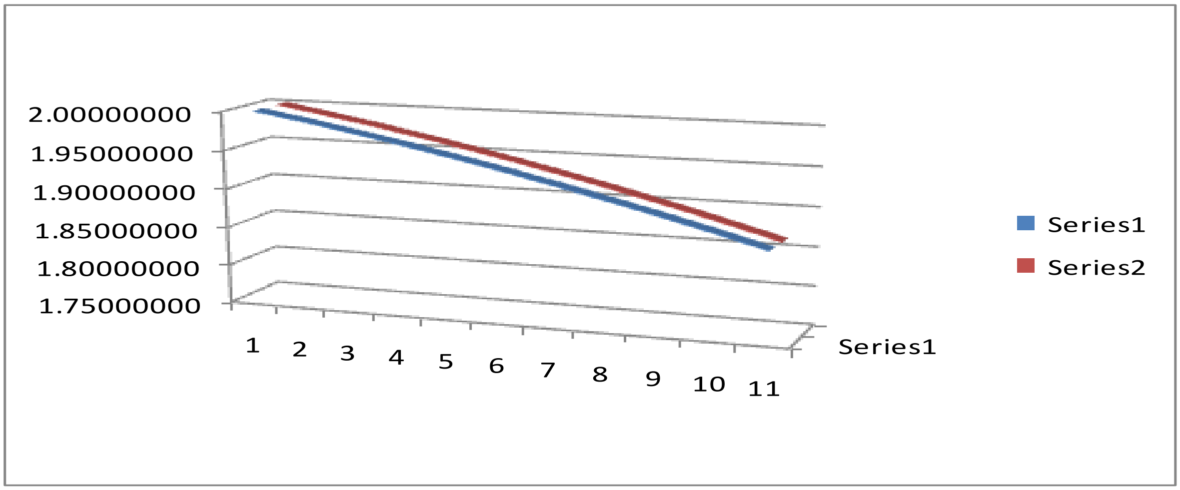


Figure 2B: The 3-D view



Example 3

We shall consider the second order differential equation of the form;

$y'' = -6y^1 - 5y, \quad y(0) = 4, \quad y'(0) = -1, \quad h = 0.01$

The results obtained are shown in the Table 3, Figures 3A and 3B below.

Table 3

X	Exact Solution	Numerical Solution	Error
0.00	4.00000000	4.00000000	0.00000000
0.01	3.98931456	3.98931503	0.00000048
0.02	3.97731566	3.97732186	0.00000620
0.03	3.96408510	3.96411610	0.00003099
0.04	3.94970179	3.94979882	0.00009704
0.05	3.93423939	3.93447471	0.00023532
0.06	3.91776776	3.91825366	0.00048590
0.07	3.90035462	3.90124989	0.00089526
0.08	3.88206244	3.88358259	0.00152016
0.09	3.86295199	3.86537480	0.00242281
0.10	3.84307981	3.84675431	0.00367451

Figure3A

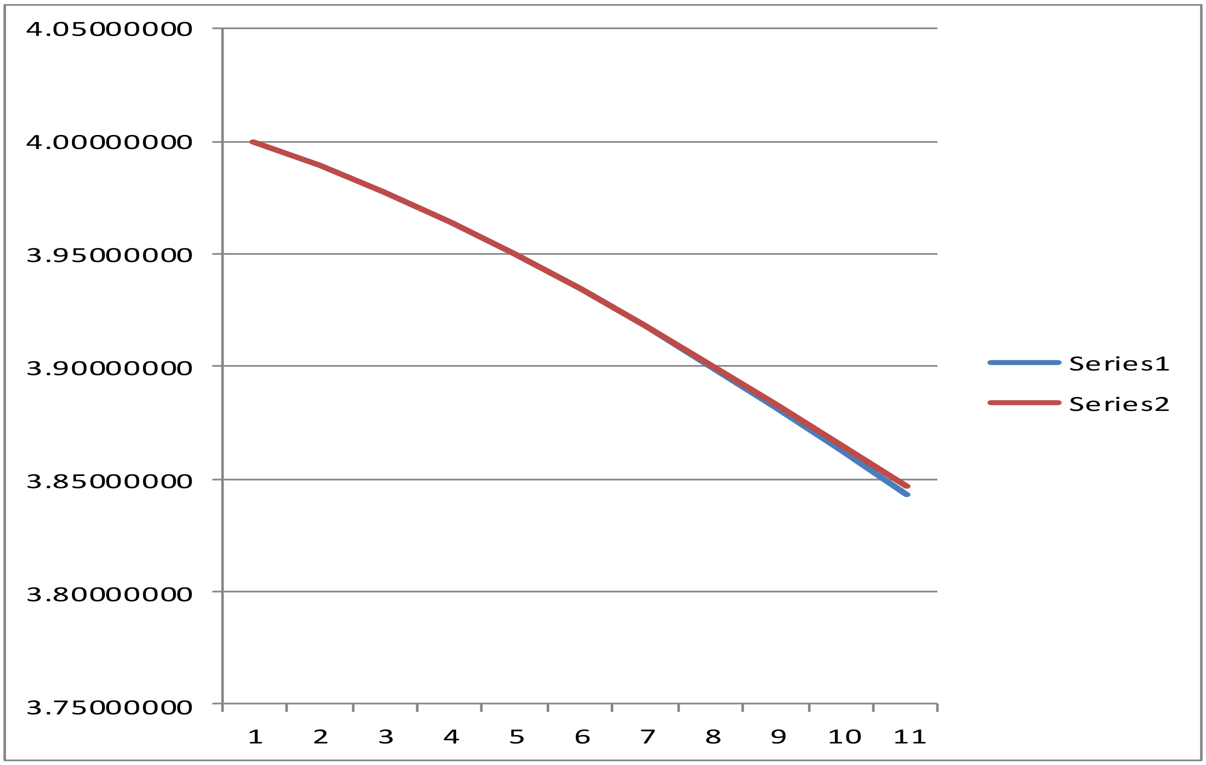
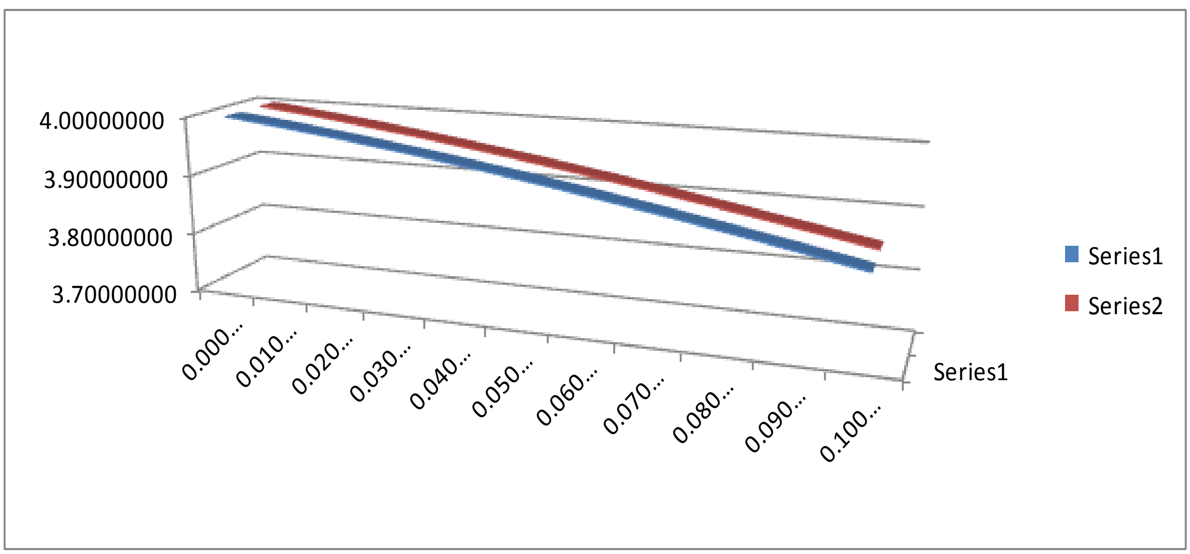


Figure 3B: The 3-D view



DISCUSSION OF RESULTS

From the Tables above, we can see that Adomian decomposition method is very close to the exact solution. Also, the results presented here indicate that the method is reliable, accurate and converges very rapidly.

CONCLUSION

It was observed that better accuracy can be obtained by accommodating more terms in our decomposition series, and that the solutions of the presented equations is stable and consistent in the interval $a \leq x < b$.

The above results are obtained using Fortran 77 programming language.

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